

Evgeny Epelbaum, RUB

XIIIth Quark Confinement and the Hadron Spectrum,  
31 Jul to 6 Aug 2018, Maynooth University

# Chiral effective field theory for few- and many-nucleon systems



# Why (precision) nuclear physics?

After discovery of Higgs boson,

the strong sector remains the only poorly understood part of the SM!

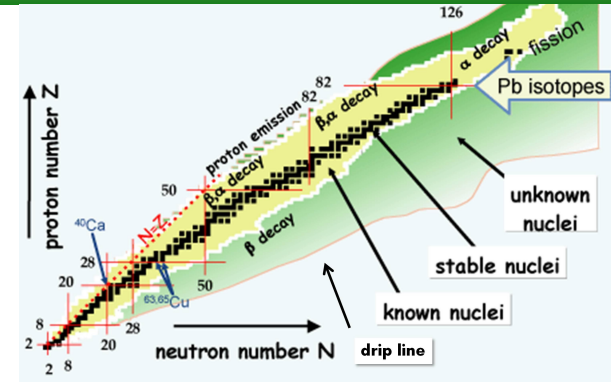
Interesting topic on its own. Some current frontiers:

- the nuclear chart and limits of stability FAIR, GANIL, ISOLDE,...
- EoS for nuclear matter (gravitational waves from n-star mergers) LIGO/Virgo,...
- hypernuclei (neutron stars) JLab, JSI/FAIR, J-PARC, MAMI,...

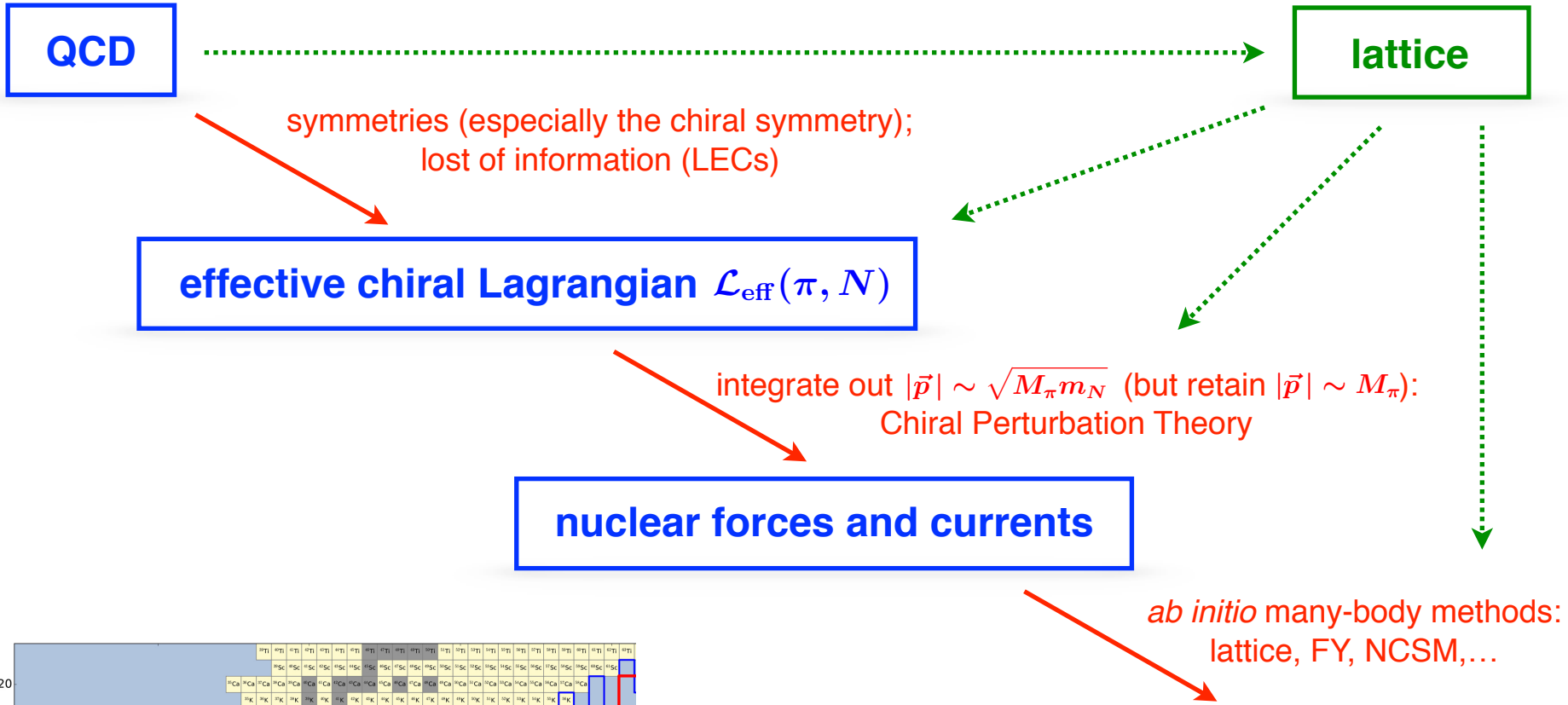
But also highly relevant for searches for BSM physics, e.g.:

- direct Dark Matter searches (WIMP-nucleus scattering)
- searches for  $0\nu\beta\beta$  decays
- searches for nucleon/nuclear EDMs
- proton radius puzzle (complementary experiments with light nuclei...)

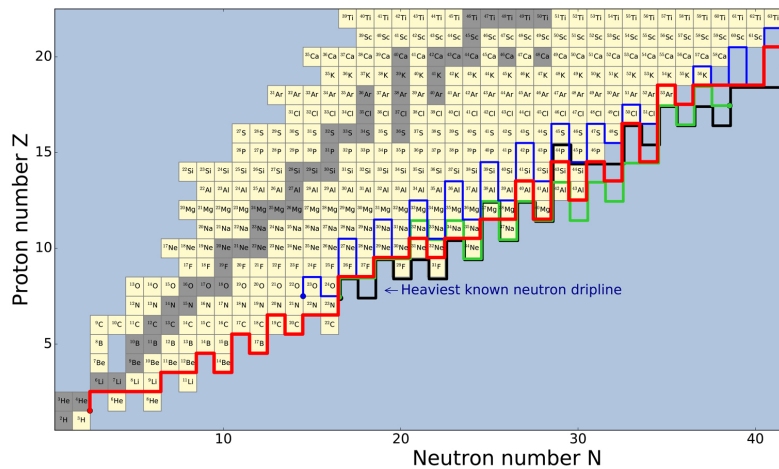
→ need a reliable approach to nuclear structure with quantified uncertainties!



# From QCD to nuclei



**nuclear structure and dynamics**



# Method of UT for nuclear forces

EE, Glöckle, Meißner, NPA 637 (1998) 107; EE, PLB 639 (2006) 456

- Begin with the  $L_{\text{eff}}[\pi, N]$  without external fields

- Canonical formalism:  $L_{\text{eff}}[\pi, N] \rightarrow H[\pi, N] = \text{---}\overset{|}{\bullet}\text{---} + \text{---}\overset{\'{y}'}{\bullet}\text{---} + \dots$



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- Apply **UT in Fock space** to decouple purely nucleonic states [model space] from the rest

$$H \rightarrow \tilde{H} = U^\dagger \left( \begin{array}{c|c} \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right) U = \begin{pmatrix} \tilde{H}_{\text{nucl}} & 0 \\ 0 & \tilde{H}_{\text{rest}} \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{\eta\text{-space } \lambda\text{-space}}$

Using Okubo's minimal parametrization of U in terms of  $A = \lambda A \eta$  leads to the

decoupling equation:  $\lambda(H - [A, H] - AHA)\eta = 0$

which is solved perturbatively employing the chiral expansion

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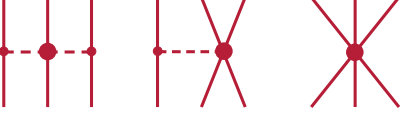

