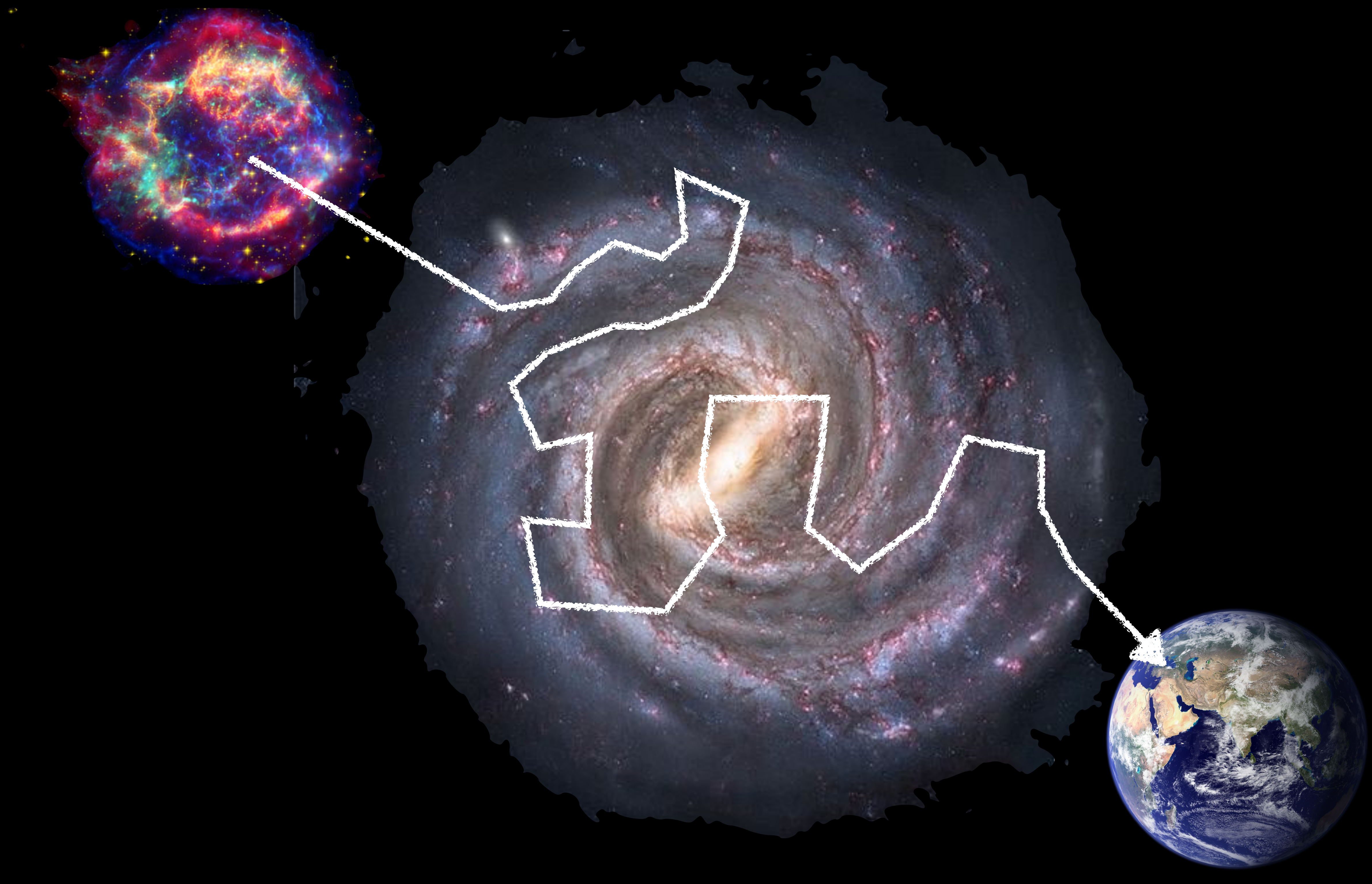
CR transport and anisotropic diffusion

Andrea Vittino
TTK, RWTH Aachen University

In collaboration with ***S.S. Cerri****,* ***D. Gaggero****,* ***C.Evoli*** *and* ***D. Grasso***
arXiv:1707.07694, JCAP10(2017)019

Three elephants in the gamma-ray sky: Loop I, the Fermi bubbles, and the Galactic center excess
Garmisch-Partenkirchen 21-24 October 2017

Charged cosmic-ray propagation



transport equation

$$\nabla \cdot (D_{xx} \nabla N_i - \vec{v}_w N_i) + \frac{\partial}{\partial p}\left[p^2 D_{pp} \frac{\partial}{\partial p}\left(\frac{N_i}{p^2}\right)\right] - \frac{\partial}{\partial p}\left[\dot{p} N_i - \frac{p}{3}\left(\vec{\nabla} \cdot \vec{v}_w\right) N_i\right] =$$

$$Q - \frac{N_i}{\tau_i^{\mathrm{f}}} + \sum_j \Gamma^{\mathrm{s}}_{j \to i}(N_j) - \frac{N_i}{\tau_i^{\mathrm{r}}} + \sum_j \frac{N_j}{\tau^{\mathrm{r}}_{j \to i}}$$

N_i = CR momentum density

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source term

N_i = CR momentum density

- Q is the **number of CR particles** injected by sources.

transport equation

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spallation/decay terms

- Q is the **number of CR particles** injected by sources.
- Spallation/decay terms model **CR creation/destruction** through **interactions with the ISM/radioactive decays**

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spatial diffusion

N_i = CR momentum density

- Q is the **number of CR particles** injected by sources.
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- The spatial diffusion of CRs is the result of their **scattering on the MHD waves** and **discontinuities of the Galactic Magnetic Field**

transport equation

advection   adiabatic energy losses

$$\nabla \cdot (D_{xx} \nabla N_i - \vec{v}_w N_i) + \frac{\partial}{\partial p}\left[p^2 D_{pp} \frac{\partial}{\partial p}\left(\frac{N_i}{p^2}\right)\right] - \frac{\partial}{\partial p}\left[\dot{p} N_i - \frac{p}{3}\left(\vec{\nabla} \cdot \vec{v}_w\right) N_i\right] =$$

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- **Advective** (or **convective**) **winds** are observed in many Galaxies. If the wind **increases with z** it generates **energy losses**.

transport equation

reacceleration

$$\nabla \cdot (D_{xx} \nabla N_i - \vec{v}_w N_i) + \frac{\partial}{\partial p}\left[p^2 D_{pp} \frac{\partial}{\partial p}\left(\frac{N_i}{p^2}\right)\right] - \frac{\partial}{\partial p}\left[\dot{p} N_i - \frac{p}{3}\left(\vec{\nabla} \cdot \vec{v}_w\right) N_i\right] =$$

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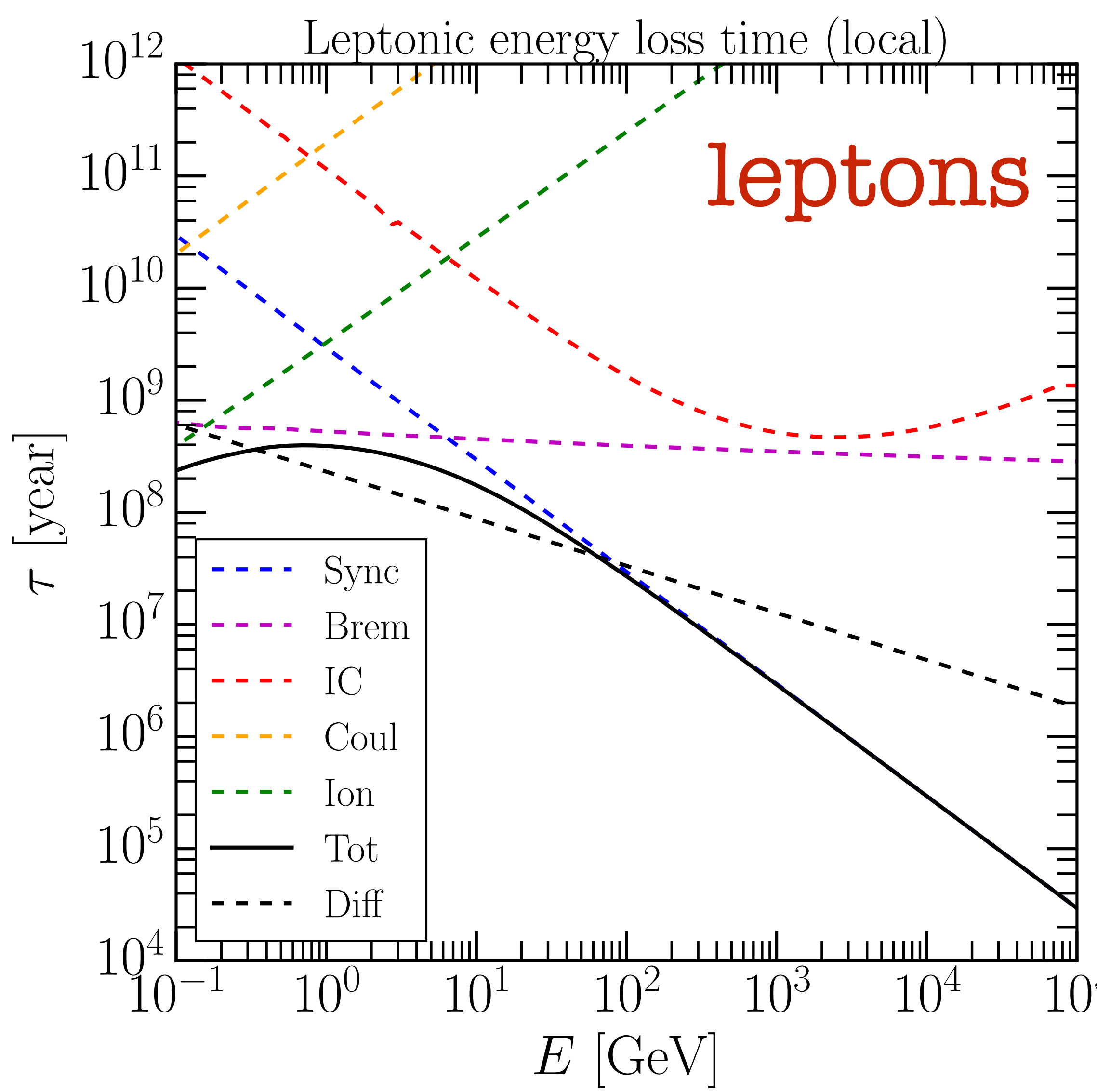
- **Advective** (or **convective**) **winds** are observed in many Galaxies. If the wind **increases with z** it generates **energy losses**.
- The **interaction of CRs with MHD waves** does not only lead to spatial diffusion, but also to **stochastic acceleration**, i.e. to a diffusion in momentum space, parametrized with the diffusion coefficient D_{pp}

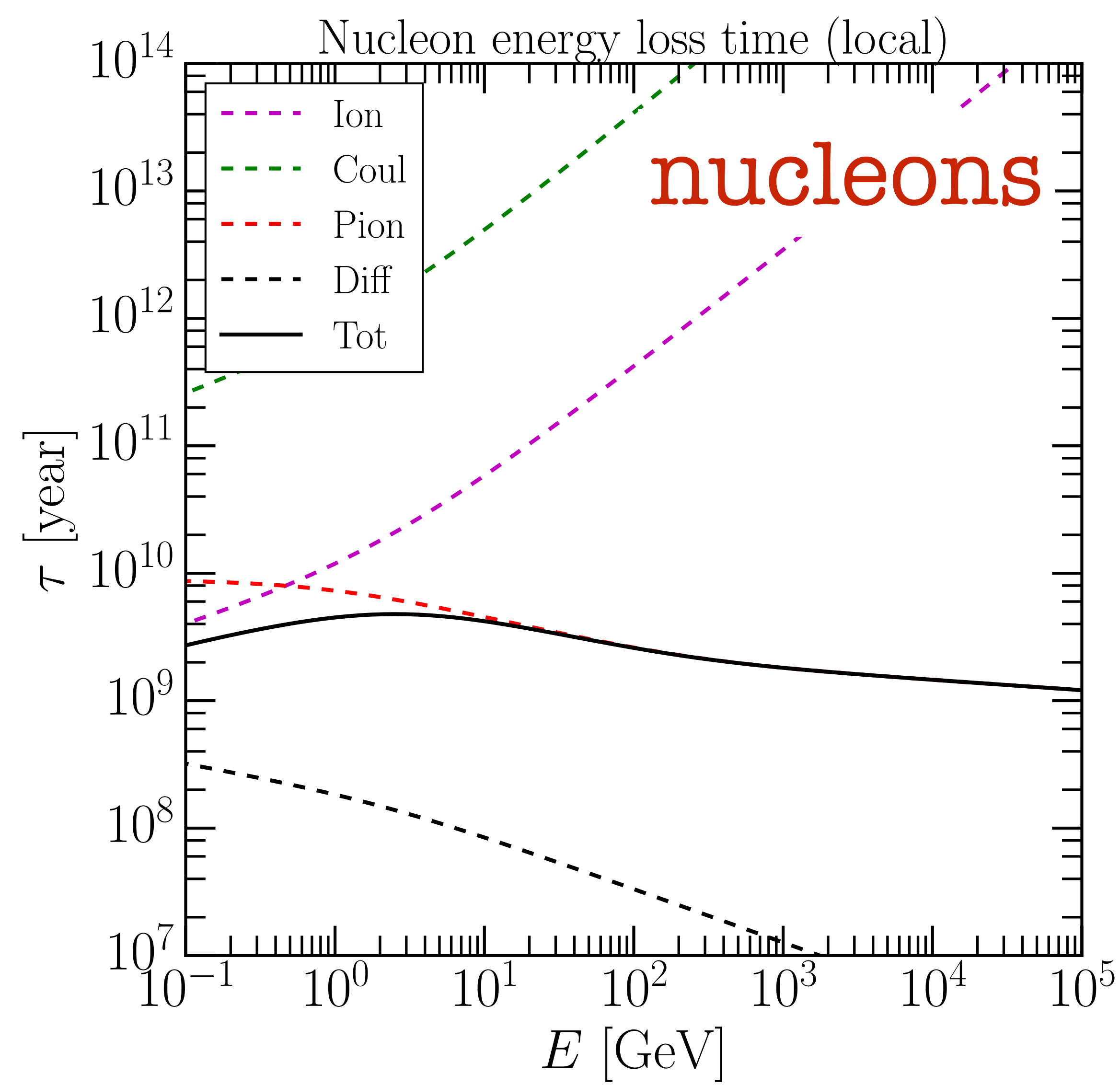
transport equation

energy losses

$$\nabla \cdot (D_{xx} \nabla N_i - \vec{v}_w N_i) + \frac{\partial}{\partial p}\left[p^2 D_{pp} \frac{\partial}{\partial p}\left(\frac{N_i}{p^2}\right)\right] - \frac{\partial}{\partial p}\left[\dot{p} N_i - \frac{p}{3}\left(\vec{\nabla} \cdot \vec{v}_w\right) N_i\right] = Q - \frac{N_i}{\tau_i^{\mathrm{f}}} + \sum_j \Gamma^{\mathrm{s}}_{j \to i}(N_j) - \frac{N_i}{\tau_i^{\mathrm{r}}} + \sum_j \frac{N_j}{\tau^{\mathrm{r}}_{j \to i}}$$

