

# Heavy flavour measurements: a theoretical perspective

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Workshop on the physics of HL-LHC  
and perspectives at HE-LHC



# Heavy Flavour in the QGP: the conceptual setup

- Description of **soft observables** based on **hydrodynamics**, assuming to deal with **a system close to local thermal equilibrium** (no matter why);
- Description of **jet-quenching** based on **energy-degradation** of **external probes** (high- $p_T$  partons);
- Description of **heavy-flavour** observables requires to employ/develop a setup (**transport theory**) allowing to deal with more general situations and in particular to describe *how particles would (asymptotically) approach equilibrium*. Initial (off-equilibrium!)  $Q\bar{Q}$  production occurs on a very short time-scale  $\tau_{Q\bar{Q}} \sim 1/2M_Q \lesssim 0.1 \text{ fm}/c \ll \tau_{\text{QGP}}$

NB At high- $p_T$  the interest in heavy flavor is no longer related to thermalization, but to the study of the **mass** and **color charge dependence** of **jet-quenching** (last part of this talk)

# Heavy quarks as probes of the QGP

A realistic study requires developing *a multi-step setup*:

- **Initial production**: pQCD + possible nuclear effects (nPDFs,  $k_T$ -broadening) → QCD event generators, validated on p-p data;
- Description of the **background medium** (initial conditions,  $T(x)$ ,  $u^\mu(x)$ ) → hydrodynamics, validated on soft hadrons;
- **HQ-medium interaction** → transport coefficients, in principle derived from QCD, but still far from a definite answer for the relevant experimental conditions;
- **Dynamics in the medium** → transport calculations, in principle rigorous under certain kinematic conditions, but require transport coefficients;
- **Hadronization**: not well under control (fragmentation in the vacuum? recombination with thermal partons? validated on what?)
  - An item of interest in itself (*change of hadrochemistry* in A-A collisions? And also in p-p?)
  - However, a source of systematic uncertainty for studies of *parton-medium interaction*;
- **Hadronic rescattering** (e.g.  $D\pi \rightarrow D\pi$ ), from effective Lagrangians, but no experimental data on relevant cross-sections

# Transport theory: general setup

# Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution  $f_Q(t, \mathbf{x}, \mathbf{p})^1$ :

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}}$$

Neglecting  $\mathbf{x}$ -dependence and mean fields:  $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k} \left[ \underbrace{w(\mathbf{p} + \mathbf{q}, \mathbf{q}) f_Q(\mathbf{p} + \mathbf{q})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{q}) f_Q(\mathbf{p})}_{\text{loss term}} \right]$$

$w(\mathbf{p}, \mathbf{q})$ : HQ transition rate  $\mathbf{p} \rightarrow \mathbf{p} - \mathbf{q}$

<sup>1</sup>Approach adopted by [Catania](#), [Nantes](#), [Frankfurt](#), [LBL](#), [groups](#)

# From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*<sup>2</sup> (Landau)

$$C[f_Q] \approx \int d\mathbf{q} \left[ q^i \frac{\partial}{\partial p^i} + \frac{1}{2} q^i q^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{q}) f_Q(t, \mathbf{p})]$$

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the Boltzmann equation reduces to the Fokker-Planck equation

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{q} q^i w(\mathbf{p}, \mathbf{q}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p) p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{q} q^i q^j w(\mathbf{p}, \mathbf{q}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = (\delta^{ij} - \hat{p}^i \hat{p}^j) B_0(p) + \hat{p}^i \hat{p}^j B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*,  
directly derived from the scattering matrix

<sup>2</sup>In a relativistic gauge plasma  $q \sim m_D \sim gT$



# Approach to equilibrium in the FP equation

The FP equation can be viewed as a **continuity equation** for the phase-space distribution of the kind  $\partial_t \rho(t, \vec{p}) + \vec{\nabla}_p \cdot \vec{J}(t, \vec{p}) = 0$

$$\frac{\partial}{\partial t} \underbrace{f_Q(t, \mathbf{p})}_{\equiv \rho(t, \vec{p})} = \frac{\partial}{\partial p^i} \underbrace{\left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}}_{\equiv -J^i(t, \vec{p})}$$

admitting a **steady solution**  $f_{\text{eq}}(p) \equiv e^{-E_p/T}$  when the current vanishes:

$$A^i(\vec{p}) f_{\text{eq}}(p) = - \frac{\partial B^{ij}(\vec{p})}{\partial p^j} f_{\text{eq}}(p) - B^{ij}(\mathbf{p}) \frac{\partial f_{\text{eq}}(p)}{\partial p^j}.$$

One gets then a **constraint linking the three transport coefficients**, which are not independent, but obey the **Einstein fluctuation-dissipation relation**

$$A(p) = \frac{B_1(p)}{TE_p} - \left[ \frac{1}{p} \frac{\partial B_1(p)}{\partial p} + \frac{d-1}{p^2} (B_1(p) - B_0(p)) \right],$$

quite involved due to the **momentum dependence** of the transport coefficients (**measured HQ's are relativistic particles!**)

# The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark arising from the pQCD Monte Carlo simulation of the initial  $Q\bar{Q}$  production: the **Langevin equation**

$$\frac{\Delta p^i}{\Delta t} = - \underbrace{\eta_D(\mathbf{p}) p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}},$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t) \rangle = 0 \quad \langle \xi^i(\mathbf{p}_t) \xi^j(\mathbf{p}_{t'}) \rangle = b^{ij}(\mathbf{p}) \frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) \equiv \kappa_L(\mathbf{p}) \hat{p}^i \hat{p}^j + \kappa_T(\mathbf{p}) (\delta^{ij} - \hat{p}^i \hat{p}^j)$$

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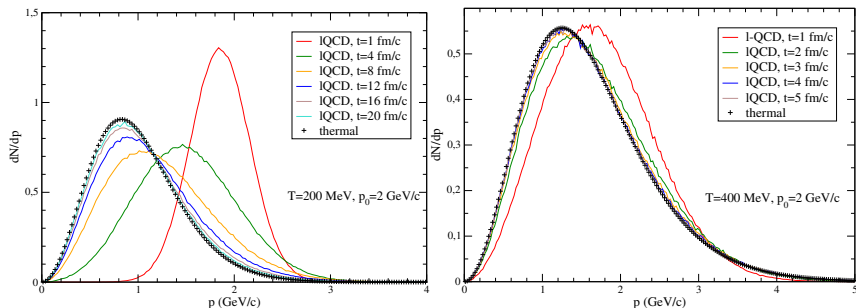
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**Transport coefficients** related to the FP ones:

- **Momentum diffusion**:  $\kappa_T(p) = 2B_0(p)$  and  $\kappa_L(p) = 2B_1(p)$
- **Friction** term, in the *Ito pre-point discretization scheme*,

$$\eta_D^{\text{Ito}}(p) = A(p) = \frac{B_1(p)}{TE_p} - \left[ \frac{1}{p} \frac{\partial B_1(p)}{\partial p} + \frac{d-1}{p^2} (B_1(p) - B_0(p)) \right]$$

# A first check: thermalization in a static medium



(Test with a sample of  $c$  quarks with  $p_0=2 \text{ GeV}/c$ ).

For  $t \gg 1/\eta_D$  one approaches a relativistic Maxwell-Jüttner distribution

$$f_{\text{MJ}}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T K_2(M/T)}, \quad \text{with} \quad \int d^3p f_{\text{MJ}}(p) = 1$$

The larger  $\kappa$  ( $\kappa \sim T^3$ ), the faster the approach to thermalization.

# Transport coefficients

# Transport coefficients: non-perturbative definition

One consider the **non-relativistic limit** of the Langevin equation for a HQ

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t) \xi^j(t') \rangle = \delta^{ij} \delta(t - t') \kappa$$

in which the strength of the noise is given by a single number, the **momentum-diffusion coefficient**  $\kappa$ . Hence, in the  $p \rightarrow 0$  limit:

$$\kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t) \xi^i(0) \rangle_{\text{HQ}} \approx \frac{1}{3} \int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t) F^i(0) \rangle_{\text{HQ}}}_{\equiv D^>(t)}$$

For a static ( $M = \infty$ ) HQ the **force** is due to the **color-electric field**:

$$\mathbf{F}(t) = g \int d\mathbf{x} Q^\dagger(t, \mathbf{x}) t^a Q(t, \mathbf{x}) \mathbf{E}^a(t, \mathbf{x})$$

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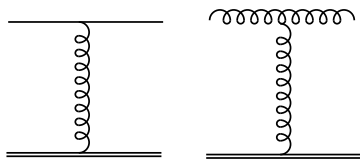
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The above non-perturbative definition, referring to the  $M \rightarrow \infty$  limit, is the starting point for a thermal-field-theory evaluation based on

- **weak-coupling** calculations (up to NLO);
- gauge-gravity duality ( $\mathcal{N} = 4$  SYM)
- **lattice-QCD** simulations

# HQ momentum diffusion: weak-coupling calculation



In the  $M \rightarrow \infty$  limit the HQ exchange momentum  $q^\mu = (0, \vec{q})$ , with  $q \sim gT$ , with the medium partons. The exchanged soft gluon is dressed by the Debye mass  $m_D \sim gT$ , which screens IR divergences

$$\kappa^{\text{LO}} \equiv \frac{g^4 C_F}{12\pi^3} \int_0^\infty k^2 dk \int_0^{2k} \frac{q^3 dq}{(q^2 + m_D^2)^2} \\ \times \left[ N_c n_B(k)(1+n_B(k)) \left( 2 - \frac{q^2}{k^2} + \frac{q^4}{4k^2} \right) + N_f n_F(k)(1-n_F(k)) \left( 2 - \frac{q^2}{2k^2} \right) \right]$$

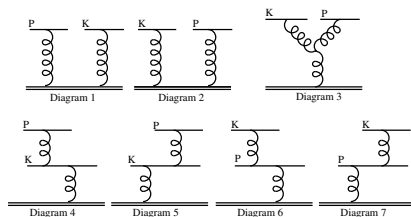
Under the assumption that  $q \ll k \sim T$  one can “expand” the results in a weak-coupling series

$$\kappa = \frac{C_F g^4 T^3}{18\pi} \left( \left[ N_c + \frac{N_f}{2} \right] \left[ \ln \frac{2T}{m_D} + \xi \right] + \frac{N_f \ln 2}{2} + \mathcal{O}(g) \right)$$

with the structure  $\kappa \sim g^4 T^3 (\# \ln(1/g) + \# + \mathcal{O}(g))$ , clearly meaningful only as long as  $g \ll 1$ .



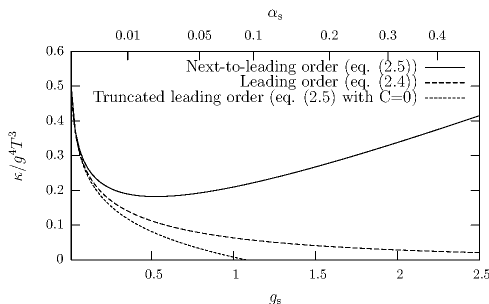
# HQ momentum diffusion: weak-coupling calculation



The weak-coupling expansion for  $\kappa$  receives  $\mathcal{O}(g)$  corrections of various origin (S. Caron-Huot and G.D. Moore, JHEP 0802 (2008) 081):

- one part is contained in the **unexpanded tree-level result**, arising from the region  $k \sim gT$  in which  $n_B(k) \sim T/k \sim 1/g$  and the approximation  $q \ll k$  no longer holds;
- another part arises from a **NLO correction to the screened gluon propagator**, which can be easily inserted in the tree-level result;
- a last part comes from **overlapping scatterings**. Having a total scattering rate  $\sim g^2 T$  and the duration of a single scattering  $\sim 1/q \sim 1/gT$  entails that a **fraction  $\mathcal{O}(g)$  of scattering events** overlap with each other (see diagrams).

# HQ momentum diffusion: weak-coupling calculation



Collecting together the various terms one gets, for  $N_f = N_c = 3$ ,

$$\kappa = \frac{16\pi}{3} \alpha_s^2 T^3 \left( \ln \frac{1}{g} + 0.07428 + 1.9026g + \mathcal{O}(g^2) \right)$$

which shows that, for realistic values of the coupling  $\alpha_s \sim 0.3$ , NLO corrections to  $\kappa$  are positive and large: **what's the guidance provided by weak-coupling calculations** if NLO corrections are so large?

# HQ momentum diffusion from lattice-QCD ( $N_f = 0!$ )

The ( $p \rightarrow 0$ ) HQ momentum-diffusion coefficient

$$\kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t) \xi^i(0) \rangle_{\text{HQ}} = \frac{1}{3} \int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t) F^i(0) \rangle_{\text{HQ}}}_{\equiv D^>(t)}$$

is given by the  $\omega \rightarrow 0$  limit of the FT of the electric-field correlator  $D^>$ . In a thermal ensemble, from the periodicity of the bosonic fields, one has  $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega}) D^>(\omega)$ , so that

$$\kappa \equiv \lim_{\omega \rightarrow 0} \frac{D^>(\omega)}{3} = \lim_{\omega \rightarrow 0} \frac{1}{3} \frac{\sigma(\omega)}{1 - e^{-\beta\omega}} \underset{\omega \rightarrow 0}{\sim} \frac{1}{3} \frac{T}{\omega} \sigma(\omega)$$

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On the lattice one evaluates then the *euclidean electric-field correlator* ( $t = -i\tau$ )

$$D_E(\tau) = - \frac{\langle \text{Re Tr}[U(\beta, \tau) g E^i(\tau, \mathbf{0}) U(\tau, 0) g E^i(0, \mathbf{0})] \rangle}{\langle \text{Re Tr}[U(\beta, 0)] \rangle}$$

and from the latter one extract the *spectral density* according to

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

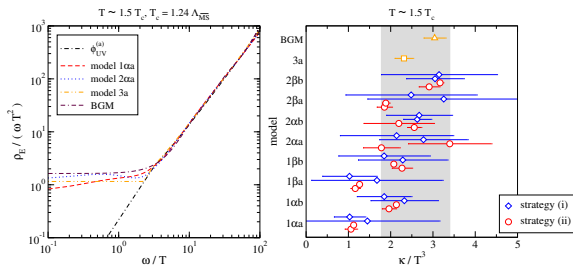
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The direct extraction of the spectral density from the euclidean correlator

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

is a ill-posed problem, since the latter is known for a limited set ( $\sim 20$ ) of points  $D_E(\tau_i)$ , and one wish to obtain a fine scan of the the spectral function  $\sigma(\omega_j)$ . A direct  $\chi^2$ -fit is not applicable. Possible strategies:

- Bayesian techniques (Maximum Entropy Method)
- Theory-guided ansatz for the behaviour of  $\sigma(\omega)$  to constrain its functional form (A. Francis *et al.*, PRD 92 (2015), 116003)



From the different ansatz on the functional form of  $\sigma(\omega)$  one gets a systematic uncertainty band:

$$\kappa/T^3 \approx 1.8 - 3.4$$

# Collisional broadening in the non-static case

In the case of experimental interest HQ's have a large but finite mass and most of the  $p_T$ -bins for which data are available refer to quite fast, or even relativistic, HF hadrons: **extending the estimates for the HQ transport coefficients to finite momentum** is mandatory to provide theoretical predictions relevant for the experiment.

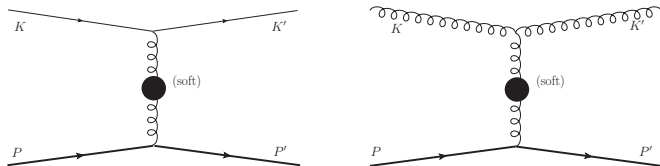
The effect of **2 → 2 collisions** can be included in an **“improved” tree-level** calculation ([W.M. Alberico et al., EPJC 73 \(2013\) 2481](#)) with an *Intermediate cutoff*  $|t|^* \sim m_D^2$ <sup>3</sup> separating the contributions of

- **hard collisions** ( $|t| > |t|^*$ ): kinetic pQCD calculation
- **soft collisions** ( $|t| < |t|^*$ ): Hard Thermal Loop approximation (*resummation of medium effects*)

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<sup>3</sup>Similar strategy for the evaluation of  $dE/dx$  in [S. Peigne and A. Peshier, Phys.Rev.D77:114017 \(2008\)](#)

# Transport coefficients $\kappa_{T/L}(p)$



When the exchanged 4-momentum is **soft** the **t-channel gluon feels the presence of the medium** and **requires resummation**.

The *blob* represents the **dressed gluon propagator**, which has longitudinal and transverse components:

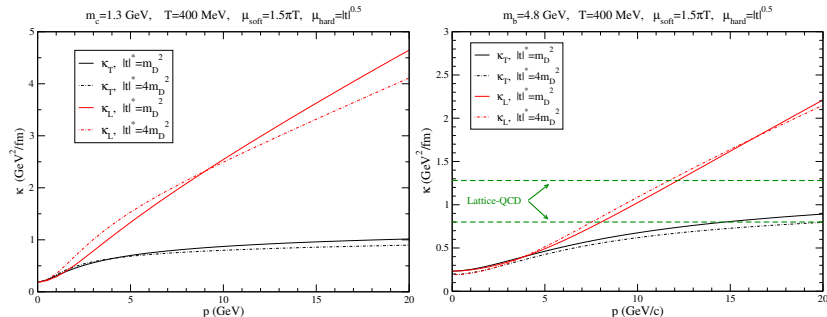
$$\Delta_L(z, q) = \frac{-1}{q^2 + \Pi_L(z, q)}, \quad \Delta_T(z, q) = \frac{-1}{z^2 - q^2 - \Pi_T(z, q)},$$

where *medium effects* are embedded in the **HTL gluon self-energy**.

NB In the corresponding **static calculation** only **longitudinal gluon** exchange, dressed simply by a **Debye mass**, without any energy and momentum dependence

# Transport coefficients: numerical results

Combining together the hard and soft contributions...



...the dependence on the intermediate cutoff  $|t|^*$  is very mild!

NB Notice, in the case of **charm**, the **strong momentum-dependence** of  $\kappa_L$ , much milder in the case of beauty, for which  $\kappa_L \approx \kappa_T$  up to 5 GeV

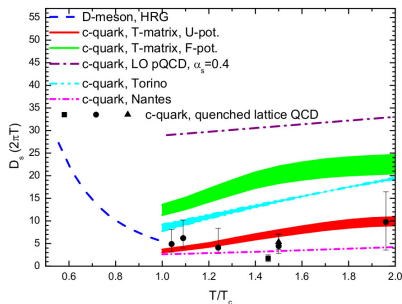


# Spatial diffusion coefficient $D_S$

In the *non-relativistic* limit an excess of HQ's initially placed at the origin will diffuse according to

$$\langle \vec{x}^2(t) \rangle \underset{t \rightarrow \infty}{\sim} 6D_S t \quad \text{with} \quad D_S = \frac{2T^2}{\kappa}.$$

For a **strongly interacting** system spatial diffusion is **very small!** Theory calculations for  $D_S$  have been collected (F. Prino and R. Rapp, JPG 43 (2016) 093002) and are often used by the experimentalists to summarize the difference among the various models (BUT **momentum dependence, not captured by  $D_S$ , is important!**)



- lattice-QCD

$$(2\pi T)D_S^{IQCD} \approx 3.7 - 7$$

- $\mathcal{N} = 4$  SYM:

$$(2\pi T)D_S^{SYM} = \frac{4}{\sqrt{g_{SYM}^2 N_c}} \approx 1.2$$

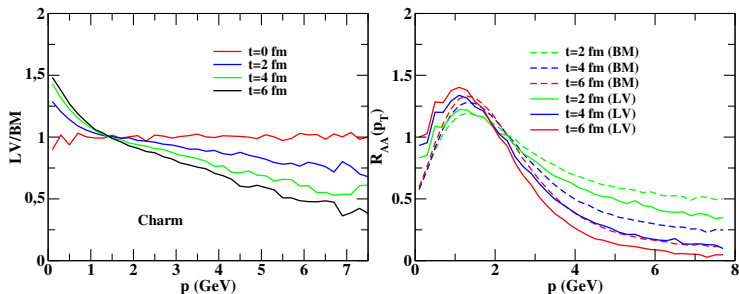
for  $N_c = 3$  and  $\alpha_{SYM} = \alpha_s = 0.3$ .

# The Langevin/FP approach: a critical perspective

Although the Langevin approach is a very convenient numerical tool and allows one to establish a link between observables and transport coefficients derived from QCD...

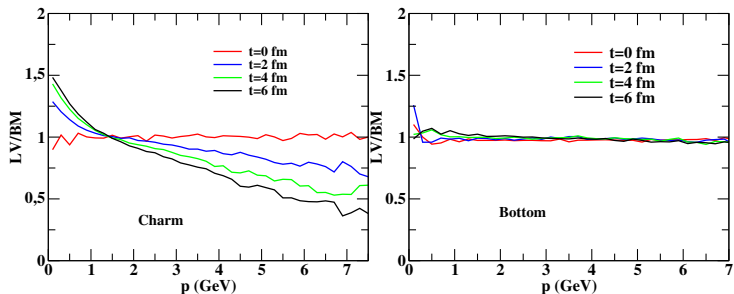
# The Langevin/FP approach: a critical perspective

Although the Langevin approach is a very **convenient numerical** tool and allows one to establish a **link between observables and transport coefficients derived from QCD...** it is nevertheless based on a *soft-scattering expansion* of the collision integral  $\mathcal{C}[f]$  truncated at second order (friction and diffusion terms), which may be *not always justified*, in particular for charm, possibly affecting the final  $R_{AA}$  (V. Greco *et al.*, [Phys.Rev. C90 \(2014\) 044901](#))



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For beauty on the other hand Langevin  $\equiv$  Boltzmann!

# From quarks to hadrons

In the presence of a medium, rather than fragmenting like in the vacuum (e.g.  $c \rightarrow cg \rightarrow c\bar{q}q$ ), HQ's can hadronize by **recombining with light thermal quarks** (or even *diquarks*) from the medium. This has been implemented in several ways in the literature:

- $2 \rightarrow 1$  (or  $3 \rightarrow 1$  for baryon production) coalescence of partons close in phase-space:  $Q + \bar{q} \rightarrow M$
- String formation:  $Q + \bar{q} \rightarrow \text{string} \rightarrow \text{hadrons}$
- Resonance formation/decay  $Q + \bar{q} \rightarrow M^* \rightarrow Q + \bar{q}$

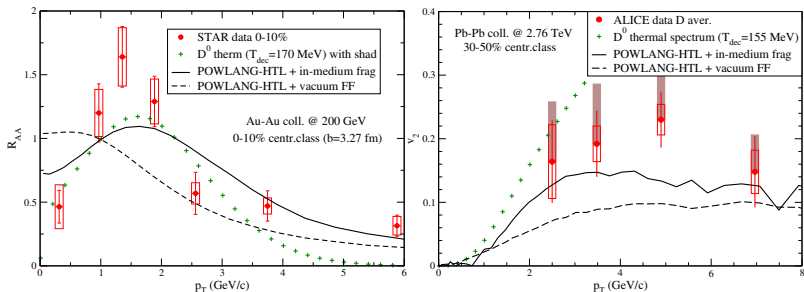
**In-medium hadronization** may affect the  $R_{AA}$  and  $v_2$  of final D-mesons due to the **collective (radial and elliptic) flow of light quarks**.

Furthermore, it can change the **HF hadrochemistry**, leading for instance to an enhanced production of strange particles ( $D_s$ ) and baryons ( $\Lambda_c$ ): **no need to excite heavy  $s\bar{s}$  or diquark-antidiquark pairs from the vacuum** as in elementary collisions, a lot of **thermal partons available nearby!**

Selected results will be shown in the following.

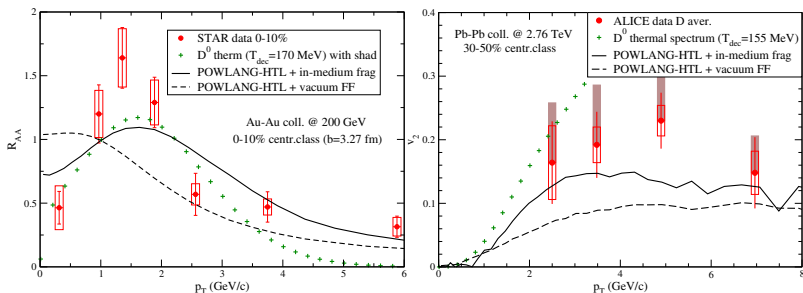
# From quarks to hadrons: *kinematic* effect on $R_{AA}$ and $v_2$

Experimental D-meson data show a **peak in the  $R_{AA}$**  and a **sizable  $v_2$**  one would like to interpret as a signal of *charm radial flow and thermalization* (green crosses: kinetic equilibrium, decoupling from FO hypersurface)



# From quarks to hadrons: *kinematic* effect on $R_{AA}$ and $v_2$

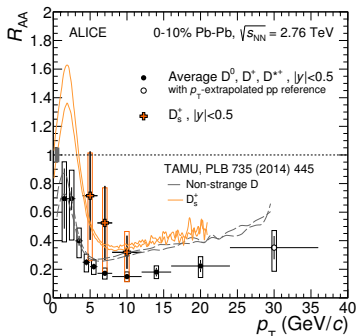
Experimental D-meson data show a **peak in the  $R_{AA}$**  and a **sizable  $v_2$**  one would like to interpret as a signal of *charm radial flow and thermalization* (green crosses: kinetic equilibrium, decoupling from FO hypersurface)



However, at least part of the effect might be due to the **radial and elliptic flow of the light partons** from the medium **picked-up at hadronization** (POWLANG results [A.B. et al., in EPJC 75 \(2015\) 3, 121](#)).

# From quarks to hadrons: HF hadrochemistry

The abundance of strange quarks in the plasma can lead e.g. to an enhanced production of  $D_s$  mesons wrt p-p collisions via  $c + \bar{s} \rightarrow D_s$



ALICE data for  $D$  and  $D_s$  mesons ([JHEP 1603 \(2016\) 082](#)) compared with TAMU-model predictions ([M- He et al., PLB 735 \(2014\) 445](#))

Langevin transport simulation in the QGP + hadronization modeled via

$$\left(\partial_t + \vec{v} \cdot \vec{\nabla}\right) F_M(t, \vec{x}, \vec{p}) = - \underbrace{(\Gamma/\gamma_p) F_M(t, \vec{x}, \vec{p})}_{M \rightarrow Q + \bar{q}} + \underbrace{\beta(t, \vec{x}, \vec{p})}_{Q + \bar{q} \rightarrow M}$$

$$\text{with } \sigma(s) = \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$$



## Some new predictions

In the following, some new predictions by the **POWLANG setup**<sup>4</sup> will be shown, mostly focused on

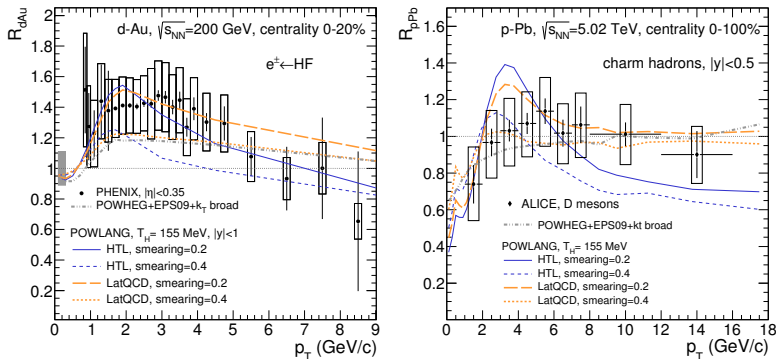
- HF observables in small systems
- Higher flow harmonic ( $v_2$ ,  $v_3$ )

and compared to recent **experimental data**

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<sup>4</sup>A.B. *et al.*, EPJC 75 (2015) no.3, 121 and JHEP 1603 (2016) 123 + work in progress

# Heavy-flavor in small systems: model predictions

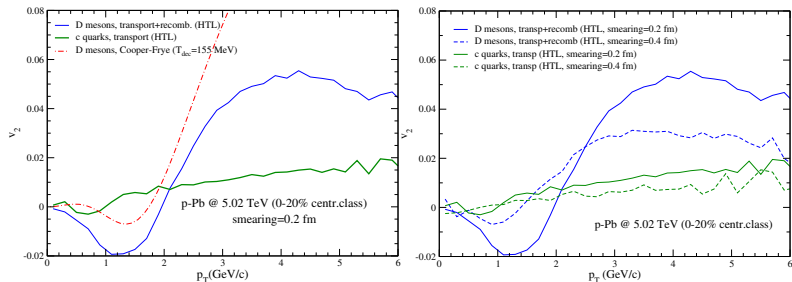


We display our predictions<sup>5</sup>, with different initializations (source smearing) and transport coefficients (HTL vs IQCD), compared to

- HF-electron  $R_{dAu}$  by PHENIX at RHIC (left panel)
- D-mesons  $R_{pPb}$  by ALICE at the LHC (right panel)

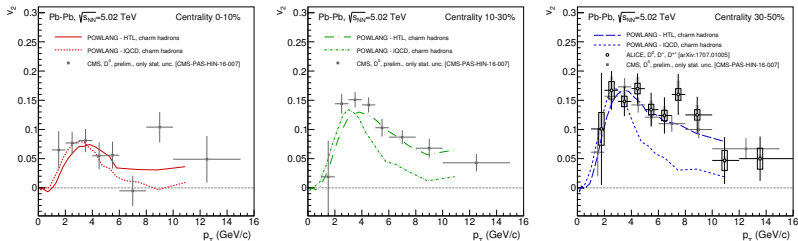
<sup>5</sup>A.B. et al., JHEP 1603 (2016) 123

# Non-vanishing elliptic flow?



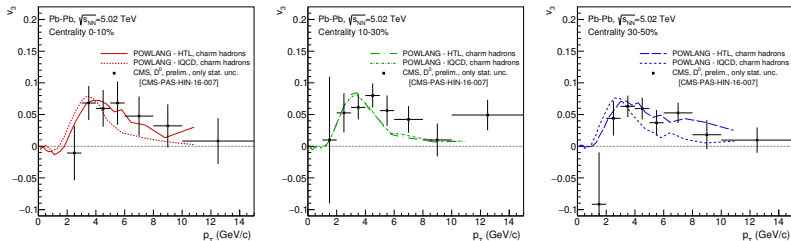
We also predict a non-vanishing  $v_2$  of charmed hadrons, arising mainly from the elliptic flow inherited from the light thermal partons

# New results at 5.02 TeV: $D$ -meson $v_2$ and $v_3$ in Pb-Pb



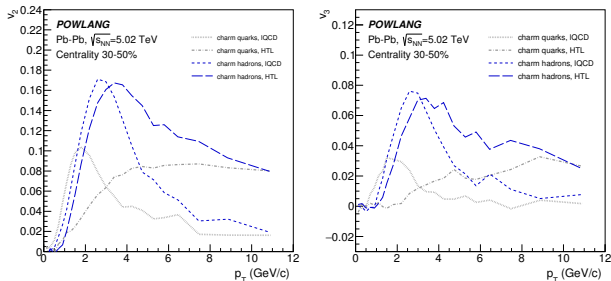
- CMS data for  $D$ -meson  $v_{2,3}$  satisfactory described;
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# Status and perspectives of transport calculations

Theory-to-experiment comparison allows one to draw some robust qualitative conclusions: **c-quarks interact significantly with the medium formed in heavy-ion collision, which affects both their propagation in the plasma and their hadronization.** As a result, HF-hadron spectra are **quenched** at high- $p_T$ , while at low- $p_T$  they display signatures of **radial, elliptic and triangular flow.**

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- **Charm** measurements **down to  $p_T \rightarrow 0$ : flow/thermalization** and **total cross-section** (of relevance for charmonium suppression!)
- $D_s$  and  $\Lambda_c$  measurements: change in **hadrochemistry** and **total cross-section**
- **Beauty** measurements in AA via exclusive hadronic decays: **better probe**, due to  $M \gg \Lambda_{\text{QCD}}, T$  (initial production, evaluation of transport coefficients and Langevin dynamics under better control)
- **Charm in p-A** collisions: which relevance for high-energy atmospheric muons/neutrinos (Auger and IceCube experiments)? Possible initial/**final-state nuclear effects**? Better measurements are needed!



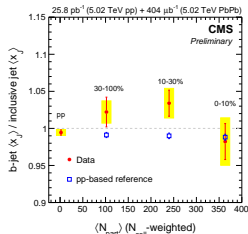
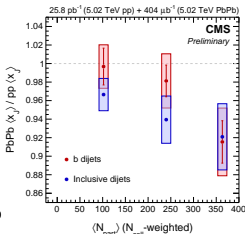
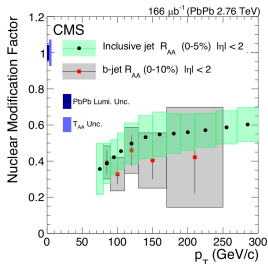
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The **challenge** is to become more quantitative, with the **extraction of HF transport coefficients** from the data (like  $\eta/s$  in hydrodynamics), goal for which beauty is the golden channel

# The high-energy frontier: b-tagged jets



b-tagging of jets/dijets allows in principle the study of the color-charge dependence of jet-quenching ( $C_F$  vs  $C_A$  for the Casimir factor). However nuclear-modification-factor and dijet asymmetry look similar for inclusive (gluon dominated) and b-tagged jets. Better study of gluon-splitting contribution (but should be negligible for back-to-back dijets!) is necessary. Error bars need to be reduced!