

# $B_s \rightarrow K$ form factors from QCD Light-Cone Sum Rules

Aleksey Rusov  
University of Siegen, Germany

Workshop "Implications of LHCb measurements and future prospects", CERN, 8 – 10 November 2017

based on [ArXiv:1703.04765](https://arxiv.org/abs/1703.04765) (published in JHEP)

In collaboration with Alexander Khodjamirian



# Introduction

- Exclusive semileptonic  $B_s \rightarrow K l \nu_l$  decay provides an additional source for the extraction of the CKM matrix element  $|V_{ub}|$
  - FCNC rare semileptonic  $B_s \rightarrow K l^+ l^-$  decays are an additional probe of New Physics
- One needs an accurate calculation of the corresponding  $B_s \rightarrow K$  form factors
- We derive them using the **Light-Cone Sum Rules** method

# The definition of the form factors

$$\begin{aligned}\langle K(p) | \bar{q} \gamma^\mu b | B_s(p+q) \rangle &= f_{B_s K}^+(q^2) \left[ 2p^\mu + \left( 1 - \frac{m_{B_s}^2 - m_K^2}{q^2} \right) q^\mu \right] \\ &+ f_{B_s K}^0(q^2) \frac{m_{B_s}^2 - m_K^2}{q^2} q^\mu \\ \langle K(p) | \bar{q} \sigma^{\mu\nu} q_\nu b | B_s(p+q) \rangle &= \frac{if_{B_s K}^T(q^2)}{m_{B_s} + m_K} \left[ 2q^2 p^\mu + \left( q^2 - (m_{B_s}^2 - m_K^2) \right) q^\mu \right]\end{aligned}$$

- Involved in the semileptonic  $\bar{B}_s^0 \rightarrow K^+ \ell^- \bar{\nu}_\ell$  and  $\bar{B}_s^0 \rightarrow K^0 \ell^+ \ell^-$  decays
- Needed in non-leptonic  $B_s$  decays (within QCDF approach)

$q = u, d$ , **Isospin symmetry** is assumed:

$$f_{B_s^0 K^+}^{(+,0,T)}(q^2) = f_{B_s^0 K^0}^{(+,0,T)}(q^2)$$

# Correlation function

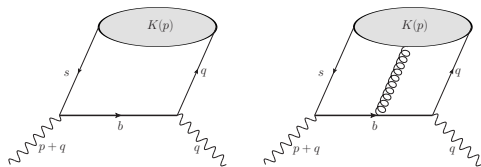
- Starting object used for calculation of  $B_s \rightarrow K$  form factors in LCSR — Correlation function

$$\begin{aligned} F_{B_s K}^\mu(p, q) &= i \int d^4x e^{iqx} \langle K(p) | T \{ \bar{q}(x) \Gamma^\mu b(x), m_b \bar{b}(0) i \gamma_5 s(0) \} | 0 \rangle \\ &= \begin{cases} F_{B_s K}(q^2, (p+q)^2) p^\mu + \tilde{F}_{B_s K}(q^2, (p+q)^2) q^\mu, & \Gamma^\mu = \gamma^\mu \\ F_{B_s K}^T(q^2, (p+q)^2) [q^2 p^\mu - (q \cdot p) q^\mu], & \Gamma^\mu = -i \sigma^{\mu\nu} q_\nu \end{cases} \end{aligned}$$

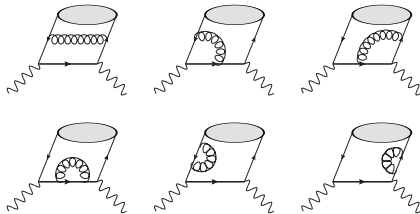
- Region of light-cone dominance ( $x^2 \sim 0$ )
  - $q^2 \ll m_b^2$
  - $(p+q)^2 \ll m_b^2$

# Relevant OPE diagrams

- At leading order in  $\alpha_s$  expansion



- Radiative corrections to the relevant hard-scattering kernels



# The current accuracy of the OPE result

$$F_{B_s K}^{(T)}(q^2) \Rightarrow \text{OPE} = \left( T_0^{(2)} + (\alpha_s/\pi) T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)} \right) \otimes \varphi_K^{(2)} \\ + \frac{\mu_K}{m_b} \left( T_0^{(3)} + (\alpha_s/\pi) T_1^{(3)} \right) \otimes \varphi_K^{(3)} + T_0^{(4)} \otimes \varphi_K^{(4)} + \langle \bar{q}q \rangle \left( T_0^{(5)} \otimes \varphi_K^{(2)} + \frac{\mu_K}{m_b} T_0^{(6)} \otimes \varphi_K^{(3)} \right)$$

$$\mu_K = \frac{m_K^2}{m_s + m_q}, \quad \varphi_K^{(k)} = \text{AF} + \text{non-asympt. corrections}$$