N_i = CR momentum density





transport equation

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N_i = CR momentum density

We consider a **simplified version** of the transport equation, where **only spatial diffusion** is taken into account

$$Q + \vec{\nabla}\cdot\left(D_{xx}\vec{\nabla}N\right) = 0$$

good to describe the **propagation of protons** (with energies not too low)

CR spatial diffusion

What is the **standard picture** of spatial diffusion?

$$Q + \vec{\nabla} \cdot \left(D_{xx} \vec{\nabla} N \right) = 0$$

Spatial diffusion is typically described in terms of a **spatially independent** coefficient, usually a **power-law in rigidity**:

$$D_{xx} \propto (p/Z)^{\delta}$$

Over the years, models of CR propagation based on this assumption have been **successfully adopted** to reproduce **many experimental data**

CR spatial diffusion

What is the **distribution of protons in the Galaxy** in this standard scenario?

$$Q + \vec{\nabla} \cdot \left(D_{xx} \vec{\nabla} N \right) = 0$$

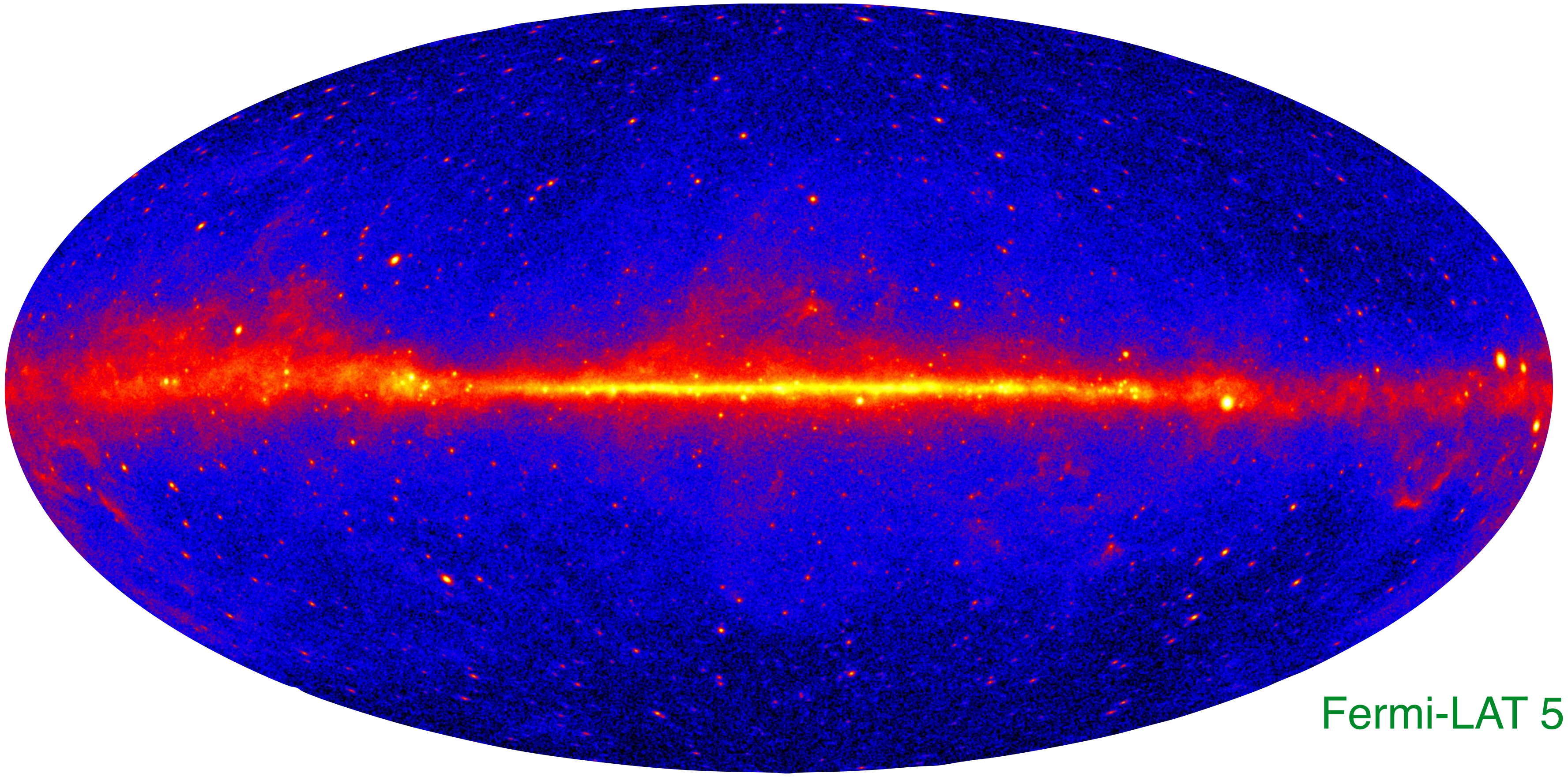
with $D_{xx} \propto p^{\delta}$ and $Q \propto p^{-\alpha_{\rm inj}}$

$$N \propto p^{-(\delta + \alpha_{\rm inj})}$$

In the **standard scenario** of CR spatial diffusion, the proton spectrum has the **same spectral index everywhere in the Galaxy**

CR protons in the gamma-ray sky

Proton distribution can be inferred from observations of the **gamma-ray** sky



Fermi-LAT 5 years map

$$p + H_{\rm ISM} \rightarrow \pi^0 \rightarrow \gamma\gamma$$

By mapping this emission in a certain region of the Galaxy, we can get a map of the **CR proton distribution** in that region

Gamma-ray observations

The **Fermi-LAT Collaboration** has recently **measured** the **proton spectral index across the Galactic plane** at different distances from the Galactic center

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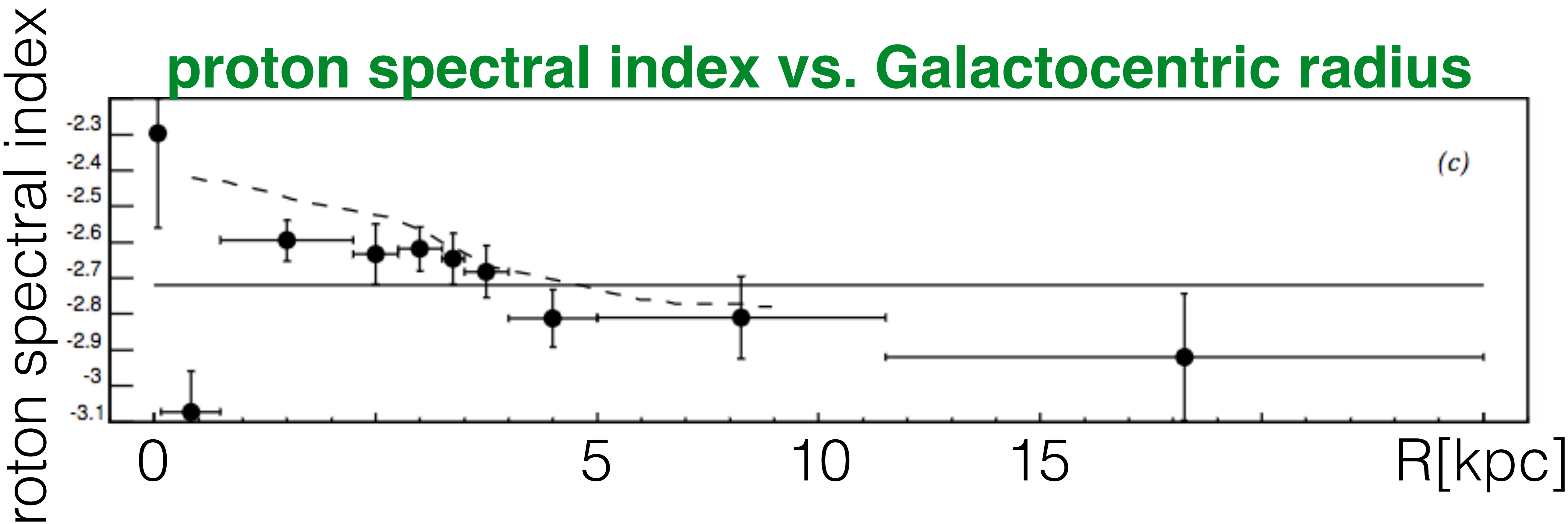

taken from **Acero et al. (Fermi-LAT Coll.), ApJS 223 26 (2016)**

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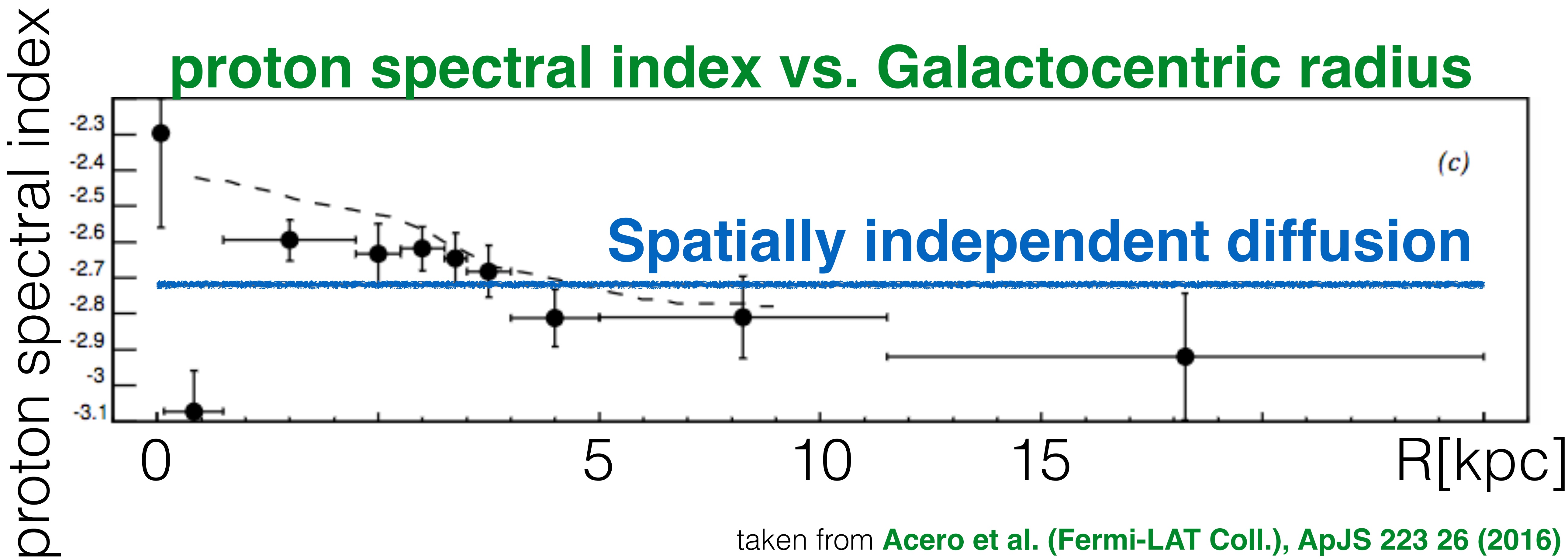

