Prospects for data-driven analyses of the decay $B \rightarrow K^*ll$

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Implications of LHCb measurements and future prospects (CERN)
Introduction: the $B \to K^{*}\mu\mu$ decay

- Historically a key channel for indirect search for NP

Historically a key channel for indirect search for NP

  - coherently in all $b \to s \mu\mu$ decay modes


Are these hints of NP...? or is there a problem with the understanding of QCD?

In this talk we present prospects for present and future experiments to disentangle the two effects!
Controlling the charm loop

- EFT → general model-independent parametrization of NP

\[ \mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \frac{4G_F}{\sqrt{2}} \sum_i C_i(\mu) \mathcal{O}_i(\mu) \]

- possible sources of NP included in \( C_i \)
- Non-local hadronic contribution from SM
  - difficult to reliably quantify from first principles

- Parametrization introduced in arxiv:1707.07305v1
  [see talk by J. Virto]
  - The angular distributions can be described by
  the \( q^2 \)-dependent \( K^* \) amplitudes (\( \lambda = \perp, ||, 0 \))

\[ A_{\lambda}^{L,R} = N_{\lambda} \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_B M_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_B} \mathcal{H}_\lambda(q^2) \right] \right\} \]

Wilson coeff.  Non-local contribution

both will be extracted from data...
The parametrization (in a nutshell)

How to parametrize the correlator $\mathcal{H}$?

- Taylor expansion around $z = 0$ [after mapping: $q^2 \rightarrow z(q^2)$]
- Extract the poles: $\mathcal{H}_\lambda(z) = \frac{1 - zz^*_J/\psi}{z - z_J/\psi} \frac{1 - zz^*_\psi(2S)}{z - z_\psi(2S)} \hat{\mathcal{H}}_\lambda(z)$

$\hat{\mathcal{H}}_\lambda(z)$ analytic within unit circle $|z| = 1$

$\hat{\mathcal{H}}_\lambda(z) = \left[ \sum_k \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)$

What do we know about the correlator $\mathcal{H}$?

- **Experimental constraints:**
  - $B \rightarrow K^*\{J/\psi, \psi(2S)\}$ branching ratio by Belle and angular observables by BaBar and LHCb
- **Theory constraints:**
  - the ratios $\mathcal{H}_\lambda(q^2)/\mathcal{F}_\lambda(q^2)$ are theoretically computed for $q^2 < 0$: $\{-1, -3, -5, -7\}$ GeV$^2$

We can use these information as priors for the $\alpha_k^{(\lambda)}$ parameters in our fit

**TABLE I.** Mean values and standard deviations (in units of $10^{-4}$) of the prior PDF for the parameters $\alpha_k^{(\lambda)}$.

<table>
<thead>
<tr>
<th>$k$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re[\alpha_k^{(\lambda)}]</td>
<td>-0.06 ± 0.21</td>
<td>-6.77 ± 0.27</td>
<td>18.96 ± 0.59</td>
</tr>
<tr>
<td>Re[\alpha_k^{(\lambda)}]</td>
<td>-0.35 ± 0.62</td>
<td>-3.13 ± 0.41</td>
<td>12.20 ± 1.34</td>
</tr>
<tr>
<td>Re[\alpha_k^{(0)}]</td>
<td>0.05 ± 1.52</td>
<td>17.26 ± 1.64</td>
<td>-</td>
</tr>
<tr>
<td>Im[\alpha_k^{(\lambda)}]</td>
<td>-0.21 ± 2.25</td>
<td>1.17 ± 3.58</td>
<td>-0.08 ± 2.24</td>
</tr>
<tr>
<td>Im[\alpha_k^{(0)}]</td>
<td>-0.04 ± 3.67</td>
<td>-2.14 ± 2.46</td>
<td>6.03 ± 2.50</td>
</tr>
<tr>
<td>Im[\alpha_k^{(0)}]</td>
<td>-0.05 ± 4.99</td>
<td>4.29 ± 3.14</td>
<td>-</td>
</tr>
</tbody>
</table>
Amplitude fit

We perform a \(q^2\)-unbinned amplitude fit:

- signal \(p.d.f. = \frac{1}{\Gamma} \frac{d^4 \Gamma}{dq^2 d\Omega}\)

