Molecules of hadrons bound by pion exchange: application to LHCb pentaquarks

Tim Burns

Swansea University

8 November 2017

[T.B. & E.Swanson (ongoing)]
$P_c(4380)$ and $P_c(4450)$

<table>
<thead>
<tr>
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<td>Assignments</td>
<td></td>
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<td>$\Lambda_c^+(1P) \bar{D}^0$</td>
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<td>$4457.09 \pm 0.35$</td>
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<tr>
<td>$\chi_c 1\rho$</td>
<td>$(udu)(c\bar{c})$</td>
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| $\Sigma^*_c + \bar{D}^0$     | $(udc)(u\bar{c})$ | 4382.3 ± 2.4
| $\Sigma_c^+ \bar{D}^*0$     | $(udc)(u\bar{c})$ | 4459.9 ± 0.5
| $\Lambda_c^+(1P) \bar{D}^0$ | $(udc)(u\bar{c})$ | 4457.09 ± 0.35
| $\chi_{c1p}$                 | $(udu)(c\bar{c})$ | 4448.93 ± 0.07
Hidden-charm molecules

- Yang, Sun, He, Liu, Zhu (2011)
- Wu, Molina, Oset, Zou, Xiao, Nieves, Uchino, Liang, Roca, Magas, Feijoo, Ramos, ... (2010-2016)
- He (2015)
- Shimizu, Suenaga, Harada (2016)
- Yamaguchi, Santopinto (2016)
- Yang, Ping (2015)
- Ortega, Entem, Fernandez (2016)
- ...

One-pion exchange potential

Potential mixes particles and angular momenta, e.g.

- $\Lambda_c \bar{D}(^2S_{1/2}) \rightarrow \Sigma_c \bar{D}^*(^4D_{1/2})$

but the pattern is driven by diagonal blocks of fixed particles.

Note

- $\Lambda_c \Lambda_c \pi$ vertex is forbidden (isospin)
- $\bar{D} \bar{D} \pi$ vertex is forbidden (spin-parity)
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One-pion exchange potential

Quark models and heavy-quark chiral Lagrangians give

\[ V(\vec{r}) = \left[ V_C(r) \vec{\Sigma}_1 \cdot \vec{\Sigma}_2 + V_T(r) S_{12}(\hat{r}) \right] \vec{T}_1 \cdot \vec{T}_2 \]
One-pion exchange potential

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\]

Coefficients are model-independent; e.g. \( \Sigma_c \bar{D}^* \) in \( 1/2(3/2^-) \):

\[
\begin{align*}
\langle 4 S_{3/2} | & \quad | 2 D_{3/2} \rangle & \quad | 4 D_{3/2} \rangle \\
\langle 4 S_{3/2} | & \quad -\frac{8}{3} V_C & \quad -\frac{8}{3} V_T & \quad -\frac{16}{3} V_T \\
\langle 2 D_{3/2} | & \quad -\frac{8}{3} V_T & \quad +\frac{16}{3} V_C & \quad +\frac{8}{3} V_T \\
\langle 4 D_{3/2} | & \quad -\frac{16}{3} V_T & \quad +\frac{8}{3} V_T & \quad -\frac{8}{3} V_C
\end{align*}
\]

and relative strengths fixed by HQ symmetry.
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\[
\begin{array}{ccc}
|4S_{3/2}\rangle & |2D_{3/2}\rangle & |4D_{3/2}\rangle \\
\langle 4S_{3/2}| & -\frac{8}{3} V_C & -\frac{8}{3} V_T & -\frac{16}{3} V_T \\
\langle 2D_{3/2}| & -\frac{8}{3} V_T & +\frac{16}{3} V_C & +\frac{8}{3} V_T \\
\langle 4D_{3/2}| & -\frac{16}{3} V_T & +\frac{8}{3} V_T & -\frac{16}{3} V_C \\
\end{array}
\]

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Central and tensor potentials with form factor cutoff
One-pion exchange potential

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\langle 4D_{3/2} \rangle & -\frac{16}{3} V_T & +\frac{16}{3} V_C & +\frac{16}{3} V_T \\
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Central and tensor potentials with form factor cutoff

Larger isospin \( \Longrightarrow \) weaker potential; e.g. \( V_{I=3/2} = -\frac{1}{2} V_{I=1/2} \)
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Central and tensor potentials with form factor cutoff

Larger isospin \( \implies \) weaker potential; e.g. \( V_{I=3/2} = -\frac{1}{2} V_{I=1/2} \)