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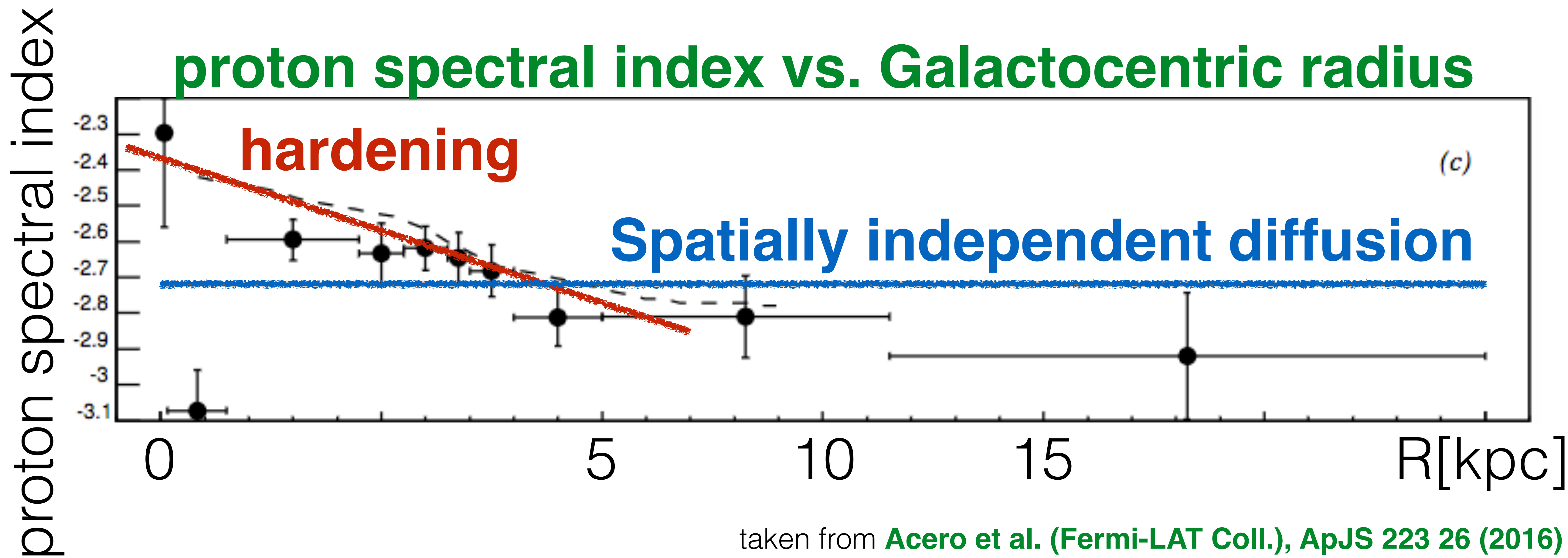

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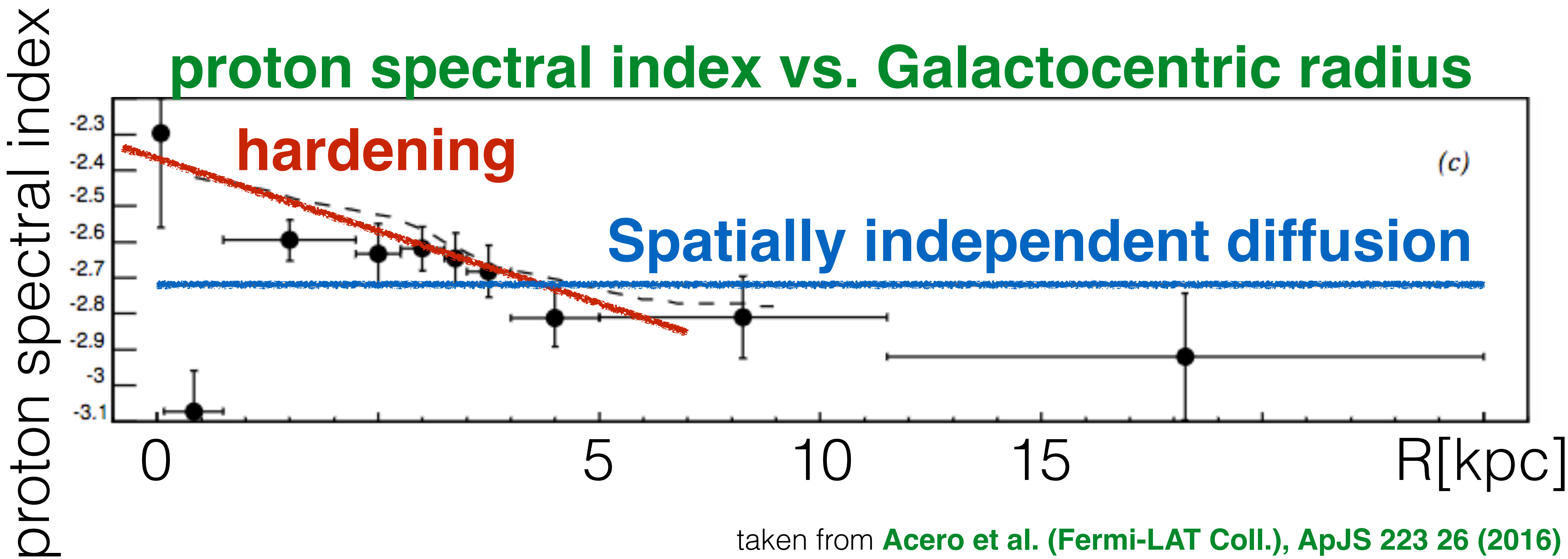


taken from **Acero et al. (Fermi-LAT Coll.), ApJS 223 26 (2016)**

This measurement might represent **a possible challenge** to the standard picture of CR spatial diffusion

Our message

The **aim of this talk** is to illustrate that a **hardening** compatible with the one **observed by Fermi-LAT** can be obtained **if CR spatial diffusion** is assumed to be **anisotropic**

What is anisotropic diffusion?

The Galactic magnetic field is defined as the **sum** of a **regular** and a **turbulent** component:

$$\vec{B} = \vec{B}_0 + \delta\vec{B}$$

regular component   turbulent component

As a result of the **interaction with the turbulent component**, CRs **diffuse**.

$D_{||}$ describes diffusion in a direction parallel to $\vec{B}_0$

$D_{\perp}$ describes diffusion in a direction perpendicular to $\vec{B}_0$

Diffusion is **anisotropic** if $D_{||} \neq D_{\perp}$

Theoretical motivations for anisotropic diffusion

As a **reference theoretical framework** for CR transport modelling, one often considers the **quasi-linear theory** (QLT) of pitch-angle scattering in a random magnetic field.

$$\frac{D_\perp}{D_{||}} \sim \frac{\delta B^2}{B_0^2}$$

δB = turbulent fluctuations

B_0 = regular field

$\delta B / B_0 \sim 1$ **at ~100 pc** (if turbulence is injected by Supernova explosion)

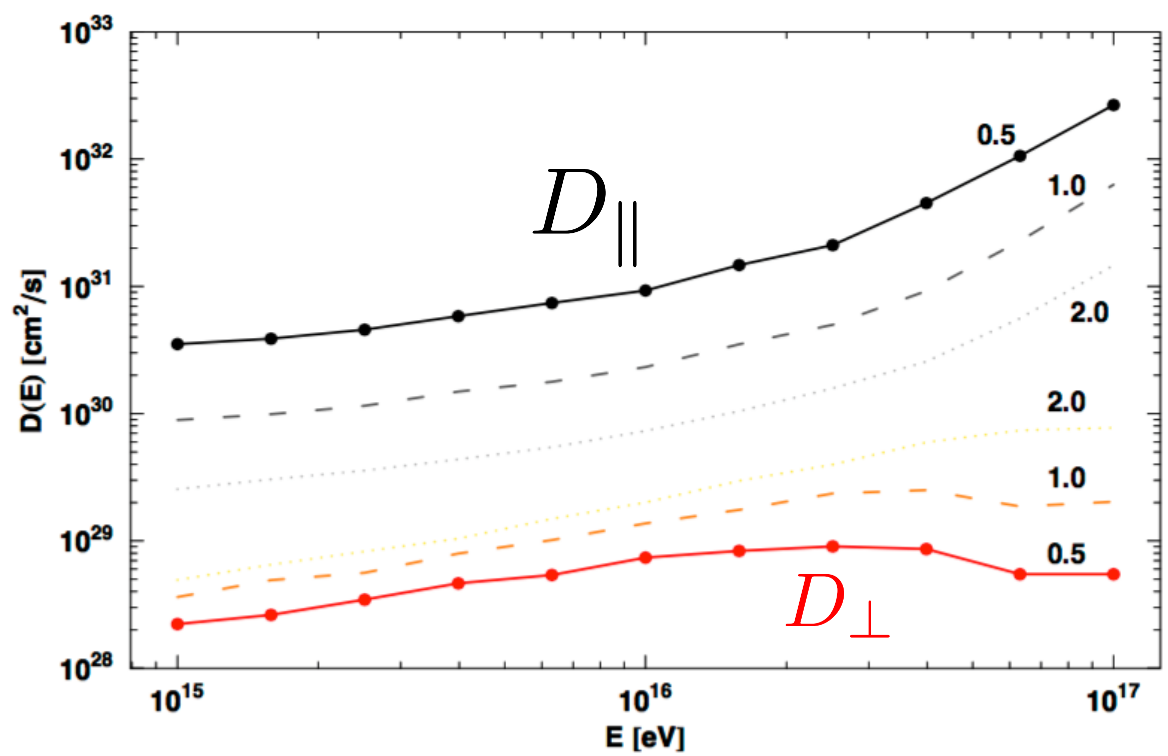
this is the regime where QLT applies

$\delta B / B_0 \ll 1$ if the particle Larmor radius is << 100 pc. **This is the case for GeV - TeV particles**

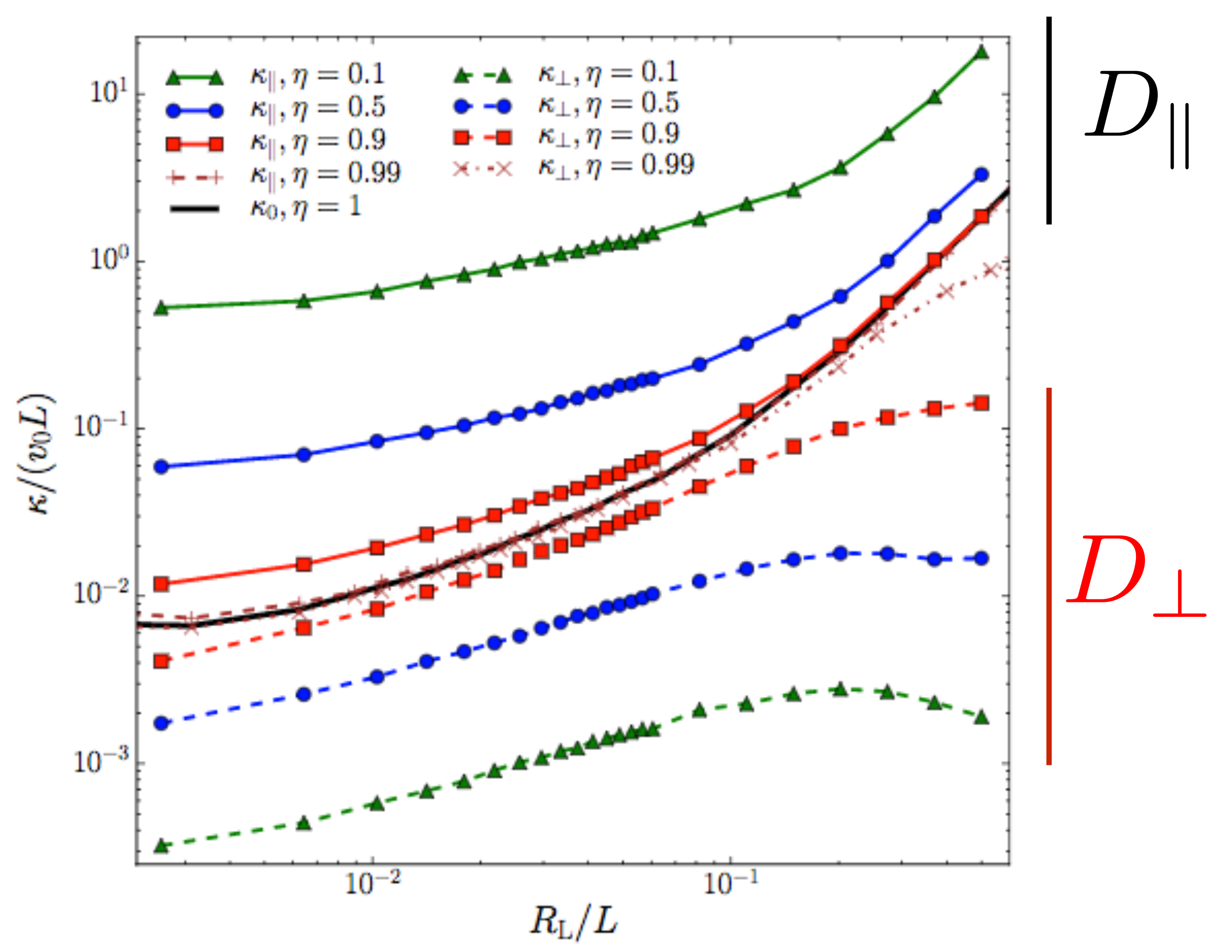
QLT provides a guideline to model CR transport of GeV-TeV particles. In its domain of applicability it predicts a **highly anisotropic diffusion.**

Theoretical motivations for anisotropic diffusion

Several effects **complicate** the scenario predicted by QLT and enhance perpendicular transport (e.g. field line random walk). However, **test particle simulations** show that such effects **do not alter the global picture**, that is the **need for anisotropic diffusion**



De Marco, Blasi, Stanev, JCAP 6 2007



Snodin et al. ,MNRAS 457 (2016)

Our model

We implement a model of anisotropic diffusion in DRAGON2, the new version of the DRAGON code (**Evoli, Gaggero, Vittino et al., JCAP 1702 (2017) no.02, 015**)

Anisotropic diffusion will be one of the key features of DRAGON2!

