Parton Distributions at the HL-LHC and HE-LHC

Lucian Harland-Lang*, University of Oxford
Jun Gao, Shanghai Jiao Tong University
Juan Rojo, VU Amsterdam and NIKHEF

Workshop on the physics of HL-LHC and perspectives at HE-LHC
CERN, 1 November 2017

*speaker
Outline

• PDFs at the HL-LHC/HE-LHC - overview.
• Recent Progress in PDF fitting.
• Theoretical uncertainties:
  ‣ Parametric.
  ‣ Missing higher orders.
• Role of electroweak corrections:
  ‣ Corrections to cross sections.
  ‣ Photon-initiated processes.
  ‣ EW PDFs.
• PDFs with resummation:
  ‣ High $x$ - threshold.
  ‣ Low $x$ - BFKL.
• PDFs at a high energy ep/eA collider.
PDFs at the HL-LHC

New parton distribution functions from a global analysis of quantum chromodynamics

Sayipjamal Dulat,1, 2, * Tie-Jun Hou,3, † Jun Gao,4, 1 Marco Guzzi,5, 1 Joey Huston,2, ‡ Pavel Nadolsky,3, ** Jon Pumplin,7, †† Carl Schmidt,2, †† Daniel Stump,2, ‡‡ and C.-P. Yuan,†††

1 School of Physics Science and Technology, Xinyang University, Xinyang, Xinyang 830046 China
2 Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824 U.S.A.
3 Department of Physics, Southern Methodist University, Dallas, TX 75275-0188, U.S.A.
4 High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, U.S.A.
5 School of Physics & Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom

Abstract

We present NNPDF3.0, the first set of parton distribution functions (PDFs) of the LHC era: MMHT 2014 PDFs

Parton distributions in the LHC era:

MMHT 2014 PDFs


a Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK
b Institute for Particle Physics Phenomenology, Durham University, Durham, DH1 3LE, UK

DESY 16-179
DO-TH 1613
January 2017

Parton Distribution Functions, \(\alpha_s\), and Heavy-Quark Masses for LHC Run II

S. Alekhiina,b, J. Blümleina, S. Mochb and R. Plačakyeb

a II. Institut für Theoretische Physik, Universität Hamburg
Luruper Chaussee 149, D–22761 Hamburg, Germany
b Institute for High Energy Physics
142281 Protvino, Moscow region, Russia
c Deutsches Elektronensynchrotron DESY
Platanenallee 6, D–15738 Zeuthen, Germany

• What will we need for `PDFs at the HL-LHC/HE-LHC'?
PDFs at the HL-LHC

- **HL-LHC**: precision machine \( \Rightarrow \) need **high precision PDFs** to control theory uncertainties for searches and precision measurements.
  - Higher statistics \( \Rightarrow \) increased coverage at **high** \( x \) where PDFs currently less well constrained.
  - Must systematically account for contributions/uncertainties often omitted before: **EW corrections, theoretical uncertainties**…
  - **NNLO** will be the standard. **New tools** for fitting?

- **High precision data**
  - Opportunity for PDF fitting:
    - Rarer processes increasingly abundant use in fits.
    - Increasingly differential information.

**Figure 53**: The gluon PDF at high scale of \( Q = 100 \) GeV with the corresponding fits where the PDF uncertainties are quite large, leading to significant theoretical uncertainties for the PDFs at the HL-LHC.

**NNLO**, \( \alpha_s = 0.118 \), \( Q = 100 \) GeV
PDFs at the HE-LHC

- **HE-LHC**: many/all of previous considerations apply here as well. In addition, higher energy reach:

  - Higher $Q^2$ coverage $\rightarrow$ role of e.g. **EW corrections** could be larger.
  - Increased coverage in lower $x$, higher sensitivity to e.g. **resummation** effects?

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![Kinematics of HE-LHC at $\sqrt{s}=27$ TeV](image-url)
PDF luminosities

- **HE-LHC**: impact on $gg$ luminosity not dramatic.

- **Lower masses** - uncertainties increased somewhat.

- **Higher masses** - increased reach, uncertainties somewhat lower.
• Picture similar for other luminosities, e.g. $q\bar{q}$
• Much recent progress in PDFs - this talk will only cover a small amount.

