$B \to D^{(*)}$ hadronic form factors and impact on $R(D^{(*)})$ predictions

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based on works with Dante Bigi and Paolo Gambino
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Importance and Status of $|V_{cb}|$

- V_{cb} plays an important role in the Unitarity Triangle.
  ➥ We want to overconstrain the triangle as a new physics test.
- V_{cb} goes into the prediction of ε_K via

$$\varepsilon_K \propto x\, |V_{cb}|^4 + \ldots .$$

- V_{cb} goes into the predictions of flavor changing neutral currents.
- The ratio

$$\left|\frac{V_{ub}}{V_{cb}}\right|$$

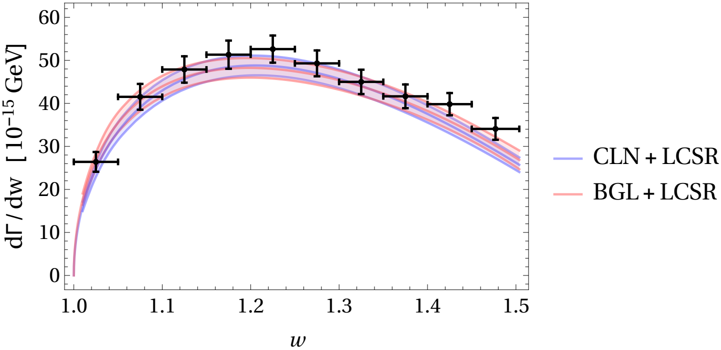
directly constrains one side of the Unitarity Triangle.

Status: HFLAV V_{cb} averages [HFLAV, 1612.07233v2]

$|V_{cb}| = (42.19 \pm 0.78) \cdot 10^{-3}$	from $B \to X_c l \nu$
$|V_{cb}| = (39.05 \pm 0.47_{\text{exp}} \pm 0.58_{\text{th}}) \cdot 10^{-3}$	from $B \to D^* l \nu$
$|V_{cb}| = (39.18 \pm 0.94_{\text{exp}} \pm 0.36_{\text{th}}) \cdot 10^{-3}$	from $B \to D l \nu$

Recent (preliminary) Belle data for $B \to D^* l\nu$

- First time w and angular deconvoluted distributions independent of parametrization.
  - ➥ Possible to use different parametrizations.



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}, \; q^2 = (p_B - p_{D^*})^2$$

Model independent form factor parametrization

[Boyd Grinstein Lebed (BGL), hep-ph/9412324, hep-ph/9504235, hep-ph/9705252]

Boyd Grinstein Lebed parametrization

$$f_i(z) = \frac{1}{B_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n ,$$

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} , \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} .$$

- $0 < z < 0.056$ for $B \to D^* l\nu \Rightarrow$ truncation at $N = 2$ enough, $z^3 \sim 10^{-4}$.
- $B_i(z)$: "Blaschke factor": removes poles.
- $\phi_i(z)$: phase space factors.

- Limit of massless leptons:
  3 form factors V_4 (vector), A_1 and A_5 (axial vector).
- Massive lepton $m_\tau \neq 0$: additional form factor P_1 (pseudoscalar).

Unitarity Constraints

[Boyd Grinstein Lebed 1994, 1997]

- Use dispersion relations to relate physical semileptonic region

$$m_l^2 \leq q^2 \leq (m_B - m_D)^2 \,, \qquad q^2 \equiv (p_B - p_{D^*})^2 \,,$$

to pair-production region beyond threshold

$$q^2 \geq (m_B + m_D)^2 \,, \quad \text{with poles at } q^2 = m_{B_c}^2 \,.$$

- Constrain form factors in pair-production region with pert. QCD.
- Translate constraint to semileptonic region using analyticity.

(Weak) Unitarity Conditions

- Vector current: $\sum_{i=0}^{\infty} \left(a_n^{V_4}\right)^2 \leq 1 \,.$
- Axial vector current: $\sum_{i=0}^{\infty} \left(\left(a_n^{A_1}\right)^2 + \left(a_n^{A_5}\right)^2\right) \leq 1 \,.$

Additional Information on Form Factors

Lattice QCD (LQCD)

[FNAL/MILC 1403.0635, HPQCD 1612.06716]

Normalization for the $|V_{cb}|$ extraction with form factor value

$$A_1(w = 1) = 0.902(12).$$

Light Cone Sum Rules (LCSR)

