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Higgs couplings with or without SMEFT

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Workshop on the physics of HL-LHC, and perspectives at HE-LHC

Collider no-lose

- $Sp\bar{p}S$ Find W and Z
- LEP Weakly vs strongly coupled EW breaking
- Tevatron Find top
- LHC Find Higgs
- HE LHC ?

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No-lose for HE-LHC

- Currently, there is no solid indication of a new physical scale within the reach of the next high-energy collider

- I will argue however the HE LHC is sure to settle one outstanding question about electroweak symmetry

- Namely, whether electroweak symmetry is realized linearly or non-linearly

Linear vs non-linear

Two mathematical formulations for theories with SM spectrum

Linear

$SU(3)_c \times SU(2)_L \times U(1)_Y$

$$H \to LH, \qquad L \in SU(2)_L$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}$$

Non-linear

$SU(3)_c \times U(1)_{em}$

$$U \to g_L U g_Y^\dagger, \qquad h \to h$$

$$U = \exp(2i\varphi^a T^a / v)$$

Practical difference consists in correlations between interactions terms with different number of Higgs bosons h predicted by linear formulation

SM EFT Approach to BSM

Basic assumptions

- Much as in SM, relativistic QFT with linearly realized SU(3)xSU(2)xU(1) local symmetry spontaneously broken by VEV of Higgs doublet field

$$H \to LH, \qquad L \in SU(2)_L$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \dots \\ v + h(x) + \dots \end{pmatrix}$$

- SM EFT Lagrangian expanded in inverse powers of Λ, equivalently in operator dimension D

$$v \ll \Lambda \ll \Lambda_L$$

$$\mathcal{L}_{\rm SM\ EFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda_L}\mathcal{L}^{D=5} + \frac{1}{\Lambda^2}\mathcal{L}^{D=6} + \frac{1}{\Lambda_L^3}\mathcal{L}^{D=7} + \frac{1}{\Lambda^4}\mathcal{L}^{D=8} + \dots$$

Lepton number or B-L violating, hence too small to probed at present and near-future colliders

Generated by integrating out heavy particles with mass scale Λ
In large class of BSM models that conserve B-L, D=6 operators capture leading effects of new physics on collider observables at E << Λ

Subleading wrt D=6 if Λ high enough

Buchmuller,Wyler (1986)

HEFT

for review see e.g.
LHCHXSWG 1610.07922

Introduce triplet of Goldstone field φ via unitary matrix U:

$$U = \exp(2i\varphi^a T^a / v)$$

Transformation of U under SU(2)LxU(1) implies
electroweak symmetry acts non-linearly on φ:

$$U \to g_L U g_Y^\dagger, \qquad h \to h$$

Higgs boson is perfect singlet
under electroweak symmetry!

Lagrangian organized in derivative expansion:

$$\mathcal{L}_{\rm HEFT} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

Arbitrary polynomial of h
allowed to multiply
each term in $\mathcal{L}_{\rm HEFT}$!

$$\mathcal{L}_2 = -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\psi=q_L,l_L,u_R,d_R,e_R} \bar\psi i \not{D}\psi$$
$$+\frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\,(1+F_U(h)) + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$
$$-v\left[\bar q_L\left(Y_u + \sum_{n=1}^{\infty} Y_u^{(n)}\left(\frac{h}{v}\right)^n\right)UP_+q_R + \bar q_L\left(Y_d + \sum_{n=1}^{\infty} Y_d^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-q_R\right.$$
$$\left.+\bar l_L\left(Y_e + \sum_{n=1}^{\infty} Y_e^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-l_R + \text{h.c.}\right]$$

$$\mathcal{L}_4 = a_1 g' g\langle U T_3 B_{\mu\nu} U^\dagger W^{\mu\nu}\rangle + i a_2 g'\langle U T_3 B_{\mu\nu}U^\dagger[V^\mu, V^\nu]\rangle - i a_3 g\langle W_{\mu\nu}[V^\mu, V^\nu]\rangle$$
$$+a_4\langle V_\mu V_\nu\rangle\langle V^\mu V^\nu\rangle + a_5\langle V_\mu V^\mu\rangle\langle V_\nu V^\nu\rangle + \frac{e^2}{16\pi^2}c_{\gamma\gamma}\frac{h}{v}F_{\mu\nu}F^{\mu\nu} + \frac{g^{hh}}{v^4}(\partial_\mu h\partial^\mu h)^2$$
$$+\frac{d^{hh}}{v^2}(\partial_\mu h \partial^\mu h)\langle D_\nu U^\dagger D^\nu U\rangle + \frac{e^{hh}}{v^2}(\partial_\mu h\partial^\nu h)\langle D^\mu U^\dagger D_\nu U\rangle + \ldots$$

$$D_\mu U = \partial_\mu U + i g W_\mu U - i g' B_\mu U T_3$$

$$F_U(h) = \sum_{n=1}^{\infty} f_{U,n}\left(\frac{h}{v}\right)^n, \qquad V(h) = v^4\sum_{n=2}^{\infty} f_{V,n}\left(\frac{h}{v}\right)^n$$

Example: Higgs self-interactions

$$\mathcal{L}_{\rm SM} \supset m^2|H|^2 - \lambda|H|^4$$

$$\to -\frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4$$

SMEFT

HEFT

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{c_6 (H^\dagger H)^3}{M^2}$$

$$\mathcal{L} \supset -\frac{m_h^2}{2v}(1+\delta\lambda_3)\,h^3 - \frac{m_h^2}{8v^2}(1+\delta\lambda_4)\,h^4 - \frac{\lambda_5}{v}h^5 - \frac{\lambda_6}{v^2}h^6$$

$$\delta\lambda_3 = \frac{2v^4 c_6}{m_h^2 M^2} \quad \delta\lambda_4 = \frac{12 v^4 c_6}{m_h^2 M^2} \quad \lambda_5 = \frac{3v^2 c_6}{4M^2} \quad \lambda_6 = \frac{v^2 c_6}{8M^2}$$

Correlations

$$\mathcal{L} \supset -c_3 \frac{m_h^2}{2v}h^3 - c_4\frac{m_h^2}{8v^2}h^4 - c_5\frac{1}{v}h^5 - c_6\frac{1}{v^2}h^6 + \ldots$$

No correlations

Difference of fundamental scale

- The correlations are hardly a smoking gun to distinguish two formalism as multi-Higgs interactions terms in Lagrangian can rarely be measured in practice. Moreover, particular HEFT couplings could be correlated by accident

- However, there is a difference in the underlying fundamental scale which, for HEFT, should lead to observable consequences when colliders probe energies E >> 4 π

SMEFT

Λ

Can be smoothly taken to "infinity"

HEFT

$4\pi v$

Has fixed numerical value of few TeV

SMEFT vs HEFT

AA,Rattazzi, unpublished

HEFT = SMEFT + non-analytic terms

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2} f_h(h) \partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4} f_1(h) {\rm Tr}[\partial_\mu U^\dagger \partial_\mu U] + v^2 f_2(h) \left({\rm Tr}[U^\dagger \partial_\mu U \sigma_3] \right)^2 + \ldots$$

$$U = \exp(2i\varphi^a T^a / v)$$

One can always re-express non-linear Lagrangian in linear language by replacing:

$$U \to \frac{\left(\tilde{H}, H\right)}{\sqrt{H^\dagger H}}$$

$$h \to \sqrt{2 H^\dagger H} - v$$

After this substitution, Lagrangian has linearly realized electroweak symmetry but contains terms that are non-analytic at H=0

However, a non-analytic term is in fact an infinite series of higher-order interactions, in this suppressed merely by low scale v = 246 GeV

Example: brutal triple Higgs deformation

Given Lagrangian for Higgs boson h, one can always uplift it to manifestly SU(3)xSU(2)xU(1) invariant form replacing

$$h \to \sqrt{2H^\dagger H} - v$$

$$\frac{m_h^2}{2}h^2 + \frac{m_h^2}{2v}\left(1+\delta\lambda_3\right)h^3 + \frac{m_h^2}{8v^2}h^4$$

$$\Lambda_3 = \frac{m_h^2}{2v}\delta\lambda_3$$

$$\to m^2 H^\dagger H + \lambda(H^\dagger H)^2 + 3\Lambda_3 v^2 (2H^\dagger H)^{1/2} + \Lambda_3 (2H^\dagger H)^{3/2}$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}$$

$$V \supset \frac{3vm_h^2}{2}\delta\lambda_3((h+v)^2+G^2)^{1/2} + \frac{m_h^2}{2v}\delta\lambda_3((h+v)^2+G^2)^{3/2}.$$

$$G^2 = 2G_+G_- + G_z^2.$$

Non-analytic terms lead to infinite series of n-point Goldstone and Higgs boson interactions

$$\mathcal{L} \supset \mathcal{L}_{G^2} + \mathcal{L}_{G^4} + \mathcal{L}_{G^6} + \dots$$

$$\mathcal{L}_{G^2} = -m_h^2\left(2G_+G_- + G_z^2\right)\left[\frac{h}{2v} + \frac{1+3\delta\lambda_3}{4}\frac{h^2}{v^2} - \frac{3\delta\lambda_3}{4}\frac{h^3}{v^3} + \dots\right]$$

$$\mathcal{L}_{G^4} = -m_h^2\left(2G_+G_- + G_z^2\right)^2\left(\frac{1}{8v^2} + \frac{3\delta\lambda_3}{8}\frac{h}{v^3} - \frac{15\delta\lambda_3}{16}\frac{h^2}{v^4} + \dots\right)$$

$$\dots$$

Consequence: in deformed SM with $\delta\lambda 3 \neq 0$,

VV→n x h, VV→ VV + n x h, …., lose unitarity near scale $4\pi v$

Unitarity primer

S matrix unitarity $S^\dagger S = 1$

symmetry factor for n-body final state

implies relation between forward scattering amplitude, and elastic and inelastic production cross sections

$$2\mathrm{Im}\mathcal{M}(p_1,p_2 \to p_1,p_2) = S_2 \int d\Pi_2 |\mathcal{M}_{\mathrm{el.}}(p_1,p_2 \to k_1,k_2)|^2 + \sum S_n \int d\Pi_n |\mathcal{M}_{\mathrm{inel.}}(p_1,p_2 \to k_1 \ldots k_n)|^2$$

Initial and final 2-body state can be projected to partial waves

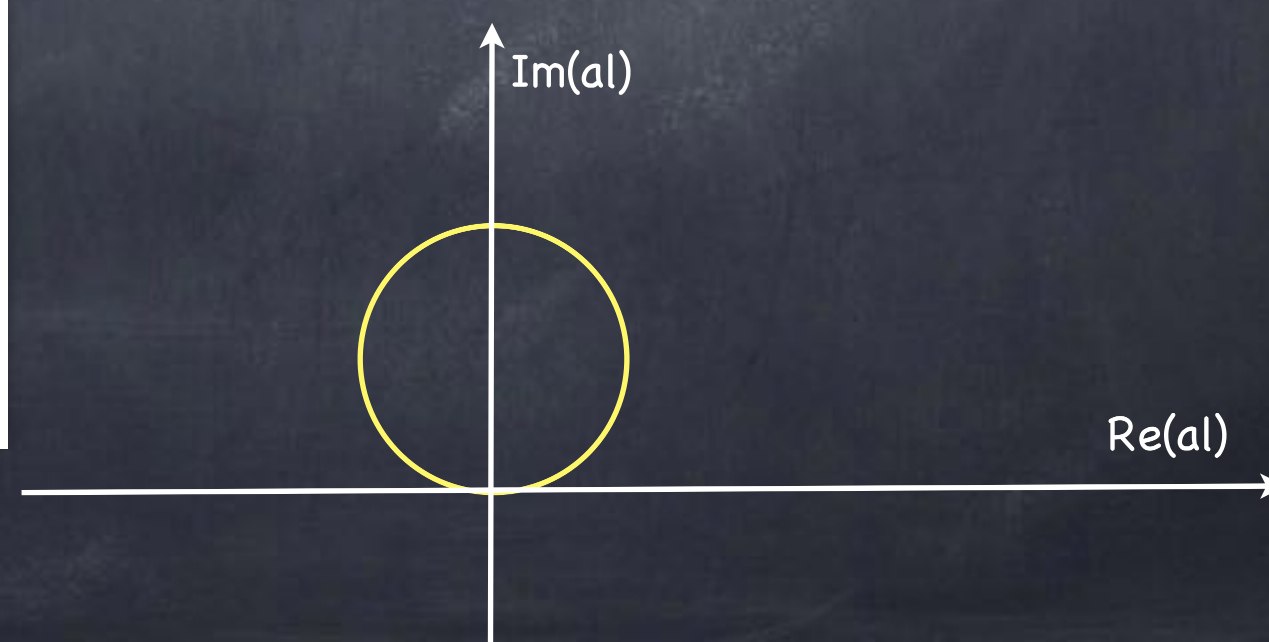
$$2\mathrm{Im}\, a_l = |a_l|^2 + \sum_{n\in\mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(E,0,l,m \to \{n\})|^2.$$

$$(\mathrm{Re}\, a_l)^2 + (\mathrm{Im}\, a_l - 1)^2 = R_l^2, \qquad R_l = \sqrt{1 - \sum_{n\in\mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(E,0,l,m \to \{n\})|^2}$$

This implies perturbative unitarity constraints on elastic and inelastic amplitudes

$$|\mathrm{Re}\, a_l| \le 1$$

$$\sum_{n\in\mathrm{inel.}} S_n \int d\Pi_n |\mathcal{M}(E,0,l,m \to \{n\})|^2 :$$

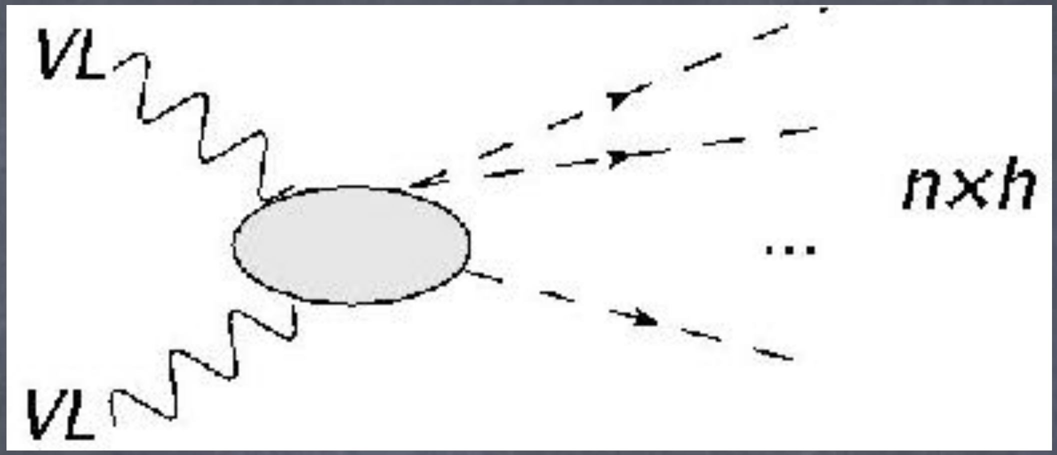


Multi-Higgs with HEFT-deformed Higgs cubic

Higgs potential with Goldstones

$$V \supset \frac{3vm_h^2}{2}\delta\lambda_3((h+v)^2+G^2)^{1/2}+\frac{m_h^2}{2v}\delta\lambda_3((h+v)^2+G^2)^{3/2}.$$

$$G^2 = 2G_+G_- + G_z^2.$$



Expanding to leading order in G^2

$$V \supset \delta\lambda_3 \frac{3m_h^2 v}{2}\frac{G^2}{h+v} = \delta\lambda_3 \frac{3m_h^2}{2} G^2 \sum_{n=0}^{\infty}\left(\frac{-h}{v}\right)^n .$$