• For more details, see arXiv:1709.04922 (To appear in Physics Reports).

The Structure of the Proton in the LHC Precision Era

Jun Gao\textsuperscript{a}, Lucian Harland-Lang\textsuperscript{b}, Juan Rojo\textsuperscript{c,d}

\textsuperscript{a}Institute of Nuclear and Particle Physics, Shanghai Key Laboratory for Particle Physics and Cosmology, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, China

\textsuperscript{b}Department of Physics and Astronomy, University College London, WC1E 6BT, United Kingdom

\textsuperscript{c}Department of Physics and Astronomy, VU University, De Boelelaan 1081, 1081HV Amsterdam, The Netherlands

\textsuperscript{d}Nikhef, Science Park 105, NL-1098 XG Amsterdam, The Netherlands

Abstract

We review recent progress in the determination of the parton distribution functions (PDFs) of the proton,
Progress in PDF fits
Progress in PDF fits

- Global groups busily updating fits to include the plentiful and precise new LHC data. **ABMP16, NNPDF3.1** released recently, **MMHT17/18** and **CT17** on their way.

New datasets in NNPDF3.1

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Data taking</th>
<th>Motivation</th>
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<tbody>
<tr>
<td>Combined HERA inclusive data</td>
<td>Run I+II</td>
<td>quark singlet and gluon</td>
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<tr>
<td>ATLAS inclusive W, Z rap 7 TeV</td>
<td>2011</td>
<td>strangeness</td>
</tr>
<tr>
<td>ATLAS inclusive jets 7 TeV</td>
<td>2011</td>
<td>large-x gluon</td>
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<tr>
<td>ATLAS low-mass Drell-Yan 7 TeV</td>
<td>2010+2011</td>
<td>small-x quarks</td>
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<tr>
<td>ATLAS Z pT 7,8 TeV</td>
<td>2011+2012</td>
<td>medium-x gluon and quarks</td>
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<td>ATLAS and CMS tt differential 8 TeV</td>
<td>2012</td>
<td>large-x gluon</td>
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<tr>
<td>CMS Z (pT,η) 2D xsecs 8 TeV</td>
<td>2012</td>
<td>medium-x gluon and quarks</td>
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<td>small-x and large-x quarks</td>
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<td>CMS W asymmetry 8 TeV</td>
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<td>quark flavor separation</td>
</tr>
<tr>
<td>CMS 2.76 TeV jets</td>
<td>2012</td>
<td>medium and large-x gluon</td>
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<tr>
<td>LHCb W,Z rapidity dists 7 TeV</td>
<td>2011</td>
<td>large-x quarks</td>
</tr>
<tr>
<td>LHCb W,Z rapidity dists 8 TeV</td>
<td>2012</td>
<td>large-x quarks</td>
</tr>
</tbody>
</table>

CT17p — data to be included

- Previous LHC and HERA 1 data included in CT14 will be superseded by updated Run 1 and HERA 1+2 data; adding new LHC data, especially on Z boson pT and top quark differential distributions
  - Combined HERA1+2 DIS [1506.06042] update
  - LHCb 7 TeV Z, W muon rapidity dist. [1505.07024] update
  - LHCb 8 TeV Z rapidity dist. [1503.00963] update
  - ATLAS 7 TeV inclusive jet [1410.8857] update
  - CMS 7 TeV inclusive jet (extended y range) [1406.0324] update
  - ATLAS 7 TeV Z pT dist. [1406.3660] new
  - LHCb 13 TeV Z rapidity dist. [1607.06495] update
  - CMS 8 TeV Z pT and rapidity dist. (double diff) [1504.03511] new
  - CMS 8 TeV W, muon asymmetry dist. [1603.01803] update
  - ATLAS 7 TeV W/Z lepton(s) rapidity dist. [1612.03016] update
  - CMS 7,8 TeV tT differential distributions new
  - ATLAS 7,8 TeV tT differential distributions new

MMHT (2016 fit)