- LO twist 2, 3, 4  $q\bar{q}$  and  $\bar{q}qG$  terms

[V.Belyaev, A.Khodjamirian, R.Rückl (1993); V.Braun, V.Belyaev, A.Khodjamirian, R.Rückl (1996)]

- NLO  $O(\alpha_s)$  twist 2 (collinear factorization)

[A.Khodjamirian, R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997)]

- NLO  $O(\alpha_s)$  twist 3 (coll. factorization for asympt. DA)

[P. Ball, R. Zwicky (2001); G.Duplancic, A.Khodjamirian, B.Melic, Th.Mannel, N.Offen (2007) ]

- Part of NNLO  $O(\alpha_s^2\beta_0)$  twist 2 [A. Bharucha (2012)]

- LO twist 5 and twist 6 in factorization approximation [A.V. Rusov (2017)]

# LCSR: a general scheme

- Transform the OPE result to the dispersion form (in  $(p+q)^2$  variable):

$$F_{B_s K}^{(T)(\text{OPE})}(q^2, (p+q)^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} ds \frac{\text{Im} F_{B_s K}^{(T)(\text{OPE})}(q^2, s)}{s - (p+q)^2}$$

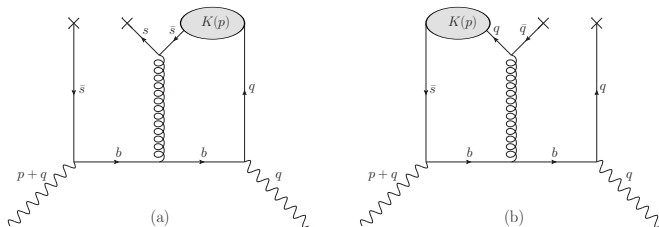
- Matching to the **hadronic dispersion relation** (isolating  $B_s$ -meson state)
- Applying the **quark hadron duality**
- Applying the **Borel transform**
- Finally

$$f_{B_s K}^+(q^2) = \frac{e^{m_{B_s}^2/M^2}}{2m_{B_s}^2 f_{B_s}} \frac{1}{\pi} \int_{m_b^2}^{s_0^{B_s}} ds \text{Im} F_{B_s K}^{(\text{OPE})}(q^2, s) e^{-s/M^2}$$

$$f_{B_s K}^T(q^2) = \frac{(m_{B_s} + m_K) e^{m_{B_s}^2/M^2}}{2m_{B_s}^2 f_{B_s}} \frac{1}{\pi} \int_{m_b^2}^{s_0^{B_s}} ds \text{Im} F_{B_s K}^{T(\text{OPE})}(q^2, s) e^{-s/M^2}$$

# Factorizable twist-5 and twist-6 contributions

A.V. Rusov, 1705.01929 [hep-ph]



- An estimate of the twist-5 and twist-6 contributions is obtained in the **factorization** approximation

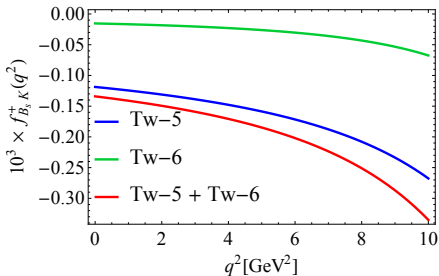
[V.M. Braun, A. Khodjamirian, M. Maul (2000)], LCSR for  $F_\pi(Q^2)$ ;

[S.S. Agaev, V.M. Braun, N. Offen, F.A. Porkert (2011)], LCSR for  $F_{\pi^0\gamma^*\gamma}(Q^2)$



# Factorizable twist-5 and twist-6 contributions

A.V. Rusov, 1705.01929 [hep-ph]



$$f_{B_s K}^+(0) = 0.336 \pm 0.023$$

[A. Khodjamirian, A.V. Rusov (2017)]

$f_{B_s K}^+(0)$	0.336
Tw2 LO	47.0%
Tw2 NLO	8.8%
Tw3 LO	47.1%
Tw3 NLO	-3.9%
Tw4 LO	1.0%
Tw5 LO-fact	-0.039%
Tw6 LO-fact	-0.005%

Higher twist contributions are strongly suppressed!

