Direct fits of Wilson coefficients using $B^0 \to K^{*0}\mu^+\mu^-$

Based on [arXiv:1708.04474, T. Hurth, CL, N. Mahmoudi]

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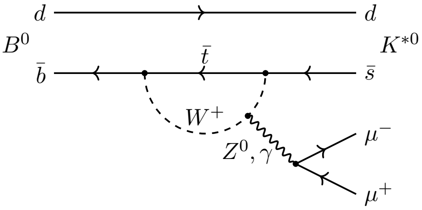
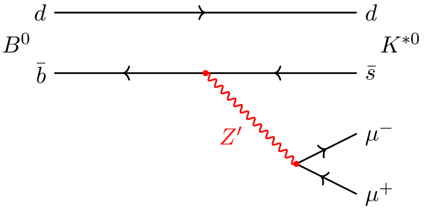
Outline

1. Introduction to the decay $B^0 \to K^{*0}\mu^+\mu^-$
2. Conventional q^2-binned approach
3. Method to directly determine Wilson coefficients from data
4. Validation and performance comparison
5. Conclusions

The rare decay $B^0 \to K^{*0}\mu^+\mu^-$



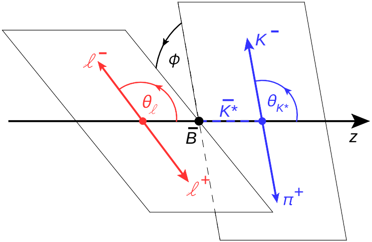


- $b \to s$ flavour changing neutral current
  Forbidden at tree-level and only occurs as loop-process in the SM
- New heavy particles can give significant contributions, affect branching fraction, angular observables
- Model independent description in EFT ($\Lambda_{\text{NP}} \gg m_W \gg m_b > \Lambda_{\text{QCD}}$):

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{\text{tb}} V_{\text{ts}}^* \sum_i \mathcal{C}_i \mathcal{O}_i \qquad \Delta\mathcal{H}_{\text{NP}} = \frac{\kappa}{\Lambda_{\text{NP}}^2}\mathcal{O}_i$$

Wilson coefficient ($\mathcal{C}_i$) — Local operator ($\mathcal{O}_i$)

The $B^0 \to K^{*0}[\to K^+\pi^-]\mu^+\mu^-$ angular distribution



- Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$
- Four-differential decay rate $\frac{d^4\Gamma(\overset{(-)}{B}{}^0 \to \overset{(-)}{K}{}^{*0}\mu^+\mu^-)}{d\vec{\Omega}\, dq^2} = \sum_i \overset{(-)}{I}_i(q^2) f_i(\vec{\Omega})$ with $\overset{(-)}{I}(q^2)$ bilinear comb. of decay amp. and $f_i(\vec{\Omega})$ angular terms[1]
- Conventional apporach: Measure q^2-integrated angular observables

$$S_i(q^2_{\min}, q^2_{\max}) = \frac{\int_{q^2_{\min}}^{q^2_{\max}} I_i(q^2) + \bar{I}_i(q^2)\, dq^2}{\int_{q^2_{\min}}^{q^2_{\max}} \frac{d\Gamma(q^2)}{dq^2} + \frac{d\bar{\Gamma}(q^2)}{dq^2}\, dq^2}$$

[1] See backup for details

The q^2-binned $B^0 \to K^{*0}\mu^+\mu^-$ angular distribution

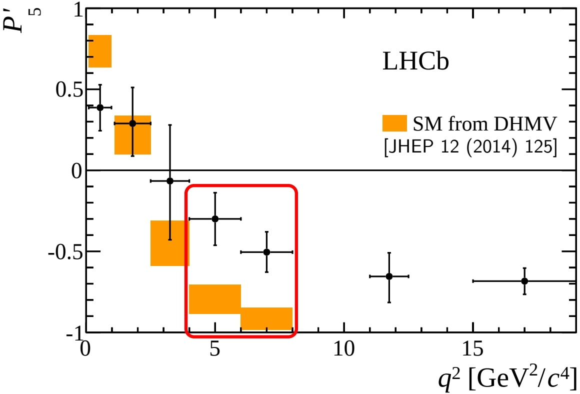
- Angular distribution in bins of q^2 given by

