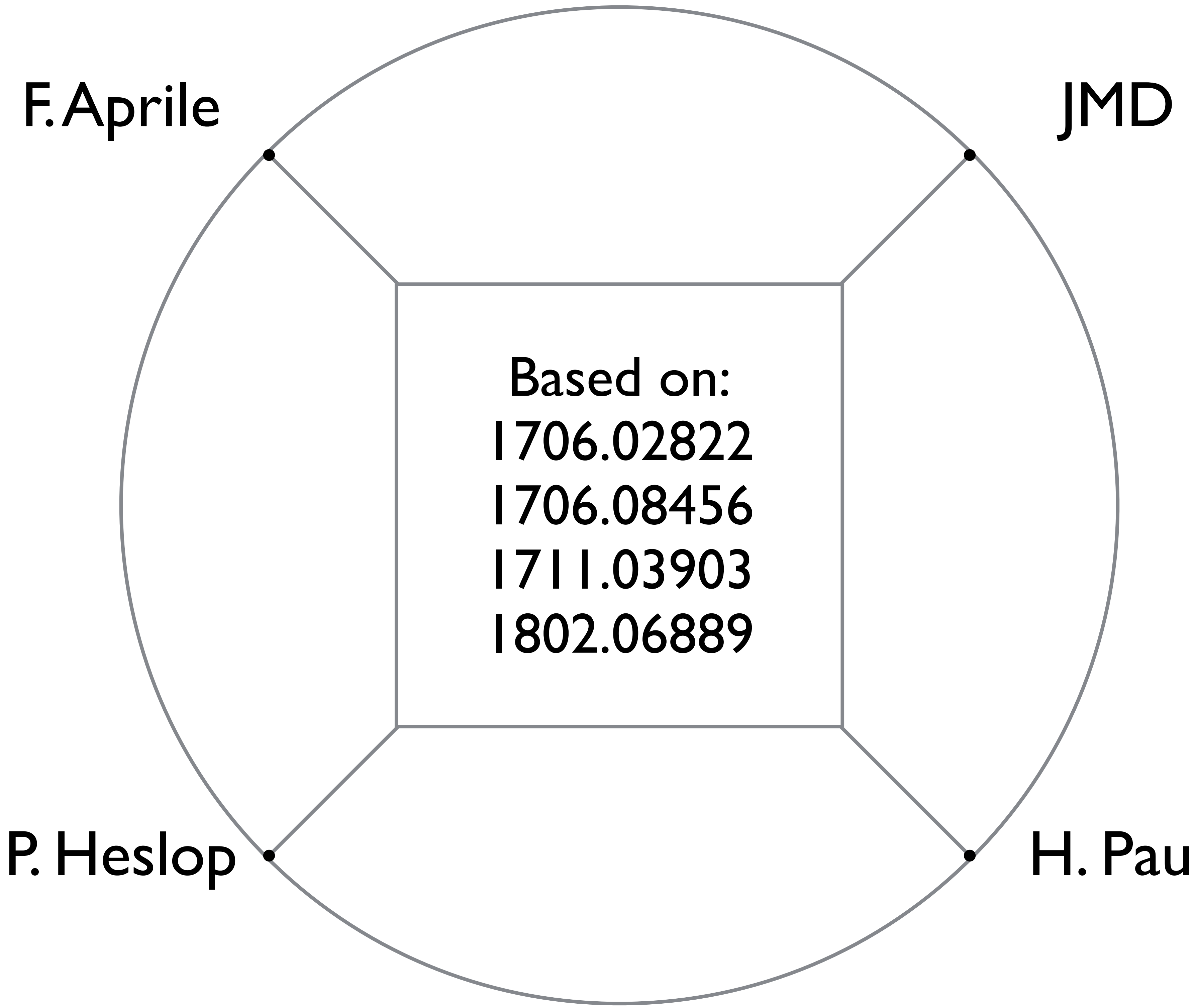
James Drummond

UNIVERSITY OF Southampton

Quantum Gravity from Conformal Field Theory

Amplitudes 2018, SLAC


F. Aprile
JMD
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P. Heslop
H. Paul

Gravity Amplitudes

Can we study gravitational scattering on curved backgrounds?

AdS space a natural candidate - constant negative curvature.

AdS/CFT: CFT techniques can be deployed to study the problem.

AdS/CFT is a strong/weak duality.

Normally it is used to describe a strongly coupled gauge theory in terms of a weakly coupled gravity theory.

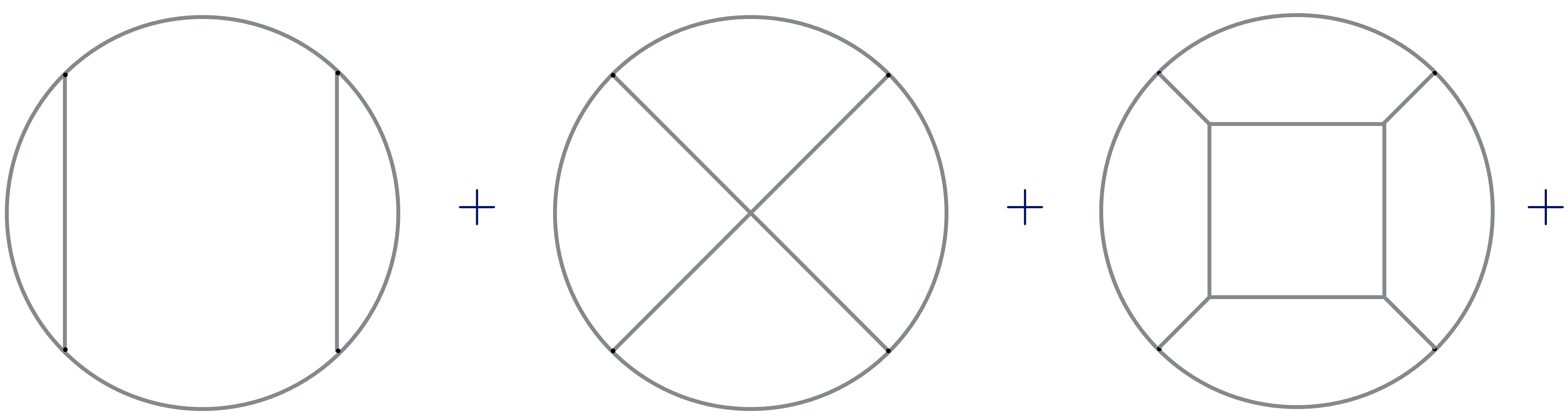
Even weakly coupled AdS gravity becomes more tractable from a boundary perspective.

Gravity Amplitudes as boundary correlators

Elementary gravitons/matter are single-trace operators.

AdS/CFT: bulk AdS amplitudes are boundary CFT correlators.

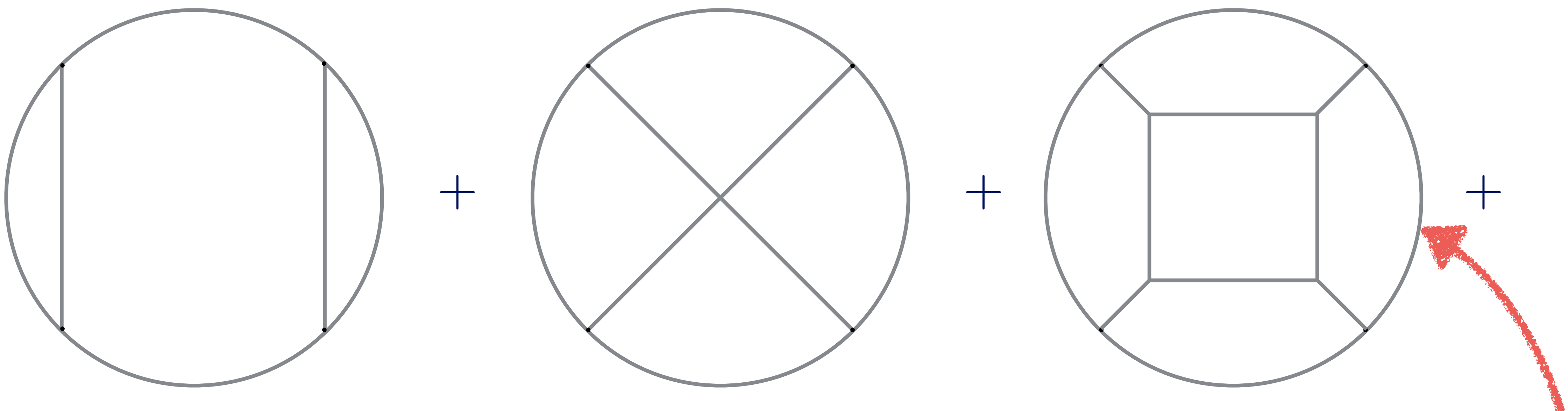
Their structure is governed by properties of multi-particle bound states which correspond to multi-trace operators.