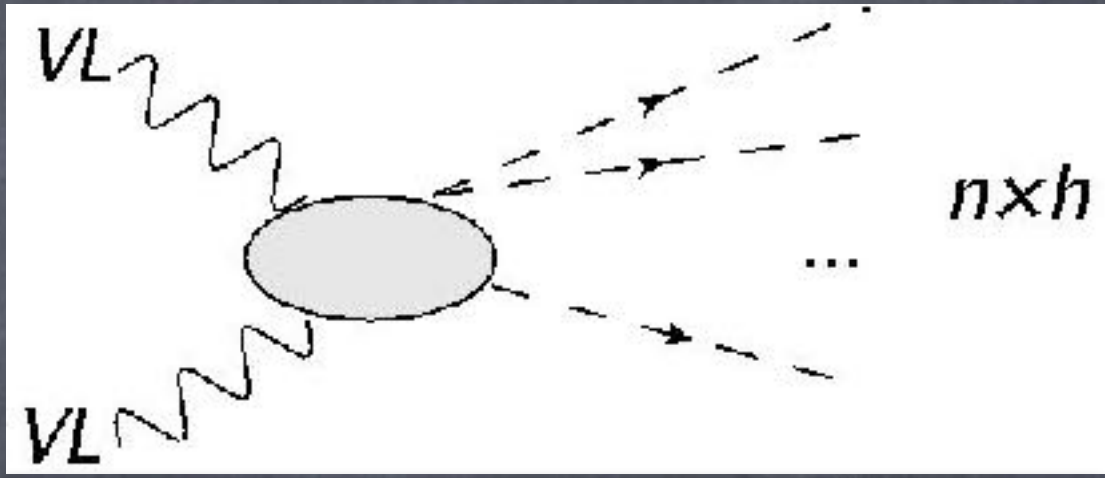
s-wave isospin-0 amplitude

$$|\mathcal{M}([GG]_{I=0}^{l=0} \to h^n)| \equiv |\mathcal{M}_n| = \frac{1}{4\sqrt{\pi}}\delta\lambda_3 \frac{3\sqrt{3}n!m_h^2}{v^n}.$$

Multi-Higgs production amplitudes are only suppressed by scale v, leading to unitarity loss at some scale above v

Multi-Higgs with HEFT-deformed Higgs cubic

Perturbative unitarity bound on non-elastic amplitude



$$\sum_{n=2}^{\infty} S_n \int d\Pi_n |\mathcal{M}_n|^2 \bigg|_{\sqrt{s}=\Lambda_*} = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\Lambda_*) |\mathcal{M}_n|^2 \sim \pi^2,$$

$$|\mathcal{M}([GG]_{I=0}^{l=0} \to h^n)| \equiv |\mathcal{M}_n| = \frac{1}{4\sqrt{\pi}} \delta\lambda_3 \frac{3\sqrt{3} n! m_h^2}{v^n}.$$

Sum over n Higgs bosons exponentiates

$$\begin{aligned} \pi^2 &\sim \frac{27\delta\lambda_3^2 m_h^4}{16\pi} \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\Lambda_*^{2n-4}}{2(n-1)!(n-2)!(4\pi)^{2n-3}} \frac{(n!)^2}{v^{2n}} \\ &= \frac{27\delta\lambda_3^2 m_h^4}{128\pi^2 v^4} \sum_{n=2}^{\infty} \frac{n\Lambda_*^{2n-4}}{(n-2)!(4\pi v)^{2n-4}} = \frac{27\delta\lambda_3^2 m_h^4}{128\pi^2 v^4} \left(2 + \frac{\Lambda_*^2}{(4\pi v)^2}\right) \exp\left(\frac{\Lambda_*^2}{(4\pi v)^2}\right) \end{aligned}$$

$$\frac{\Lambda_*}{4\pi v} \sim 2 \log^{1/2} \left(\frac{4\pi v}{m_h |\delta\lambda_3|^{1/2}} \right)$$

For any observable cubic Higgs deformations, new physics must enter at scale <=few*4π v to regulate multi-Higgs amplitudes!

Collider no-lose

- $Sp\bar{p}S$ Find W and Z
- LEP Weakly vs strongly coupled EW breaking
- Tevatron Find top
- LHC Find Higgs
- HE LHC Linear vs non-linear EW symmetry