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K + \tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_\ell - F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + \tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi + S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

- $F_{\mathrm{L}}, A_{\mathrm{FB}}, S_i$ combinations of K^{*0} spin amplitudes depending on Wilson coefficients $C_7^{(\prime)}$, $C_9^{(\prime)}$, $C_{10}^{(\prime)}$ and form factors
- Perform ratios of observables where form factors cancel at LO
  Example: $P_5' = \frac{S_5}{\sqrt{F_{\mathrm{L}}(1-F_{\mathrm{L}})}}$ [S. Descotes-Genon *et al.*, JHEP, 05 (2013) 137]

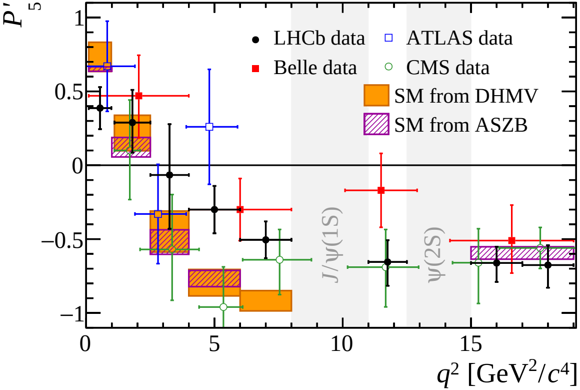
P_5' anomaly

[JHEP 02 (2016) 104]



- In q^2 bins $[4.0, 6.0]$ and $[6.0, 8.0]\,\text{GeV}^2/c^4$ local deviations of $2.8\,\sigma$ and $3.0\,\sigma$
- Global $B^0 \to K^{*0}\mu^+\mu^-$ analysis finds deviation corresponding to $3.4\,\sigma$
- [JHEP 02 (2016) 104] [ATLAS-CONF-2017-023] [CMS-PAS-BPH-15-008] [PRL 118 (2017) 111801]

P_5' anomaly



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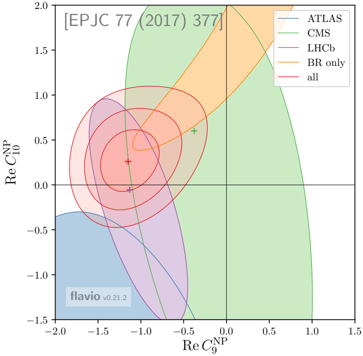
Conventional q^2 binned approach

- Global fits include q^2-binned angular obs. S_m ($P_m^{(\prime)}$) and correlations
- Typically construct χ^2 from $b \to s$ data[2]

$$\chi^2 = \left(S_m^{\rm obs} - S_m^{\rm th}(\mathcal{C}_i, \vec{\lambda}^{\rm th})\right)^T {\rm Cov}^{-1}(S_m^{\rm obs}, S_n^{\rm obs}) \left(S_n^{\rm obs} - S_n^{\rm th}(\mathcal{C}_i, \vec{\lambda}^{\rm th})\right)$$

- Theory nuisance parameters $\vec{\lambda}^{\rm th}$
- Resulting confidence regions depending on WC



- Significance around 5σ, hadronic uncertainties under discussion
- Many other global fits:
  [arxiv:1704.05340]
  [JHEP 06 (2016) 092]
  [NPB 909 (2016) 737]
  ...

[2] LHCb published $B^0 \to K^{*0}\mu^+\mu^-$ on HepData: [hepdata.net/record/ins1409497]

Proposed q^2-unbinned direct fit method

- Use $\frac{d^4\Gamma(\overset{(-)}{B}{}^0 \to \overset{(-)}{K}{}^{*0}\mu^+\mu^-)}{d\vec{\Omega}\, dq^2} = \sum_i \overset{(-)}{I}_i(q^2|\mathcal{C}_i, \vec{\lambda}^{\text{th}}) f_i(\vec{\Omega})$ directly as PDF in likelihood fit ("(decay) amplitude analyis") at low[3] q^2
- Use Implementation of $d^4\Gamma/d\vec{\Omega}\, dq^2$ from [EOS] and [SuperIso]