Our model

Transport equation:

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial N}{\partial x_j}\right) + Q$$

energy losses, reacceleration and advection are neglected

We consider the **two-dimensional case** (cylindrical coordinates):

$$\frac{\partial N}{\partial t} = D_{RR}\frac{\partial^2 N}{\partial R^2} + D_{zz}\frac{\partial N}{\partial z^2} + 2D_{Rz}\frac{\partial^2 N}{\partial R \partial z} + u_R\frac{\partial N}{\partial R} + u_z\frac{\partial N}{\partial z} + Q$$

- The source term Q is modelled as in **Lorimer et al., MNRAS 372 (2006)**
- The **diffusion coefficient** is built in order to account for a possible anisotropy

Diffusion tensor

We define the **diffusion tensor** as:

$$D_{ij} \equiv D_\perp \delta_{ij} + \left(D_\parallel - D_\perp\right) b_i b_j \,, \qquad b_i \equiv \frac{B_i}{|\vec{B}|}$$

B is the *i*-th component of the regular magnetic field

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With:

$$D_\parallel = D_{0\parallel} \left(\frac{p}{Z}\right)^{\delta_\parallel} \quad \text{and} \quad D_\perp = D_{0\perp} \left(\frac{p}{Z}\right)^{\delta_\perp} \equiv \epsilon_D \, D_{0\parallel} \left(\frac{p}{Z}\right)^{\delta_\perp}$$

diffusion parallel to *B* — diffusion perpendicular to *B*

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diffusion parallel to *B* — diffusion perpendicular to *B*

Different rigidity scaling:

- For **parallel diffusion**, we assume $\delta_\parallel = 0.3$
- For **perpendicular diffusion**, we assume $\delta_\perp \in [0.5, 0.7]$ in agreement with a low-energy extrapolation of the results of test particle numerical simulations

Diffusion tensor

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diffusion parallel to *B*

diffusion perpendicular to *B*

Different normalization:

We consider $\epsilon_D \in [0.01, 1]$ (we go **from the isotropic case** to a **relatively strong anisotropy**, once again based on a low-energy extrapolation of test particle numerical simulations)

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We define the **diffusion tensor** as:

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B is the *i*-th component of the regular magnetic field

With:

$D_{\parallel} =$ ... $\left(\frac{p}{Z}\right)^{\delta_{\perp}}$

diffusion

In the treatment of anisotropic diffusion, the key point is the definition of the Galactic magnetic field

Different normalization:

We consider $\epsilon_D \in [0.01, 1]$ (we go **from the isotropic case** to a **relatively strong anisotropy**, once again based on a low-energy extrapolation of test particle numerical simulations)

Galactic magnetic field

$$B = B_{\text{halo}} + B_{\text{disk}} + B_{\text{poloidal}}$$

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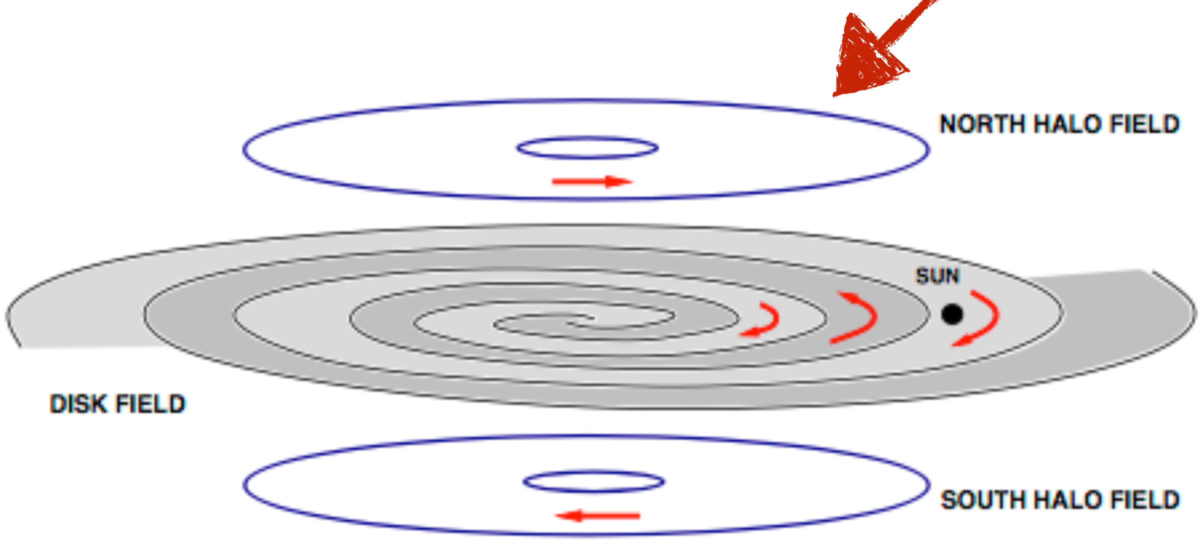

Pshirkov et al. ApJ 738 (2011)

Purely azimuthal field

Galactic magnetic field

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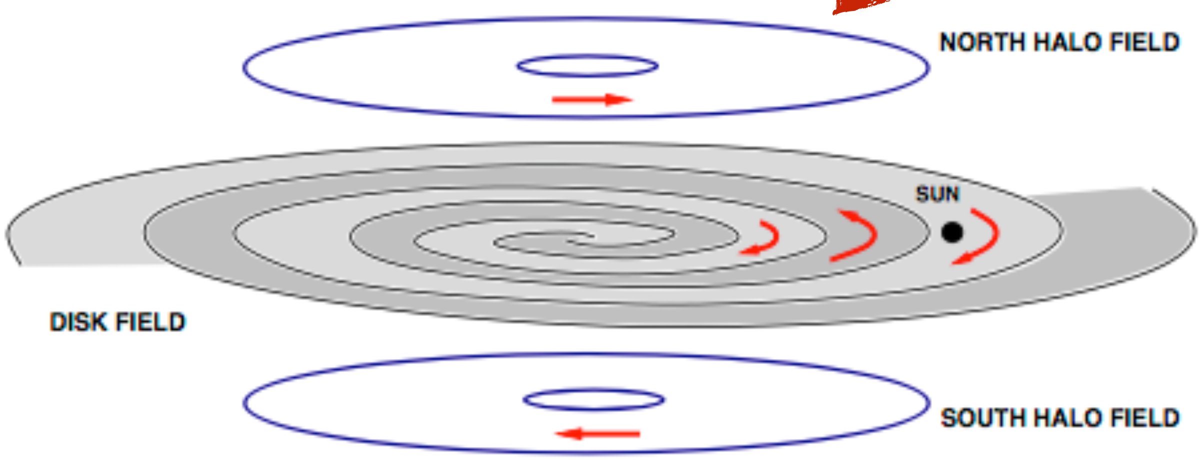

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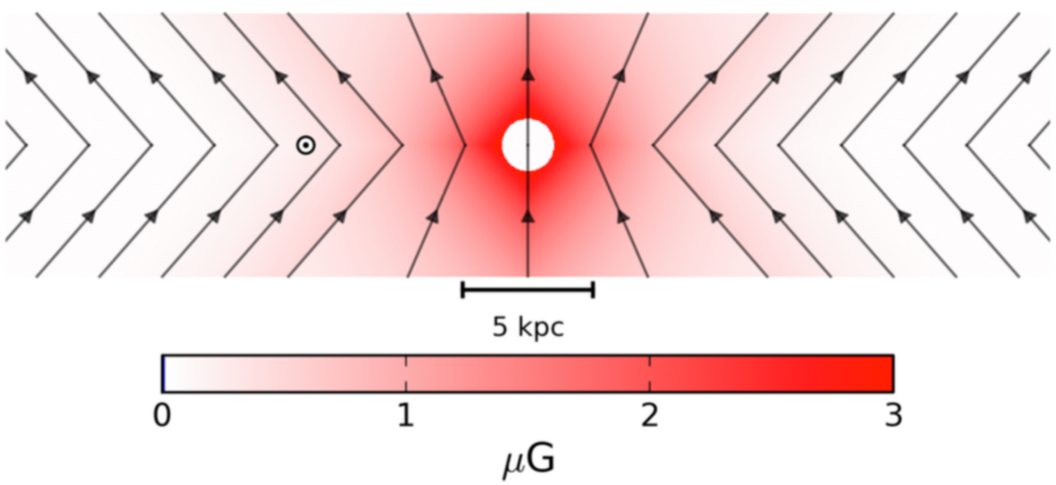
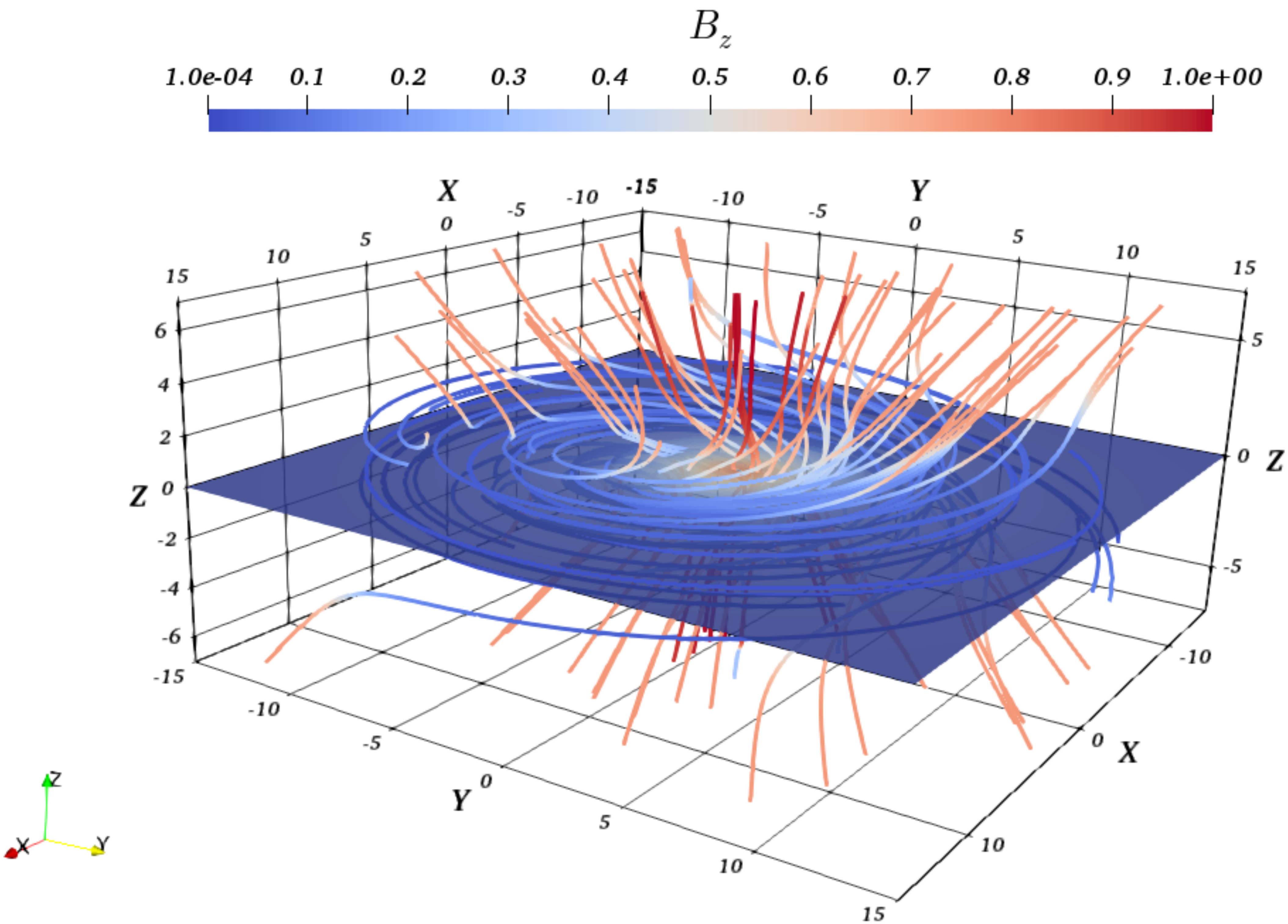


Jansson and Farrar ApJ 757 (2012)

Purely azimuthal field

X-shaped component (in the (r,z) plane)