[Faller Khodjamirian Klein Mannel 0809.0222]

$$A_1(w_{\max}) = 0.65(18)\,, \qquad R_1(w_{\max}) = 1.32(4)\,, \qquad R_2(w_{\max}) = 0.91(17)\,.$$

Heavy Quark Effective Theory and QCD sum rules (HQET)

[Bernlochner Ligeti Papucci Robinson 1703.05330, Caprini Lellouch Neubert hep-ph/9712417, Luke Phys.Lett B252,447 (1990),

Neubert Rieckert Nucl. Phys. B382, 97 (1992) Neubert hep-ph/9306320, Ligeti Neubert Nir hep-ph/9209271, 9212266, 9305304]

- Important constraints for all $B^{(*)} \to D^{(*)}$ form factors.
- In the heavy quark limit $m_{c,b} \gg \Lambda_{\text{QCD}}$ all $B^{(*)} \to D^{(*)}$ form factors either vanish or are proportional to 1 Isgur-Wise (IW) function.
- NLO corrections at $\mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ known, expressible with 3 subleading IW functions, which are extracted using QCDSRs.

How large are the theoretical uncertainties due to unknown NNLO corrections, $\mathcal{O}(\alpha_s^2, \frac{\Lambda_{\rm QCD}^2}{m_{c,b}^2}, \alpha_s \frac{\Lambda_{\rm QCD}}{m_{c,b}})$?

- Reliable estimate from NLO corrections complicate: At zero recoil several form factors protected from NLO power corrections through Luke's theorem [Luke Phys.Lett B252,447 (1990)]
- Protection does not apply to NNLO corrections.
- The form factors which are not protected by Luke's theorem do have NLO corrections up to 60%.

$$\frac{V_6(w)}{V_1(w)} = 1.0\,, \qquad \text{(LO)}$$

$$\frac{V_6(w)}{V_1(w)} = 1.58(1 - 0.18(w-1) + \ldots)\,. \qquad \text{(NLO)}$$

Compare LQCD and HQET results: Difference from beyond NLO corrections

$$\frac{S_1(w)}{V_1(w)}\bigg|_{\text{LQCD}} \approx 0.975(6) + 0.055(18)w_1, \qquad \frac{S_1(w)}{V_1(w)}\bigg|_{\text{HQET}} \approx 1.021(30) - 0.044(64)w_1$$

$$\frac{A_1(1)}{V_1(1)}\bigg|_{\text{LQCD}} = 0.857(15), \qquad \frac{A_1(1)}{V_1(1)}\bigg|_{\text{HQET}} = 0.966(28)$$

$$\frac{S_1(1)}{A_1(1)}\bigg|_{\text{LQCD}} = 1.137(21), \qquad \frac{S_1(1)}{A_1(1)}\bigg|_{\text{HQET}} = 1.055(2), \qquad (w_1 = w - 1)$$

Deviations of $5\% - 13\%$.

Taking everything into account

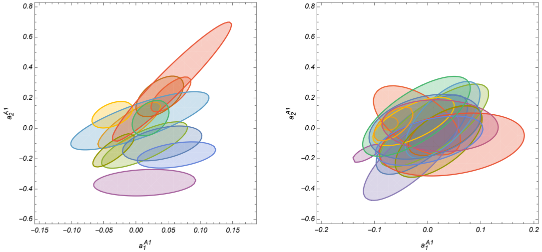
NNLO corrections as large as $\mathcal{O}(10\% - 20\%)$ are natural. They cannot be neglected for robust tests of the SM and reliable extractions of V_{cb}.