- Cut hadronic expansion at order \(z^2\):
  - 8 complex parameters \(\alpha^{(0)}_{\perp,\parallel,0}, \alpha^{(1)}_{\perp,\parallel,0}, \alpha^{(2)}_{\perp,\parallel}\)
    (\(\hat{H}_0\) must vanish for \(q^2 = 0\))

- two \(q^2\) ranges: \([1.1, 9.0]\) & \([10.0, 13.0]\) GeV\(^2\)
  - also tried \([1.1, 8.0]\) & \([11.0, 12.5]\) GeV\(^2\)

  current analysis ranges:
  - LHCb: \([1.1, 8.0]\) & \([11, 12.5]\) GeV\(^2\) [1]
  - \([1.1, 8.8]\) & \([10.1, 12.9]\) GeV\(^2\) [2]
  - Belle: \([1.1, 8.0]\) & \([10.1, 12.9]\) GeV\(^2\) [3]

Additional constraints to the fit:

- the BR is gaussian constrained separately in the two regions [4]:
- multi-variate gaussian priors on \(\alpha^{(k)}\)
- multi-variate gaussian priors on form factors [5]
- gaussian constraint to CKM parameters [6]

\[
\frac{d\Gamma}{dq^2} = \sum_{\chi=L,R} \sum_{\lambda=\perp,\parallel,0} |A_{\lambda,\chi}|^2
\]

39 nuisance parameters + free-floating \(C_i\)
We define a “BenchMark-Point” (BMP) scenario for NP: \( C_9^{\text{BMP}} = C_9^{\text{SM}} - 1 \)

- \( C_7 \) and \( C_{10} \) fixed at their SM values
- generate 4\( k \) toys and perform the fit
- extract the sensitivity to non-SM \( C_9 \) for present and future experiments (LHCb and Belle-II)
  - no experimental effects (resolution*, efficiency…)
  - no S-wave pollution
    [see Langenbruch et al.’s work, arXiv:1708.04474v1]

\[ \text{Sensitivity to be understood as upper bound (stat. only)} \]

*assuming a “naive” constant resolution for \( \sqrt{q^2} \) of 9 MeV [PRD95,071101(2017)]
has a negligible impact on the sensitivity
Prior knowledge on the $\alpha^{(k)}_\lambda$ parameters

So far we used theoretical and experimental prior knowledge on the $\alpha^{(k)}_\lambda$ parameters

❖ When using priors the fit is able to extract additional information
❖ uncertainties a-posteriori are smaller than a-priori
Prior knowledge on the $\alpha_{\lambda}^{(k)}$ parameters

So far we used theoretical and experimental prior knowledge on the $\alpha_{\lambda}^{(k)}$ parameters

- When using priors the fit is able to extract additional information
- Uncertainties a-posteriori are smaller than a-priori

**Can we perform the analysis without any prior?**

Yes!

When removing the priors the fit converges and we are still able to disentangle hadronic from NP effects

<table>
<thead>
<tr>
<th>with priors</th>
<th>without priors</th>
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<tbody>
<tr>
<td>$\sigma(C_9) = 0.13$</td>
<td>$\sigma(C_9) = 0.19$</td>
</tr>
</tbody>
</table>
Sensitivity to higher order in $z$

So far we cut the expansion of $\hat{H}_\lambda(z)$ after $z^2$

- This introduces a model dependence
- Priors for $z^3$ would require complete NP analysis

Still we try to study effects of expansion up to $z^3$:

- Toys generated with $\alpha_{\lambda}^{(3)} = 0$ \quad but $\alpha_{\lambda}^{(3)}$ free-floating in the fit
- Fitted without theory priors on $\alpha_{\lambda}^{(k)}$

We find:

- $\sigma(\alpha_{\lambda}^{3}) \sim 5 \cdot 10^{-3}$ \quad No sensitivity for $\alpha_{\lambda}^{(3)} < 5 \cdot 10^{-3}$

- $C_9|_{z^3} - C_9|_{z^2} = 0.17$

- $\sigma(C_9)|_{z^3} = 0.69 \quad vs \quad \sigma(C_9)|_{z^2} = 0.17$

$C_9$ resolution worsen by a factor 3 with the $z^3$ terms

Possible solution: combined NP analysis of theory results, hadronic decays and $B \to K^*\mu\mu$
NP sensitivity: projection

Assuming the parametrization of [arxiv:1707.07305] is good description of nature

Motivation to perform combined theory+experimental analysis in LHCb/Belle-II soon

Crucial to study higher order of $z$ in data
Recent non-standard measurements of LFU ratios ($R_K, R_{K^*}$) suggest a possible LFU violating NP explanation to $b \rightarrow s \mu \mu$ anomalies.