Galactic magnetic field



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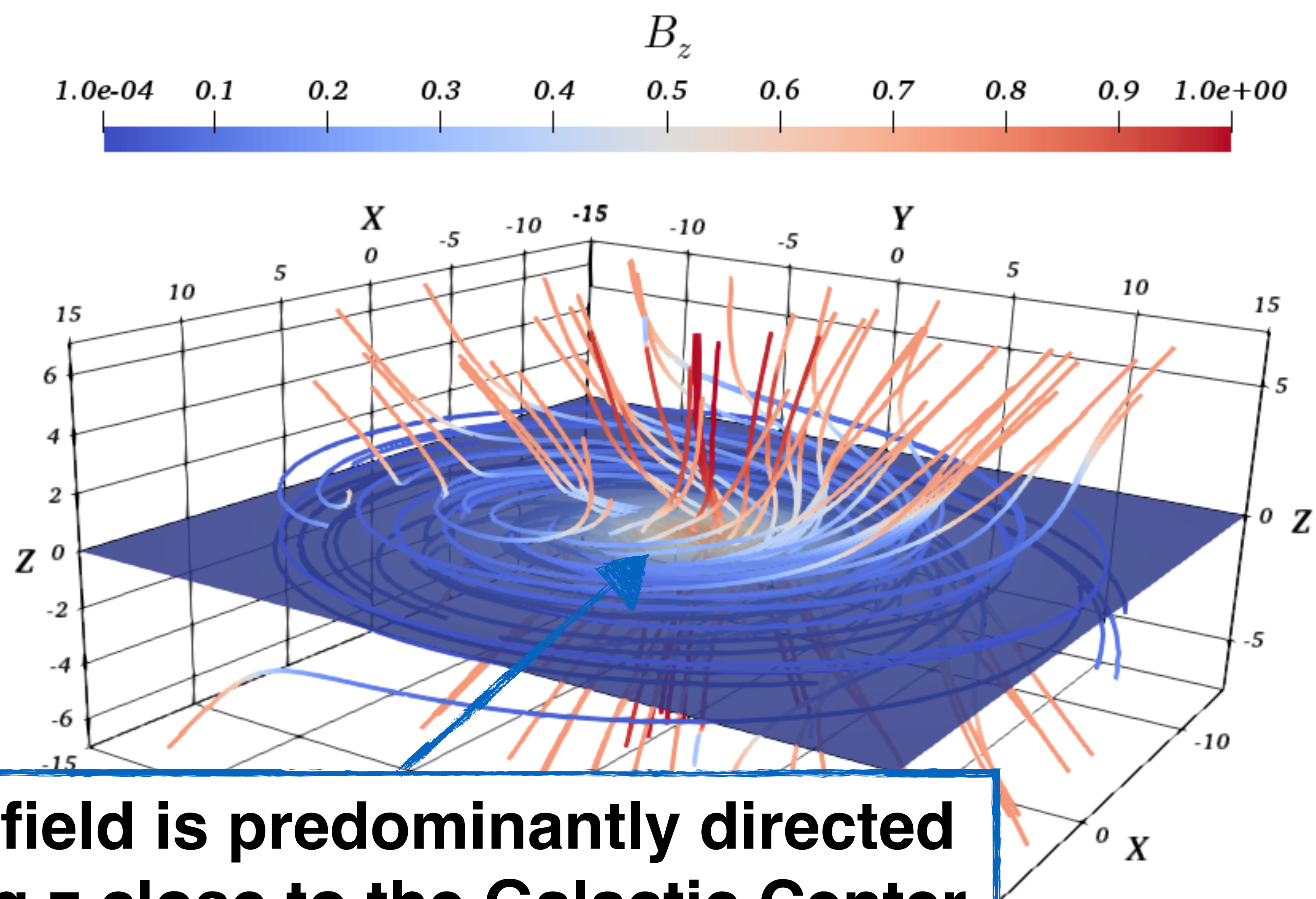

The field is predominantly directed along z close to the Galactic Center and predominantly azimuthal at large distances from it

Results : Spectral index profile

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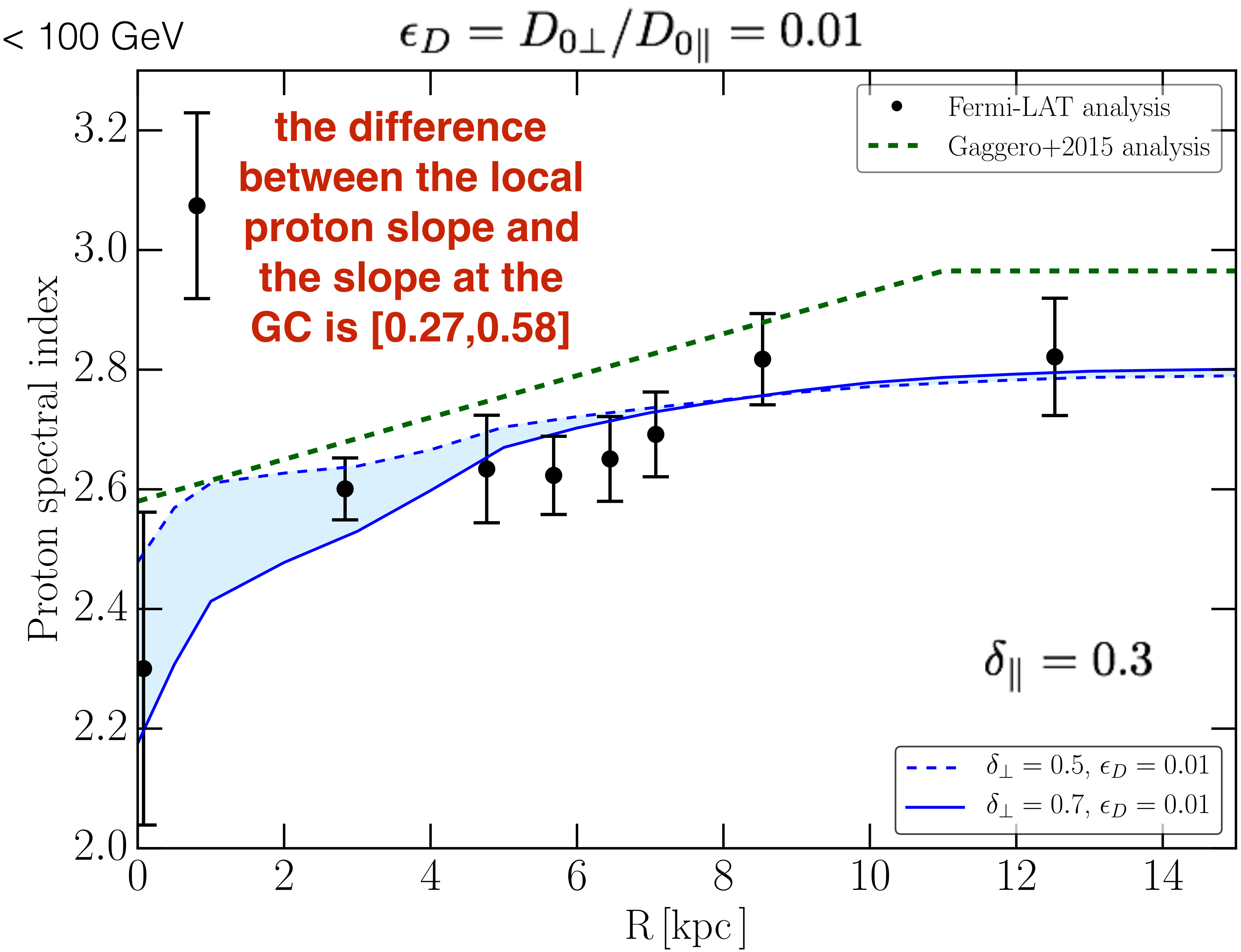
10 GeV < E_p < 100 GeV



The injection index varies from one curve to the other, in order to have a fixed local slope

Results : Spectral index profile

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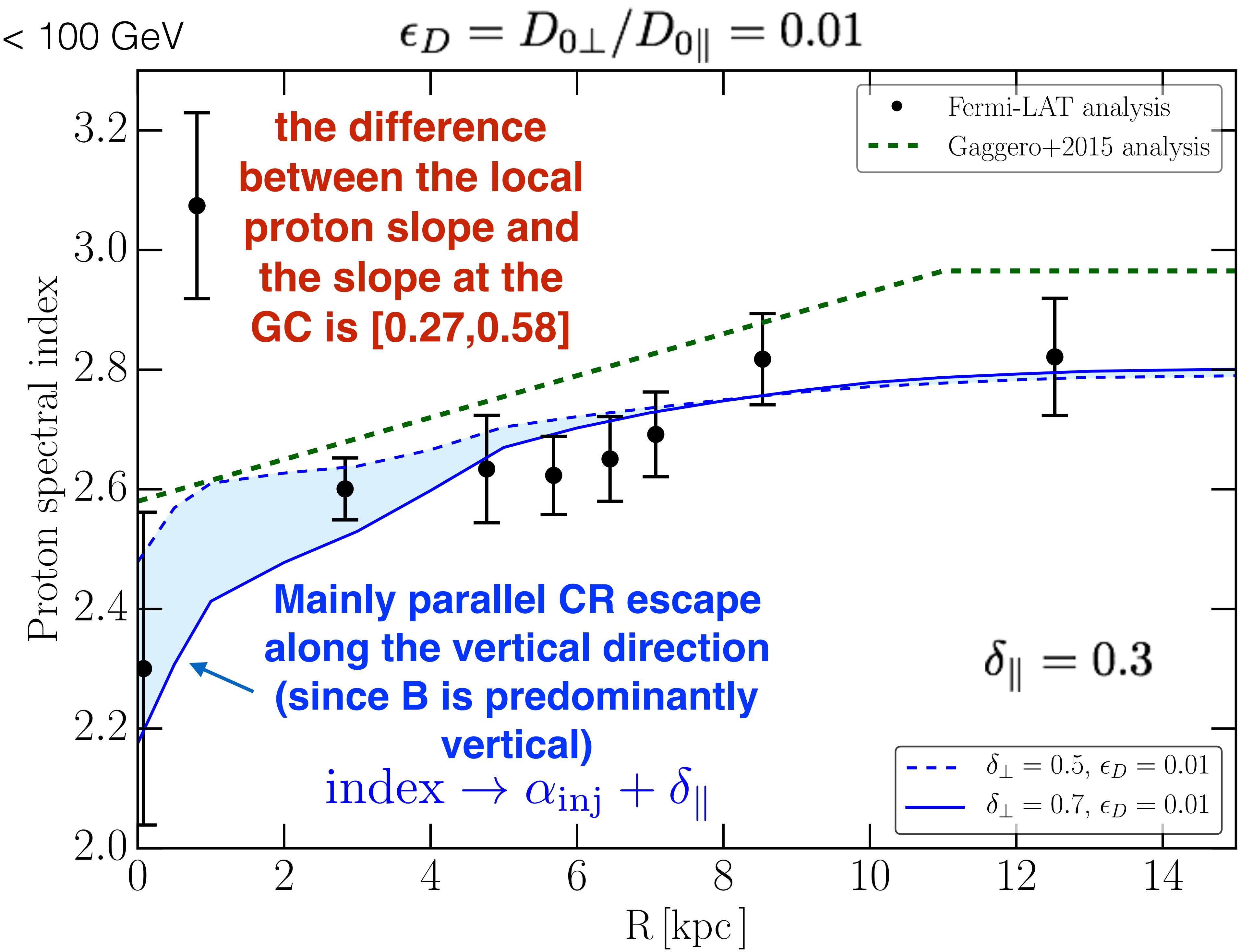

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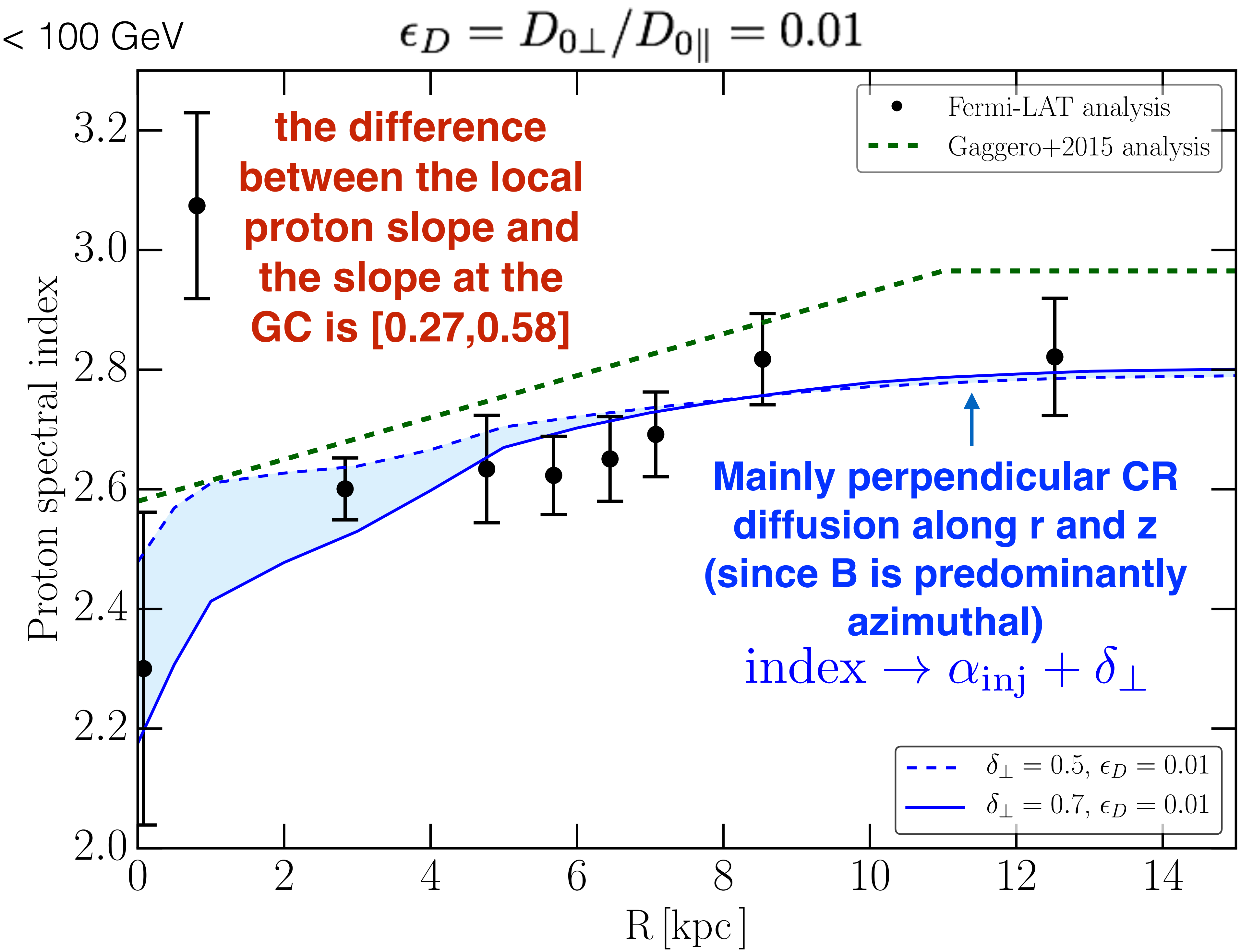