Strong Unitarity Constraints and HQET Input

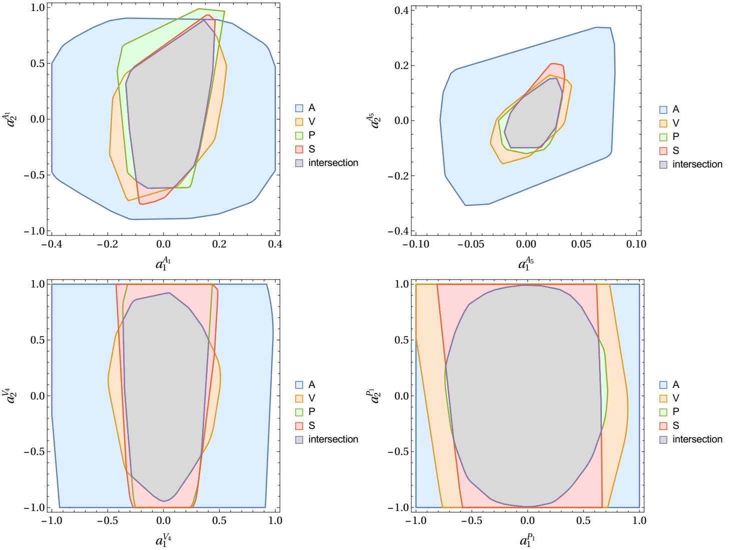
- Use HQET information on further $b \to c$ channels:
  $B \to D$, $B^* \to D$, $B^* \to D^*$, to relate them to $B \to D^*$.
- Make the unitarity bounds stronger: [BGL, hep-ph/9705252]

$$\sum_{i=1}^{H} \sum_{n=0}^{\infty} b_{in}^2 \leq 1 \,. \qquad \text{for } S, P, V, A \text{ currents}$$

- Vary QCDSR parameters + higher order corrections: obtain many different unitarity bounds.
- Take their envelope as side condition in the fit.



Allowed regions for BGL parameters from strong unitarity constraints



Different Method: Use strong unitarity/HQET to eliminate parameters and obtain simplified parametrization

Caprini Lellouch Neubert parametrization as used in exp. analyses

$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right),$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2, \quad \text{HQET: } R_1(1) = 1.27$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2 \quad \text{HQET: } R_2(1) = 0.80$$

- Theoretical uncertainties of HQET results for slope and curvature of form factor ratios R_1 and R_2 are set to zero.
- Relation of curvature and slope of axial form factor A_1 is fixed to HQET central value.
- Uncertainties on fixed parameters never included in exp. analyses.
- At current exp. precision these cannot be longer neglected.

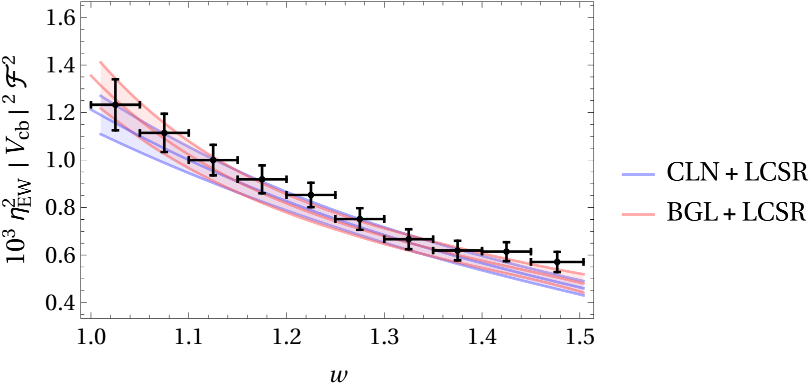
Central values of V_{cb} differ by 3.6% (with LCSR) and 5.6% (wo LCSR)

[Bigi Gambino Schacht 1703.06124 and 1707.09509, “BGL weak” agreeing with Grinstein Kobach, 1703.08170]

Fit	BGL weak	BGL weak	BGL strong	BGL strong
LCSR	$\times$	$\checkmark$	$\times$	$\checkmark$
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36
$\lvert V_{cb}\rvert$	$0.0424\,(18)$	$0.0413\,(14)$	$0.0415\,(13)$	$0.0406\,\binom{+12}{-13}$

Fit	CLN	CLN
LCSR	$\times$	$\checkmark$
χ^2/dof	35.4/37	35.9/40
$\lvert V_{cb}\rvert$	$0.0393(12)$	$0.0392(12)$

Main reason for deviation

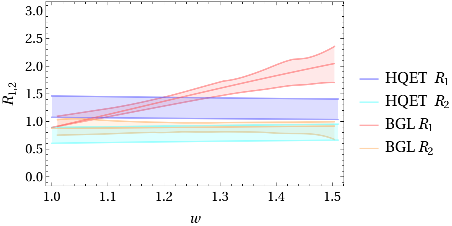


- CLN fit has limited flexibility of slope.
  ➡ CLN band underestimates all three low recoil points.
- Extrapolation near $w = 1$ crucial: Lattice input for V_{cb} extraction.
- CLN fit with free floating $R_{1,2}$ slopes (wo LCSR): $|V_{cb}| = 0.0415(19)$.
- Intrinsic uncertainties of CLN fit can no longer be neglected.

