Long-distance effects in $D^0 \to h^+h^-\ell^+\ell^-$

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LHCb Implications Workshop, November 9th, 2017

Motivation

- FCNC processes suppressed in the SM: window to new physics. D physics has a very efficient GIM suppression but long-distance dominated.

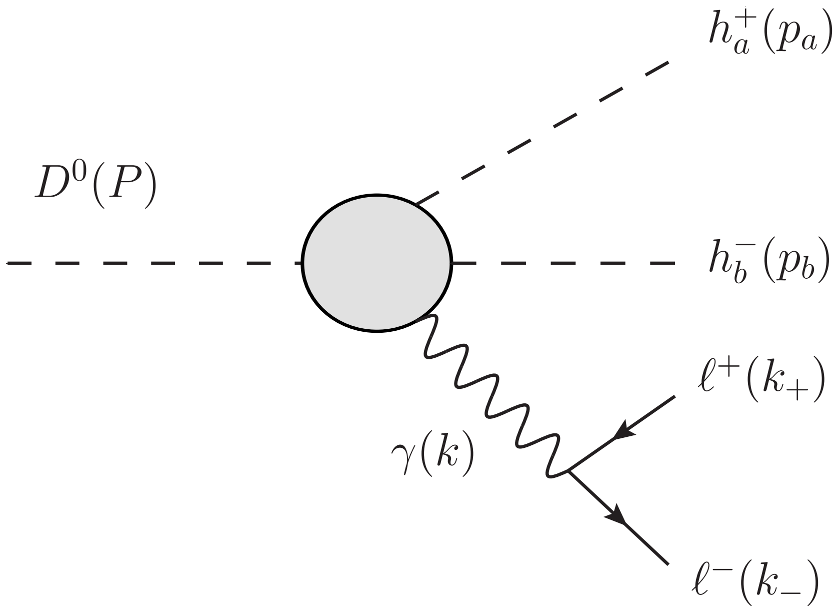
- As opposed to B and K physics, no meaningful expansion works. EFT language not very useful: estimation of hadronic form factors through lattice or sum rules.

- Many intermediate states allowed by phase space. Resonant contributions dominate.

- Rich kinematics.

- Some of the channels have already been measured. [arXiv:1510.08367; 1707.08377]

Long distances



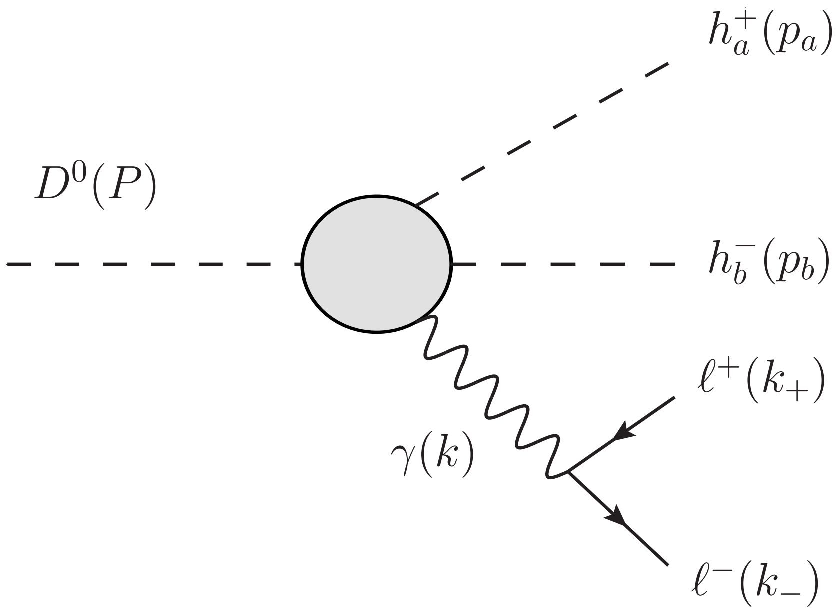
- Bremsstrahlung: pure QED effects, calculable with Low's theorem:

$$\mathcal{M}_b(D^0 \to h_1^+ h_2^- \gamma) = 2e \left[\frac{p_1 \cdot \epsilon}{2p_1 \cdot q + q^2} - \frac{p_2 \cdot \epsilon}{2p_2 \cdot q + q^2} \right] \mathcal{M}(D^0 \to h_1^+ h_2^-)$$

- Resonant contributions (dominant effects). [Burdman et al'02]

- Charge radius (not discussed in this talk).

Long distances



Generic kinematical parametrization:

$$\mathcal{M}_{ab} = \frac{e}{k^2}\left[\bar{u}(k_-)\gamma^\mu v(k_+)\right] H^\mu_{ab}(p_a, p_b; k)$$

with

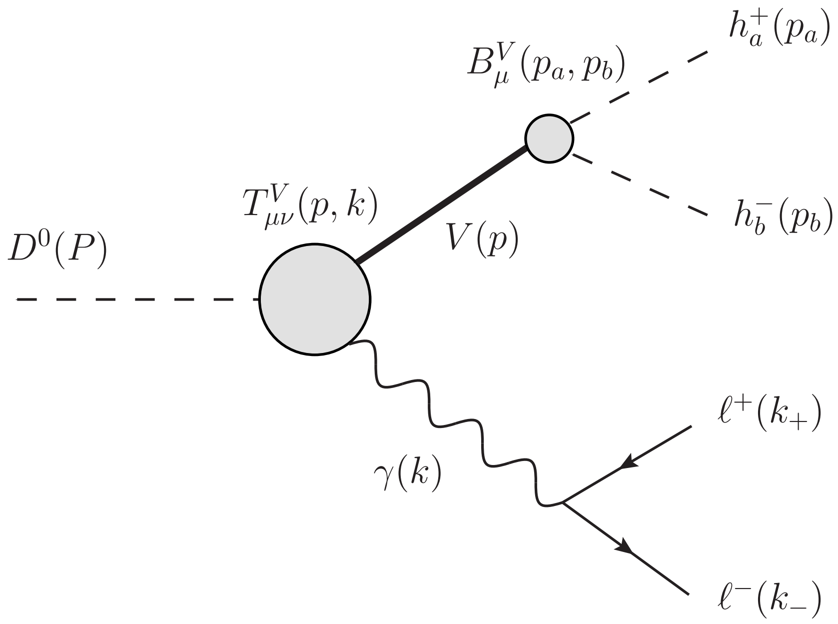
$$H^\mu_{ab}(p_a, p_b; k) = F_1^{(ab)} p_a^\mu + F_2^{(ab)} p_b^\mu + F_3^{(ab)} \epsilon^{\mu\nu\lambda\rho} p_{a\nu} p_{b\lambda} k_\rho$$

Matrix elements:

$$\sum_{\text{spins}} |\mathcal{M}_{ab}|^2 = \frac{2e^2}{q^4}\left[\sum_i^3 |F_i^{(ab)}|^2 T_{ii} + 2\text{Re}\sum_{i<j}^3 (F_i^{(ab)})^* F_j^{(ab)} T_{ij}\right]$$

Resonant contribution

[Cappiello, OC, D'Ambrosio'12]



$$H^\mu_{ab}(p_a,p_b;k) = \langle h_a^+ h_b^- \gamma^* |\mathcal{H}| D^0\rangle \varepsilon^\mu_\gamma = \sum_k \langle h_a^+ h_b^- |\mathcal{H}| V_k\rangle \frac{\varepsilon^\mu_\gamma}{P_k(p^2)} \langle V_k \gamma^* |\mathcal{H}| D^0\rangle$$

where

$$\langle h_a^+ h_b^- |\mathcal{H}| V\rangle \equiv B^\mu_V(p_a,p_b)\varepsilon^V_\mu(p); \qquad \langle V\gamma^* |\mathcal{H}| D^0\rangle \equiv T^{\mu\nu}_V(p,k)\varepsilon^{V*}_\mu(p)\varepsilon^{\gamma*}_\nu(k)$$

Relevant kinematic invariants:

$$B^\mu_V(p_a,p_b) = b^V(p^2)(p_a-p_b)^\mu; \qquad T^V_{\mu\nu}(p,k) = t^V_1 g_{\mu\nu} + t^V_2 k_\mu p_\nu + t^V_3 \epsilon_{\mu\nu\lambda\rho} p_\lambda k_\rho$$

Weak form factors

At hadronic scales,

$$\mathcal{H}_{\Delta c=1} = \sum_{j=1,2} \left[\mathcal{H}_j^{CF} + \mathcal{H}_j^{SCS} + \mathcal{H}_j^{DCS} \right]$$

where

$$\mathcal{H}_j^{CF} = \frac{G_F}{\sqrt{2}} \left[\lambda_{sd} C_j^{(sd)} Q_{sd}^{(j)} \right] \qquad (K^-\pi^+)$$

$$\mathcal{H}_j^{SCS} = \frac{G_F}{\sqrt{2}} \left[\lambda_d C_j^{(d)} Q_d^{(j)} + \lambda_s C_j^{(s)} Q_s^{(j)} \right] \qquad (\pi^+\pi^-; K^+K^-)$$

$$\mathcal{H}_j^{DCS} = \frac{G_F}{\sqrt{2}} \left[\lambda_{ds} C_j^{(ds)} Q_{ds}^{(j)} \right] \qquad (K^+\pi^-)$$

and

$$Q_{sd}^{(1)} = (\bar{s}\gamma^\mu c)_L (\bar{u}\gamma_\mu d)_L; \qquad Q_{sd}^{(2)} = (\bar{u}\gamma_\mu c)_L (\bar{s}\gamma^\mu d)_L$$

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Weak form factors

MAIN ASSUMPTIONS:

(i) Photon leg dominated by (lowest-lying) vector exchange:

$$\langle R_i \gamma^*(k)|\mathcal{H}|R_j\rangle = \sum_{V=\rho,\omega,\phi} \frac{\langle \gamma^* |\mathcal{H}_{\rm EM}|V\rangle}{P_V(k^2)} \langle R_i V|\mathcal{H}|R_j\rangle$$

(ii) Factorization of weak matrix elements (current-current operators)

$$\langle R^+R^-|J^\mu_{ik}J^{jc}_\mu|D^0\rangle = \langle R^+|J^\mu_{ik}|0\rangle\langle R^-|J^{jc}_\mu|D^0\rangle + \langle R^-|J^\mu_{ik}|0\rangle\langle R^+|J^{jc}_\mu|D^0\rangle$$

The weak tensor $T_{\mu\nu}$ can then be related to the $D^0 \to V$ transitions:

$$\langle V(p)|J^{\bar{u}c}_\mu|D^0(P)\rangle = P_0^V(k^2) k\cdot\varepsilon^* \frac{(m_D^2-m_V^2)}{k^2}k^\mu + A_1^V(k^2)(m_D^2-m_V^2)\left[\varepsilon^{*\mu} - \frac{k\cdot\varepsilon^*}{k^2}k^\mu\right]$$
$$+ A_2^V(k^2) k\cdot\varepsilon^*\left[p_+^\mu - \frac{(m_D^2-m_V^2)}{k^2}k^\mu\right] + iV^V(k^2)\epsilon^{\mu\nu\lambda\rho}p_{+\nu}k_\lambda\varepsilon^*_\rho$$

The form factors are determined using single pole exchange, e.g. [Wirbel et al'85]

$$V(k^2) \sim \frac{h_{V1}m_{V1}^2}{m_{V1}^2-k^2}$$

with $m_{V1} = 2110$ MeV and residues measured from $D^0 \to K^*$.