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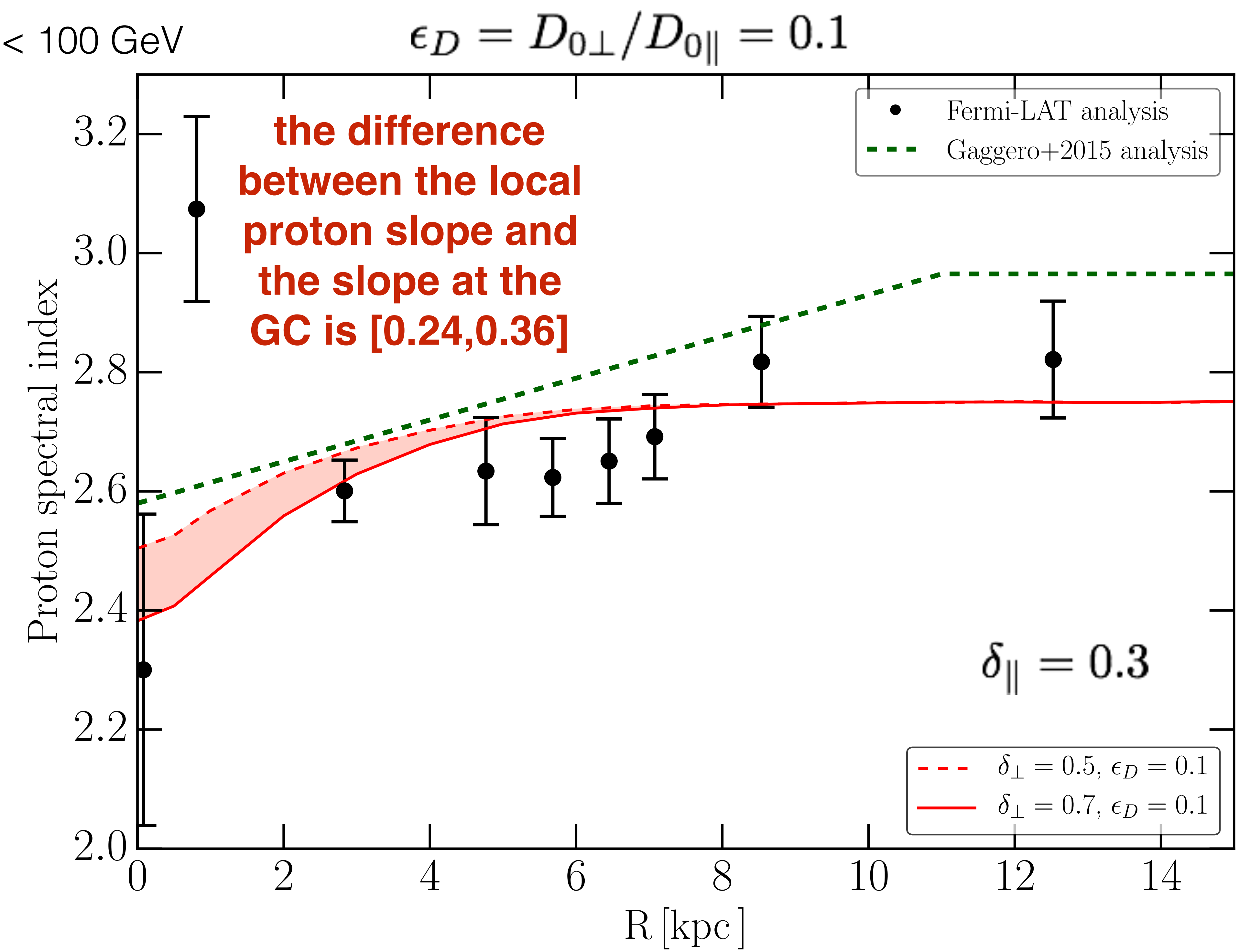

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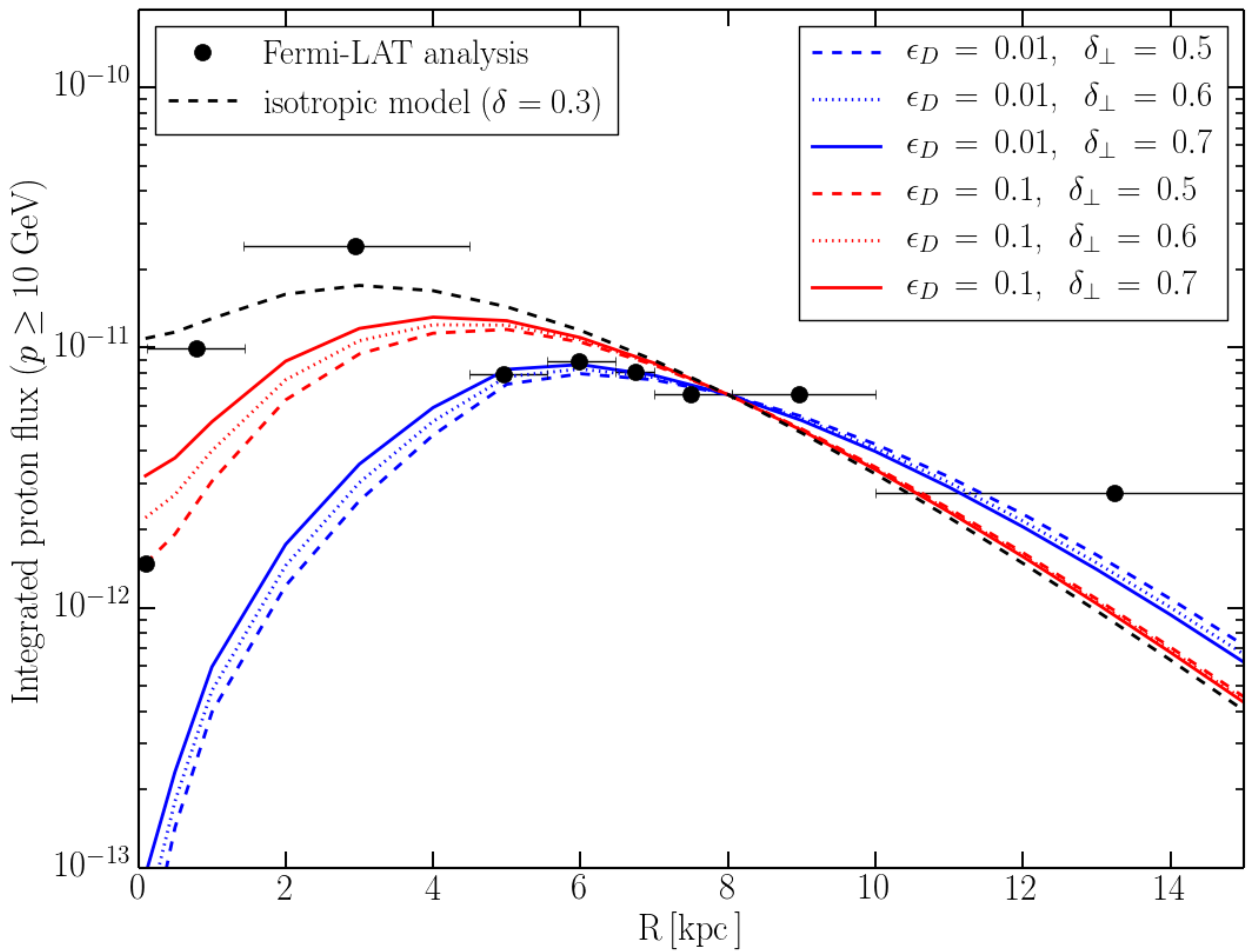
Results : Proton density



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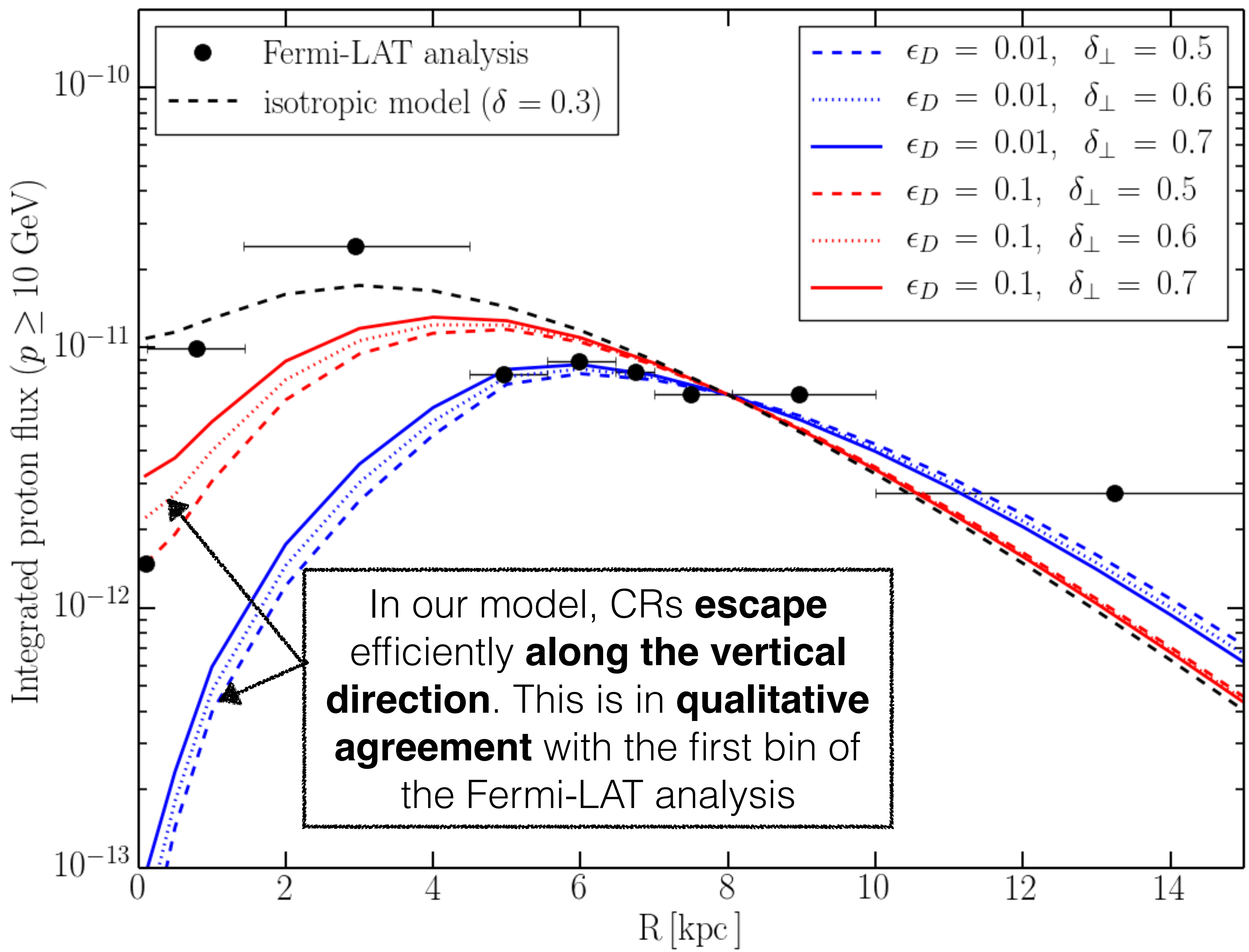

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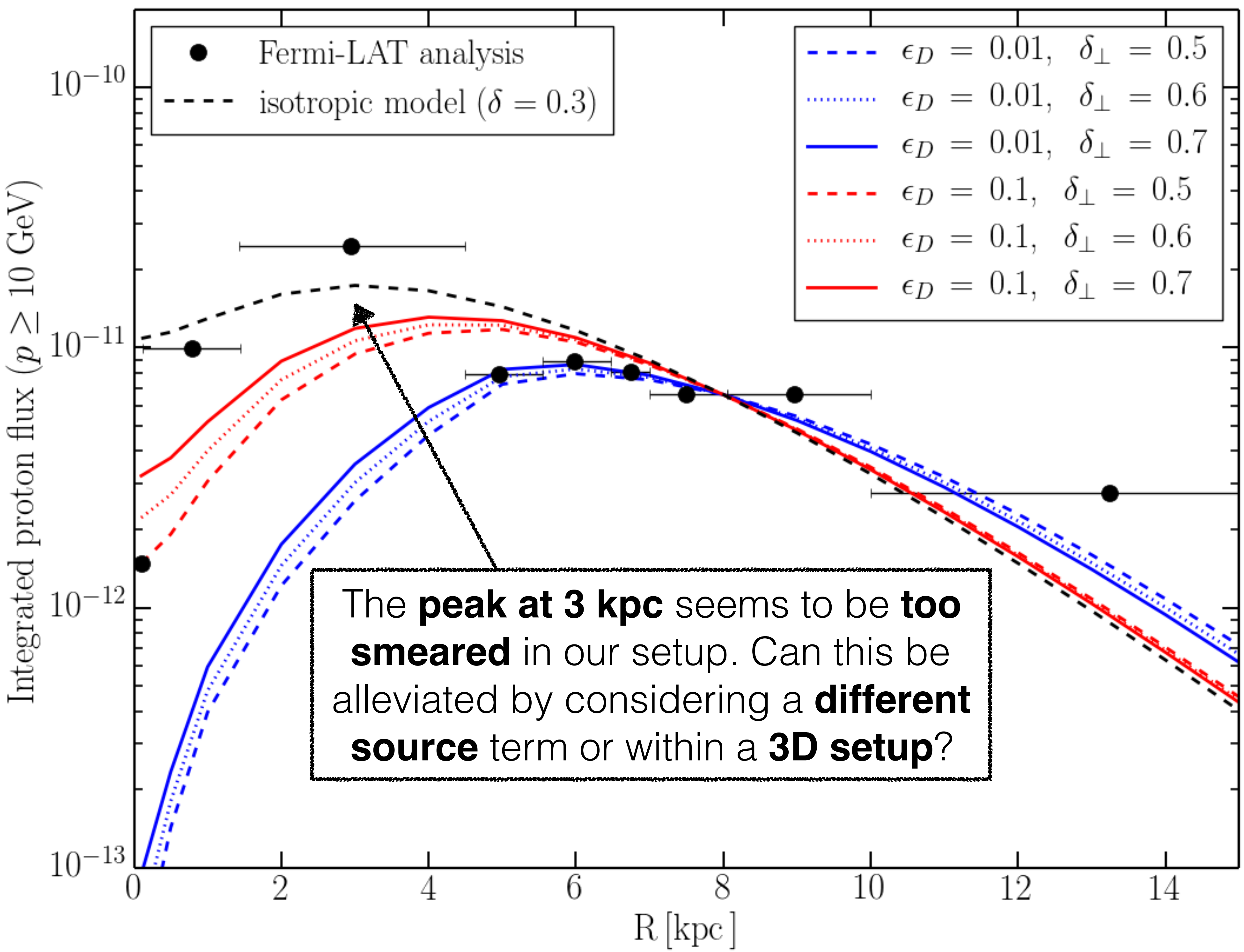


Conclusions / outlook

In this talk we have discussed **anisotropic diffusion and its possible signatures** in the gamma-ray diffuse emission

In particular, we have illustrated how, within the framework of a 2D treatment of CR transport, **anisotropic diffusion can lead to the hardening** of the proton spectrum that is inferred from **gamma-ray observations**.