✓ Pros:

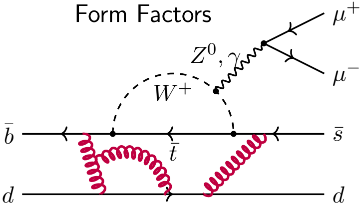
- No loss of q^2-information } Expect higher statistical power
- q^2 and angles used per-event } Expect higher statistical power
- Use of all events in a single fit $\rightarrow$ Expect higher fit stability
- Automatic inclusion of (also non-linear) experimental correlations

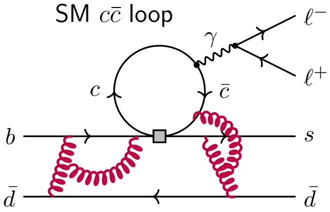
✗ Cons:

- Relies on calculation
- Requires unbinned efficiency corrected data

[3] For high q^2 OPE relies on integration over q^2

Theory nuisance parameters[4]





Form factors

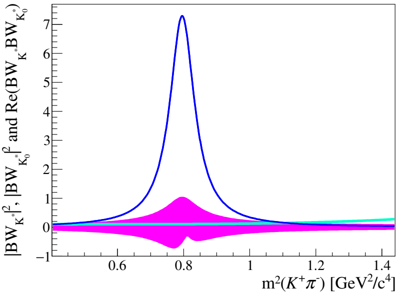
- $V(q^2)$, $A_{0,1,2}(q^2)$ and $T_{1,2,3}(q^2)$ from combined fit of LCSR [JHEP 08 (2016) 098] and lattice [PRD 89 (2014) 094501]
- Use covariance matrix as multivariate Gaussian constraint

Subleading corrections

- Non-fact. subleading $\Lambda_{\mathrm{QCD}}/m_b$ corrections following [JHEP 12 (2014) 125, NPB 909 (2016) 737]
- Factors $(1 + a_{0,\parallel,\perp} + b_{0,\parallel,\perp} \frac{q^2}{6\,\mathrm{GeV}^2})$ to corr. hadronic terms
- $\mathrm{Re}/\mathrm{Im}(a_i)$ $(\mathrm{Re}/\mathrm{Im}(b_i))$ constrained around 0 with $\sigma = \pm 0.1$ (± 0.25)

[4] full list in backup

S-wave contribution



- Use $m_{K\pi}$ distribution to separate P-wave and S-wave contributions
- Parameterisation [NPB 868 (2013) 368]

$$\mathcal{BW}_{K^*}(m_{K\pi}^2) = \frac{\sqrt{m_{K^*}\Gamma_{K^*}/\pi}}{m_{K^*}^2 - m_{K\pi}^2 - i m_{K^*}\Gamma_{K^*}}$$

$$\mathcal{BW}_{K_0^*}(m_{K\pi}^2) = \mathcal{N}_m \left[-\frac{g_\kappa}{(m_\kappa - i\Gamma_\kappa/2)^2 - m_{K\pi}^2} + \frac{1}{(m_{K_0^*} - i\Gamma_{K_0^*}/2)^2 - m_{K\pi}^2} \right]$$

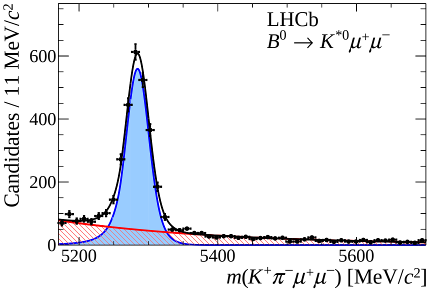
- $K^+\pi^-$ system can be in S-wave conf. (non-resonant decay/scalar resonance)
- Four-differential decay rate needs to be modified

$$\frac{d^4\Gamma(\overline{B}^0 \to K^-\pi^+\mu^+\mu^-)}{d\vec{\Omega}\, dq^2} = (1 - F_S) \left.\frac{d^4\Gamma(\overline{B}^0 \to \overline{K}^{*0}\mu^+\mu^-)}{d\vec{\Omega}\, dq^2}\right|_{\rm P-wave} + \sum_{i=10}^{17} I_i(q^2) f_i(\vec{\Omega})$$