- LFUV, if exist, must be observed also in angular analysis.
- We can perform a simultaneous fit to muons & electrons
  - all parameters shared except for Wilson coefficients.

We define:

$$\Delta C_9 = C_9^{(\mu)} - C_9^{(e)}$$

What are the prospects for a measurement of a LFU violating $\Delta C_9$?

We investigate the NP scenario:

$$C_9^{(\mu)} + 1 = C_9^{(e)} = C_9^{SM} \quad \text{(NP only in muons)}$$
Lepton Flavour Universality test

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$B \rightarrow K^* ee$ decay:

- worse experimental resolution due to bremsstrahlung

LHCb  
- analysis limited to the $q^2$ range: $[1.1, 7.0]$ GeV$^2$

Belle-II
- optimistic case:
  - same number of $\mu$ and $e$ in the region between $J/\psi$ and $\psi(2S)$
- pessimistic case:
  - no $e$ in the region between $J/\psi$ and $\psi(2S)$
  - sensitivity lowered by $\sim 1.5\sigma$ at 50 ab$^{-1}$
ΔC⁹: LHCb/Belle-II sensitivity

**LHCb:**  \( \varepsilon_e \sim \frac{1}{10} \varepsilon_\mu \)

![Graph showing the sensitivity of LHCb and Belle-II](image)

- Smearing of electron resolution (compatible with [JHEP 08, 055 (2017)])
  - results in a shift of the value of \( C_9(e) \) but doesn’t affect \( \Delta C_9 \) resolution

**Belle-II:**  \( \varepsilon_e \sim \varepsilon_\mu \)

![Graph showing the sensitivity of Belle-II](image)

Sensitivity of fit with \( z^3 \) under investigation

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**ΔC₉: future prospect**

- Prospect for ΔC₉ based on $B \rightarrow K^{*\mu\mu}$ and $B \rightarrow K^{*ee}$ expected yield in LHCb and Belle-II.
- Belle-II dashed line assumes the inclusion of $B \rightarrow K^{*±ll}$ decays (roughly double the number of events).

![Graph showing ΔC₉ statistical significance over time](image-url)

- LHCb
- Belle-II
- Belle-II (incl. $B \rightarrow K^{*ll}$)

**Future Prospects**

- **2020**: LHCb Run2
- **2025**: LHCb Run3, Belle-II (50ab⁻¹), LHCb Upgrade (500ab⁻¹)
- **2030**: LHCb Upgrade (500ab⁻¹)

*Fit with z²*
Combined $C_9$, $C_{10}$ fits

- Many models allow NP contribution in both $C_9$ and $C_{10}$
- Prospect for a combined fit to $C_9$, $C_{10}$ using (only) the $B \to K^*\mu\mu$ and $B \to K^*ee$ expected yield for LHCb-RunII and Belle-II
  - Projected $C_{10}$ sensitivity to $B_s \to \mu\mu$ is overlaid as reference
- Assumption: NP only in $C_9^{(\mu)}$

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**LHCb Run-II**

**Belle-II [50ab$^{-1}$]**

- fit with $z^2$
Conclusion

- We presented the prospects for future simultaneous determinations of NP shifts to $C_{9}$ and non-local hadronic effects in the decay $B \rightarrow K^{*}\mu\mu$
  - Results crucially dependent on present knowledge of hadronic correlator from arxiv:1707.07305
  - The current dataset available at the LHCb experiment might provide a large statistical significance to NP, assuming our NP scenario
  - Capability of extracting information on the hadronic correlators from data-only, without any priors

- Motivation to perform combined theory+experimental analysis in LHCb/Belle-II soon

- Simultaneous fit to $B \rightarrow K^{*}\mu\mu$ and $B \rightarrow K^{*}ee$ decays can allow to explore LFU breaking hypothesis
backup
Problem with priors for $z^3$ coefficients

- theory at $q^2 < 0$
- $B \rightarrow K^*\mu\mu$ data (C9 dependent)
- $B \rightarrow K^*\{J/\psi, \psi(2S)\}$

- priors for $z^2$ are constructed from:
  - theory
  - hadronic
  - NP independent