In future, we plan to study the anisotropic diffusion process in a **full 3D setup**, in order to capture an even **wider phenomenology** (e.g. the parallel escape along the spiral arms).

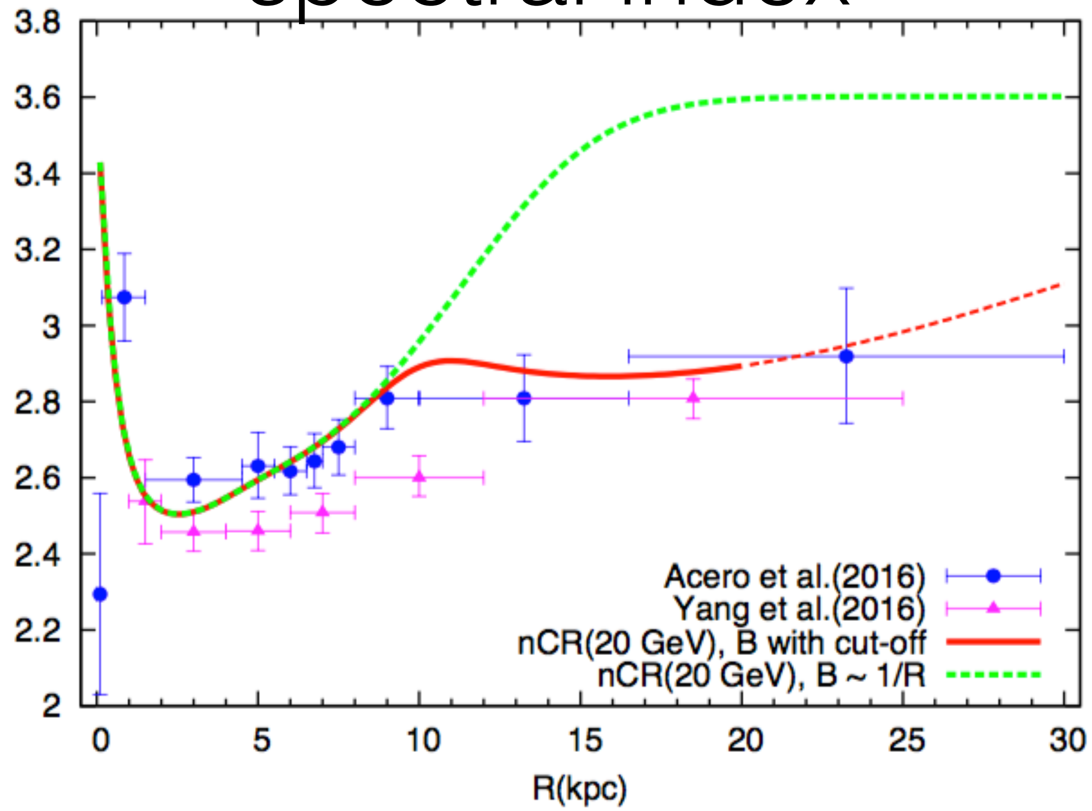
thank you for your attention!

extra slides

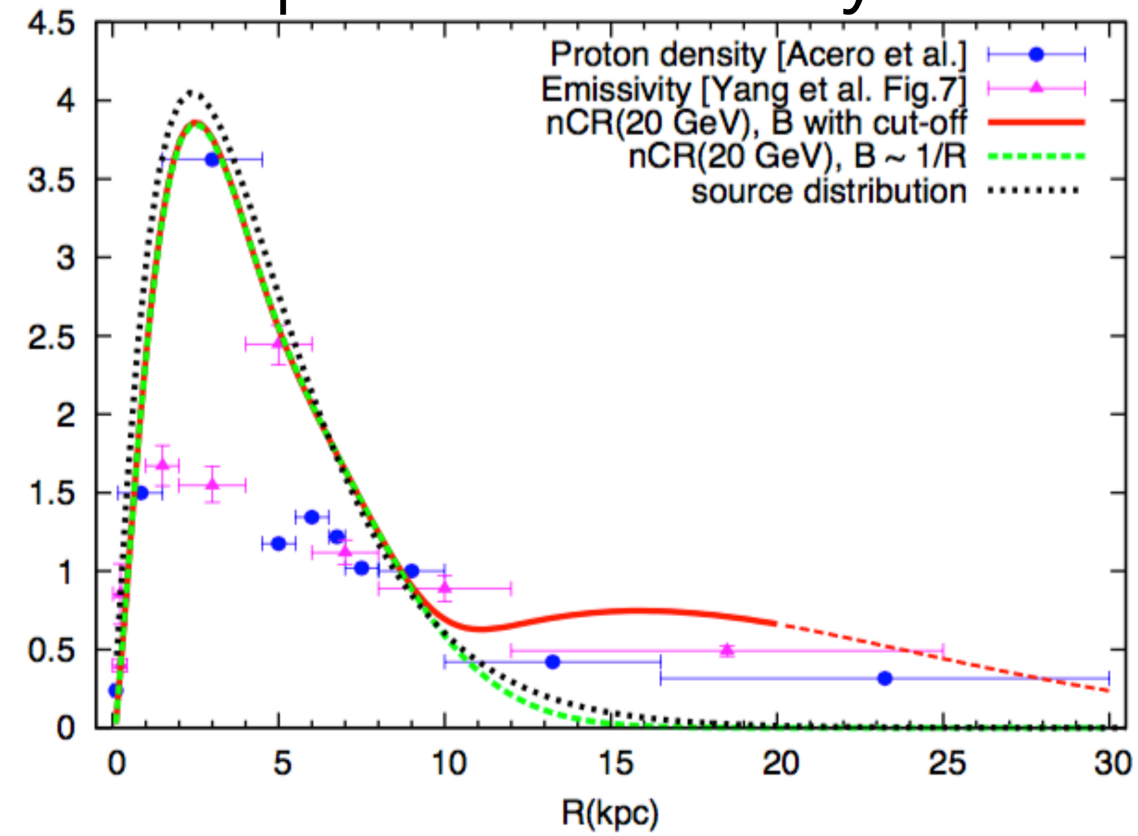
An alternative model

It was proposed in **Recchia, Blasi, Morlino, MNRAS 462 (2016)**

spectral index



proton density

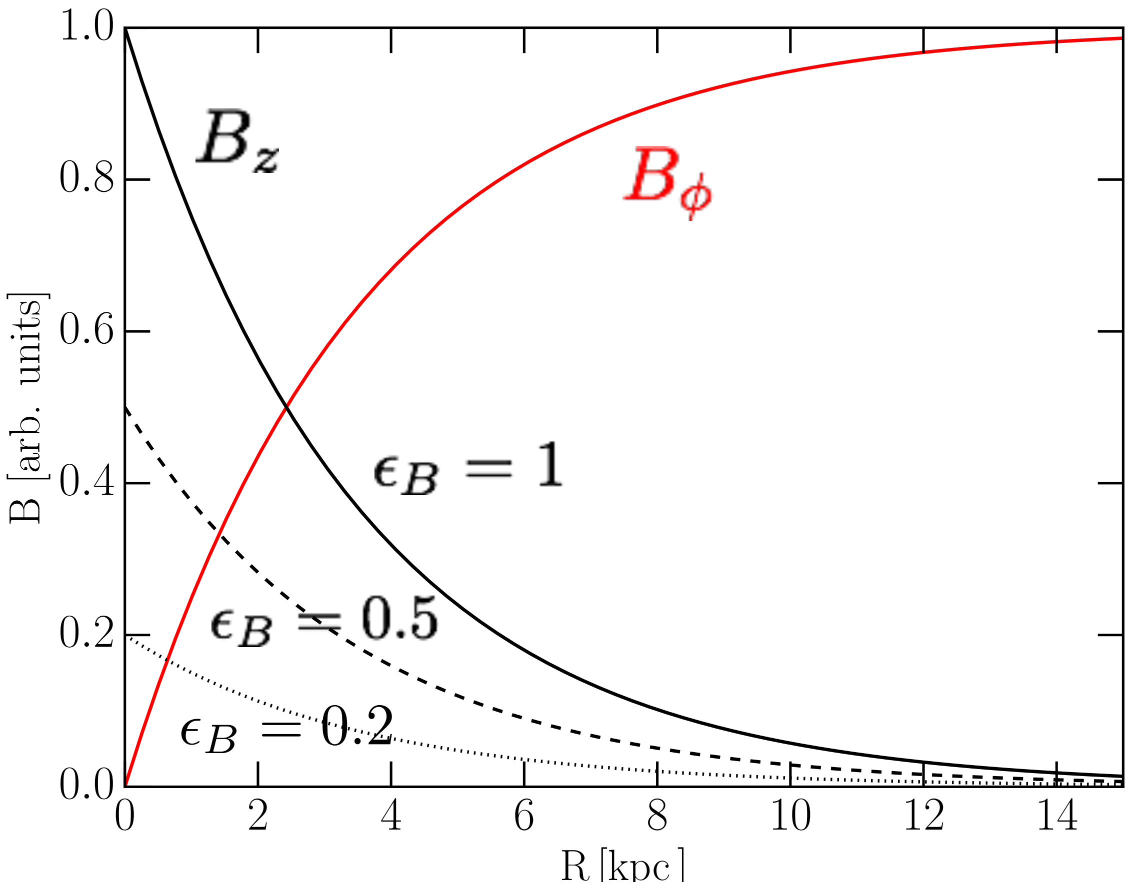


- CRs diffuse/advect in self-generated Alfvén waves below ~ 50 GeV. One can have a harder CR spectrum if advection dominates (which is the case in the inner Galaxy)
- This effect is expected to be present only below 50 GeV, while our model predicts an hardening that extends also to higher energies.

Study of a toy model

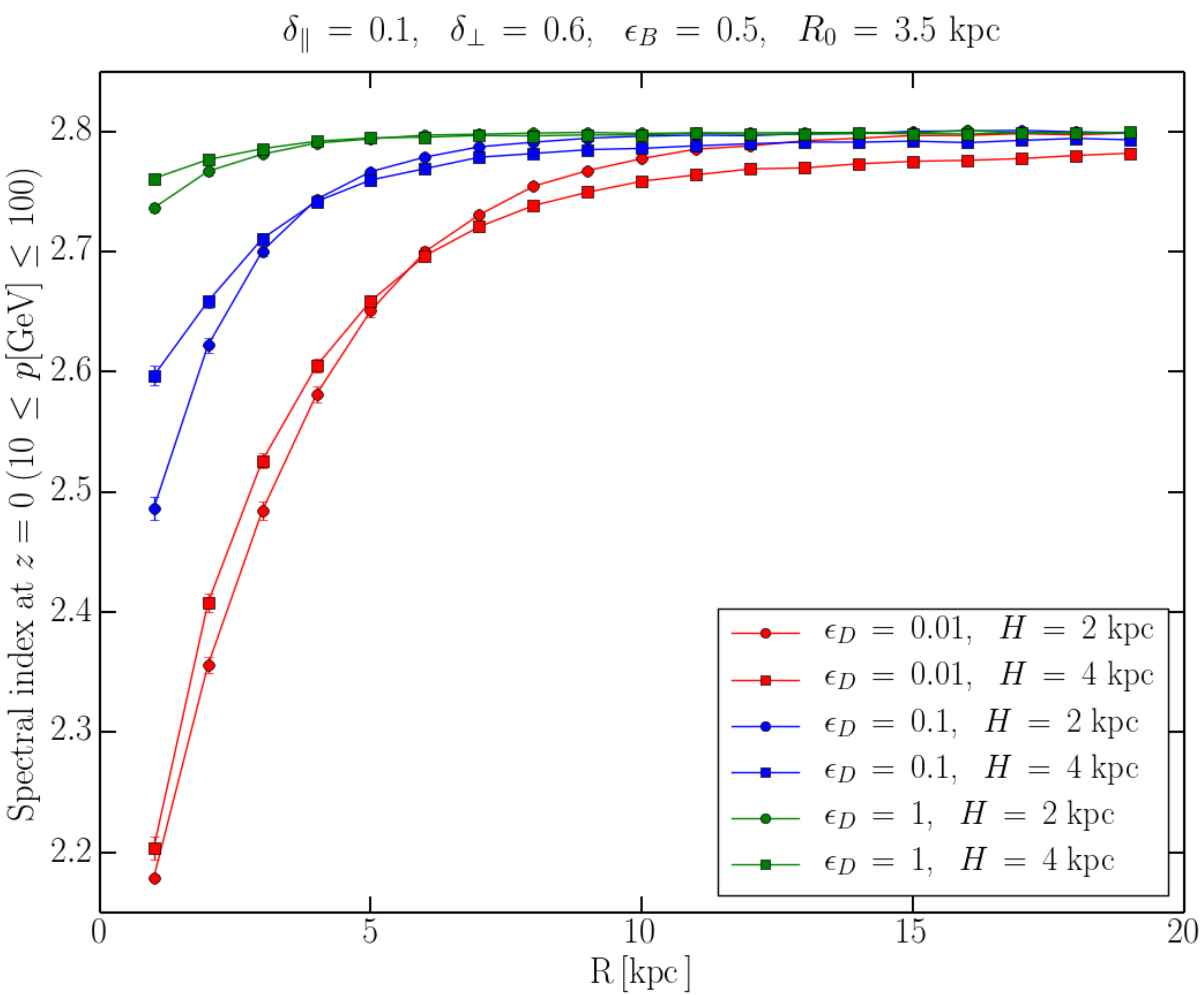
We start by considering a toy model for the Galactic magnetic field:

$$
\begin{aligned}
B_r &= 0 \\
B_\phi &= B_{0,\phi}\left(1 - e^{-r/R_0}\right) \\
B_z &= B_{0,z}\, e^{-r/R_0} \equiv \epsilon_B\, B_{0,\phi}\, e^{-r/R_0}
\end{aligned}
$$