# Results for the $B_s \rightarrow K$ form factors

A. Khodjamirian, A.V. Rusov, 1703.04765 [hep-ph]

LCSR results are fitted to the  $z$ -expansion (BCL parametrization)

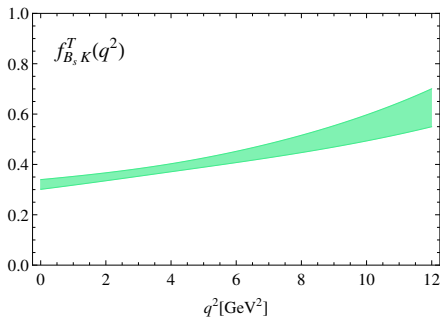
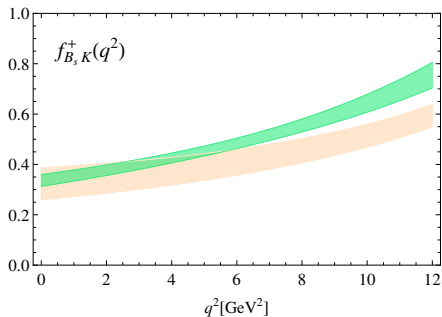
$$f_{B_s K}^{+,T}(q^2) = \frac{f_{B_s K}^{+,T}(0)}{1 - q^2/m_{B_s}^2} \left\{ 1 + b_{1(B_s K)}^{+,T} \left[ z(q^2) - z(0) + \frac{1}{2} (z(q^2)^2 - z(0)^2) \right] \right\}$$

$$z(q^2) = \frac{\sqrt{(m_{B_s} + m_K)^2 - q^2} - \sqrt{(m_{B_s} + m_K)^2 - t_0}}{\sqrt{(m_{B_s} + m_K)^2 - q^2} + \sqrt{(m_{B_s} + m_K)^2 - t_0}}, \quad t_0 = (m_{B_s} + m_K)(\sqrt{m_{B_s}} - \sqrt{m_K})^2$$

$f_{B_s K}^+(0)$	$b_{1(B_s K)}^+$	Correlation
$0.336 \pm 0.023$	$-2.53 \pm 1.17$	0.79
$f_{B_s K}^T(0)$	$b_{1(B_s K)}^T$	Correlation
$0.320 \pm 0.019$	$-1.08 \pm 1.53$	0.74

# Results for the $B_s \rightarrow K$ form factors

A. Khodjamirian, A.V. Rusov, 1703.04765 [hep-ph]



Lattice QCD results: C.M. Bouchard et al., hep-lat: 1406.2279

# Applications

A. Khodjamirian, A.V. Rusov, 1703.04765 [hep-ph]

- The partially integrated  $\bar{B}_s^0 \rightarrow K^+ \ell^- \bar{\nu}_\ell$  decay width

$$\Delta\zeta_{B_s K}[0, 12 \text{ GeV}^2] \equiv \frac{1}{|V_{ub}|^2} \int_0^{12 \text{ GeV}^2} dq^2 \frac{d\Gamma(\bar{B}_s \rightarrow K^+ \ell \bar{\nu}_\ell)}{dq^2} = 7.03_{-0.69}^{+0.73} \text{ ps}^{-1}$$

- The  $CP$ -averaged  $q^2$ -binned branching fraction of  $\bar{B}_s^0 \rightarrow K^0 \ell^+ \ell^-$

$$\mathcal{B}_{B_s K}[q_1^2, q_2^2] \equiv \frac{1}{2} \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \left[ \frac{dB(\bar{B}_s^0 \rightarrow K^0 \ell^+ \ell^-)}{dq^2} + \frac{dB(B_s^0 \rightarrow \bar{K}^0 \ell^+ \ell^-)}{dq^2} \right]$$

$$\mathcal{B}_{B_s K}[1.0 \text{ GeV}^2, 6.0 \text{ GeV}^2] = (4.38_{-0.57}^{+0.62} \pm 0.28) \times 10^{-8} \text{ GeV}^{-2}$$

- $q^2$ -binned direct  $CP$ -asymmetry in  $\bar{B}_s^0 \rightarrow K^0 \ell^+ \ell^-$  decays

$$\mathcal{A}_{B_s K}[1.0 \text{ GeV}^2, 6.0 \text{ GeV}^2] = -0.04_{-0.03}^{+0.01}$$

# Conclusion

- $B_s \rightarrow K$  form factors at large recoil from LCSR are predicted
- The partially integrated  $\bar{B}_s^0 \rightarrow K^+ l^- \bar{\nu}_l$  decay width is calculated (to be used for the  $|V_{ub}|$  extraction)
- The  $CP$ -averaged  $q^2$ -binned branching fraction and  $q^2$ -binned direct  $CP$ -asymmetry in  $\bar{B}_s^0 \rightarrow K^0 l^+ l^-$  decays are predicted (accounting nonlocal effects)
- Experimental data on  $\bar{B}_s^0 \rightarrow K^0 l^+ l^-$  and  $\bar{B}_s^0 \rightarrow K^+ l^- \bar{\nu}_l$  decays are anticipated