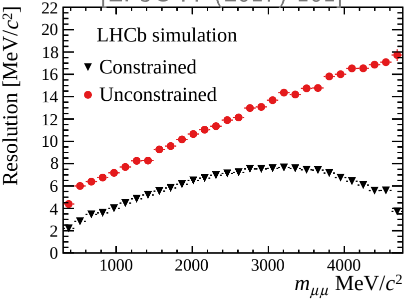
$$\frac{d^4\Gamma(\overline{B}^0 \to K^-\pi^+\mu^+\mu^-)}{d\vec{\Omega}\, dq^2} \to \frac{d^5\Gamma(\overline{B}^0 \to K^-\pi^+\mu^+\mu^-)}{d\vec{\Omega}\, dq^2\, dm_{K\pi}^2}$$

Experimental nuisance parameters

[JHEP 02 (2016) 104]

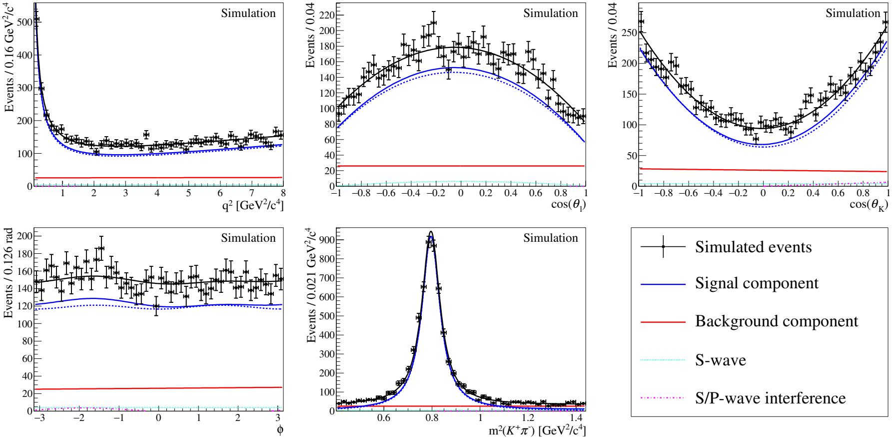


[EPJC 77 (2017) 161]



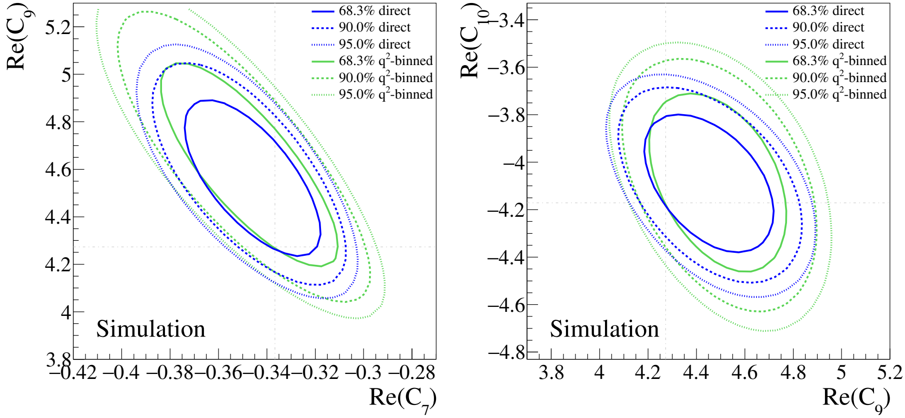
- Model background:
  - Exponential in $m_{K\pi\mu\mu}$
  - 1^{st} order polynomials in $\vec{\Omega}$, q^2, $m_{K\pi}$
- Acceptance effect, for simplicity: flat
- Performed toys to check q^2 resolution, found to be negligible

Single pseudoexperiment (low q^2)