- priors for $z^3$ are constructed from:
  - theory
  - hadronic
  - $B \rightarrow K^*\mu\mu$
  - NP dependent
Intro: parametrization

- The differential decay rate can be expressed by separating the angular and $q^2$ dependencies

$$\frac{8\pi}{3} \frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} =$$

$$\left( J_{1s} + J_{2s} \cos 2\theta_\ell + J_{6s} \cos \theta_\ell \right) \sin^2\theta_K$$
$$+ \left( J_{1c} + J_{2c} \cos 2\theta_\ell + J_{6c} \cos \theta_\ell \right) \cos^2\theta_K$$
$$+ \left( J_3 \cos 2\phi + J_9 \sin 2\phi \right) \sin^2\theta_K \sin^2\theta_\ell$$
$$+ \left( J_4 \cos \phi + J_8 \sin \phi \right) \sin 2\theta_K \sin 2\theta_\ell$$
$$+ \left( J_5 \cos \phi + J_7 \sin \phi \right) \sin 2\theta_K \sin \theta_\ell,$$

- The angular observable can be described by the $q^2$-dependent $K^*$ amplitudes ($\lambda = \perp, ||, 0$)

$$A^{L,R}_\lambda = N_\lambda \left\{ \left( C_9 + C_{10} \right) F_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 F_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$
\[\frac{4}{3} J_1 = \frac{(2 + \beta^2)}{4} \left[ |A_L| + |A_R| + (L \to R) \right] + \frac{4 m^2}{q^2} \text{Re} \left( A_L^* A_R^* + A_L A_R^* \right) + 4 \beta^2 \left( |A_0| \right) + 4 (4 - 3 \beta^2) \left( |A_0| \right) + 8 \sqrt{2} \frac{m\beta}{q^2} \text{Re} \left( A_L^* A_R^* A_{\perp} + (A_L^* + A_R A_{\perp}^*) \right),\]

\[\frac{4}{3} J_2 = \frac{(2 + \beta^2)}{4} \left[ |A_L| + |A_R| + (L \to R) \right] - 16 \left( |A_0| \right) + 8 \left( |A_0| \right) + 8 \left( |A_{\perp}| \right),\]

\[\frac{4}{3} J_3 = \frac{(2 + \beta^2)}{4} \left[ |A_L| - |A_R| + (L \to R) \right] + 16 \left( |A_0| \right) - 16 \left( |A_0| \right) + 8 \left( |A_{\perp}| \right),\]

\[\frac{4}{3} J_4 = \frac{(2 + \beta^2)}{4} \left[ |A_L| - |A_R| + (L \to R) \right] - 8 \sqrt{2} \left( A_{10} A_{\parallel}^* + A_{\perp} A_{\parallel}^* \right),\]

\[\frac{4}{3} J_5 = \frac{(2 + \beta^2)}{4} \left[ |A_L| - |A_R| + (L \to R) \right] - 8 \sqrt{2} \left( A_{10} A_{\parallel}^* + A_{\perp} A_{\parallel}^* \right),\]

\[\frac{4}{3} J_6 = 2 \beta \left( |A_L| + |A_R| + (L \to R) \right) + 4 \sqrt{2} \frac{m\beta}{q^2} \left( |A_L^* + A_R^*| A_0 - |A_L^* - A_R^*| A_{\perp} \right),\]

\[\frac{4}{3} J_7 = 2 \beta \left( |A_L| + |A_R| + (L \to R) \right) + 4 \sqrt{2} \frac{m\beta}{q^2} \left( |A_L^* + A_R^*| A_0 - |A_L^* - A_R^*| A_{\perp} \right),\]

\[\frac{4}{3} J_8 = 2 \beta \left( |A_L| + |A_R| + (L \to R) \right) - 4 \sqrt{2} \frac{m\beta}{q^2} \left( |A_L^* + A_R^*| A_0 - |A_L^* - A_R^*| A_{\perp} \right),\]

\[\frac{4}{3} J_9 = 2 \beta \left( |A_L| + |A_R| + (L \to R) \right),\]

\[\frac{4}{3} J_10 = 2 \beta \left( |A_L| + |A_R| + (L \to R) \right),\]
### Projected number of events