For channels with S-waves, binding is driven by coefficient of \( V_C(r) \) (and unlike for \( NN \), this can be positive...)
<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>1/2-</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Σ<em>D</em></td>
<td>+ 20/3</td>
<td>+ 8/3</td>
<td>- 4</td>
</tr>
<tr>
<td>ΣD*</td>
<td>+ 16/3</td>
<td>- 8/3</td>
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Critical form factor

\[ \Sigma_c D^* \quad \frac{1}{2}^- \quad [+16/3] \]

\[ \Sigma_c^* D^* \quad \frac{3}{2}^- \quad [+20/3] \]

\[ \Sigma_c D^* \quad \frac{3}{2}^- \quad [+8/3] \]

\[ \Sigma_c D^* \quad \frac{5}{2}^- \quad [-8/3] \]

\[ \Sigma_c^* D^* \quad \frac{5}{2}^- \quad [-4] \]

NN \( \theta(1^-) \)
Critical form factor

\[ \Sigma_c D^* - \frac{1}{2}^- \ [+16/3] \]

(I=3/2 states)

\[ \Sigma_c^* D^* - \frac{1}{2}^- \ [+20/3] \]

\[ \Sigma_c D^* - \frac{3}{2}^- \ [+8/3] \]

\[ \Sigma_c^* D^* - \frac{3}{2}^- \ [-8/3] \]

\[ \Sigma_c D^* - \frac{5}{2}^- \ [-4] \]

NN, 0(1^-)
Conventional wisdom

“Molecules exist close to S-wave thresholds”
Conventional wisdom

“Molecules exist close to S-wave thresholds”

▶ Only for certain quantum numbers
▶ “Few” states exist ⇔ all are “near” threshold
▶ “Many” states exist ⇔ some are more deeply bound
▶ Pattern is driven by coefficient of $V_C$, due to $\pi$ exchange, and understood in fixed-particle basis... but is more general.
Conventional wisdom

“Molecules exist close to S-wave thresholds”
- Only for certain quantum numbers
- “Few” states exist $\Leftrightarrow$ all are “near” threshold
- “Many” states exist $\Leftrightarrow$ some are more deeply bound
- Pattern is driven by coefficient of $V_C$, due to $\pi$ exchange, and understood in fixed-particle basis... but is more general.

P-wave states? (e.g. $1/2^+$, $3/2^+$, $5/2^+$)
- Possible, but with much larger cut-off, implying many more states overall
- Pattern is driven by coefficient of $V_T$.
- States near wrong thresholds for $P_c(4380)$, $P_c(4450)$. 

The potential matrices (central + tensor) are directly related.

Predict loosely bound $0(5/2^-)$ $\Xi_c^* \bar{D}^*$ state, observable in $\Lambda_b \to J/\psi \Lambda \eta$, and $\Xi_b \to J/\psi \Lambda K^-$ (LHCb run II).
Isospin mixing: $P_c(4380)$ and $P_c(4450)$

\[
\begin{align*}
\text{uudc}\bar{c} &= \left\{ \begin{array}{c}
(\text{udc})(u\bar{c}) = \Sigma_c^+ \bar{D}^0 \\
(\text{uuc})(d\bar{c}) = \Sigma_c^{++} D^-
\end{array} \right. \\
\end{align*}
\]

Isospin-conserving interactions give $|l, l_3\rangle$ eigenstates,

\[
\begin{pmatrix}
|\frac{1}{2}, \frac{1}{2}\rangle \\
|\frac{3}{2}, \frac{1}{2}\rangle
\end{pmatrix} = \begin{pmatrix}
-\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}}
\end{pmatrix} \begin{pmatrix}
|\Sigma_c^+ \bar{D}^0\rangle \\
|\Sigma_c^{++} D^-\rangle
\end{pmatrix}
\]

but only if the masses $\Sigma_c^+ = \Sigma_c^{++}$ and $\bar{D}^0 = D^-$. 

Otherwise, isospin is not a good quantum number.
Isospin mixing: $P_c(4380)$ and $P_c(4450)$

\[
\begin{align*}
P_c(4380) &= 4380 \pm 8 \pm 29 \\
\Sigma_c^{*+} \bar{D}^0 &= 4382.3 \pm 2.4 \\
\Sigma_c^{*++} D^- &= 4387.5 \pm 0.7 \\
P_c(4450) &= 4449 \pm 1.7 \pm 2.5 \\
\Sigma_c^+ \bar{D}^{*0} &= 4459.9 \pm 0.5 \\
\Sigma_c^{++} D^{*-} &= 4464.24 \pm 0.23
\end{align*}
\]

The $P_c$ states have mixed isospin:

\[|P_c\rangle = \cos \phi |\frac{1}{2}, \frac{1}{2}\rangle + \sin \phi |\frac{3}{2}, \frac{1}{2}\rangle\]