Form factors

One eventually finds

$$t_1^V(p^2,k^2) = -ie\xi_2^V(m_D+m_\rho)\left[J^V(k^2)\hat{A}_1(p^2) + \delta_{V,\rho}W(k^2)\hat{A}_1(k^2)\right]$$

$$t_2^V(p^2,k^2) = \frac{2ie\xi_2^V}{m_D+m_\rho}\left[J^V(k^2)\hat{A}_2(p^2) + \delta_{V,\rho}W(k^2)\hat{A}_2(k^2)\right]$$

$$t_3^V(p^2,k^2) = -\frac{2e\xi_2^V}{m_D+m_\rho}\left[J^V(k^2)\hat{V}(p^2) + \delta_{V,\rho}W(k^2)\hat{V}(k^2)\right]$$

with

$$J^V(k^2) = k^2\left(\frac{f_\rho}{m_\rho P_\rho(k^2)} + \frac{f_\omega}{3m_\omega P_\omega(k^2)}\right)f_V m_V$$

$$W(k^2) = k^2\left(\frac{f_\rho^2}{P_\rho(k^2)} + \frac{f_\omega^2}{3P_\omega(k^2)} - \frac{\xi_2^s}{\xi_2^d}\frac{\sqrt{2}f_\phi^2}{3P_\phi(k^2)}\right)$$

The strong parameters b_V can be determined from $V \to e^+e^-$ decay (experimental input).

$$b_\rho \sim 6.0; \qquad b_{K^*} \sim 5.4; \qquad b_\phi \sim 4.5$$

Form factors

In terms of the hadronic form factors:

$$F_{1V}^{(ab)}(t,b) = \left[(t_2^V [k\cdot(p_b - p_a)] - t_1^V) + \frac{m_a^2 - m_b^2}{m_V^2}(t_2^V p\cdot k + t_1^V) \right] \frac{b_V}{P_V(p^2)}$$

$$F_{2V}^{(ab)}(t,b) = \left[(t_2^V [k\cdot(p_b - p_a)] + t_1^V) + \frac{m_a^2 - m_b^2}{m_V^2}(t_2^V p\cdot k + t_1^V) \right] \frac{b_V}{P_V(p^2)}$$

$$F_{3V}^{(ab)}(t,b) = 2t_3^V \frac{b_V}{P_V(p^2)}$$

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- t_i from $D^0 \to V$ transitions.

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$$F_{3V}^{(ab)}(t,b) = 2 t_3^V \frac{b_V}{P_V(p^2)}$$

- t_i from $D^0 \to V$ transitions.

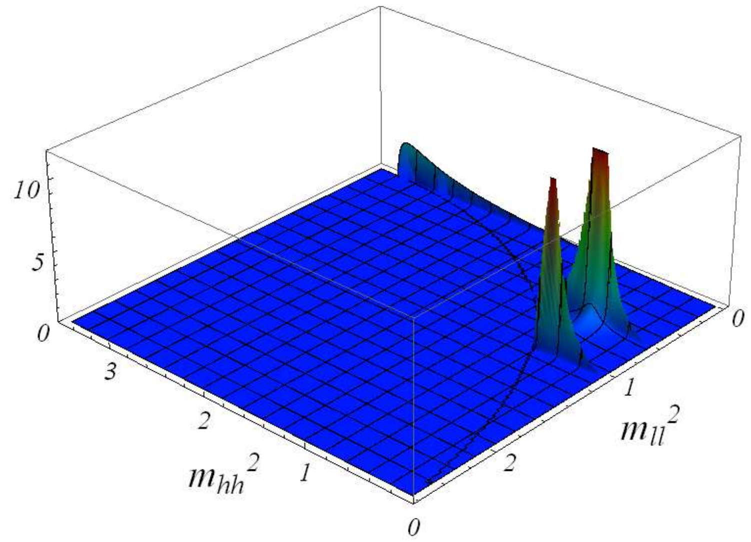
- b_V from $V \to e^+ e^-$ decays.