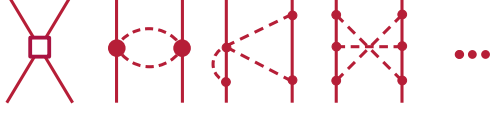

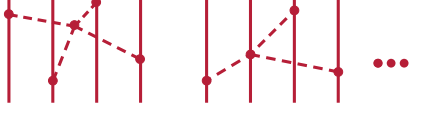
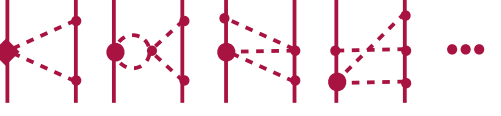


decoupling equation:  $\lambda(H - [A, H] - AHA)\eta = 0$

which is solved perturbatively employing the chiral expansion

- Apply all possible **additional UTs on the  $\eta$ -subspace** consistent with a given chiral order [6 angles  $\alpha_i$  for static N<sup>3</sup>LO contributions]
- **Renormalizability** of the potentials [all 1/(d-4) poles must be canceled by the c.t. from  $L_{\text{eff}}$ ]  
→ fixes some of the  $\alpha_i$  and leads to unique (static) expressions

For more details see: EE, *Nuclear Forces from Chiral Effective Field Theory: A Primer*, arXiv:1001.3229[nucl-th]

# Chiral expansion of nuclear forces [W-counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO ( $Q^0$ )			
	Weinberg '90		
NLO ( $Q^2$ )			
	Ordonez, van Kolck '92		
N <sup>2</sup> LO ( $Q^3$ )			
	Ordonez, van Kolck '92	van Kolck '94; EE et al. '02	
N <sup>3</sup> LO ( $Q^4$ )			
	Kaiser '00 - '02	Bernard, EE, Krebs, Meißner, '08, '11	EE '06
N <sup>4</sup> LO ( $Q^5$ )			
	Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12, '13 (short-range loop contrib. still missing)	still have to be worked out

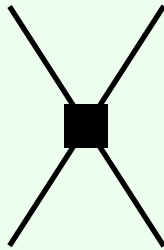
— A similar program is being pursued for in chiral EFT with explicit  $\Delta(1232)$  DOF

# Application 1: A new generation of chiral NN potentials

- semi-local, coordinate-space-regularized up to  $N^4LO$   
EE, Krebs, Meißner, EPJA 51 (2015) 53; PRL 115 (2015) 122301
- semi-local, momentum-space-regularized up to  $N^4LO^+$   
Reinert, Krebs, EE, EPJA 54 (2018) 88
- nonlocal, momentum-space-regularized up to  $N^4LO^+$   
Entem, Machleidt, Nosyk, PRC 96 (2017) 024004

# The long and short of nuclear forces

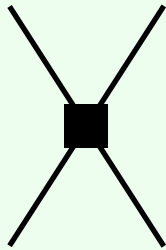
- Short-range interactions have to be tuned to experimental data. In the isospin limit, one has according to NDA:



LO [ $Q^0$ ]:	2 operators (S-waves)
NLO [ $Q^2$ ]:	+ 7 operators (S-, P-waves and $\varepsilon_1$ )
N <sup>2</sup> LO [ $Q^3$ ]:	no new terms
N <sup>3</sup> LO [ $Q^4$ ]:	+ 12 operators (S-, P-, D-waves and $\varepsilon_1, \varepsilon_2$ )
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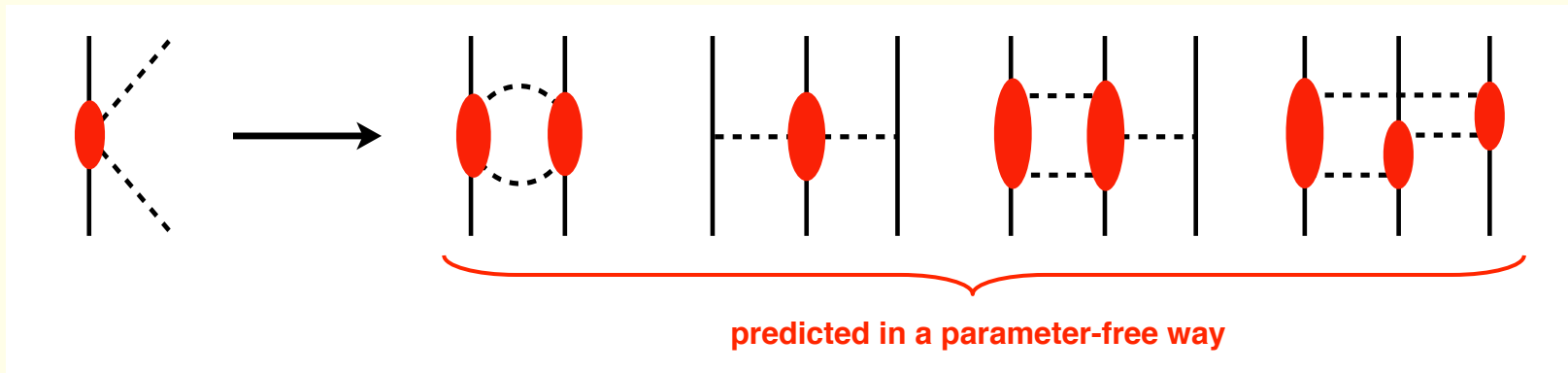
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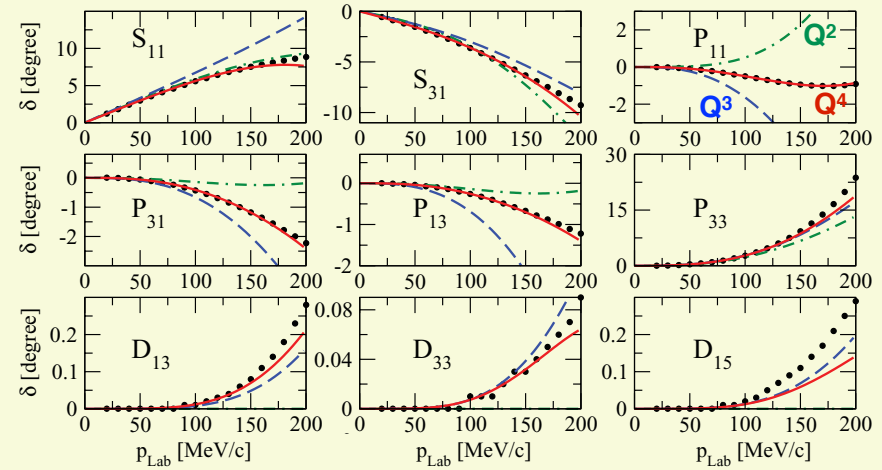
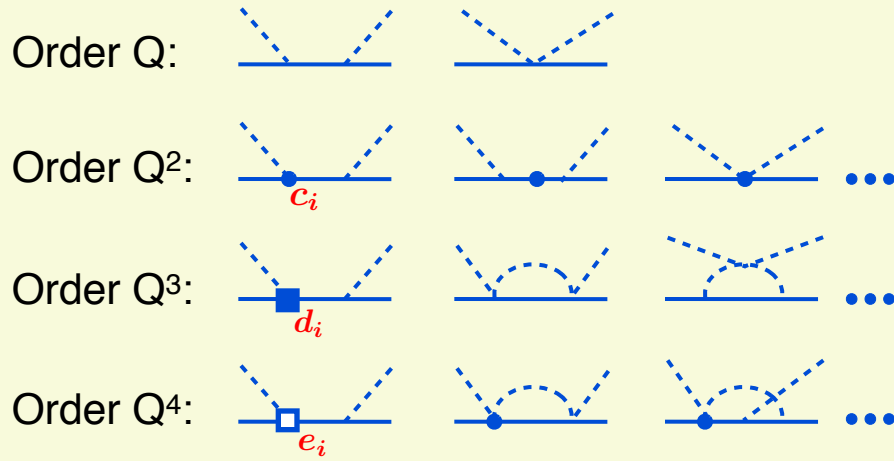
- The long-range part of nuclear forces and currents is **completely determined** by the chiral symmetry of QCD + experimental information on  $\pi N$  scattering