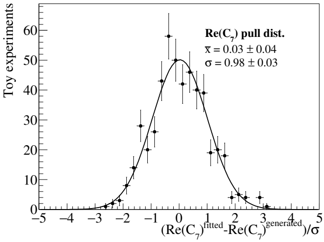
- Generate simulated events (yield corresponding LHCb Run 1+2 exp.)
- Fit pseudoexperiments to validate the method and study its sensitivity

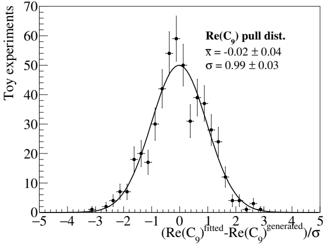
Single pseudoexperiment

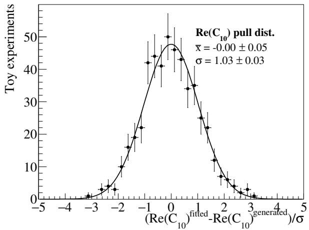


- Direct fit more precise than q^2-binned approach of the same toy
- For quantitative statements we study large ensemble of 500 pseudoexperiments

Validation via Pseudoexperiments







	pull mean	pull width
Single Wilson coefficients		
$\mathrm{Re}(C_7)$	0.03 ± 0.04	0.98 ± 0.03
$\mathrm{Re}(C_9)$	-0.02 ± 0.04	0.99 ± 0.03
$\mathrm{Re}(C_{10})$	-0.00 ± 0.05	1.03 ± 0.03
Pairs of Wilson coefficients		
$\mathrm{Re}(C_7)$	0.03 ± 0.05	1.02 ± 0.03
$\mathrm{Re}(C_9)$	-0.04 ± 0.05	1.02 ± 0.03
$\mathrm{Re}(C_9)$	-0.01 ± 0.04	0.97 ± 0.03
$\mathrm{Re}(C_{10})$	0.03 ± 0.05	1.01 ± 0.03

- Pull distributions important for fit validation $p_i = \frac{x_{\mathrm{fit}} - x_{\mathrm{gen}}}{\sigma(x_{\mathrm{fit}})}$
- Pull distributions of direct fit method centered around 0 with width 1
  $\rightarrow$ Direct method unbiased, errors estimated correctly

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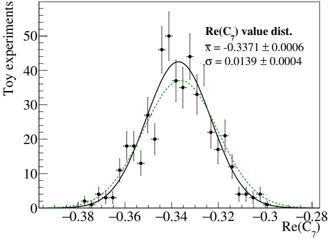
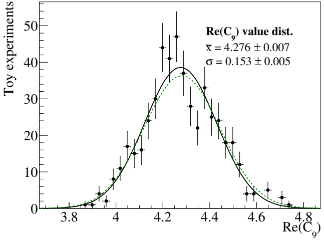
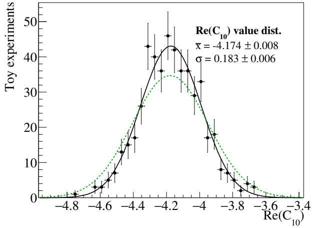
Sensitivity to Wilson coefficients