<table>
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<tr>
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<th>NLO $\chi^2_{new}$</th>
<th>NNLO $\chi^2_{pred}$</th>
<th>NNLO $\chi^2_{new}$</th>
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<td>$\sigma_T^{H}$ Tevatron +CMS+ATLAS</td>
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<td>19.6</td>
<td>20.5</td>
<td>14.7</td>
<td>15.5</td>
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<tr>
<td>LHCb 7 TeV $W + Z$</td>
<td>33</td>
<td>50.1</td>
<td>45.4</td>
<td>46.5</td>
<td>42.9</td>
</tr>
<tr>
<td>LHCb 8 TeV $W + Z$</td>
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<td>77.0</td>
<td>58.9</td>
<td>62.6</td>
<td>59.0</td>
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<td>LHCb 8TeV $e$</td>
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<tr>
<td>CMS 8 TeV $W$</td>
<td>22</td>
<td>32.6</td>
<td>18.6</td>
<td>34.9</td>
<td>20.5</td>
</tr>
<tr>
<td>CMS 7 TeV $W + c$</td>
<td>10</td>
<td>8.5</td>
<td>10.0</td>
<td>8.7</td>
<td>8.0</td>
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<tr>
<td>D0 e asymmetry</td>
<td>13</td>
<td>22.2</td>
<td>21.5</td>
<td>27.3</td>
<td>25.8</td>
</tr>
<tr>
<td>total</td>
<td>3738/3405</td>
<td>4375.9</td>
<td>4336.1</td>
<td>3741.5</td>
<td>3723.7</td>
</tr>
</tbody>
</table>
LHC impact

- These LHC data, combined with new NNLO theory, are now playing a significant role in constraining the PDFs.

- **Differential top**: increased sensitivity vs. total cross section.
- Fits performed with latest NNLO theory. Impact on gluon at high $x$.

M. Czakon et al., JHEP 1704 (2017) 044

- High precision ATLAS $W, Z$ data at 7 TeV. Sensitive to strange component.
- Impact on proton strangeness significant. Some movement towards flavour symmetric sea.


LHL et al., arXiv:1708.00047

$(s + s)/(u + d)$ (NNLO), $Q^2 = 1.9 \text{ GeV}^2$
**LHC impact**


- **Inclusive jets** now calculated at NNLO → another PDF handle. LHC jet constraint on gluon consistent with e.g. $t\bar{t}$.
- Some stability when including full theory seems to be present.

J. Gao, “Progress on CTEQ-TEA PDFs”, DIS2017

**CT17p best-fit vs. CT14 HERA2**

- **$Z$ boson $p_{\perp}$ distribution.** Sensitive to gluon at high $x$. New NNLO calculation allows constraints on PDFs at this order.

LHC questions

- However, **challenges** for including these increasingly precise data!

![Graph showing comparisons between NLO, NNLO, and NNLO+EW theories with data.](image)

- **ATLAS jets.** Systematics dominated. By eye, description looks good, but issues describing data across all rapidities.

<table>
<thead>
<tr>
<th>$\chi^2$/ndf</th>
<th>$P^\text{jet, max}_T$</th>
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<tbody>
<tr>
<td>$P_T &gt; 70$ GeV</td>
<td>$R = 0.4$</td>
</tr>
<tr>
<td>CT14</td>
<td>$R = 0.6$</td>
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<tr>
<td>HERAPDF2.0</td>
<td></td>
</tr>
<tr>
<td>NNPDF3.0</td>
<td></td>
</tr>
<tr>
<td>MMHT2014</td>
<td></td>
</tr>
</tbody>
</table>

To achieve acceptable $\chi^2$ for $Zp_T$ data, include additional $\sigma^{\text{uncorr}}$ due to missed theory/exp. uncertainties.

![Table showing comparisons between ATLAS, 7 TeV LHC jets and different PDF sets.](image)

- All bins
- NLO
- NNLO
- NNLO+EW

<table>
<thead>
<tr>
<th>Bin</th>
<th>Order</th>
<th>$N_{\text{dat}}$</th>
<th>$\chi^2_{\text{d.o.f. (NN30)}}$</th>
<th>$\chi^2_{\text{d.o.f. (CT14)}}$</th>
<th>$\chi^2_{\text{d.o.f. (MMHT14)}}$</th>
<th>$\chi^2_{\text{d.o.f. (ABMP16)}}$</th>
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<td>All bins</td>
<td>NLO</td>
<td>42</td>
<td>9.9</td>
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<td>42</td>
<td>4.9</td>
<td>6.7</td>
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<td>42</td>
<td>3.7</td>
<td>5.2</td>
<td>5.6</td>
<td>12.</td>
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Boughezal et al., JHEP 1707 (2017) 130
Theoretical uncertainties
**Parametric uncertainties**

- Due to input of $\alpha_S$ and **heavy quark mass**, especially the later requires further investigation.
- **MMHT** study - PDFs with different input charm pole mass $\pm 0.15$ GeV.

![Gluon (NLO), percentage difference at $Q^2 = 10^4$ GeV$^2$](image)

![Light quarks (NLO), percentage difference at $Q^2 = 10^4$ GeV$^2$](image)


- **Impact on benchmark cross sections** can be as large/larger than PDF uncertainties.
- Increases at low $x$, i.e. **higher energies**.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$</th>
<th>PDF unc.</th>
<th>$m_c$ var.</th>
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<tbody>
<tr>
<td>$W^+$ LHC (14 TeV)</td>
<td>12.5</td>
<td>+0.22 (+1.8%)</td>
<td>+0.091 (+0.73%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.18 (-1.4%)</td>
<td>-0.12 (-0.93%)</td>
</tr>
<tr>
<td>$W^-$ LHC (14 TeV)</td>
<td>9.3</td>
<td>+0.15 (+1.6%)</td>
<td>+0.064 (+0.69%)</td>
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<tr>
<td></td>
<td></td>
<td>-0.14 (-1.5%)</td>
<td>-0.075 (-0.81%)</td>
</tr>
<tr>
<td>$Z$ LHC (14 TeV)</td>
<td>2.06</td>
<td>+0.035 (+1.7%)</td>
<td>+0.021 (+1.03%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.030 (-1.5%)</td>
<td>-0.025 (-1.2%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$</th>
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<th>$m_c$ var.</th>
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<tr>
<td>Higgs Tevatron (1.96 TeV)</td>
<td>0.87</td>
<td>+0.024 (+2.7%)</td>
<td>-0.0060 (-0.68%)</td>
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<tr>
<td></td>
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<td>-0.030 (-3.4%)</td>
<td>+0.0070 (+0.79%)</td>
</tr>
<tr>
<td>Higgs LHC (7 TeV)</td>
<td>14.6</td>
<td>+0.21 (+1.4%)</td>
<td>+0.025 (+0.17%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.29 (-2.0%)</td>
<td>-0.019 (-0.13%)</td>
</tr>
<tr>
<td>Higgs LHC (14 TeV)</td>
<td>47.7</td>
<td>+0.63 (+1.3%)</td>
<td>+0.27 (+0.57%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.88 (-1.8%)</td>
<td>-0.22 (-0.48%)</td>
</tr>
</tbody>
</table>
Missing higher orders

- Uncertainties due to missing higher-order corrections (MHOUs).
- Scale variations around certain nominal choice by a factors of two (widely used). Ambiguities in the nominal scale choice, underestimation issues...
- More advanced approaches:
  - **Cacciari-Houdeau**: uniform prior pdf and Bayesian approach to determine confidence interval from behaviour of known perturbative coefficients.
  - Series acceleration method: e.g., **Passarino-David**, a uniform pdf assumed between known perturbative result and the approximate full result.

*Figure 75:* The cross section for Higgs boson production in gluon fusion calculated at increasing perturbative orders. At each order the theoretical uncertainty is shown for using scale variation (red circles), the C-H method (blue crosses), and the Scaled Parameter (green squares); at N^3LO the Passarino-David uncertainty based on series acceleration method is also shown (purple diamonds).

Illustration for Higgs production in gluon fusion at LHC 8 TeV up to approximate N3LO

Easy for a total rate; need to generate for correlations in differential observables

Missing higher orders - PDFs

- MHOU associated with the global analysis of PDFs, in e.g. QCD splitting kernels and matrix elements.

- **Global approaches** - comparing the outcome PDFs for fit at different orders. Gives ~ upper limit on MHOU. Can be comparable to PDF uncertainties.

- **Other approaches**: compare PDFs with different scale choices or more sophisticated treatment of the MHOU as correlated systematics.

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R. Ball et al., JHEP 1504 (2015) 040

The NNLO standard - tools
Tools

• **APPLgrid** and **FastNLO**: tools for storing NLO calculations to grid, for fast convolution with arbitrary PDF set.
• Essential for precise PDF fits with NLO theory.
• Huge progress in **NNLO calculations**, but very computationally expensive - not directly usable in PDF fits.
• Simple (PDF dependent) **K-factors** generally used.

\[ K_{\text{NNLO}} = \frac{\sigma_{\text{NNLO}}^i}{\sigma_{\text{NLO}}^i} \]

• Moving beyond this: the **APPLfast** project.

![Graph showing NNLO/NLO K-factors for inclusive jet production](image-url)
Tools

- **APPLfast** will make fast **NNLOJET** (IPPP, Zurich, ETH et al.) calculation feasible for inclusive $W$, $Z$, $Z +$ jet, $H +$ jet, inclusive jets in $ep$ and $pp$...
- Compute once (~ 200K CPU hours)- after that in much less than a second!

C. Gwenlain, DIS2017

<table>
<thead>
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<td>—</td>
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<td>248196 h</td>
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</table>

3 x 11710 grids/tables + all NNLOJET output! 
Final 3 files for analysis are $O(10\text{MB})$ each

- This will become the standard in the future.
- **Benchmarking** exercises (are fitters using comparable NLO, NNLO input grids?) and centralising to avoid ‘double counting’ should be looked at.
Electroweak corrections
Electroweak effects - cross sections

- **EW corrections** can contribute at same level as NNLO QCD corrections:
  \[ \alpha_s(M_Z)^2 \sim \alpha_{EM}(M_Z) \]

- Particularly at **large \( p_\perp \)**/invariant mass due to the Sudakov logarithms. Can be crucial when confronting precision data at HL-LHC.

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**pT of top quark at LHC 13 TeV**

35.8 fb\(^{-1}\) (13 TeV)

1. High mass Drell-Yan at LHC 13 TeV

2. E\(_T\) of direct photon at LHC 8 TeV

Czakon et al., arXiv:1705.04105

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• In certain regions EW corrections must to be included in PDF fits for a good description of the data; impact on PDFs can be significant.
The photon PDF

- Potentially important part of EW corrections is due to initial-state photon contributions. Requires inclusion via photon PDF.
- Recap: 2016 studies indicated that contribution to DY, \(WW\), \(t\bar{t}\) … could be large at high mass, with large uncertainty.
- Issue for high precision HL-LHC, or higher HE-LHC energies? No!

\[ \frac{dL}{d\ln M_X^2}, \sqrt{s} = 13 \text{ TeV} \]

\[ \frac{d\sigma}{d\mu} \text{ [fb/TeV]}, \sqrt{s} = 13 \text{ TeV} \]


\[ \gamma\gamma - \text{this work} \]
\[ \gamma\gamma - \text{NNPDF} \]
\[ gg \]
\[ qq \]
\[ gg \]
• ‘Agnostic’ approach of **NNPDF3.0QED** far **too conservative**. Photon PDF known to high precision in terms of measured $e p$ scattering.


• Put on truly quantitative footing by **LUXqed** set. Photon PDF **completely determined** in terms of (well known) $F_2$ and $F_L$ structure functions.

\[
x f_{\gamma/p}(x, \mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{\mu^2}{1-z}}^{\frac{\mu^2}{Q^2}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \right. \\
\left. \left[ \left( z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(\frac{x}{z}, Q^2) - z^2 F_L \left( \frac{x}{z}, Q^2 \right) \right] \\
- \alpha^2(\mu^2) z^2 F_2 \left( \frac{x}{z}, \mu^2 \right) \right\}, \tag{6}
\]

A. Manohar et al., arXiv:1708.01256
• Conclusion: photon PDF known to % level precision across relevant $x$.
• Moved beyond era of large photon PDF uncertainties. Photon has gone from being the poorest to the best constrained parton!

• Ongoing work: implementation into global PDF framework.
Electroweak effects - PDFs

- **EW corrections** to PDFs also important at high energies.
- Recent numerical study for unpolarized proton with generalized DGLAP evolution (LO) in the approximation of exact $SU(3) \times SU(2) \times U(1)$ symmetry.

**full SM evolution vs. QCD only for quarks and gluon**

- **Left handed**
- **Right handed**

---

C. W. Bauer et al., JHEP 1708 (2017) 036
Interesting comparisons on **EW boson PDF** luminosities at 14 and 27 TeV proton proton colliders are also available.  

Chen et al., arXiv:1611.00788
PDFs with resummation
PDFs with resummation

• Higher **HL-LHC** integrated lumi will improve coverage at large $x$.
• **Threshold resummation** can be used there to improve perturbative expansion, construct approx. higher results…

• At **HE-LHC**, improved coverage of the small $x$ region enhances the need for high-energy resummation.
• Small $x$ (**high-energy/BFKL**) resummation might also be relevant for precision LHC phenomenology at Run II+III and in HL phase.
• PDFs and matrix elements should be treated consistently.

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**Kinematics of HE-LHC at $\sqrt{s}=27$ TeV**

- **HE-LHC 27 TeV**
- **LHC 13 TeV**

Plot by J. Rojo, Oct 2017
PDFs with large-$x$ resummation

- Close to a production threshold ($x = 1$) terms of the form $\alpha_s^k \ln^r (1 - x)$ are enhanced and need to be resummed to all orders.
- Working in Mellin space, a resummed partonic cross section can be constructed that includes terms of the type $\alpha_s^k \ln^p N$ to all orders.

$$
\sigma(N, Q^2) = \int_0^1 dx \, x^{N-2} \sigma(x, Q^2) = \sum_{a,b} \mathcal{L}_{ab}(N, Q^2) \hat{\sigma}_{ab}(N, Q^2, \alpha_s)
$$

$$
\hat{\sigma}^{(\text{res})}_{ab}(N, Q^2, \alpha_s) = \sigma^{(\text{born})}_{ab}(N, Q^2, \alpha_s) \, C^{(\text{res})}_{ab}(N, \alpha_s).
$$

$$
C^{(N\text{-soft})}(N, \alpha_s) = g_0(\alpha_s) \exp \mathcal{S}(\ln N, \alpha_s),
$$

$$
\mathcal{S}(\ln N, \alpha_s) = \left[ \frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots \right]
$$

- Threshold-resumed partonic cross-sections available for many processes: **DIS, DY, jets, top quark pair**…
• A variant of the **NNPDF3.0** fit with (N)NLL threshold resummation was produced using the resummed DIS+DY partonic cross sections from the TROLL code

\[
\sigma_{\text{N}j\text{LO} + \text{N}^k\text{LL}} = \sigma_{\text{N}j\text{LO}} + \sigma_{\text{LO}} \times \Delta_j K_{\text{N}^k\text{LL}}
\]

M. Bonvini et al., JHEP 1509 (2015) 191

• Main effect is **suppression** of the large- \(x\) PDFs to compensate increase of resummed partonic cross-sections
Impact

- Large $x$ resummed PDFs can be combined with threshold-resummed cross sections for e.g. high-mass SUSY production.
- Shift in central values comparable with PDF errors: important for limit settings (and even more in the case of discovery!)

PDFs with small-$x$ resummation

• Perturbative fixed-order QCD calculations extremely successful in describing a wealth of data from $pp$ and $ep$ collisions.

• However, theoretical reasons to go beyond DGLAP: $\ln(1/x)$ become dominant at small $x$ and need to be resummed to all orders.