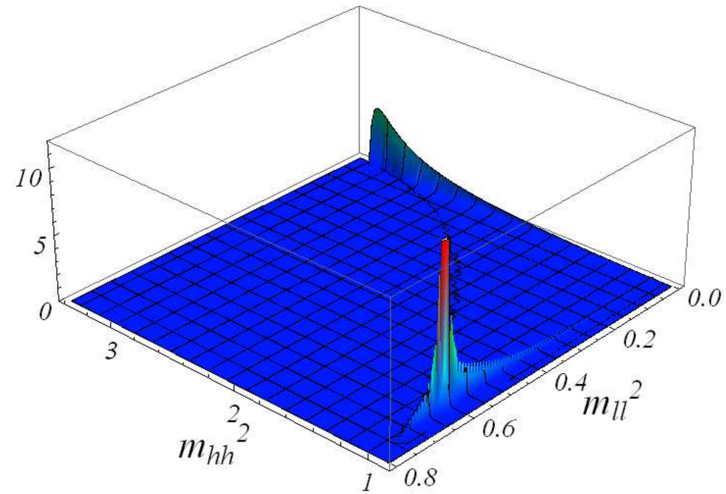
Dalitz plots

$$F_1^{(\pi\pi)} = \frac{2ie}{2q\cdot p_1 + q^2}\mathcal{M}_{(D\to\pi\pi)} + b_\rho \frac{t_2^\rho[q\cdot(p_1-p_2)] + t_1^\rho}{P_\rho(p^2)}$$

$$F_2^{(\pi\pi)} = -\frac{2ie}{2q\cdot p_2 + q^2}\mathcal{M}_{(D\to\pi\pi)} + b_\rho \frac{t_2^\rho[q\cdot(p_1-p_2)] - t_1^\rho}{P_\rho(p^2)}$$

$$F_3^{(\pi\pi)} = -2b_\rho \frac{t_3^\rho}{P_\rho(p^2)}$$





Bremsstrahlung and resonant regions far apart: interference negligible.

Branching ratios

Decay mode	Bremsstrahlung	Direct emission (E)	Direct emission (M)
$D^0 \to K^-\pi^+e^+e^-$	$9.9 \cdot 10^{-6}$	$6.2 \cdot 10^{-6}$	$4.8 \cdot 10^{-7}$
$D^0 \to \pi^+\pi^-e^+e^-$	$5.3 \cdot 10^{-7}$	$1.3 \cdot 10^{-6}$	$1.3 \cdot 10^{-7}$
$D^0 \to K^+K^-e^+e^-$	$5.4 \cdot 10^{-7}$	$1.1 \cdot 10^{-7}$	$5.0 \cdot 10^{-9}$
$D^0 \to K^+\pi^-e^+e^-$	$3.7 \cdot 10^{-8}$	$1.7 \cdot 10^{-8}$	$1.3 \cdot 10^{-9}$
$D^0 \to K^-\pi^+\mu^+\mu^-$	$8.6 \cdot 10^{-8}$	$6.2 \cdot 10^{-6}$	$4.8 \cdot 10^{-7}$
$D^0 \to \pi^+\pi^-\mu^+\mu^-$	$5.6 \cdot 10^{-9}$	$1.3 \cdot 10^{-6}$	$1.3 \cdot 10^{-7}$
$D^0 \to K^+K^-\mu^+\mu^-$	$3.3 \cdot 10^{-9}$	$1.1 \cdot 10^{-7}$	$5.0 \cdot 10^{-9}$
$D^0 \to K^+\pi^-\mu^+\mu^-$	$3.3 \cdot 10^{-10}$	$1.7 \cdot 10^{-8}$	$1.3 \cdot 10^{-9}$

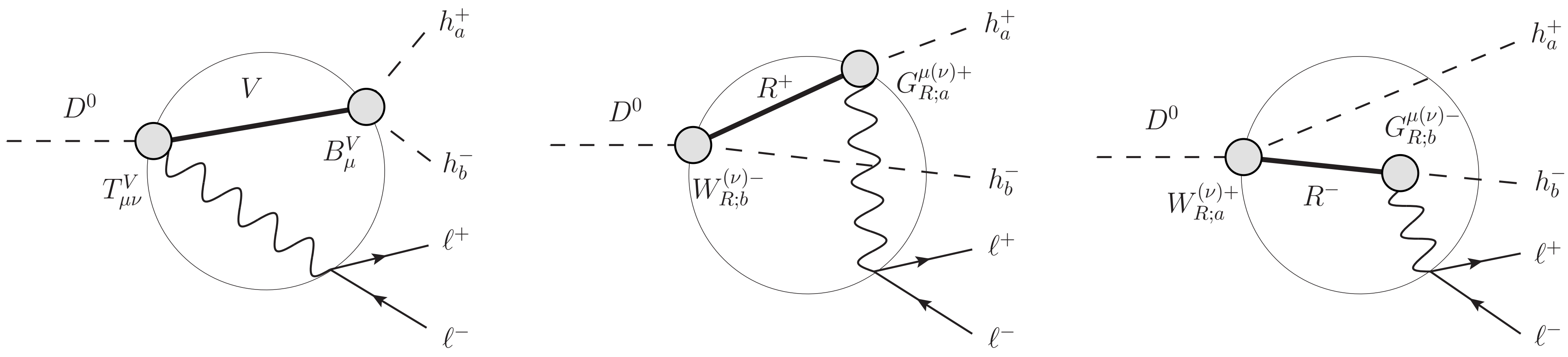
- Experimental results: [arXiv:1510.08367; arXiv:1707.08377]

$$\mathcal{B}(D^0 \to K^-\pi^+\mu^+\mu^-) = (4.17 \pm 0.42) \cdot 10^{-6}$$
$$\mathcal{B}(D^0 \to \pi^+\pi^-\mu^+\mu^-) = (9.64 \pm 0.48 \pm 0.51 \pm 0.97) \cdot 10^{-7}$$
$$\mathcal{B}(D^0 \to K^+K^-\mu^+\mu^-) = (1.54 \pm 0.27 \pm 0.09 \pm 0.16) \cdot 10^{-7}$$

- Good agreement overall. Approximations capture the bulk of the effect.

Additional resonant contributions

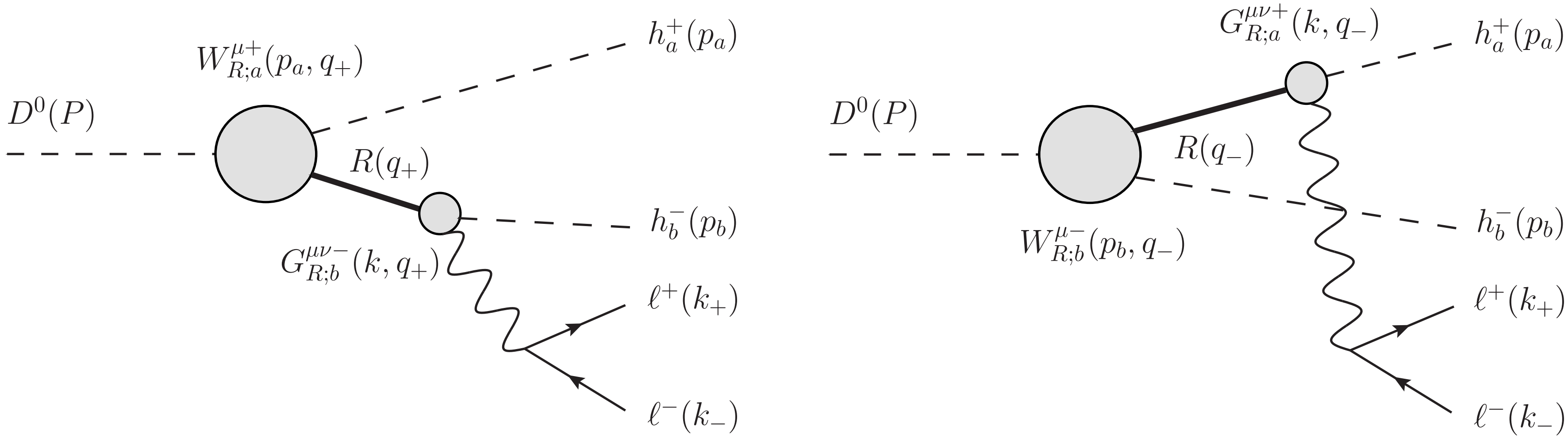
[OC, D'Ambrosio'17?]