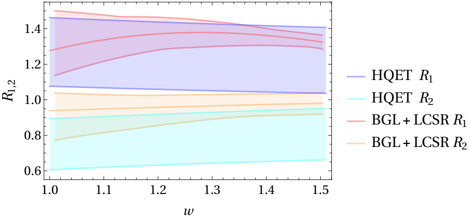
Comparison of $R_{1,2}$ fits with HQET+QCDSR

[Bigi Gambino Schacht 1703.06124, Bernlochner Ligeti Papucci Robinson 1708.07134]

"BGL strong" without LCSR.



"BGL strong" with LCSR.



- Fits for R_2 in good agreement with HQET+QCDSR. Same goes for R_1 with LCSR.
- R_1 without LCSR well compatible with HQET only at small/moderate recoil. At large w clear tension with both HQET and LCSR.
  ➡ Fit without LCSR appears somewhat disfavored.
- Lattice will compute A_1 and $R_{1,2}$ and settle the story.

Lepton Flavor Nonuniversality:

Anatomy of $R(D^*) \equiv \frac{\int_1^{w_{\tau,\max}} dw\, d\Gamma_\tau/dw}{\int_1^{w_{\max}} dw\, d\Gamma/dw}$

Differential decay rate for $B \to D^* \tau \nu_\tau$

[BGL, hep-ph/9705252]

$$\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw}$$

$$\frac{d\Gamma_{\tau,1}}{dw} = \left(1 - m_\tau^2/q^2\right)^2 \left(1 + m_\tau^2/(2q^2)\right) \frac{d\Gamma}{dw}$$

$$\frac{d\Gamma_{\tau,2}}{dw} = |V_{cb}|^2 m_\tau^2 \times \text{kinematics} \times P_1(z)^2$$

- $d\Gamma/dw$: Measured differential decay rate of $B \to D^* l \nu$ with $m_l = 0$, depends on axial vector form factors A_1, A_5 and vector form factor V_4.
- P_1 : Additional unconstrained pseudoscalar form factor.
- $d\Gamma_{\tau,2}/dw$ contributes $\sim 10\%$ to $R(D^*)$.
- Common normalization/notation:

$$R_0 = \frac{P_1}{A_1} = 1 \quad \text{in heavy quark limit}$$

[BGL, hep-ph/9705252]

Standard $R(D^*)$ Calculation: Normalizing P_1 on A_1

- NLO HQET result for $R_0 = P_1/A_1$. [Bernlochner Ligeti Papucci Robinson 1703.05330]
- Estimate of NNLO uncertainty as 15% of P_1 central value (enters quadratically).
- Our result with LCSR and strong unitarity bounds:

$$R_{\tau,1}(D^*) = 0.232 \qquad R_{\tau,2}(D^*) = 0.026\,,$$
$$R(D^*) = 0.258(5)(^{+8}_{-7}) = 0.258\left(^{+10}_{-9}\right)\,.$$

- Our result without LCSR and strong unitarity bounds:

$$R_{\tau,1}(D^*) = 0.232\,, \qquad R_{\tau,2}(D^*) = 0.025\,,$$
$$R(D^*) = 0.257(5)(^{+8}_{-7}) = 0.257\left(^{+10}_{-8}\right)\,.$$

BGL expansion + enforcing a constraint at $q^2 = 0$

Use $N = 2$ BGL expansion

$$P_1(w) = \frac{\sqrt{r}}{(1+r)B_{0^-}(z)\phi_{P_1}(z)} \sum_{n=0}^{2} a_n^{P_1} z^n \,,$$

with 3 unknowns $a_0^{P_1}, a_1^{P_1}, a_2^{P_1}$ and 3 constraints:

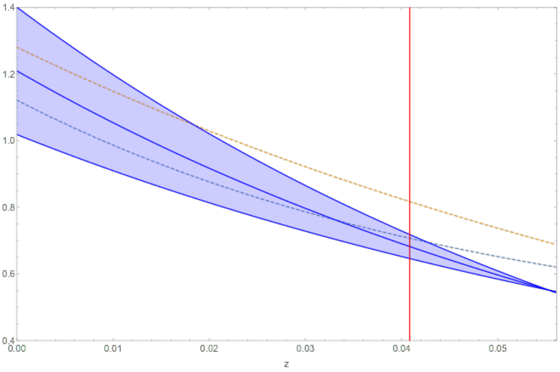
- Kinematical endpoint relation $P_1(w_{\rm max}) = A_5(w_{\rm max})$, with fit result for $A_5(w_{\rm max})$.
- HQET result $P_1(1) = 1.21 \pm 0.06 \pm 0.18$.
  - 1st error: Parametric NLO error.
  - 2nd error: Estimate of the NNLO uncertainty as 15% of central value.
- Strong unitarity.