The perturbative expansion is an expansion in $\frac{1}{N^2}$

Gravity Amplitudes as boundary correlators

Why not just compute the bulk diagrams?



Not simple - but see Ellis' talk

Moreover there are often infinitely many fields running in the loop!

Concrete AdS/CFT examples often have AdS x M.

Tower of Kaluza-Klein modes present.

For this talk: $\mathcal{N}=4\ \mathrm{SYM}\quad\sim\quad \mathrm{IIB\ superstrings\ on\ AdS}_5\times S^5$

Half-BPS operators

Natural operators to consider: $(y^2 = 0)$

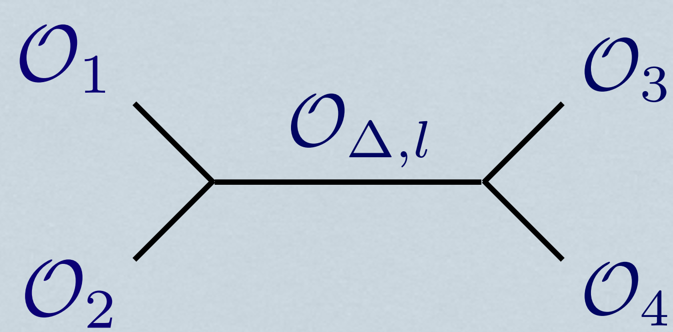
$$\mathcal{O}^{(p)}(x,y) = y^{R_1} \dots y^{R_p} \mathrm{tr}(\phi_{R_1} \dots \phi_{R_p})(x)$$

Two-point functions and three-point functions protected by SUSY.

Four-point functions non-trivial:

$$\langle p_1 p_2 p_3 p_4 \rangle = \langle \mathcal{O}^{(p_1)}(x_1,y_1) \mathcal{O}^{(p_2)}(x_2,y_2) \mathcal{O}^{(p_3)}(x_3,y_3) \mathcal{O}^{(p_4)}(x_4,y_4) \rangle$$

OPE involves exchange of unprotected operators.



Correlator depends on $\{x_i, y_i; N, g\}$ $\qquad \lambda = g^2 N$

Supergravity limit

We study the correlation functions in the supergravity limit.

$$\langle 2222 \rangle = \langle 2222 \rangle^{(0)} + a \langle 2222 \rangle^{(1)}[\lambda] + a^2 \langle 2222 \rangle^{(2)}[\lambda] + \ldots$$

$$a = \frac{1}{N^2 - 1}$$

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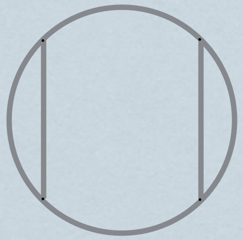
Leading term - disconnected part:

$$\langle 2222 \rangle^{(0)} = \sum \langle 22 \rangle \langle 22 \rangle$$



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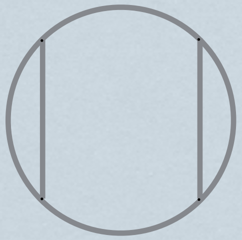
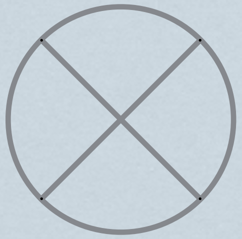


$$\langle 2222 \rangle^{(0)} = \sum \langle 22 \rangle \langle 22 \rangle$$

First correction - classical strings:



$$\langle 2222 \rangle^{(1)}[\lambda] = \langle 2222 \rangle_0^{(1)} + \lambda^{-\frac{3}{2}} \langle 2222 \rangle_2^{(1)} + \dots$$

Supergravity limit

We study the simplest correlation function in the supergravity limit.

$$\langle 2222 \rangle = \langle 2222 \rangle^{(0)} + a \langle 2222 \rangle^{(1)}[\lambda] + a^2 \langle 2222 \rangle^{(2)}[\lambda] + \dots$$

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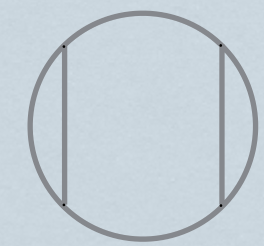

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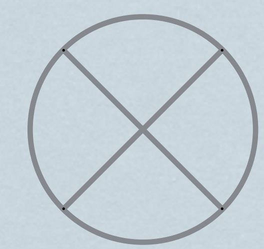

$$\langle 2222 \rangle^{(1)}[\lambda] = \langle 2222 \rangle^{(1)}_0 + \lambda^{-\frac{3}{2}} \langle 2222 \rangle^{(1)}_2 + \dots$$

Leading term - classical supergravity:

First stringy correction

Supergravity limit

We study the simplest correlation function in the supergravity limit.

$$\langle 2222\rangle = \langle 2222\rangle^{(0)} + a\langle 2222\rangle^{(1)}[\lambda] + a^2\langle 2222\rangle^{(2)}[\lambda] + \ldots$$

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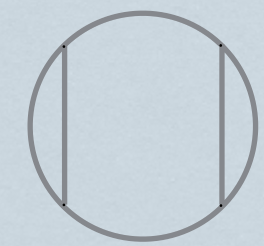
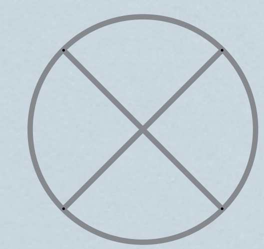
Leading term - disconnected part:

$$\langle 2222\rangle^{(0)} = \sum \langle 22\rangle\langle 22\rangle$$

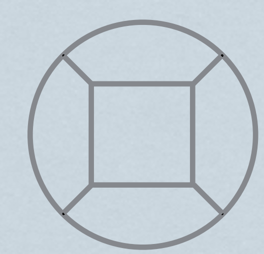


First correction - classical strings:



$$\langle 2222\rangle^{(1)}[\lambda] = \langle 2222\rangle^{(1)}_0 + \lambda^{-\frac{3}{2}}\langle 2222\rangle^{(1)}_2 + \ldots$$

Second correction - string loop:



$$\langle 2222\rangle^{(2)}[\lambda] = \langle 2222\rangle^{(2)}_0 + \ldots$$

Kinematic dependence

Propagators: $d_{ij} = \frac{y_{ij}^2}{x_{ij}^2}$ $\quad x_{ij}^2 = (x_i - x_j)^2$ $\quad y_{ij}^2 = y_i \cdot y_j$

Conformal and internal cross-ratios:

$$u = x\bar{x} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = (1-x)(1-\bar{x}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\,,$$

$$y\bar{y} = \frac{y_{12}^2 y_{34}^2}{y_{13}^2 y_{24}^2}\,, \qquad (1-y)(1-\bar{y}) = \frac{y_{14}^2 y_{23}^2}{y_{13}^2 y_{24}^2}\,.$$

Superconformal symmetry implies a decomposition:

[Eden, Petkou, Schubert, Sokatchev]
[Dolan, Osborn]

$$\langle 2222 \rangle = \langle 2222 \rangle_{\text{free}} + \langle 2222 \rangle_{\text{int}}$$

$$\langle 2222 \rangle_{\text{int}} = d_{13}^2 d_{24}^2 s(x, \bar{x}; y, \bar{y}) F(u, v)$$

$$s(x, \bar{x}; y, \bar{y}) = (x-y)(x-\bar{y})(\bar{x}-y)(\bar{x}-\bar{y})$$

Kinematic dependence

Propagators: $d_{ij} = \frac{y_{ij}^2}{x_{ij}^2}$ $\quad x_{ij}^2 = (x_i - x_j)^2$ $\quad y_{ij}^2 = y_i \cdot y_j$

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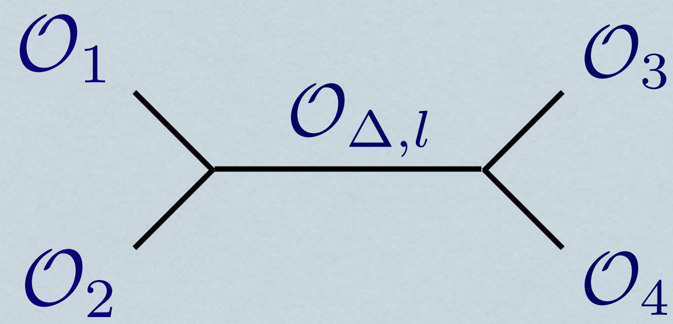
Free theory:

$$\langle 2222 \rangle_{\text{free}}^{(0)} = \Big(d_{12}^2 d_{34}^2 + d_{13}^2 d_{24}^2 + d_{14}^2 d_{23}^2 \Big)\,,$$

$$\langle 2222 \rangle_{\text{free}}^{(1)} = 4 \Big(d_{12} d_{34} d_{13} d_{24} + d_{12} d_{34} d_{14} d_{23} + d_{13} d_{24} d_{14} d_{23} \Big)$$