- Extra topologies have intermediate axials and pseudoscalars. A priori sizeable.

- Information on the form factors available: predictions possible (work in progress)

Extraction of form factors

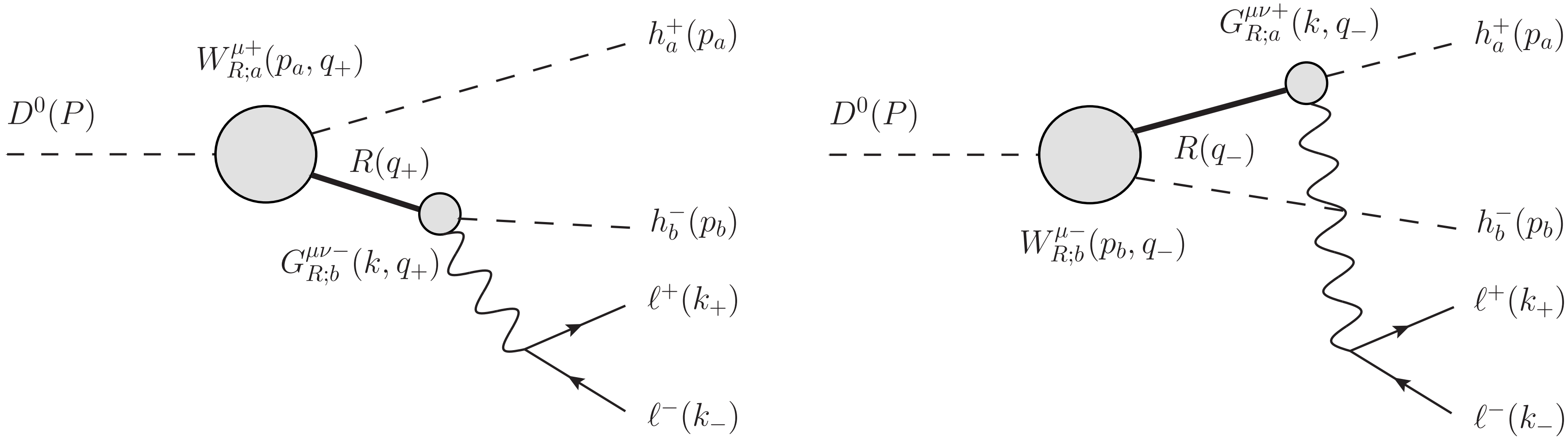


$$F_{1A}^{(ab)} = \frac{w_1^{A;a}}{P_A(q_+^2)} g_{1A;b}^- + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[g_{2A;a}^+ \left(p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g_{1A;a}^+ \frac{p_b \cdot q_-}{m_A^2} \right]$$

$$F_{2A}^{(ab)} = \frac{w_1^{A;b}}{P_A(q_-^2)} g_{1A;a}^+ + \frac{w_1^{A;a}}{P_A(q_+^2)} \left[g_{2A;b}^- \left(p_a \cdot k - \frac{(k \cdot q_+)(p_a \cdot q_+)}{m_A^2} \right) - g_{1A;b}^- \frac{p_a \cdot q_+}{m_A^2} \right]$$

$$F_{3V}^{(ab)} = \frac{w_1^{V;a}}{P_V(q_+^2)} g_{3V:b}^- - \frac{w_1^{V;b}}{P_V(q_-^2)} g_{3V:a}^+$$