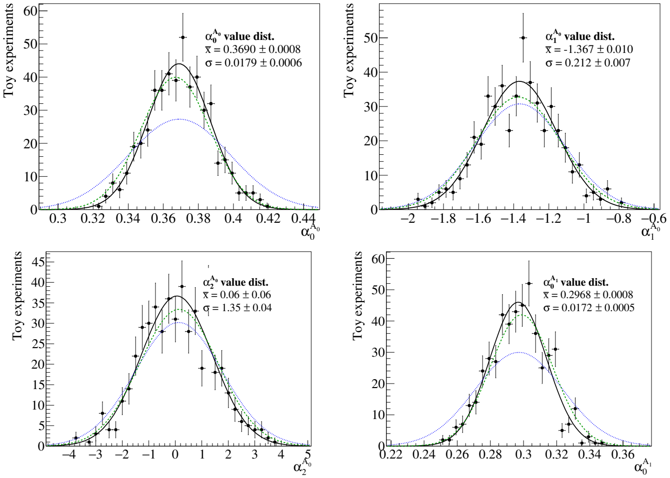




	direct	q^2-binned
Single Wilson coefficients		
$\text{Re}(C_7)$	0.0139 ± 0.0004	0.0159 ± 0.0005
$\text{Re}(C_9)$	0.1534 ± 0.0049	0.1625 ± 0.0052
$\text{Re}(C_{10})$	0.1833 ± 0.0058	0.2291 ± 0.0073
Pairs of Wilson coefficients		
$\text{Re}(C_7)$	0.0193 ± 0.0006	0.0252 ± 0.0008
$\text{Re}(C_9)$	0.2130 ± 0.0068	0.2555 ± 0.0081
$\text{Re}(C_9)$	0.1715 ± 0.0054	0.1868 ± 0.0059
$\text{Re}(C_{10})$	0.2054 ± 0.0065	0.2667 ± 0.0085

- Improved performance of direct method compared to q^2-binned approach

Form factor nuisance parameters



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- Data allows to constrain parameters beyond FF covariance matrix
- Direct method gives stronger constraints than q^2-binned approach

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Conclusions

- Presented method to directly determine Wilson coefficients from unbinned $B^0 \to K^{*0}\mu^+\mu^-$ data
- Allows more precise determination of Wilson coefficients
- Allows to further constrain FF Nuisance parameters

$\Rightarrow$ Better exploitation of the data

- Requires unbinned efficiency corrected (background subtracted) data
- Should also continue to publish (theory-ind.) q^2-binned results

$I_i(q^2)$ and angular terms $f_i(\vec{\Omega})$

i	$I_i(q^2)$	$f_i(\vec{\Omega})$
1s	$\frac{3}{4}\left[|\mathcal{A}_\parallel^L|^2+|\mathcal{A}_\perp^L|^2+|\mathcal{A}_\parallel^R|^2+|\mathcal{A}_\perp^R|^2\right]$	$\sin^2\theta_K$
1c	$|\mathcal{A}_0^L|^2+|\mathcal{A}_0^R|^2$	$\cos^2\theta_K$
2s	$\frac{1}{4}\left[|\mathcal{A}_\parallel^L|^2+|\mathcal{A}_\perp^L|^2+|\mathcal{A}_\parallel^R|^2+|\mathcal{A}_\perp^R|^2\right]$	$\sin^2\theta_K\cos 2\theta_\ell$
2c	$-|\mathcal{A}_0^L|^2-|\mathcal{A}_0^R|^2$	$\cos^2\theta_K\cos 2\theta_\ell$
3	$\frac{1}{2}\left[|\mathcal{A}_\perp^L|^2-|\mathcal{A}_\parallel^L|^2+|\mathcal{A}_\perp^R|^2-|\mathcal{A}_\parallel^R|^2\right]$	$\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$