Operator product expansion



$$2t = \Delta - l$$

$$\langle 2222 \rangle_{\text{long}} = (d_{13}d_{24})^2 \sum_{t,l \geq 0} C_{t,l} F_{t,l}^{\text{long}}$$

Long superconformal blocks:

[Dolan, Osborn]

$$F_{t,l}^{\text{long}} = (x-y)(x-\bar{y})(\bar{x}-y)(\bar{x}-\bar{y}) G_{t,l}(x,\bar{x})$$

$$G_{t,l}(x,\bar{x}) = \frac{f_{t+l}(x) f_{t-1}(\bar{x}) - f_{t+l}(\bar{x}) f_{t-1}(x)}{x - \bar{x}}$$

$$f_\rho(x) = x^{\rho-1} {}_2F_1(\rho+2, \rho+2, 2\rho+4; x)$$

Supergravity Spectrum

In supergravity regime long single-trace operators decouple.

They cancel in the combination $\langle 2222\rangle^{(1)}_{\text{free}} + \langle 2222\rangle^{(1)}_{\text{int}}$

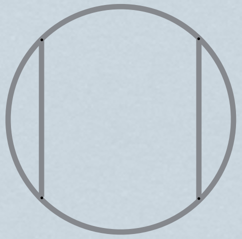
Remaining spectrum is made from products of half-BPS operators.

At leading order in large N only double-trace operators (degenerate):

At twist $2t$ $\{(\mathcal{O}_2\Box^{t-2}\partial^l\mathcal{O}_2),(\mathcal{O}_3\Box^{t-3}\partial^l\mathcal{O}_3),\ldots,(\mathcal{O}_t\Box^{0}\partial^l\mathcal{O}_t)\}$

have $(t-1)$ of them. $K_{t,l,i}$ $i=1,\ldots,t-1$

From disconnected part (large N free fields):

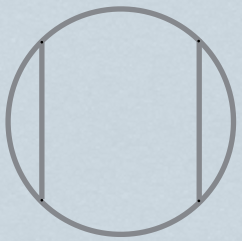


$$\sum_{i=1}^{t-1}\langle\mathcal{O}^{(2)}\mathcal{O}^{(2)}K_{t,l,i}\rangle^2 = C^{(0)}_{t,l} = \frac{2(t+l+1)!^2\, t!^2(l+1)(2t+l+2)}{(2t)!(2t+2l+2)!}$$

Supergravity Expansion

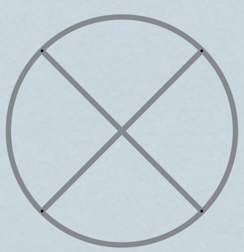
[also in Agnese's talk]

From disconnected part (large N free fields):



$$\sum_{i=1}^{t-1} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 = C^{(0)}_{t,l} = \frac{2(t+l+1)!^2 t!^2 (l+1)(2t+l+2)}{(2t)!(2t+2l+2)!}$$

Classical connected part:



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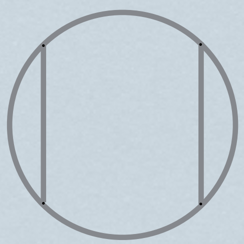

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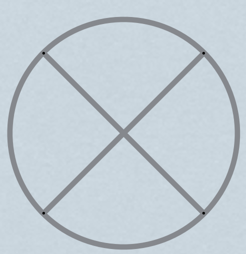

$$\langle 2222 \rangle^{(1)} = \langle 2222 \rangle^{(1)}_{\text{free}} + d_{13}^2 d_{24}^2 s(x, \bar{x}, y, \bar{y}) F^{(1)}(u,v)$$

$$F^{(1)}(u,v) = -4 \partial_u \partial_v (1 + u\partial_u + v\partial_v) \Phi^{(1)}(u,v)$$

$$\int \frac{d^4 x_0}{x_{01}^2 x_{02}^2 x_{03}^2 x_{04}^2} = \frac{\Phi^{(1)}(u,v)}{x_{13}^2 x_{24}^2}$$

Supergravity Expansion

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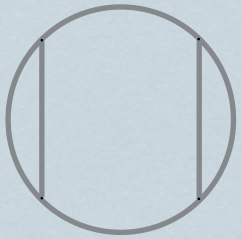

$$\sum_{i=1}^{t-1} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 = C^{(0)}_{t,l} = \frac{2(t+l+1)!^2 t!^2 (l+1)(2t+l+2)}{(2t)!(2t+2l+2)!}$$

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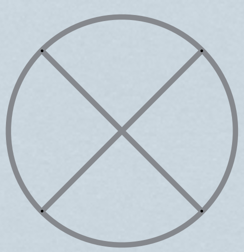
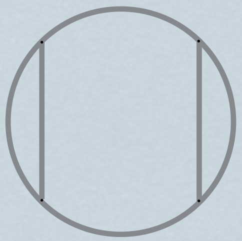


$$\sum_{i=1}^{t-1} \eta^{(1)}_{t,l,i} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 = -C^{(0)}_{t,l} \frac{(t-1)t(t+1)(t+2)}{2(l+1)(2t+l+2)}$$

log u arises because twist is: $2t \longrightarrow 2(t + a\eta^{(1)}_{t,l,i} + a^2 \eta^{(2)}_{t.l,i} + \dots)$

Mixing problem

From disconnected part (large N free fields):



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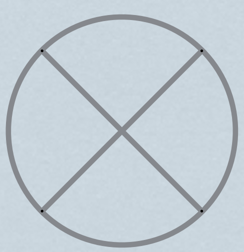
Classical connected part:



$$\sum_{i=1}^{t-1} \eta^{(1)}_{t,l,i} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 = -C^{(0)}_{t,l} \frac{(t-1)t(t+1)(t+2)}{2(l+1)(2t+l+2)}$$

Cannot extract 3-point functions and anomalous dimensions: mixing!

(except at twist 4 where there is only one operator [Dolan, Osborn])

Mixing problem

$K_{t,l,i}$ $\{(\mathcal{O}_2\Box^{t-2}\partial^l\mathcal{O}_2),(\mathcal{O}_3\Box^{t-3}\partial^l\mathcal{O}_3),\ldots,(\mathcal{O}_t\Box^0\partial^l\mathcal{O}_t)\}$

But: same operators arise in OPE of more general correlators: $\langle ppqq\rangle$

Disconnected piece simple.

Classical connected piece available in Mellin space rep. [Rastelli, Zhou]

Can play the same OPE game.

Unknowns:

Anom dims: $\eta_{t,l,i}$ $(t-1)$

3-pt. functions: $C_{ppK_{t,l,i}}=\langle\mathcal{O}_p\mathcal{O}_pK_{t,l,i}\rangle$ $(t-1)^2$

Unmixing

Organise long conformal block coefficients into matrices:

Large N free fields:

$$\mathcal{A}(t|l)\Big|_{[0,0,0]} = \begin{pmatrix} \mathcal{A}^{2222} & \mathcal{A}^{2233} & \dots & \mathcal{A}^{22tt} \\ & \mathcal{A}^{3333} & \dots & \mathcal{A}^{33tt} \\ & & \dots & \dots \\ & & & \mathcal{A}^{tttt} \end{pmatrix}$$

Classical part (log u) term:

$$\mathcal{M}(t|l)\Big|_{[0,0,0]} = \begin{pmatrix} \mathcal{M}^{2222} & \mathcal{M}^{2233} & \dots & \mathcal{M}^{22tt} \\ & \mathcal{M}^{3333} & \dots & \mathcal{M}^{33tt} \\ & & \dots & \dots \\ & & & \mathcal{M}^{tttt} \end{pmatrix}$$

Have 2 x t(t-1)/2 pieces of information.

Exactly enough information to resolve mixing!

Unmixing

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Large N free fields:

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Have 2 x t(t-1)/2 pieces of information.

Exactly enough information to resolve mixing!

Complicated algebraic solutions?

$$\mathbb{C} \cdot \mathbb{C}^T = \mathcal{A}$$

$$\mathbb{C} \cdot \eta \cdot \mathbb{C}^T = \mathcal{M}$$

Solution

$K_{t,l,i}$ $\{(\mathcal{O}_2\Box^{t-2}\partial^l\mathcal{O}_2),(\mathcal{O}_3\Box^{t-3}\partial^l\mathcal{O}_3),\dots,(\mathcal{O}_t\Box^0\partial^l\mathcal{O}_t)\}$

Anomalous dimensions:

$$\eta_{t,l,i} = -\frac{2(t-1)_4(l+t)_4}{(l+2i-1)_6}$$

(generalises to all channels)

(residual partial degeneracy)

Three-point functions:

(beautiful structure in non-degenerate cases)

$$\langle \mathcal{O}^{(2)}\mathcal{O}^{(2)}K_{t,l,i}\rangle^2 = C^{(0)}_{t,l}R_{t,l,i}a_{t,i}$$

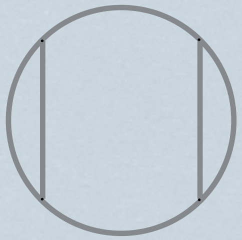
$$R_{t,l,i} = \frac{2^{1-t}(2l+3+4i)(l+i+1)_{t-i-1}(t+l+4)_{i-1}}{(\frac{5}{2}+l+i)_{t-1}}\,,$$

$$a_{t,i} = \frac{2^{(1-t)}(2+2i)!(t-2)!(2t-2i+2)!}{3(i-1)!(i+1)!(t+2)!(t-i-1)!(t-i+1)!}\,.$$

Surprisingly simple structure to resolve mixing!

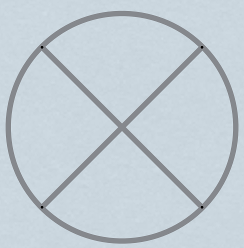
Loop predictions

From disconnected part:



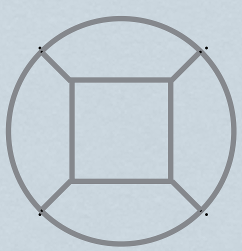
$$\sum_{i=1}^{t-1} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 = C_{t,l}^{(0)} = \frac{2(t+l+1)!^2 t!^2 (l+1)(2t+l+2)}{(2t)!(2t+2l+2)!}$$

From classical connected part (log u) term:



$$\sum_{i=1}^{t-1} \eta_{t,l,i}^{(1)} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 = -C_{t,l}^{(0)} \frac{(t-1)t(t+1)(t+2)}{2(l+1)(2t+l+2)}$$

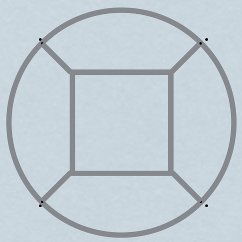
Predict one-loop (log u)^2 term:



$$\frac{1}{2} \sum_{t=2}^{\infty} \sum_{l=0}^{\infty} \sum_{i=1}^{t-1} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} K_{t,l,i} \rangle^2 (\eta_{t,l,i}^{(1)})^2 G_{t,l}(x, \bar{x})$$

Loop predictions

Predict one-loop (log u)^2 term:



$$\frac{1}{2}\sum_{t=2}^{\infty}\sum_{l=0}^{\infty}\sum_{i=1}^{t-1}\langle\mathcal{O}^{(2)}\mathcal{O}^{(2)}K_{t,l,i}\rangle^2(\eta^{(1)}_{t,l,i})^2 G_{t,l}(x,\bar{x})$$

Can resum!

$$F_2^{(2)}(u,v) = \frac{1}{uv}\Big[p(u,v)\frac{\mathrm{Li}_1(x)^2-\mathrm{Li}_1(\bar{x})^2}{x-\bar{x}} + 2\Big[p(u,v)+p\Big(\frac{1}{v},\frac{u}{v}\Big)\Big]\frac{\mathrm{Li}_2(x)-\mathrm{Li}_2(\bar{x})}{x-\bar{x}} + q(u,v)(\mathrm{Li}_1(x)+\mathrm{Li}_1(\bar{x})) + r(u,v)\frac{\mathrm{Li}_1(x)-\mathrm{Li}_1(\bar{x})}{x-\bar{x}} + s(u,v)\Big]$$

$$p(u,v) = -24uv\partial_x^2\partial_{\bar{x}}^2\left[\frac{u^2v^2(1-u-v)[(1-u-v)^4+20uv(1-u-v)^2+30u^2v^2]}{(x-\bar{x})^{10}}\right]$$

Loop predictions

Double discontinuity:

$$F_2^{(2)}(u,v) = \frac{1}{uv}\Big[p(u,v)\frac{\mathrm{Li}_1(x)^2 - \mathrm{Li}_1(\bar{x})^2}{x-\bar{x}} + 2\Big[p(u,v) + p\Big(\frac{1}{v},\frac{u}{v}\Big)\Big]\frac{\mathrm{Li}_2(x) - \mathrm{Li}_2(\bar{x})}{x-\bar{x}}$$

$$+ q(u,v)(\mathrm{Li}_1(x) + \mathrm{Li}_1(\bar{x})) + r(u,v)\frac{\mathrm{Li}_1(x) - \mathrm{Li}_1(\bar{x})}{x-\bar{x}} + s(u,v)\Big]$$

Also obtained in [Alday, Caron-Huot]

Can promote to crossing symmetric function! (Analytic perturbative bootstrap)

$$F^{(2)}(u,v) = \frac{1}{2^5}\Big[\Delta^{(8)}\frac{u^4v^2(1-u-v)}{(x-\bar{x})^7}\tilde{\phi}^{(2)}(x,\bar{x}) + \text{cross terms}\Big] + \dots$$

Only two-loop ladder integrals appearing + lower weights!

A single ambiguity - corresponds to adding a D-function.

Loop predictions

Next correction to twist 4 anomalous dimensions:

$$\eta_l^{(2)} = \begin{cases} \frac{1344(l-7)(l+14)}{(l-1)(l+1)^2(l+6)^2(l+8)} - \frac{2304(2l+7)}{(l+1)^3(l+6)^3} & l = 2, 4, \ldots \\ \frac{9}{14}\alpha + \frac{1148}{3} & l = 0 \end{cases}$$

Above formula for l=2,4 also obtained in [Alday, Bissi]

All spin formula reproduced in [Alday, Caron-Huot]

Discussion and Outlook

Have obtained supergravity loop corrections from OPE consistency.

Relied on being able to resolve double-trace mixing from classical data.

Can now extract subleading corrections to anomalous dimensions & three-point functions.

Much more to explore in terms of the mixing problem.

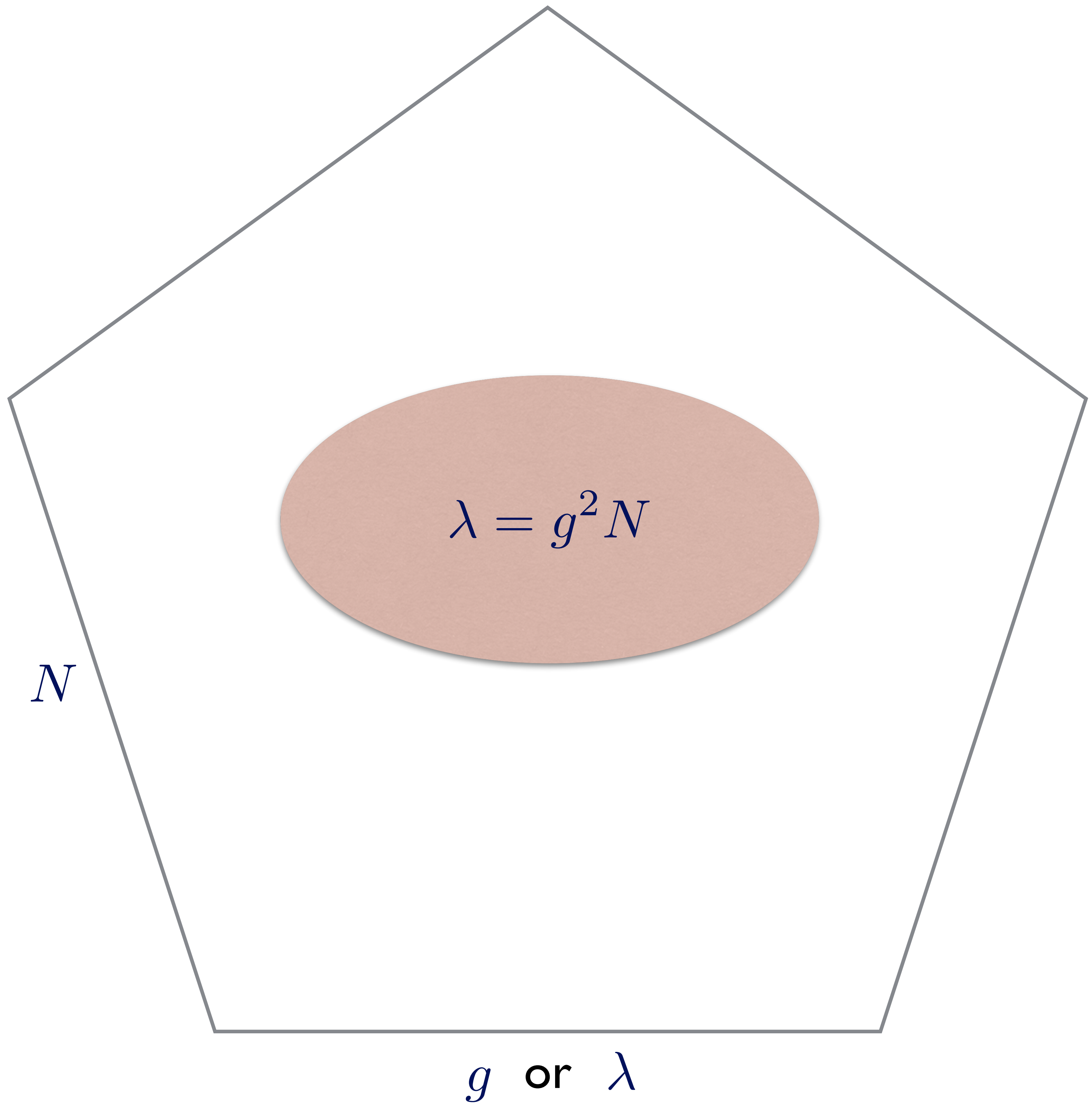
Loop corrections to higher charge correlators:

Higher loops? (Triple trace ops!) Mellin space representation?

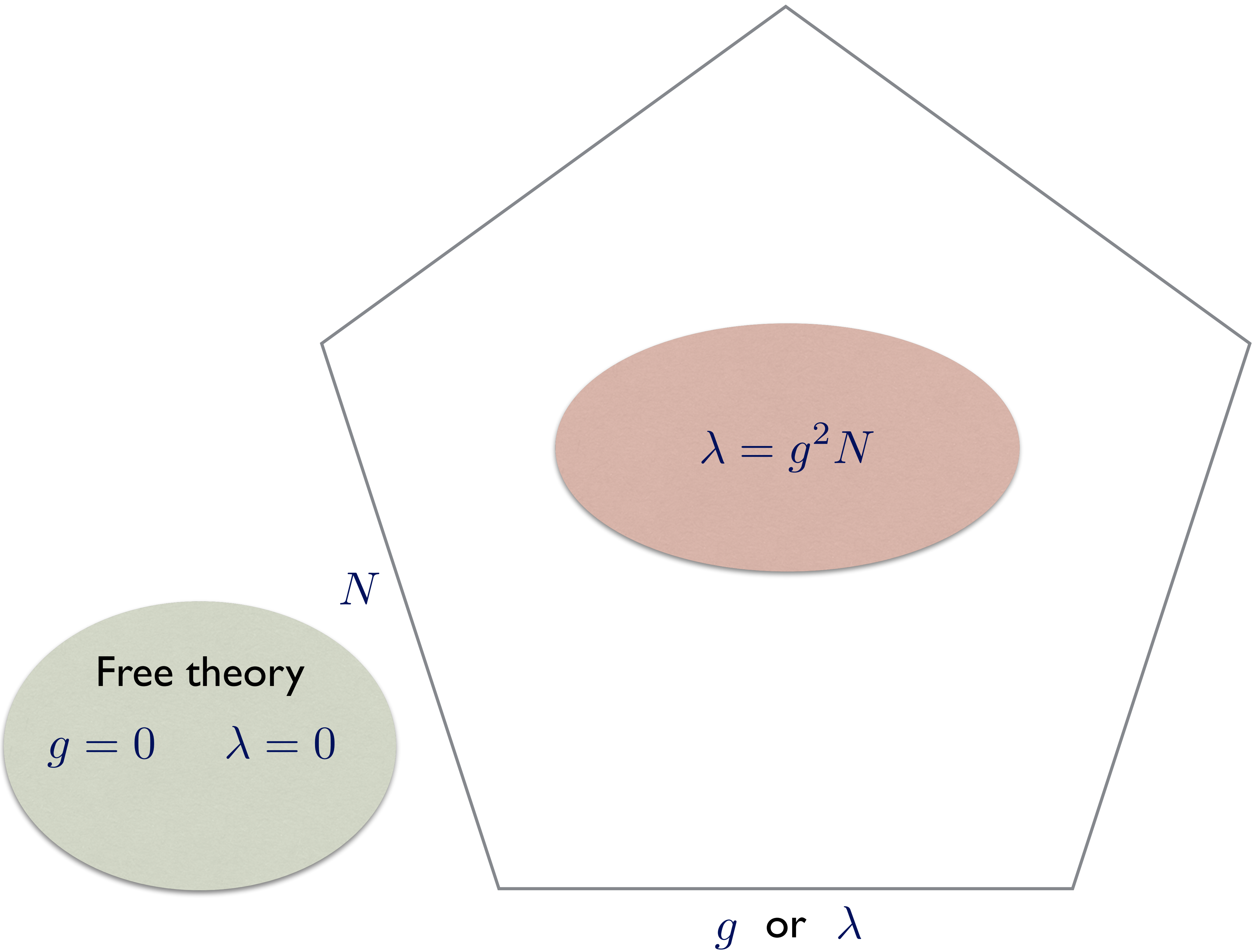
Non-planar corrections not well understood - polygonalization?