Extraction of form factors



$$F_{1A}^{(ab)} = \frac{w_1^{A;a}}{P_A(q_+^2)} g^-_{1A;b} + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[g^+_{2A;a} \left(p_b \cdot k - \frac{(k\cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g^+_{1A;a} \frac{p_b \cdot q_-}{m_A^2} \right]$$

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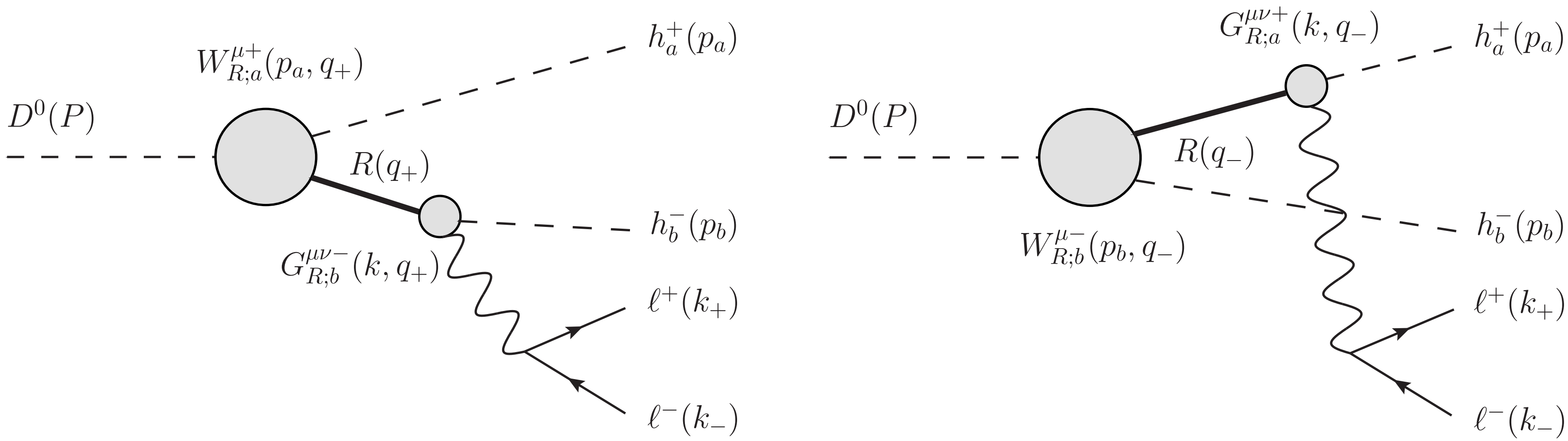
- w_1 from $D^0 \to V, A$ transitions (applying factorization).

Extraction of form factors



$$F_{1A}^{(ab)} = \frac{w_1^{A;a}}{P_A(q_+^2)} g_{1A;b}^- + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[g_{2A;a}^+ \left(p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g_{1A;a}^+ \frac{p_b \cdot q_-}{m_A^2} \right]$$

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$$F_{3V}^{(ab)} = \frac{w_1^{V;a}}{P_V(q_+^2)} g_{3V:b}^- - \frac{w_1^{V;b}}{P_V(q_-^2)} g_{3V:a}^+$$

- w_1 from $D^0 \to V, A$ transitions (applying factorization).
- g_i from VAP and VVP form factors (studied for $(g-2)_\mu$).

Conclusions

- Long distance estimate of $D^0 \to V(h^+h^-)\ell^+\ell^-$ in good agreement with experiment. Slight tension in $D^0 \to K^-\pi^+\mu^+\mu^-$ should be resolved.

- Determination of the DCS mode $D^0 \to K^+\pi^-\mu^+\mu^-$ and the modes decaying into electrons would be interesting.

- Other contributions to long distances are present, a priori as important as the one measured.

- Short distances: probes of new physics not restricted to charge asymmetry. Clean angular asymmetries exist.