# Determination of $\pi N$ LECs

## Pion-nucleon scattering up to $Q^4$ in heavy-baryon ChPT

Fettes, Meißner '00; Krebs, Gasparyan, EE '12

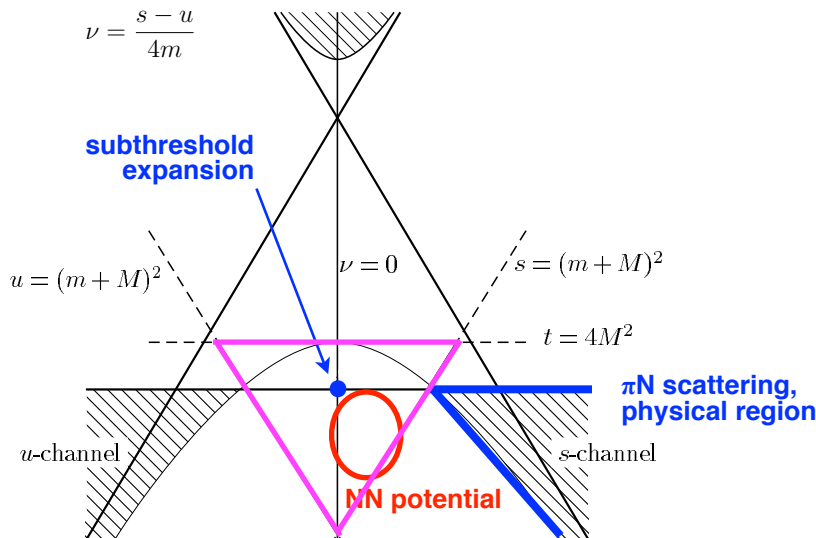
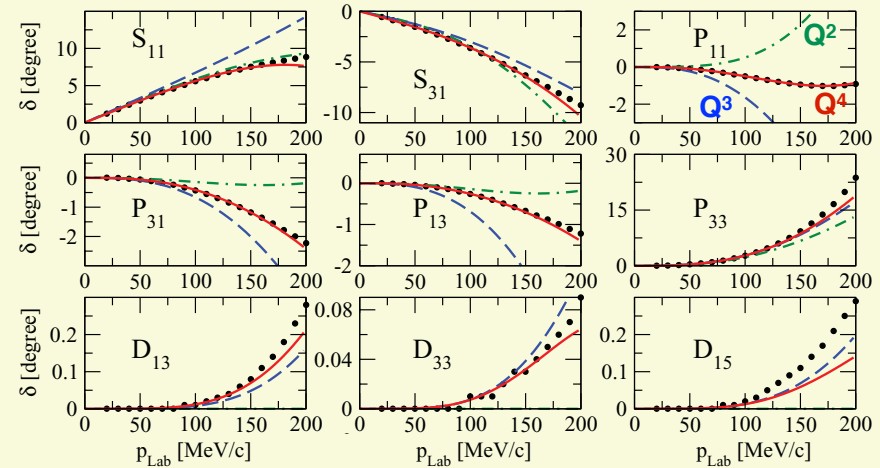
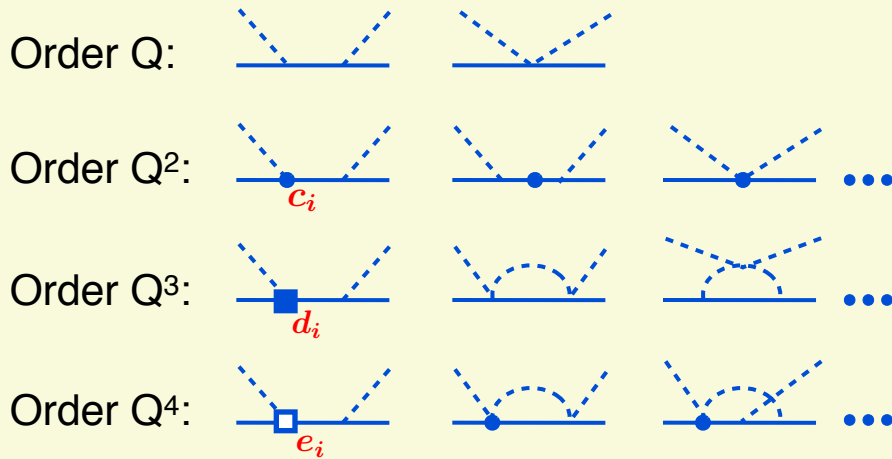




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## Matching ChPT to $\pi N$ Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- $\chi$  expansion of the  $\pi N$  amplitude expected to converge best within the Mandelstam triangle
- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT

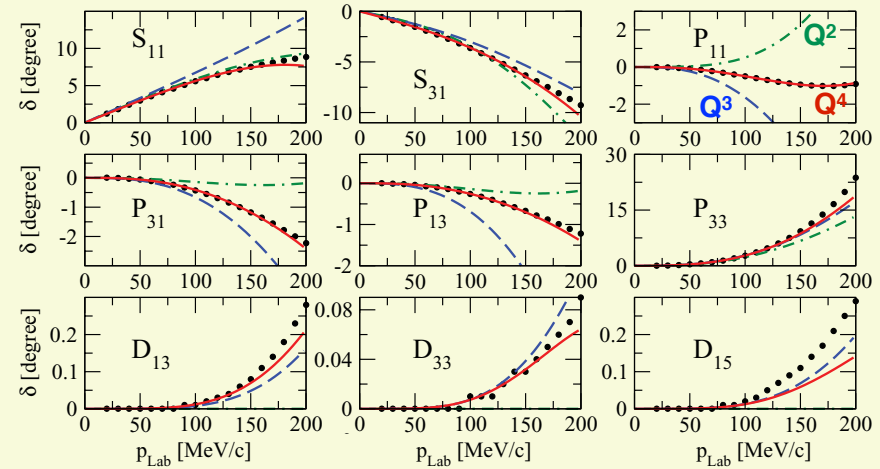
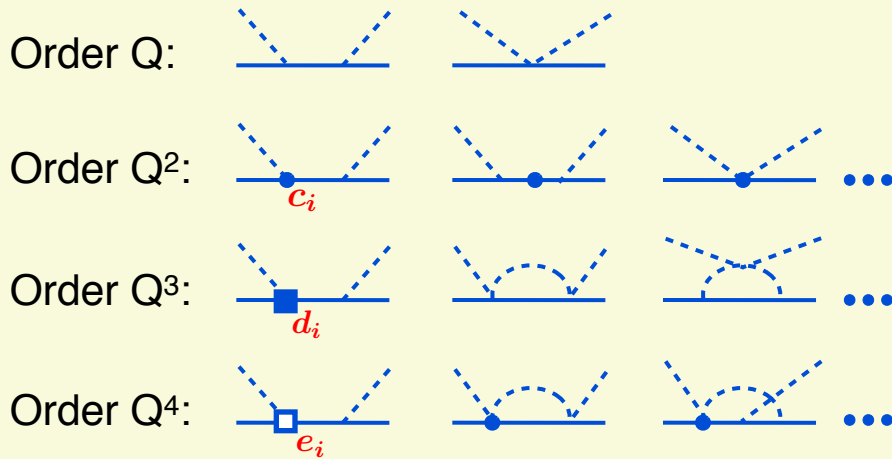
$$\bar{X} = \sum_{m,n} x_{mn} \nu^{2m+k} t^n, \quad X = \{A^\pm, B^\pm\}$$

- Closer to the kinematics relevant for nuclear forces...

# Determination of $\pi N$ LECs

## Pion-nucleon scattering up to $Q^4$ in heavy-baryon ChPT

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## Relevant LECs (in $\text{GeV}^{-n}$ ) extracted from $\pi N$ scattering

	$c_1$	$c_2$	$c_3$	$c_4$	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$
$[Q^4]_{\text{HB, NN}}$ , GW PWA	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-0.58
$[Q^4]_{\text{HB, NN}}$ , KH PWA	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-0.37
$[Q^4]_{\text{HB, NN}}$ , Roy-Steiner	-1.10	3.57	-5.54	4.17	6.18	-8.91	0.86	-12.18	1.18	-0.18
$[Q^4]_{\text{covariant}}$ , data	-0.82	3.56	-4.59	3.44	5.43	-4.58	-0.40	-9.94	-0.63	-0.90