Extra slides

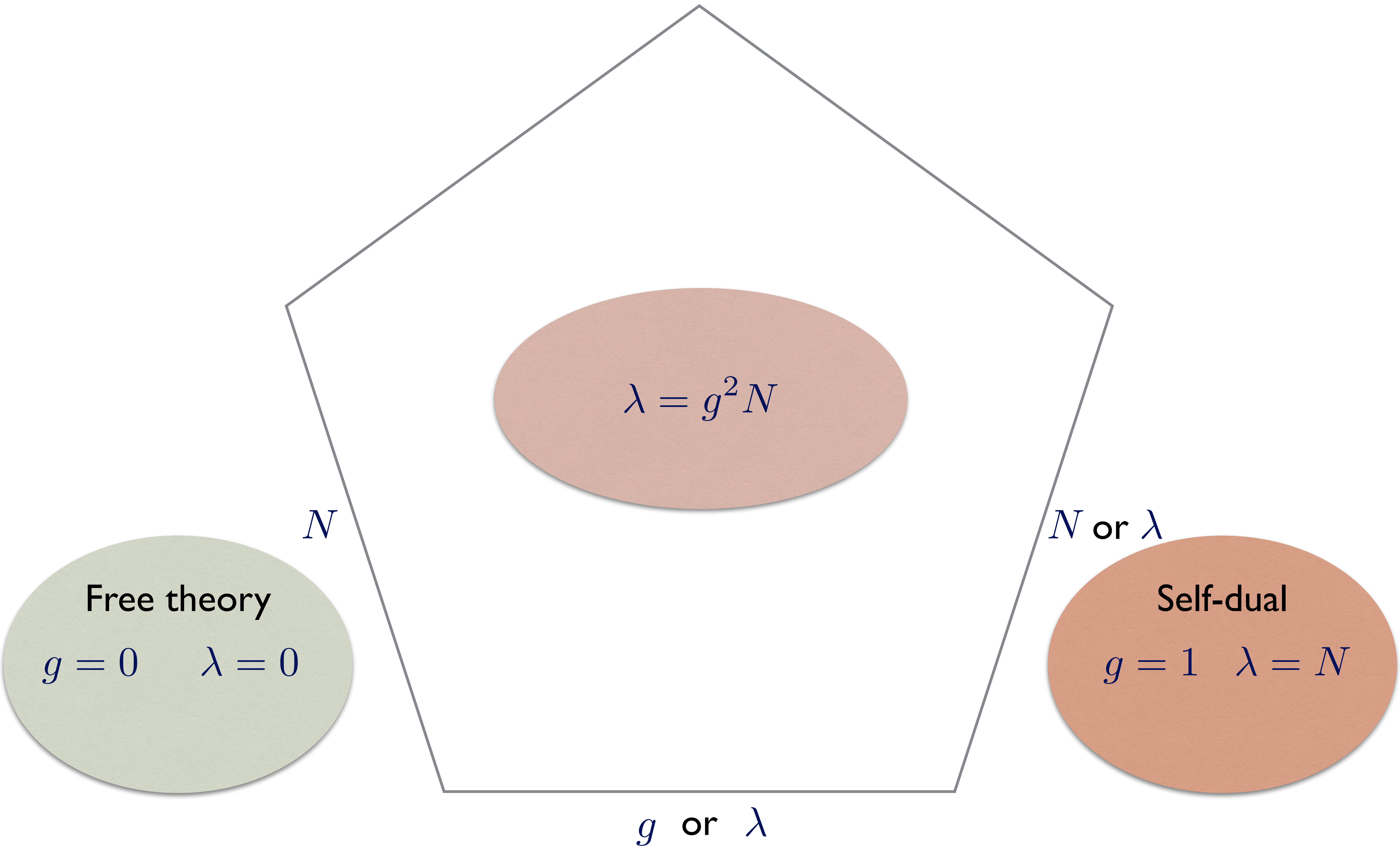
Different regimes



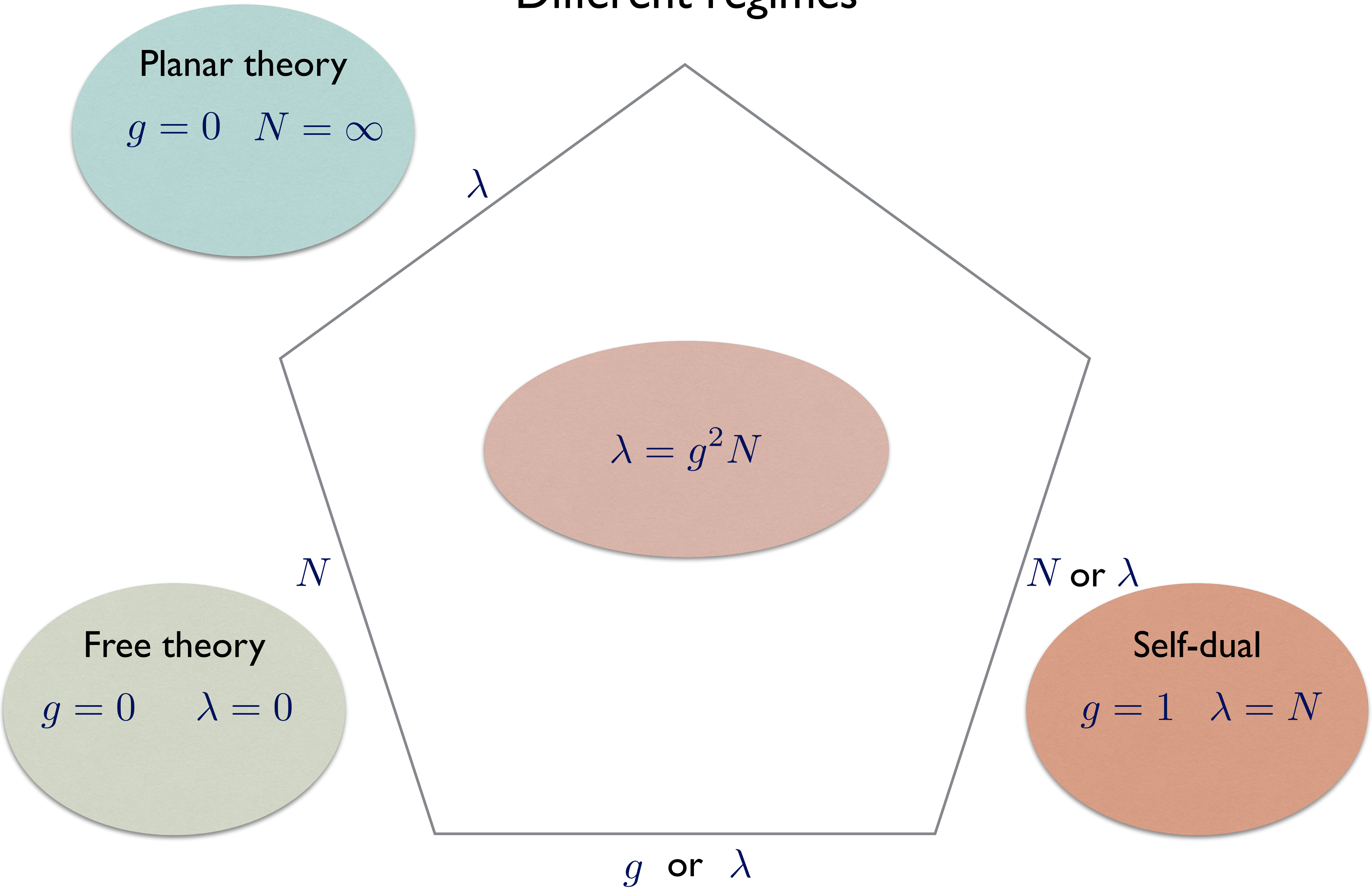
Different regimes



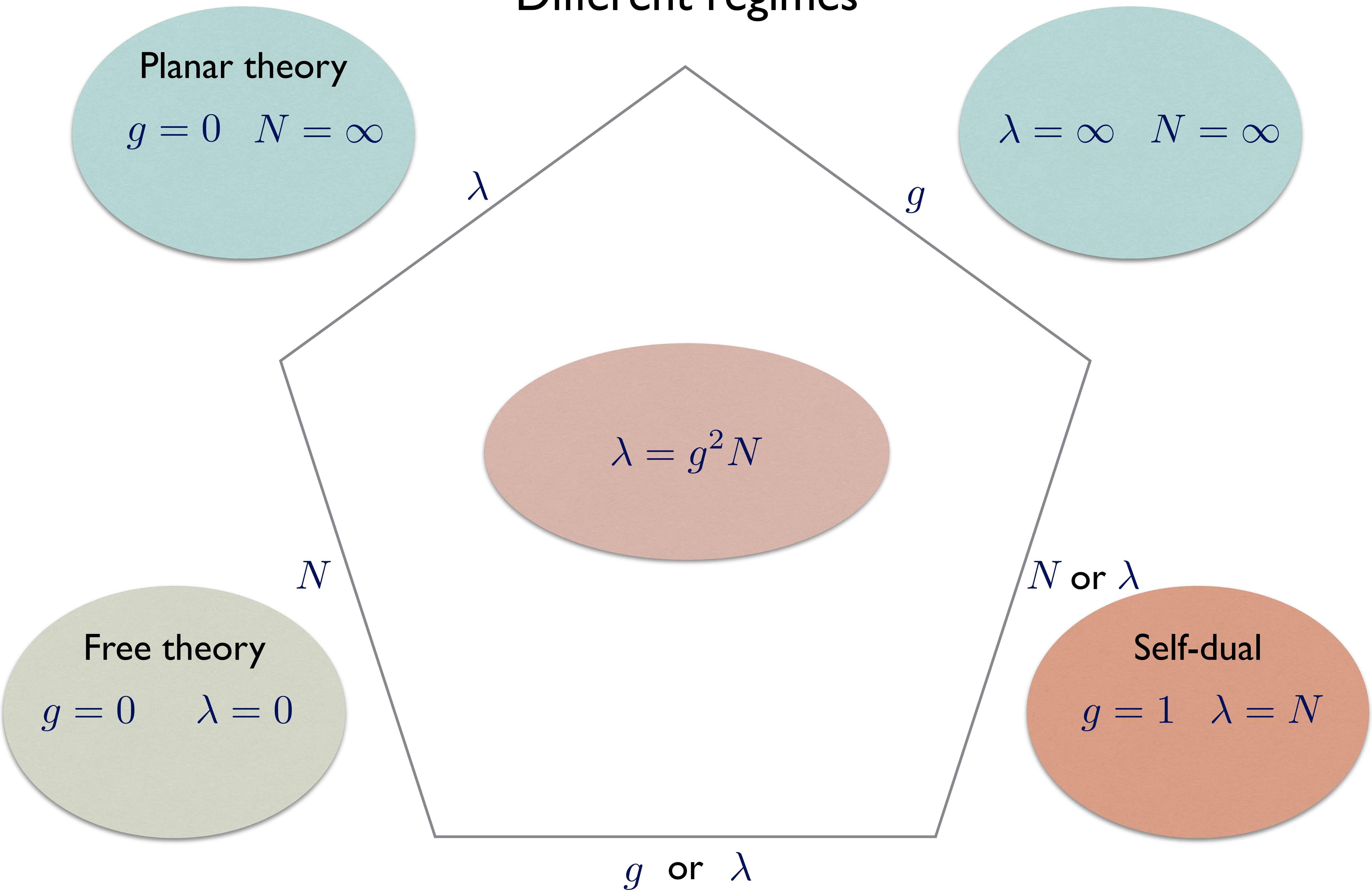
Different regimes



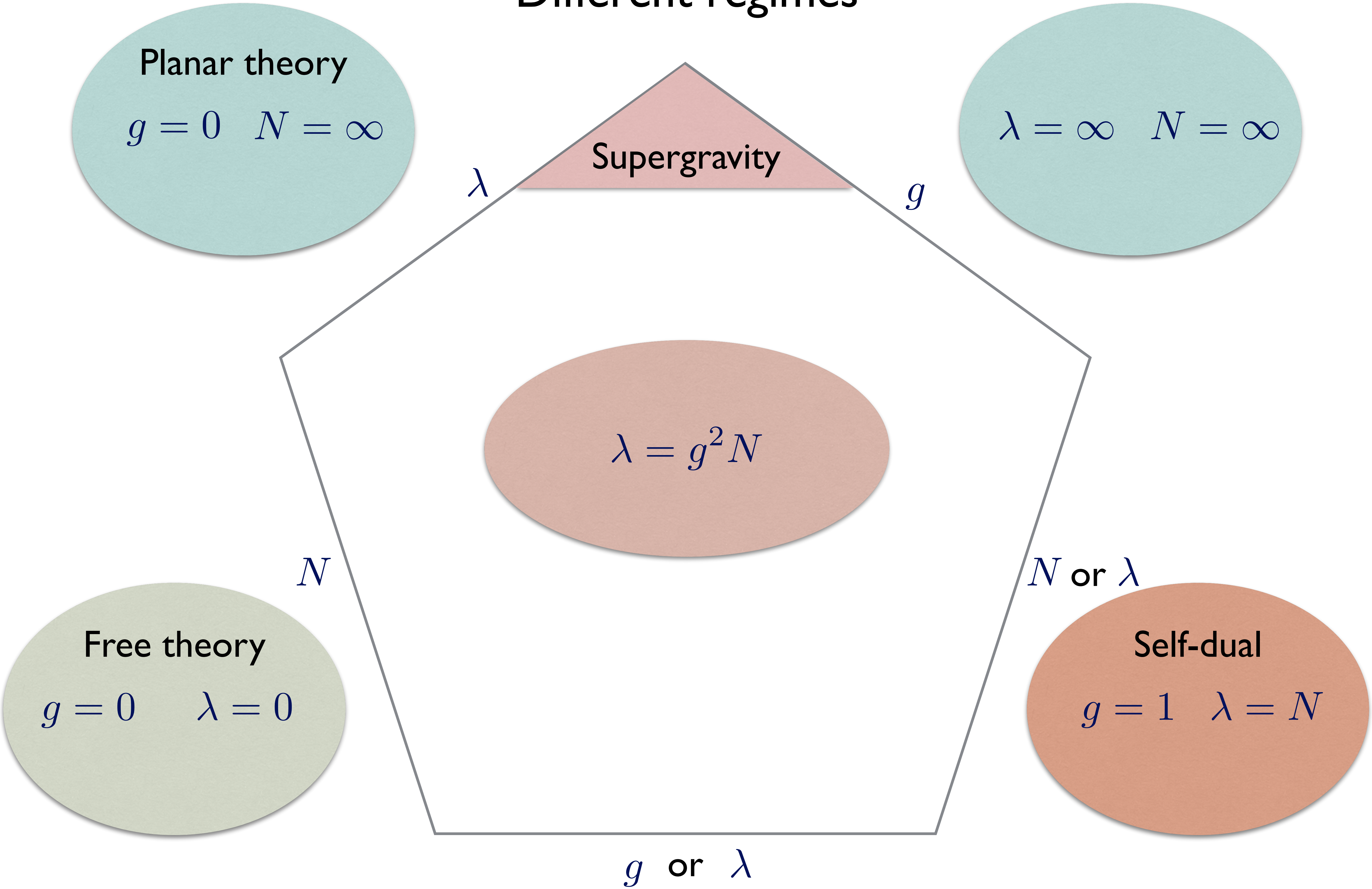
Different regimes



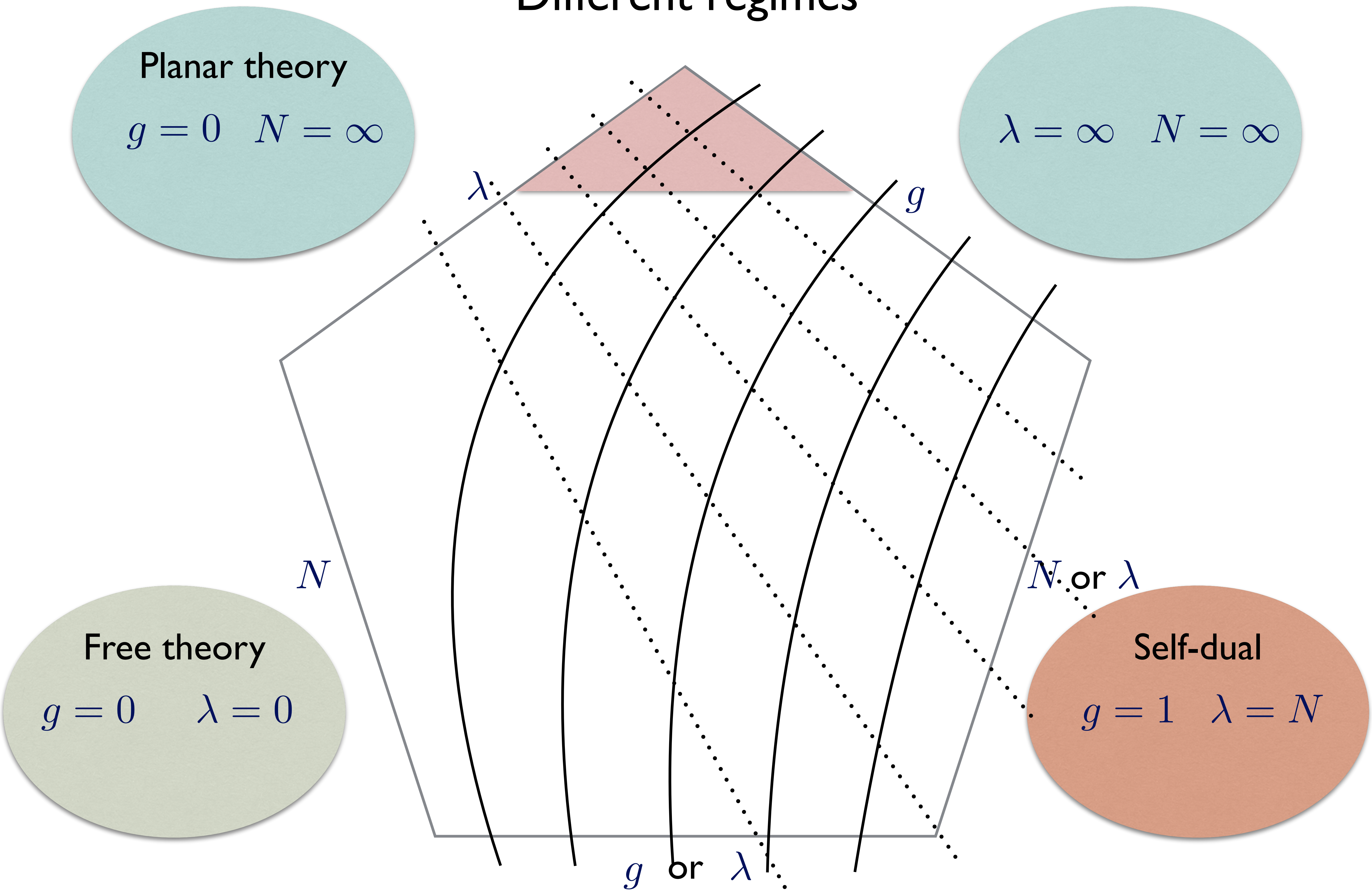
Different regimes



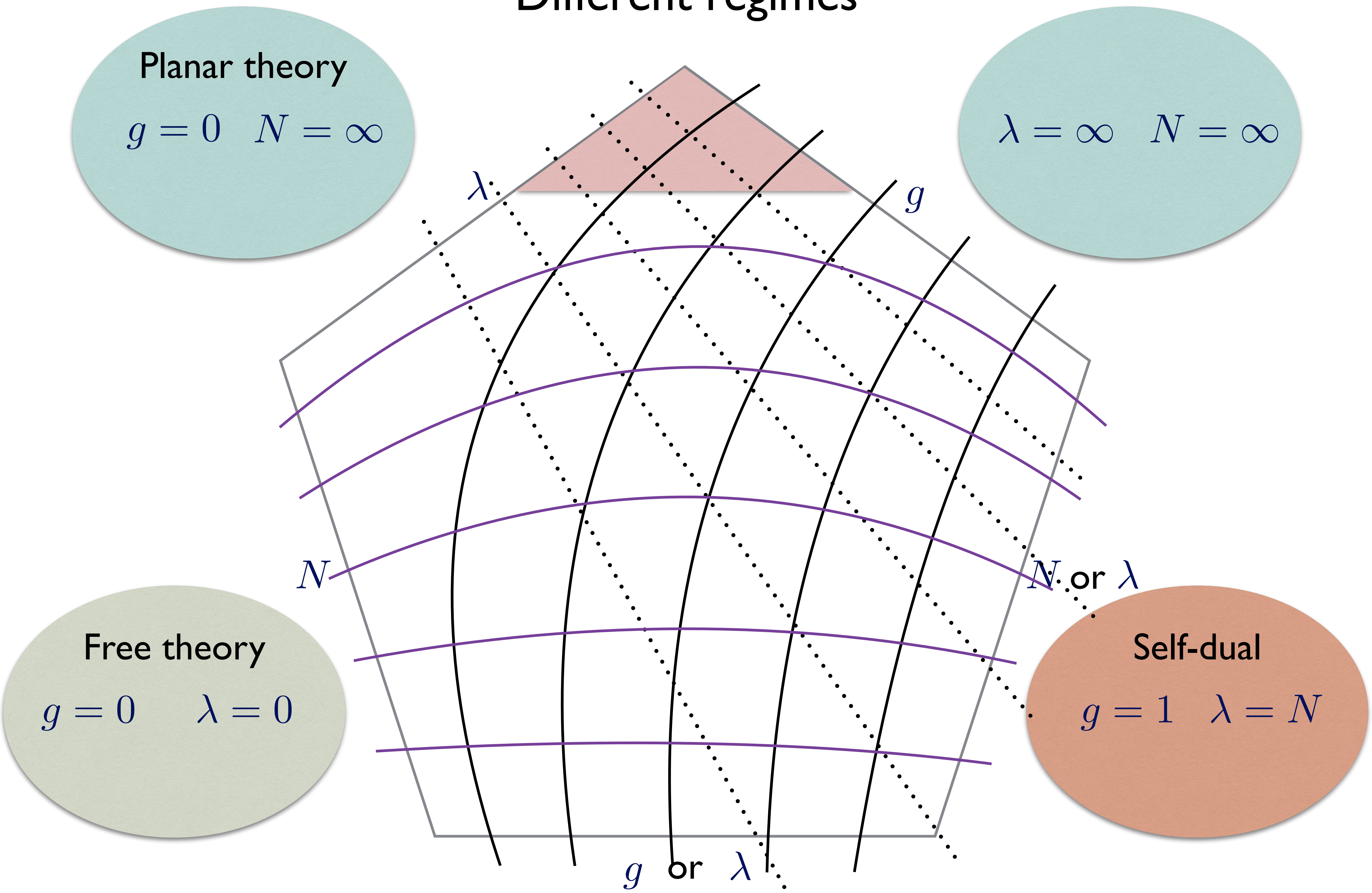
Different regimes



Different regimes



Different regimes

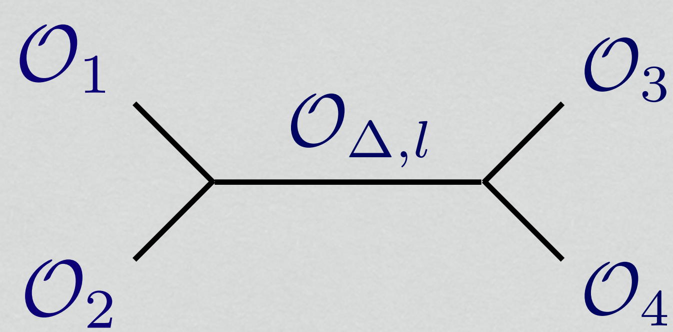


Anatomy of an N=4 correlator

$$\langle 2222\rangle = \langle 2222\rangle^{(0)}_{\text{free}} + a\langle 2222\rangle^{(1)}_{\text{free}} + a\langle 2222\rangle^{(1)}_{\text{int}} + a^2\langle 2222\rangle^{(2)}_{\text{int}} + \ldots$$

Anatomy of an N=4 correlator

$$\langle 2222 \rangle = \langle 2222 \rangle^{(0)}_{\text{free}} + a \langle 2222 \rangle^{(1)}_{\text{free}} + a \langle 2222 \rangle^{(1)}_{\text{int}} + a^2 \langle 2222 \rangle^{(2)}_{\text{int}} + \dots$$



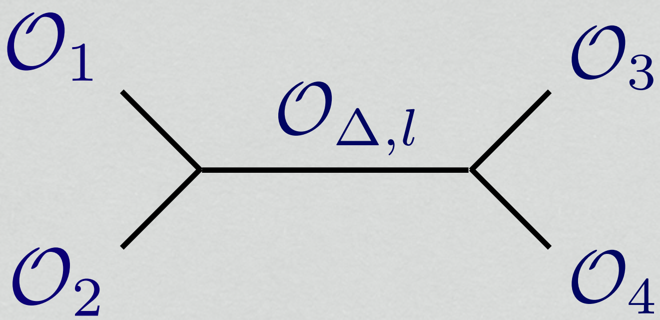
Anatomy of an N=4 correlator