They should decay also into $J/\psi \Delta^+$ and $\eta_c \Delta^+$, with weights:

\[
\begin{align*}
J/\psi p : J/\psi \Delta^+ : \eta_c \Delta^+ &= 2 \cos^2 \phi : 5 \sin^2 \phi : 3 \sin^2 \phi & [P_c(4380)] \\
J/\psi p : J/\psi \Delta^+ : \eta_c \Delta^+ &= \cos^2 \phi : 10 \sin^2 \phi : 6 \sin^2 \phi & [P_c(4450)]
\end{align*}
\]
Isospin mixing: predicted $5/2^-$ states

$$\Sigma^*_c D^* 1/2(5/2^-)$$

$$\Sigma^*_c D^* 0 = 4524.4 \pm 2.4$$
$$\Sigma^*_c D^* 0 = 4528.2 \pm 0.7$$

Mixed isospin:

$$|P\rangle = \cos \phi |\frac{1}{2}, \frac{1}{2}\rangle + \sin \phi |\frac{3}{2}, \frac{1}{2}\rangle$$

Decays:

$\rightarrow J/\psi p$: D-wave, spin flip

Reason for absence at LHCb?

$\rightarrow J/\psi \Delta$: S-wave, spin cons.

$\implies I = 3/2$ decay enhanced.
Isospin mixing: predicted $5/2^-$ states

\[
\begin{align*}
\Sigma_c^* \bar{D}^* &\quad 1/2(5/2^-) \\
\Xi_c^* \bar{D}^* &\quad 0(5/2^-)
\end{align*}
\]

\[
\begin{align*}
\Sigma_c^{*+} \bar{D}^{*0} &\quad = 4524.4 \pm 2.4 \\
\Sigma_c^{*++} D^{*-} &\quad = 4528.2 \pm 0.7 \\
\Xi_c^{*0} \bar{D}^{*0} &\quad = 4652.9 \pm 0.6 \\
\Xi_c^{*+} D^{*-} &\quad = 4656.2 \pm 0.7
\end{align*}
\]

Mixed isospin:
\[
|P\rangle = \cos \phi |\tfrac{1}{2}, \tfrac{1}{2}\rangle + \sin \phi |\tfrac{3}{2}, \tfrac{1}{2}\rangle
\]

Decays:
\[
\begin{align*}
\rightarrow J/\psi p: &\quad D\text{-wave, spin flip} \\
\rightarrow J/\psi \Lambda: &\quad D\text{-wave, spin flip} \\
\rightarrow J/\psi \Delta: &\quad S\text{-wave, spin cons.} \\
\rightarrow J/\psi \Sigma^*: &\quad S\text{-wave, spin cons.}
\end{align*}
\]

Reason for absence at LHCb?
\[
\begin{align*}
\rightarrow J/\psi \Lambda: &\quad D\text{-wave, spin flip} \\
\rightarrow \Lambda_b^0 &\quad \rightarrow J/\psi \Lambda \eta, J/\psi \Lambda \phi
\end{align*}
\]

\[
\Rightarrow I = 3/2 \text{ decay enhanced.}
\]

\[
\Rightarrow I = 1 \text{ decay enhanced.}
\]
Conclusions

- Binding in certain $I(J^P)$ only
- $3/2^- \Sigma_c \bar{D}^* = P_c(4450)$?
- $5/2^- \Sigma_c^* \bar{D}^*$ with D-wave decay

Pattern is
- easily understood
- parameter insensitive
- generic

- Predict $\Xi_c^* \bar{D}^* 0(5/2^-)$ partner
- All states are isospin mixtures
Backup slides
Central potential

\[ -C(r) \]

Yukawa

\[ \text{GeV}^{-1} \]
Central potential

\[-C(r)\]

without delta term

\[-C(r)\]

with delta term

1 GeV

2 GeV Yukawa

\[
Yukawa (GeV^{-1})^2
\]

1 GeV

2 GeV
Central potential

\(-C(r)\) without delta term

\(-C(r)\) with delta term

\(1 \text{ GeV}\)

\(2 \text{ GeV} \quad \text{Yukawa}\)

\(+C(r)\)
$\Lambda=1300$ MeV

$1/2^-$

$\Sigma^*D^*$: + 20/3

$\Sigma D^*$: + 16/3

$\Sigma^*D$: 

$\Sigma D$: 

$\Lambda D^*$: 

$\Lambda D$: 

$3/2^-$

$\Sigma^*D^*$: + 8/3

$\Sigma D^*$: - 8/3

$\Sigma^*D$: 

$\Sigma D$: 

$\Lambda D^*$: 

$\Lambda D$: 

$5/2^-$

$\Sigma^*D^*$: - 4

$\Sigma D^*$: 

$\Sigma^*D$: 

$\Sigma D$: 

$\Lambda D^*$: 

$\Lambda D$: 