<table>
<thead>
<tr>
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<th>LHCb</th>
<th>Muons</th>
<th>Electrons</th>
</tr>
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<tbody>
<tr>
<td>Run1 + 2016</td>
<td>2330</td>
<td>1200</td>
<td>265</td>
</tr>
<tr>
<td>Run2</td>
<td>4570</td>
<td>2360</td>
<td>520</td>
</tr>
<tr>
<td>Upgrade</td>
<td>40620</td>
<td>20980</td>
<td>4660</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Belle-II</th>
<th>Muons</th>
<th>Electrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ab⁻¹</td>
<td>425</td>
<td>175</td>
<td>160</td>
</tr>
<tr>
<td>12 ab⁻¹</td>
<td>1020</td>
<td>420</td>
<td>390</td>
</tr>
<tr>
<td>24 ab⁻¹</td>
<td>2045</td>
<td>840</td>
<td>780</td>
</tr>
<tr>
<td>36 ab⁻¹</td>
<td>3070</td>
<td>1260</td>
<td>1170</td>
</tr>
<tr>
<td>50 ab⁻¹</td>
<td>4260</td>
<td>1750</td>
<td>1625</td>
</tr>
</tbody>
</table>
$C_9$ bias

![Graph showing $C_9$ bias]
Prior knowledge on the $\alpha_{\lambda}^{(k)}$ parameters

So far we used prior theoretical and experimental knowledge on the $\alpha_{\lambda}^{(k)}$ parameters

Can we perform the analysis without any prior?

Yes!

- When using the theory priors the fit is able to extract additional information → uncertainties a-posteriori are smaller than a-priori
- When removing the priors the fit converges and we are still able to disentangle hadronic from NP effects

Of course we got a worse resolution in C9

LHCb Run1 + 2016 dataset

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Prior knowledge on the $\alpha^{(k)}_\lambda$ parameters

LHCb Run1 + 2016 dataset

![Graph 1](image1)

- **Theory constraints**
- **No theory constraints**

LHCb Upgrade

![Graph 2](image2)

- **Theory constraints**
- **No theory constraints**
pull of alpha parameters

\[ \text{Im}(\alpha_{\text{perp}}^{(1)}) \text{ pull} \]

\[ \mu : -0.06 \pm 0.02 \]
\[ \sigma : 1.14 \pm 0.01 \]

\[ \text{Re}(\alpha_{\text{perp}}^{(1)}) \text{ pull} \]

\[ \mu : -0.07 \pm 0.02 \]
\[ \sigma : 1.01 \pm 0.01 \]

\[ \text{Im}(\alpha_{\text{perp}}^{(0)}) \text{ pull} \]

\[ \mu : 0.09 \pm 0.02 \]
\[ \sigma : 1.16 \pm 0.01 \]

\[ \text{Re}(\alpha_{\text{perp}}^{(0)}) \text{ pull} \]

\[ \mu : -0.07 \pm 0.02 \]
\[ \sigma : 1.01 \pm 0.01 \]

\[ \text{Im}(\alpha_{\text{perp}}^{(2)}) \text{ pull} \]

\[ \mu : 0.07 \pm 0.02 \]
\[ \sigma : 1.01 \pm 0.01 \]

\[ \text{Re}(\alpha_{\text{perp}}^{(2)}) \text{ pull} \]

\[ \mu : 0.04 \pm 0.02 \]
\[ \sigma : 1.11 \pm 0.01 \]
pull of alpha parameters

\[ \text{Re}(\alpha^{(0)}_{\text{para}}) \]

\[ \mu = 0.06 \pm 0.02 \]
\[ \sigma = 1.03 \pm 0.01 \]

\[ \text{Re}(\alpha^{(1)}_{\text{para}}) \]

\[ \mu = 0.03 \pm 0.02 \]
\[ \sigma = 1.02 \pm 0.01 \]

\[ \text{Re}(\alpha^{(2)}_{\text{para}}) \]

\[ \mu = -0.02 \pm 0.02 \]
\[ \sigma = 1.02 \pm 0.01 \]

\[ \text{Im}(\alpha^{(0)}_{\text{para}}) \]

\[ \mu = 0.08 \pm 0.02 \]
\[ \sigma = 1.16 \pm 0.01 \]

\[ \text{Im}(\alpha^{(1)}_{\text{para}}) \]

\[ \mu = -0.06 \pm 0.02 \]
\[ \sigma = 1.11 \pm 0.01 \]

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pull of alpha parameters