• Small-$x$ resummation can be matched to DGLAP and included into a PDF fit

\begin{align*}
DGLAP \\
\text{Evolution in } Q^2 \\
\mu^2 \frac{\partial}{\partial \mu^2} f_i(x, \mu^2) = \int_x^1 \frac{dz}{z} P_{ij} \left( \frac{x}{z}, \alpha_s(\mu^2) \right) f_j(z, \mu^2),
\end{align*}

\begin{align*}
\text{BFKL} \\
\text{Evolution in } x \\
-x \frac{d}{dx} f_+(x, \mu^2) = \int_0^\infty \frac{dv^2}{v^2} K \left( \frac{\mu^2}{v^2}, \alpha_s \right) f_+(x, v^2)
\end{align*}

• The $N^k_{\text{LO}}$ fixed-order DGLAP splitting functions complemented with $N^{h\text{LL}}_x$ contributions from BFKL:

\[ P_{ij}^{N^k_{\text{LO}}+N^{h\text{LL}}_x}(x) = P_{ij}^{N^k_{\text{LO}}}(x) + \Delta_k P_{ij}^{N^{h\text{LL}}_x}(x), \]

ABF, CCSS, TW and others, 94-08
• **NNPDF3.1sx**: Variant of **NNPDF3.1** global fits using **NLO+NLLx** and **NNLO+NLLx** theory

• **NNLO+NLLx** theory stabilises small-$x$ gluon w.r.t. perturbative order

R.D. Ball, arXiv:1710.05935
To assess impact of small-$x$ resummation on the HERA data, compute $\chi^2$ removing data points in the region where resummation effects are expected.

- Using NNLO+NLLx theory, the $\chi^2$ trend flattens.
- Excellent fit to HERA inclusive and charm data achieved in the entire $(x, Q^2)$ region.
Implications for the HE-LHC

The use of PDFs with small (large) $x$ resummation could improve predictions for low and medium (high) invariant masses at the HE-LHC/HL-LHC.
PDFs at a high energy ep/eA collider
PDFs at a high energy eh collider

- **LHeC**: $e$ beam with $E = 60$ GeV colliding with the 7 TeV LHC $p$ in concurrent $ep$ and $pp$ operation mode during HL-LHC.

- **Expands kinematic coverage of HERA** by more than an order of magnitude at small $x$ and high $Q^2$.

- **Higher Luminosity and $Q^2$** - increased coverage at high $x$ for clean PDF determination.

- Expanded coverage crucial for **nuclear PDFs** (no ‘nuclear HERA’ available).
• LHeC provides unprecedented precision for parton structure determination of protons/nuclei compared to LHC alone.

• Improvements both at small and high $x$ (increased coverage/statistics) and for quark flavour separation (direct measurements of $s, c, b$ structure functions), with less assumptions.
• Extended coverage probes novel dynamical regimes of QCD: BFKL dynamics, saturation, non-linear effects.

• Studies with pseudo-data can be used to quantify sensitivity to non-standard QCD effects.

• Measurements of the longitudinal structure function $F_L$ represent smoking gun for such studies.

R.D. Ball, arXiv:1710.05935
• Many other important QCD/EW measurements possible: strong coupling to 0.1% (exp), 0.5% (theor), electroweak couplings….  
• Possibility for $N^3LO$ PDF extraction (given splitting functions).

• LHeC PDFs available in LHAPDF format.

• Work actively ongoing on studies/projections. Expect updates next year for input to European Strategy.
Conclusions and Outlook

• The HL-LHC/HE-LHC projects raises many new questions, challenges and opportunities for PDF fitters:
  ‣ High precision PDFs, dealing with high precision data.
  ‣ Include EW corrections systematically.
  ‣ Complete treatment of theoretical uncertainties.
  ‣ The NNLO standard - tools being developed for this. Benchmarking exercises worthwhile.
  ‣ Extended high/low $x$ reach: important of resummation?
  ‣ Increasingly precise/differential LHC data great opportunity but also challenge for PDF groups to include in fit.

• The LHeC proposal:
  ‣ Greatly extend HERA reach in $x$ and $Q^2$. Huge potential for clean and precise extraction of PDFs.
  ‣ Many other possibilities: probes of low $x$ physics, nuclear PDFs, many precicision EW/QCD measurements.
  ‣ Ongoing and active project.
Backup
PDF luminosities

NNLO, LHC 13 TeV

NNLO, LHC 27 TeV

\( \frac{L_{qq}}{L_{qg}} \) [ref] NNLO, LHC 27 TeV

\( \frac{L_{qg}}{L_{qq}} \) [ref] NNLO, LHC 13 TeV