4	$\sqrt{\frac{1}{2}}\mathrm{Re}\left(\mathcal{A}_0^L\mathcal{A}_\parallel^{L*}+\mathcal{A}_0^R\mathcal{A}_\parallel^{R*}\right)$	$\sin 2\theta_K\sin 2\theta_\ell\cos\phi$
5	$\sqrt{2}\mathrm{Re}\left(\mathcal{A}_0^L\mathcal{A}_\perp^{L*}-\mathcal{A}_0^R\mathcal{A}_\perp^{R*}\right)$	$\sin 2\theta_K\sin\theta_\ell\cos\phi$
6s	$2\mathrm{Re}\left(\mathcal{A}_\parallel^L\mathcal{A}_\perp^{L*}-\mathcal{A}_\parallel^R\mathcal{A}_\perp^{R*}\right)$	$\sin^2\theta_K\cos\theta_\ell$
7	$\sqrt{2}\mathrm{Im}\left(\mathcal{A}_0^L\mathcal{A}_\parallel^{L*}-\mathcal{A}_0^R\mathcal{A}_\parallel^{R*}\right)$	$\sin 2\theta_K\sin\theta_\ell\sin\phi$
8	$\sqrt{\frac{1}{2}}\mathrm{Im}\left(\mathcal{A}_0^L\mathcal{A}_\perp^{L*}+\mathcal{A}_0^R\mathcal{A}_\perp^{R*}\right)$	$\sin 2\theta_K\sin 2\theta_\ell\sin\phi$
9	$\mathrm{Im}\left(\mathcal{A}_\parallel^{L*}\mathcal{A}_\perp^L+\mathcal{A}_\parallel^{R*}\mathcal{A}_\perp^R\right)$	$\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi$
10	$\frac{1}{3}\left[|\mathcal{A}_S^L|^2+|\mathcal{A}_S^R|^2\right]$	1
11	$\sqrt{\frac{4}{3}}\mathrm{Re}\left(\mathcal{A}_S^L\mathcal{A}_0^{L*}+\mathcal{A}_S^R\mathcal{A}_0^{R*}\right)$	$\cos\theta_K$
12	$-\frac{1}{3}\left[|\mathcal{A}_S^L|^2+|\mathcal{A}_S^R|^2\right]$	$\cos 2\theta_\ell$
13	$-\sqrt{\frac{4}{3}}\mathrm{Re}\left(\mathcal{A}_S^L\mathcal{A}_0^{L*}+\mathcal{A}_S^R\mathcal{A}_0^{R*}\right)$	$\cos\theta_K\cos 2\theta_\ell$
14	$\sqrt{\frac{2}{3}}\mathrm{Re}\left(\mathcal{A}_S^L\mathcal{A}_\parallel^{L*}+\mathcal{A}_S^R\mathcal{A}_\parallel^{R*}\right)$	$\sin\theta_K\sin 2\theta_\ell\cos\phi$
15	$\sqrt{\frac{8}{3}}\mathrm{Re}\left(\mathcal{A}_S^L\mathcal{A}_\perp^{L*}-\mathcal{A}_S^R\mathcal{A}_\perp^{R*}\right)$	$\sin\theta_K\sin\theta_\ell\cos\phi$
16	$\sqrt{\frac{8}{3}}\mathrm{Im}\left(\mathcal{A}_S^L\mathcal{A}_\parallel^{L*}-\mathcal{A}_S^R\mathcal{A}_\parallel^{R*}\right)$	$\sin\theta_K\sin\theta_\ell\sin\phi$
17	$\sqrt{\frac{2}{3}}\mathrm{Im}\left(\mathcal{A}_S^L\mathcal{A}_\perp^{L*}+\mathcal{A}_S^R\mathcal{A}_\perp^{R*}\right)$	$\sin\theta_K\sin 2\theta_\ell\sin\phi$