➡ $R_{\tau,2}(D^*) = 0.028, \qquad R(D^*) = 0.260(5)(6) = 0.260(8)$.

Comparison of Different Normalizations for P_1

- Dashed yellow: normalized on V_1, $R(D^*) = 0.268\left(^{+15}_{-13}\right)$.
- Dashed blue: normalized on A_1, $R(D^*) = 0.258\left(^{+10}_{-9}\right)$.

- Solid blue: zero-recoil normalization to IW function and $P_1(w_{\max}) = A_5(w_{\max})$, $R(D^*) = 0.260(8)$.



Ref.	$R(D^*)$	Deviation
Experiment [HFLAV update]	0.304(13)(7)	—
2017 theory results, using new lattice and exp. data:		
[Bernlochner Ligeti Papucci Robinson 1703.05330]	0.257(3)	3.1σ
Our result [Bigi Gambino Schacht 1707.09509]	0.260(8)	2.6σ
[Jaiswal Nandi Patra 1707.09977]	0.257(5)	3.0σ
2012 theory results:		
[Fajfer Kamenik Nisandzic 1203.2654]	0.252(3)	3.5σ
[Celis Jung Li Pich 1210.8443]	0.252(2)(3)	3.4σ
[Tanaka Watanabe 1212.1878]	0.252(4)	3.4σ

Due to accounting for unkown NNLO corrections, we have a larger uncertainty as present in the literature.

Ref.	$R(D)$	Deviation
Experiment [HFLAV update]	$0.407(39)(24)$	—
2016/17 theory results, using new lattice and exp. data:		
[Bigi Gambino 1606.08030]	$0.299(3)$	2.4σ
[Bernlochner Ligeti Papucci Robinson 1703.05330]	$0.299(3)$	2.4σ
[Jaiswal Nandi Patra 1707.09977]	$0.302(3)$	2.3σ
2012 theory results:		
[Fajfer Kamenik Nisandzic 1203.2654]	$0.296(16)$	2.3σ
[Celis Jung Li Pich 1210.8443]	$0.296\binom{8}{6}(15)$	2.3σ
[Tanaka Watanabe 1212.1878]	$0.305(12)$	2.2σ

Good consensus of theory predictions. Belle data and lattice data beyond zero recoil allow for good determination of form factors, including S_1.

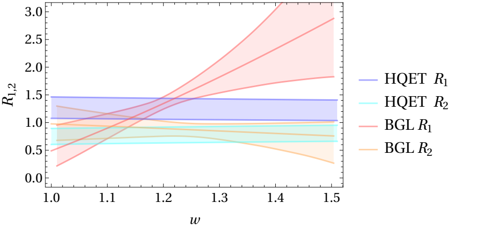
Conclusions

- Belle has new data: Deconvoluted, independent of parametrization.
- Different parametrizations imply different theoretical assumptions and different treatments of theoretical uncertainties.
- They give different results for $|V_{cb}|$, also with strong unitarity.
- In view of today's exp. precision, it is important to take into account theoretical uncertainties of HQET, including $\mathcal{O}(10\% - 20\%)$ uncertainty from unknown corrections beyond NLO.
- This is rather accomplished using the BGL parametrization with side conditions than by simplified parametrizations.
- Reanalysis of previous Belle and BaBar data is necessary. Together with future lattice data on slopes this will conclusively settle the case.
- Results: $|V_{cb}| = 40.6\left(^{+1.2}_{-1.3}\right) \cdot 10^{-3}$ (with LCSRs),
  $|V_{cb}| = 41.5(1.3) \cdot 10^{-3}$ (wo LCSRs),
  $R(D^*) = 0.260(8)$ (with and wo LCSRs).
- The $R(D^*)$ anomaly is persistent, slightly reduced to 2.6σ.

BACK-UP

Comparison of $R_{1,2}$ fits with HQET+QCDSR

"BGL weak" without LCSR:



"BGL weak" with LCSR: