



## Crab cavity studies including full detuning & offsets due to long range beam-beam interactions

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November 14, 2017



# Introduction

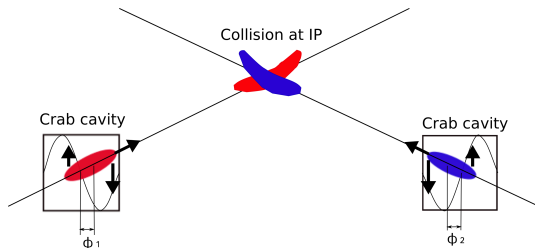
- Crab cavities are installed around IP1 & IP5 to compensate geometric reduction in luminosity due to the beam crossing angle at the IP.

Table 1: List of parameters for the baseline HL-LHC.

Proton energy at collision	7 TeV
Number of protons in a bunch $N$	$2.2 \times 10^{11}$ ppb
Number of bunch $n_b$	2748
r.m.s. bunch length ( $4\sigma_z$ )	1.2 ns
Longitudinal emittance	2.5 eVs
Transverse normalized emittance $\epsilon_{n(x,y)}$ (r.m.s)	2.5 $\mu\text{m}$
Revolution frequency	11.2455 kHz
RF frequency of main cavity	400.79 MHz
RF voltage of main cavity	16 MV/turn
Full crossing angle	510 $\mu\text{rad}$
Crab cavity voltage	3.4 MV/cavity
Crab cavity RF frequency	400.79 MHz

# Introduction

- Existing klystron power is insufficient to run the HL-LHC with the current LLRF control algorithm.
- New algorithm (full-detuning) avoids this limitation by accepting a phase modulation of accelerating RF cavity, which is operational since 2017 (Presentation by H. Timko on 16th.)
- Crab cavity (C.C) cannot follow the phase modulations (up to 100 ps pk-pk), resulting in a phase error w.r.t the individual bunch centroid.
- Phase errors at C.C creates significant sinusoidal bunch distortions at the IP.



Investigate the impact of phase modulations on the peak luminosity

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# Analytical model of bunch distributions

- 1 Define Gaussian bunch distribution at C.C:  $\rho_{c.c.}(x_{c.c.}, y_{c.c.}, z_{c.c.})$ 
  - A thin crab kick:  $\delta x'_{c.c.} = \frac{eV_1}{E_s} \sin(kz_{c.c.} + \phi)$  ( $\delta x_{c.c.} = 0$ )
- 2 Transport the bunch from C.C to IP:  $\rho_{IP}(x_{IP}, y_{IP}, z_{IP})$ 
  - $\pi/2$  phase advance between C.C to IP
  - $\alpha_{IP} = 0$
- 3 Rotate coordinate system at IP by  $\theta/2$ :  $\rho_{IP}(\tilde{x}_{IP}, \tilde{y}_{IP}, \tilde{z}_{IP})$

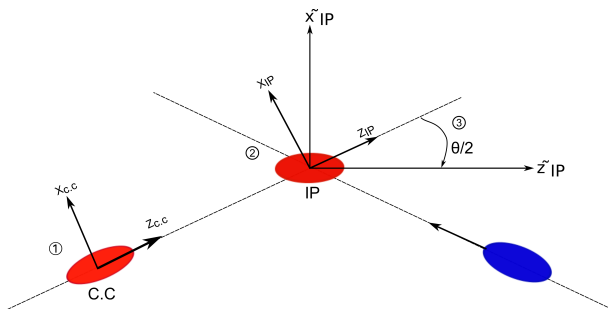


Figure 1: Coordinate system in the moving frame around IP.

# Bunch distributions without phase modulations at the IP

Table 2: Twiss parameters of baseline optics in the HL-LHC.

$\beta_{x,y}^*$ at IP5	0.20, 0.20 m
$\beta_{x,y}$ at C.C before IP5	2453, 2160 m
$\alpha_{x,y}^*$ at IP5	0, 0
$\alpha_{x,y}$ at C.C before IP5	-14.0, -36.7

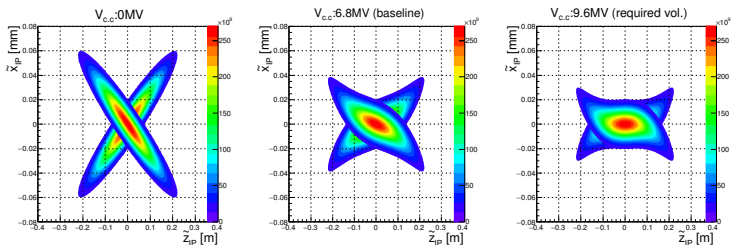


Figure 2: Bunch distributions at IP with crab voltages at 0 MV (without crab cavity), 6.8 MV (baseline) and 9.6 MV (required for full correction) without phase modulations.

# Bunch distributions at IP with coherent phase modulation

Assuming phase modulations are equal for both colliding pairs, that is correct given the symmetric filling scheme for both beams.

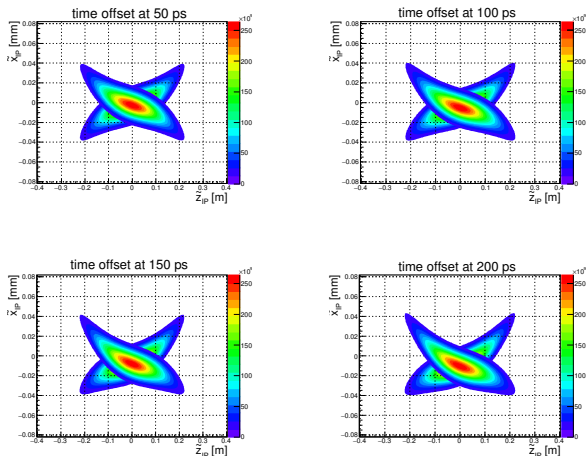


Figure 3: Bunch distributions at IP with crab voltages at 6.8 MV with phase modulations.

# Luminosity calculation with phase modulations

Translating bunches from the IP along the beam line ( $z_{IP}$ ):  $z_{IP} \pm ct$

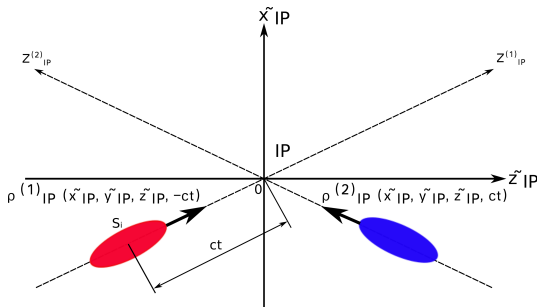


Figure 4: Schematic view of colliding pairs around the IP.

Integrated peak luminosity:

$$L = 2 \cdot \cos^2 \frac{\theta}{2} N_1 N_2 f_{rev} n_b \int \int \int \int_{-\infty}^{\infty} \rho_{IP}^{(1)}(\tilde{x}_{IP}, \tilde{y}_{IP}, \tilde{z}_{IP}, -ct) \cdot \rho_{IP}^{(2)}(\tilde{x}_{IP}, \tilde{y}_{IP}, \tilde{z}_{IP}, ct) d\tilde{x}_{IP} d\tilde{y}_{IP} d\tilde{z}_{IP} d(ct). \quad (1)$$

$N_1, N_2$ : number of protons in bunches,  $n_b$ : number of bunches

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# Linear transfer map from MADX

- Twiss parameters and first-order transfer map are created by MADX
- Transfer-map: IP1, 2 main accelerating RF cavities around IP4, one pair of C.C around IP5 and IP5
- HL-LHC V1.2 (round optics  $\beta_x^* = \beta_y^*$ )

# PYTRACK simulations

- Inject protons ( $10^5$ ) at IP1 and observe bunch distributions at IP5
- Transverse initial bunch distribution: Gaussian
- Longitudinal initial bunch profiles: Gaussian and Binomial ( $n=2.5$ ) are generated by BLongD code
- Ramping up crab voltage over 1000 turns (two synchrotron periods,  $F_s=23\text{Hz}$ ,  $F_{rev}=11\text{kHz}$ ) linearly to keep quasi-static synchrotron motion

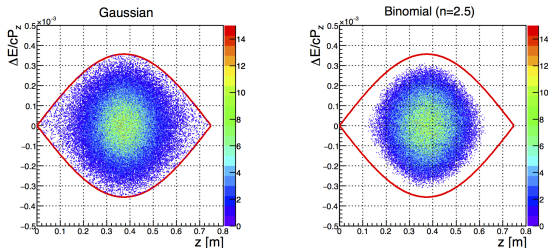


Figure 5: Longitudinal initial bunch distributions.

# Bunch distributions with coherent and incoherent phase modulations

- Coherent phase modulations: Identical filling pattern B1-B2
- Incoherent phase modulations: upper bound on what we can expect in case of different fillings B1-B2 (not planned)

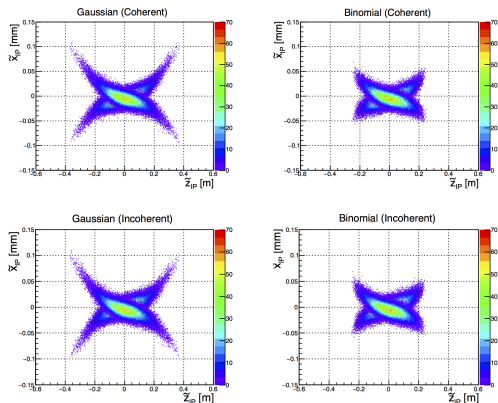


Figure 6: Bunch distributions at the IP5 with coherent ( $\phi_1 = \phi_2 = 100\text{ps} = 14.4^\circ$  @ 400.79 MHz) and incoherent ( $\phi_2 = 100\text{ps}$  and  $\phi_1 = 0$ ) phase modulations.

# Comparisons of peak luminosity

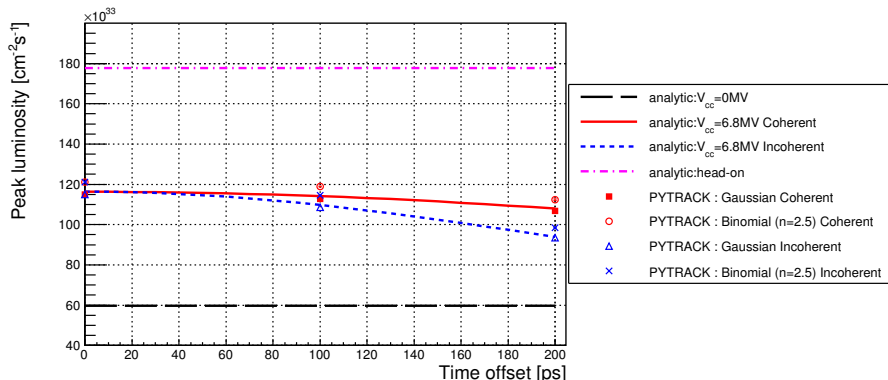


Figure 7: Peak luminosity with phase modulations in time.

- Coherent phase modulations at 100 ps: 2 % reduction (Analytical)
- Incoherent phase modulations at 100 ps: 6 % reduction (Analytical)

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# Long range beam-beam effect

Variation of the closed orbit kick along bunches:

$$\Delta x', \Delta y' \propto \frac{1}{r^2}; \quad \sigma_{x,y} \lesssim r \quad (2)$$

r: bunch separation

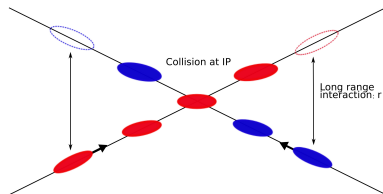


Figure 8: Long-range interactions.

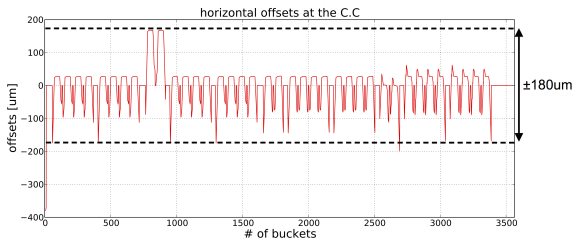


Figure 9: Horizontal bunch offset in the crab cavity.

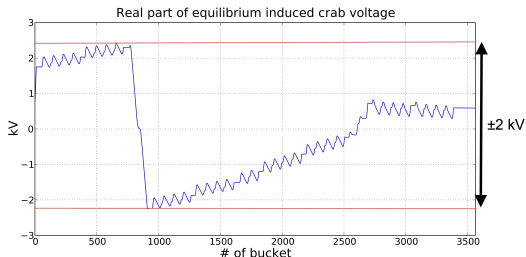
# Beam induced voltage

Induced cavity amplitude [1] after the M-th turns:

$$A_{\perp}(nT_b) = -I_{b,dc} F_b \frac{\omega^2}{c} \left(\frac{R}{Q}\right) \tau [1 - e^{-\frac{T_b}{\tau}}] \frac{1 - e^{-\frac{MN_b T_b}{\tau}}}{1 - e^{-\frac{N_b T_b}{\tau}}} \sum_{k=0}^{N_b-1} e^{-\frac{kT_b}{\tau}} \cdot x((n-k)T_b). \quad (3)$$

**Table 3:** The parameter of the crab cavity

Bunch spacing $T_b$	24.95 ns
R/Q	420 $\Omega$
Available bunches $N_b$	3564
Time constant $\tau$	400 $\mu$ s
Bunching factor $F_b$	0.82
$I_{dc}$	1.4 A



**Figure 10:** Equilibrium induced crab voltage.

- Induced voltage modulation modulates bunch by bunch crabbing angles by less than 0.06 % (2 kV/3.4 MV) :  $V_{c.c} = \frac{cE_s \tan \theta / 2}{e\omega \sqrt{\beta^*} \beta_{c.c} \sin \mu}$
- Induced power  $P_I$ :  $\frac{P_I}{P_n} = \left(\frac{V_I}{V_n}\right)^2 = \left(\frac{2kV}{3.4MV}\right)^2 \approx 0.35 \times 10^{-6}$

[1] J. Tuckmantel, Cavity-beam-transmitter interaction formula collection with derivation, CERN-ATS-Note (2011)

## Summary (Full-detuning)

- Analytical model of peak luminosity with phase modulations has been derived (should be published soon).
- Simulations have been applied by PYTRACK to compare to the analytical results, which is in good agreements.
- Reduction of peak luminosity is less than 2 % with the expected coherent phase modulation at 100 ps for both beams. In the limiting scenario of an incoherent phase modulation at 100 ps, it sums to 6 % of reductions.
- Phase modulations for the nominal HL-LHC operations are acceptable for physics runs.

## Summary (long-range interactions)

- The long-range beam-beam effects results in different transverse position of the bunches when crossing the crab cavity, which induces transient beam loading.
- Given its very high frequency, compared to the cavity bandwidth, this excitation will induce less than 0.1 % of nominal cavity voltage (3.4 MV), and have no effect on luminosity.

Back Up

# Luminosity calculation with phase modulations

$$L = \frac{\cos^2 \frac{\theta}{2} N^2 f_{\text{rev}} n_b}{\pi^{5/2} \sigma_{x1}^* \sigma_{x2}^* \sigma_{z1} \sigma_{z2} \sqrt{8(\sigma_{y1}^{*2} + \sigma_{y2}^{*2})}} \int \int \int_{-\infty}^{\infty} e^{-\left( \frac{(\tilde{x}_{IP} \cos \frac{\theta}{2} - \tilde{z}_{IP} \sin \frac{\theta}{2})^2 + \tilde{C}_1}{2\sigma_{x1}^{*2}} + \frac{(\tilde{x}_{IP} \cos \frac{\theta}{2} + \tilde{z}_{IP} \sin \frac{\theta}{2})^2 + \tilde{C}_2}{2\sigma_{x2}^{*2}} \right)} e^{-\left( \frac{(\tilde{x}_{IP} \sin \frac{\theta}{2} + \tilde{z}_{IP} \cos \frac{\theta}{2} - ct)^2}{2\sigma_{z1}^2} + \frac{(-\tilde{x}_{IP} \sin \frac{\theta}{2} + \tilde{z}_{IP} \cos \frac{\theta}{2} + ct)^2}{2\sigma_{z2}^2} \right)} d\tilde{x}_{IP} d\tilde{z}_{IP} d(ct).$$

where  $\tilde{C}_1$  and  $\tilde{C}_2$  are

$$\begin{aligned} \tilde{C}_1 = & \left( \beta^* \beta_{c.c} \frac{eV_1}{E_s} \sin \left( k \left( \tilde{x}_{IP} \sin \frac{\theta}{2} + \tilde{z}_{IP} \cos \frac{\theta}{2} - ct \right) + \phi_1 \right) \right. \\ & \left. + 2\sqrt{\beta^* \beta_{c.c}} \left( \tilde{x}_{IP} \cos \frac{\theta}{2} - \tilde{z}_{IP} \sin \frac{\theta}{2} \right) \frac{eV_1}{E_s} \sin \left( k \left( \tilde{x}_{IP} \sin \frac{\theta}{2} + \tilde{z}_{IP} \cos \frac{\theta}{2} - ct \right) + \phi_1 \right) \right) \end{aligned}$$

$$\begin{aligned} \tilde{C}_2 = & \left( \beta^* \beta_{c.c} \frac{eV_1}{E_s} \sin \left( k \left( -\tilde{x}_{IP} \sin \frac{\theta}{2} + \tilde{z}_{IP} \cos \frac{\theta}{2} + ct \right) - \phi_2 \right) \right. \\ & \left. - 2\sqrt{\beta^* \beta_{c.c}} \left( \tilde{x}_{IP} \cos \frac{\theta}{2} + \tilde{z}_{IP} \sin \frac{\theta}{2} \right) \frac{eV_1}{E_s} \sin \left( k \left( -\tilde{x}_{IP} \sin \frac{\theta}{2} + \tilde{z}_{IP} \cos \frac{\theta}{2} + ct \right) - \phi_2 \right) \right) \end{aligned}$$

# Binomial distribution

Binomial distribution:

$$F(J) = F_0 \left(1 - \frac{J}{J_0}\right)^n, \quad (4)$$

where the normalized coefficient  $F_0$ , the action  $J = \epsilon/(2\pi)$  (longitudinal emittance  $\epsilon$ ) and the  $J_0$  corresponds to the initial full longitudinal emittance ( $\epsilon_0$ )

We define the bunch length as  $4\text{-}\sigma$  which is equivalent to the full width at half maximum (FWHM)

## Induced crab voltage

The complex cavity voltage can be written as

$$V(t) = A(t) \exp(i\omega t) \quad (5)$$

Scaled generator current at time  $t$  for the case of time dependent:

$$J_g(t) = \frac{A_{\perp}(t)}{2(R/Q)} \left( \frac{1}{Q_{ext}} + \frac{1}{Q_0} - 2i \frac{\Delta\omega}{\omega} \right) + \frac{dA_{\perp}(t)}{dt} \frac{1}{\omega(R/Q)} + \frac{x(t)\omega}{c} I_{b,dc} F_b e^{-i\phi_{elec}(t)} \quad (6)$$

Assuming  $\Delta\omega = 0$  and  $\phi_{elec} = 0$ , the differential equation of complex cavity amplitude is given by the situation:  $J_g(t) = 0$ , expressed as

$$\frac{dA(t)}{dt} + \frac{A(t)}{\tau} + I_{b,dc} F_b \frac{x(t)}{c} \omega^2 \left( \frac{R}{Q} \right) = 0 \quad (7)$$

Complex cavity amplitude at time  $t$  is then derived as

$$A_{\perp}(t) = -I_{b,dc} F_b \frac{\omega^2}{c} \left( \frac{R}{Q} \right) e^{-\frac{t}{\tau}} \int_0^t x(u) e^{\frac{u}{\tau}} du \quad (8)$$