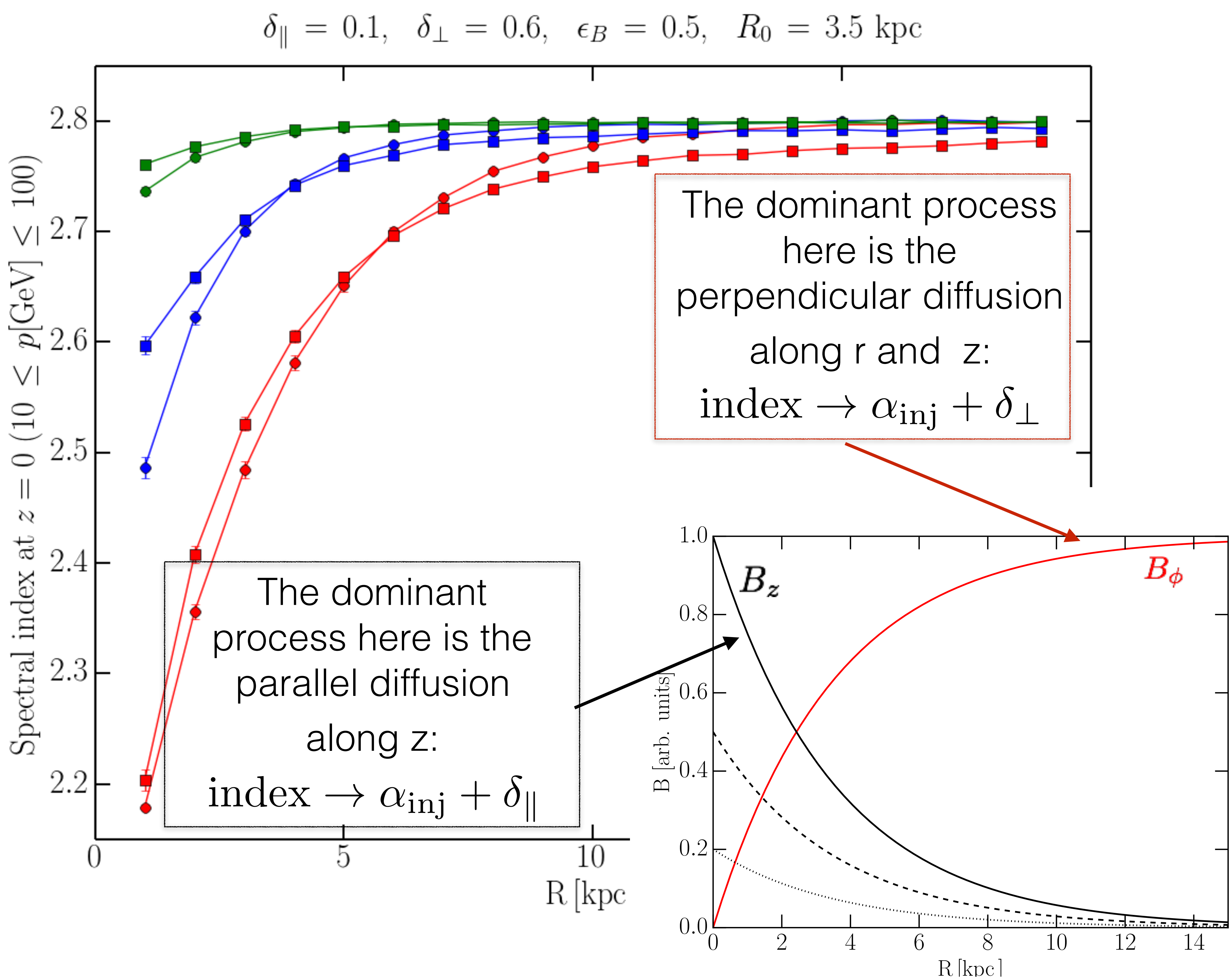


Even if our propagation setup is two-dimensional, **the magnetic field model is three-dimensional** (B_ϕ enters in the determination of $|\vec{B}|$)

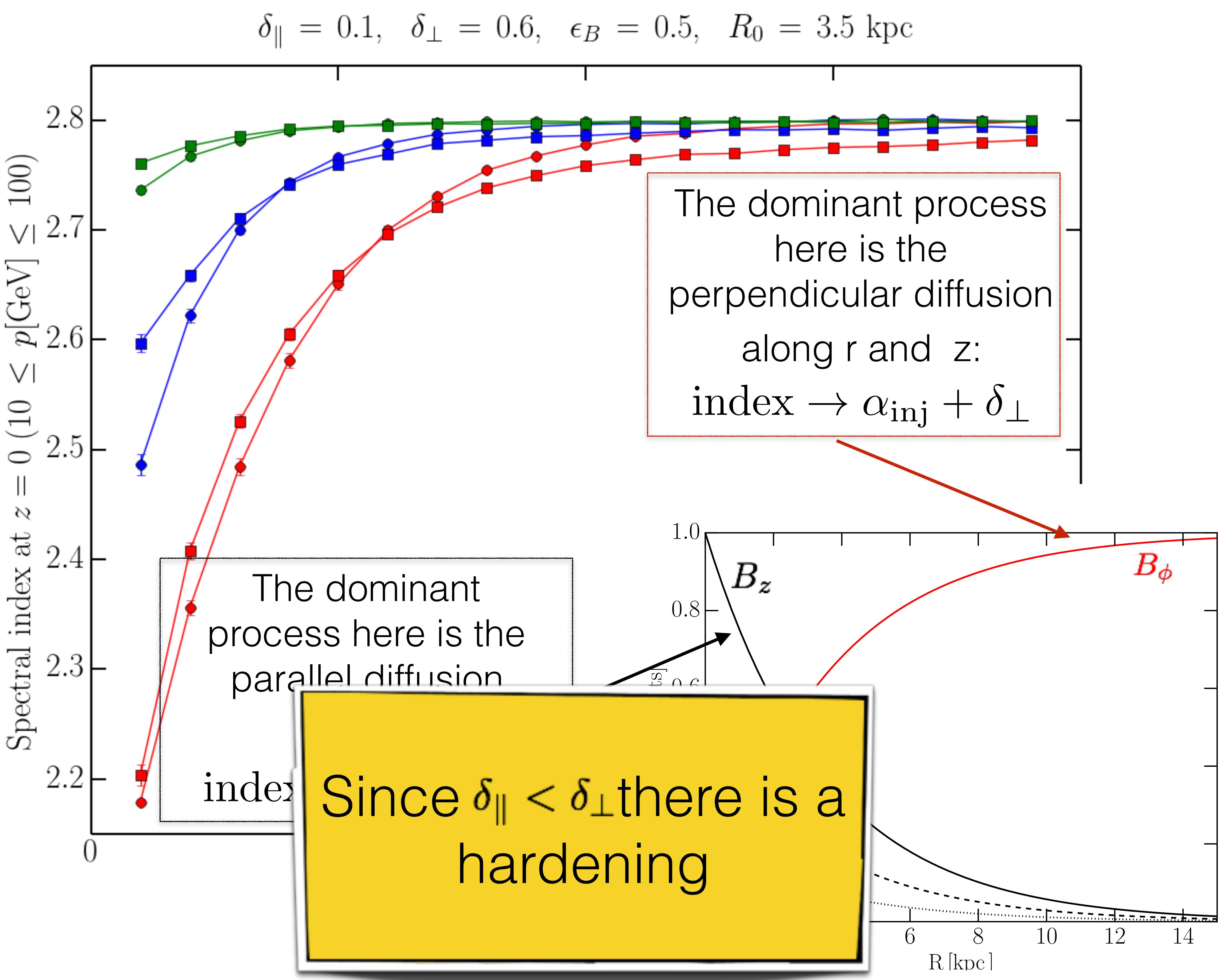
Study of a toy model



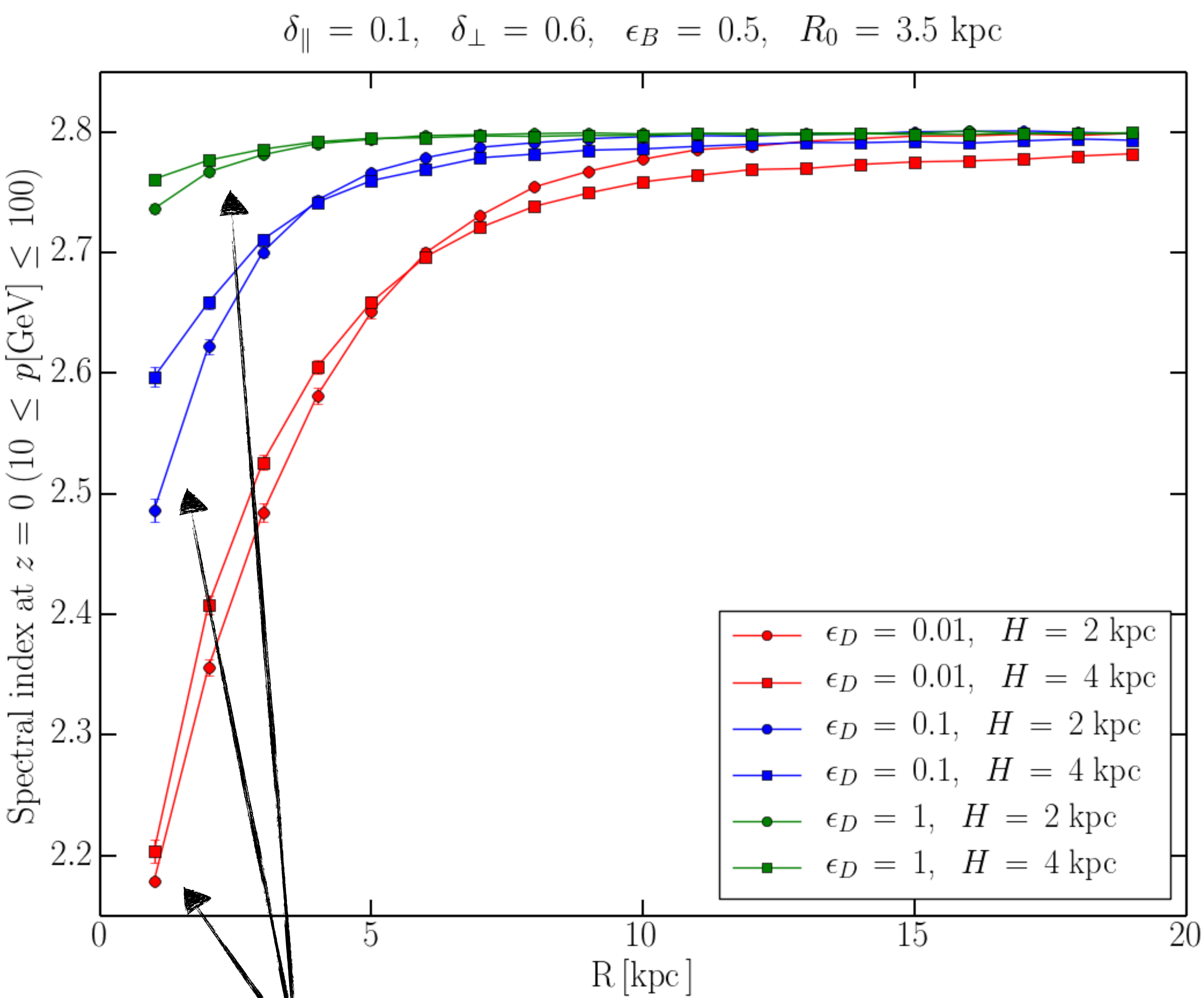
Study of a toy model



Study of a toy model



Study of a toy model



$\text{index} = \alpha_{\text{inj}} + \delta_{\parallel}$ only if ϵ_D is small (otherwise perpendicular transport is still important even at small R)