$$\langle 2222\rangle = \langle 2222\rangle^{(0)}_{\text{free}} + a\langle 2222\rangle^{(1)}_{\text{free}}$$

Protected

$$\langle 2222\rangle^{(0)}_{\text{free}} + a\langle 2222\rangle^{(1)}_{\text{free}} + a\langle 2222\rangle^{(1)}_{\text{int}} + a^2\langle 2222\rangle^{(2)}_{\text{int}} + \dots$$

Long



Unmixing

Universal factor in three-point functions:

$$(C_{ppK_{t,l,i}})^2 = \frac{(l+t+1)!^2}{(2l+2t+2)!} c_{pi}^2\,, \qquad p = 2, \ldots, t, \quad i = 1, \ldots, t-1$$

Twist 4 problem (t=2): [Dolan, Osborn]

$$(C_{22K_{t,l,1}})^2 = \mathcal{A}^{2222} \Rightarrow \qquad c_{21}^2 = \frac{4}{3}(l+1)(l+6)$$

$$\eta_1 (C_{22K_{t,l,1}})^2 = \mathcal{M}^{2222} \Rightarrow \qquad c_{21}^2 \eta_1 = -64$$

solution:

$$\eta_1 = -\frac{48}{(l+1)(l+6)}, \qquad c_{21} = \sqrt{\frac{4(l+1)(l+6)}{3}}$$

Unmixing

Twist 6: large N free fields:

$$c_{21}^2 + c_{22}^2 = \frac{2}{5}(l+1)(l+8)$$

$$c_{31}^2 + c_{32}^2 = \frac{9}{40}(l+1)(l+2)(l+7)(l+8)$$

$$c_{21}c_{31} + c_{22}c_{32} = 0$$

classical log u:

$$c_{21}^2\eta_1 + c_{22}^2\eta_2 = -96$$

$$c_{31}^2\eta_1 + c_{32}^2\eta_2 = -54(l^2+9l+44)$$

$$c_{21}c_{31}\eta_1 + c_{22}c_{32}\eta_2 = 432$$

solution!

$$\eta_1 = -\frac{240}{(l+1)(l+2)}, \qquad \eta_2 = -\frac{240}{(l+7)(l+8)},$$

$$c_{21} = -\sqrt{\frac{2(l+1)(l+2)(l+8)}{5(2l+9)}}, \qquad c_{22} = -\sqrt{\frac{2(l+1)(l+7)(l+8)}{5(2l+9)}},$$

$$c_{31} = \sqrt{\frac{9(l+1)(l+2)(l+7)^2(l+8)}{40(2l+9)}}, \qquad c_{32} = -\sqrt{\frac{9(l+1)(l+2)^2(l+7)(l+8)}{40(2l+9)}}$$

Kaluza-Klein Correlators

We have also studied the first case involving Kaluza-Klein modes:

$$\langle 2233 \rangle$$

Less crossing symmetry.

Need to consider different channels or equivalently also: $\langle 2323 \rangle$

Even and odd spins contribute.

Anomalous dimensions: $\Delta = 2t + l + 1 + 2a\eta^{(1)}_{t,l,i} + 2a^2\eta^{(2)}_{t,l,i} + \dots$

$$\eta^{(1)}_{t,l,i} = \begin{cases} -\frac{2(t-1)_2(t+2)_2(l+t)_2(l+t+3)_2}{(l+2i-1)_6} & l = 0, 2, \dots \\ -\frac{2(t-1)_2(t+2)_2(l+t)_2(l+t+3)_2}{(l+2i)_6} & l = 1, 3, \dots \end{cases}$$