Decay amplitudes

$$\mathcal{A}_{\perp}^{\mathrm{L(R)}} = \mathcal{N}\sqrt{2\lambda}\left\{\left[(\mathcal{C}_9^{\mathrm{eff}} + \mathcal{C}_9^{\prime\mathrm{eff}}) \mp (\mathcal{C}_{10}^{\mathrm{eff}} + \mathcal{C}_{10}^{\prime\mathrm{eff}})\right]\frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2}(\mathcal{C}_7^{\mathrm{eff}} + \mathcal{C}_7^{\prime\mathrm{eff}})T_1(q^2)\right\}$$

$$\mathcal{A}_{\parallel}^{\mathrm{L(R)}} = -\mathcal{N}\sqrt{2}(m_B^2 - m_{K^*}^2)\left\{\left[(\mathcal{C}_9^{\mathrm{eff}} - \mathcal{C}_9^{\prime\mathrm{eff}}) \mp (\mathcal{C}_{10}^{\mathrm{eff}} - \mathcal{C}_{10}^{\prime\mathrm{eff}})\right]\frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2}(\mathcal{C}_7^{\mathrm{eff}} - \mathcal{C}_7^{\prime\mathrm{eff}})T_2(q^2)\right\}$$

$$\mathcal{A}_{0}^{\mathrm{L(R)}} = -\frac{\mathcal{N}}{2m_{K^*}\sqrt{q^2}}\left\{\left[(\mathcal{C}_9^{\mathrm{eff}} - \mathcal{C}_9^{\prime\mathrm{eff}}) \mp (\mathcal{C}_{10}^{\mathrm{eff}} - \mathcal{C}_{10}^{\prime\mathrm{eff}})\right] \times \left[(m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*})A_1(q^2) - \lambda\frac{A_2(q^2)}{m_B + m_{K^*}}\right] + 2m_b(\mathcal{C}_7^{\mathrm{eff}} - \mathcal{C}_7^{\prime\mathrm{eff}})\left[(m_B^2 + 3m_{K^*} - q^2)T_2(q^2) - \frac{\lambda}{m_B^2 - m_{K^*}^2}T_3(q^2)\right]\right\}$$

$$\mathcal{A}_t = \frac{\mathcal{N}}{\sqrt{q^2}}\sqrt{\lambda}\left\{2(\mathcal{C}_{10}^{\mathrm{eff}} - \mathcal{C}_{10}^{\prime\mathrm{eff}}) + \frac{q^2}{m_\mu}(\mathcal{C}_P - \mathcal{C}_P^{\prime})\right\}A_0(q^2),$$

with $\mathcal{N} = G_F\alpha_{\mathrm{em}}|V_{\mathrm{tb}}V_{\mathrm{ts}}|\sqrt{\frac{q^2\sqrt{\lambda}\beta_\ell}{3\cdot 1024\pi^5 m_B^3}}$

- Depending on Wilson coefficients $\mathcal{C}_{7,9,10}^{(\prime)}$ and form factors $V(q^2), A_{0,1,2}(q^2), T_{1,2,3}(q^2)$

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Theory Nuisance parameters

Parameter	Value
CKM parameters	
A	0.807 ± 0.02
λ	0.22535 ± 0.00065
$\bar{\rho}$	0.128 ± 0.055
$\bar{\eta}$	0.375 ± 0.060
Quark masses and scales	
m_c	$(1.275 \pm 0.025)\,\mathrm{GeV}$
m_b	$(4.18 \pm 0.03)\,\mathrm{GeV}$
m_t	$173.3\,\mathrm{GeV}$
μ	$4.2\,\mathrm{GeV}$
S-wave parameters	
$\xi_{\parallel}$	0.22 ± 0.03
δ_S	$\pi\ (\in [0, +2\pi])$
$\lvert g_\kappa \rvert$	$0.1\ (\in [0, 0.2])$
$\arg(g_\kappa)$	$\pi/2\ (\in [0, +2\pi])$
Subleading corrections	
$\mathrm{Re}(a_{0,\parallel,\perp})$	0 ± 0.1
$\mathrm{Im}(a_{0,\parallel,\perp})$	0 ± 0.1
$\mathrm{Re}(b_{0,\parallel,\perp})$	0 ± 0.25
$\mathrm{Im}(b_{0,\parallel,\perp})$	0 ± 0.25
$\mathrm{Re}(c_{0,\parallel,\perp})$	0 ± 0.1
$\mathrm{Im}(c_{0,\parallel,\perp})$	0 ± 0.1

Parameter	Value
Form factor parameters	
$\alpha_0^{A_0}$	0.37 ± 0.03
$\alpha_1^{A_0}$	-1.37 ± 0.26
$\alpha_2^{A_0}$	0.13 ± 1.63
$\alpha_0^{A_1}$	0.30 ± 0.03
$\alpha_1^{A_1}$	0.39 ± 0.19
$\alpha_2^{A_1}$	1.19 ± 1.03
$\alpha_1^{A_{12}}$	0.53 ± 0.13
$\alpha_2^{A_{12}}$	0.48 ± 0.66
α_0^{V}	0.38 ± 0.03
α_1^{V}	-1.17 ± 0.26
α_2^{V}	2.42 ± 1.53
$\alpha_0^{T_1}$	0.31 ± 0.03
$\alpha_1^{T_1}$	-1.01 ± 0.19
$\alpha_2^{T_1}$	1.53 ± 1.64
$\alpha_1^{T_2}$	0.50 ± 0.17
$\alpha_2^{T_2}$	1.61 ± 0.80
$\alpha_0^{T_{23}}$	0.67 ± 0.06
$\alpha_1^{T_{23}}$	1.32 ± 0.22
$\alpha_2^{T_{23}}$	3.82 ± 2.20