Krebs, Gasparyan, EE, PRC85 (12) 054006

Hoferichter et al., PRL 115 (15) 092301

Siemens et al., PRC94 (16) 014620

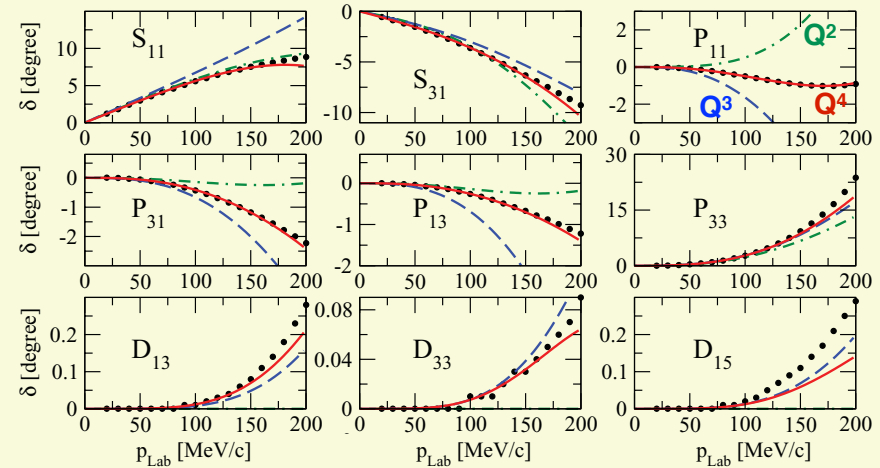
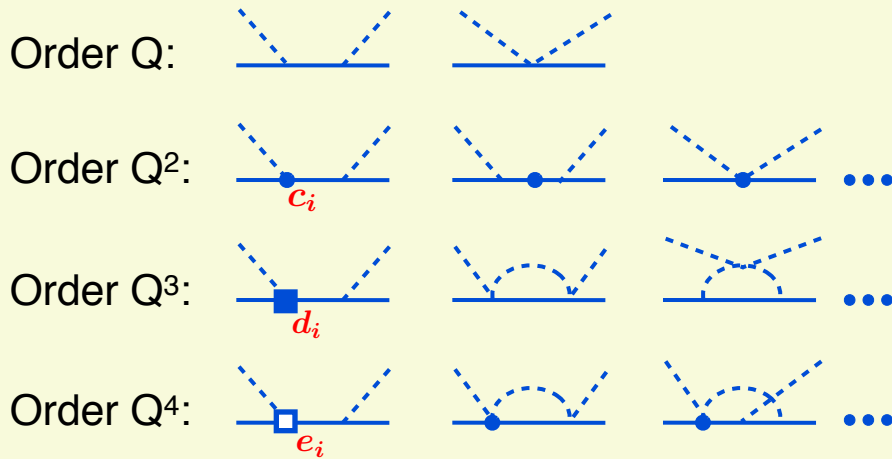
### Notice:

- some LECs show sizable correlations (especially  $c_1$  and  $c_3$ )...
- KH PWA and Roy-Steiner LECs lead to comparable results in the NN sector

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With the LECs taken from  $\pi N$ , the long-range NN force is completely fixed (parameter-free)

# Regularization

The cutoff  $\Lambda$  has to be kept finite,  $\Lambda \sim \Lambda_b$  (unless all counterterms are taken into account in the calculations) [Lepage '97; EE, Gegelia '09]. In practice, low values of  $\Lambda$  are preferred:

- many-body methods require soft interactions,
- spurious deeply-bound states for  $\Lambda > \Lambda^{\text{crit}}$  make calculations for  $\Lambda > 3$  unfeasible...
  - it is crucial to employ a regulator that minimizes finite- $\Lambda$  artifacts!

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**Nonlocal:**  $V_{1\pi}^{\text{reg}} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^2 + M_\pi^2} \longrightarrow \frac{1}{\vec{q}^2 + M_\pi^2} \underbrace{\left(1 - \frac{p'^4 + p^4}{\Lambda^4} + \mathcal{O}(\Lambda^{-8})\right)}_{\text{affect long-range interactions...}}$

EE, Glöckle, Meißner '04;  
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**Local:**  $V_{1\pi}^{\text{reg}} \propto \frac{e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}}}{\vec{q}^2 + M_\pi^2} \longrightarrow \frac{1}{\vec{q}^2 + M_\pi^2} \left(1 + \text{short-range terms}\right)$

[inspired by Thomas Rijken] Reinert, Krebs, EE '18;

→ does not affect long-range physics at any order in  $1/\Lambda^2$ -expansion

- Application to  $2\pi$  exchange does not require re-calculating the corresponding diagrams:

$$V(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{q^2 + \mu^2} + \dots \xrightarrow{\text{reg.}} V_\Lambda(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{q^2 + \mu^2} e^{-\frac{\mu^2}{2\Lambda^2}} + \dots$$

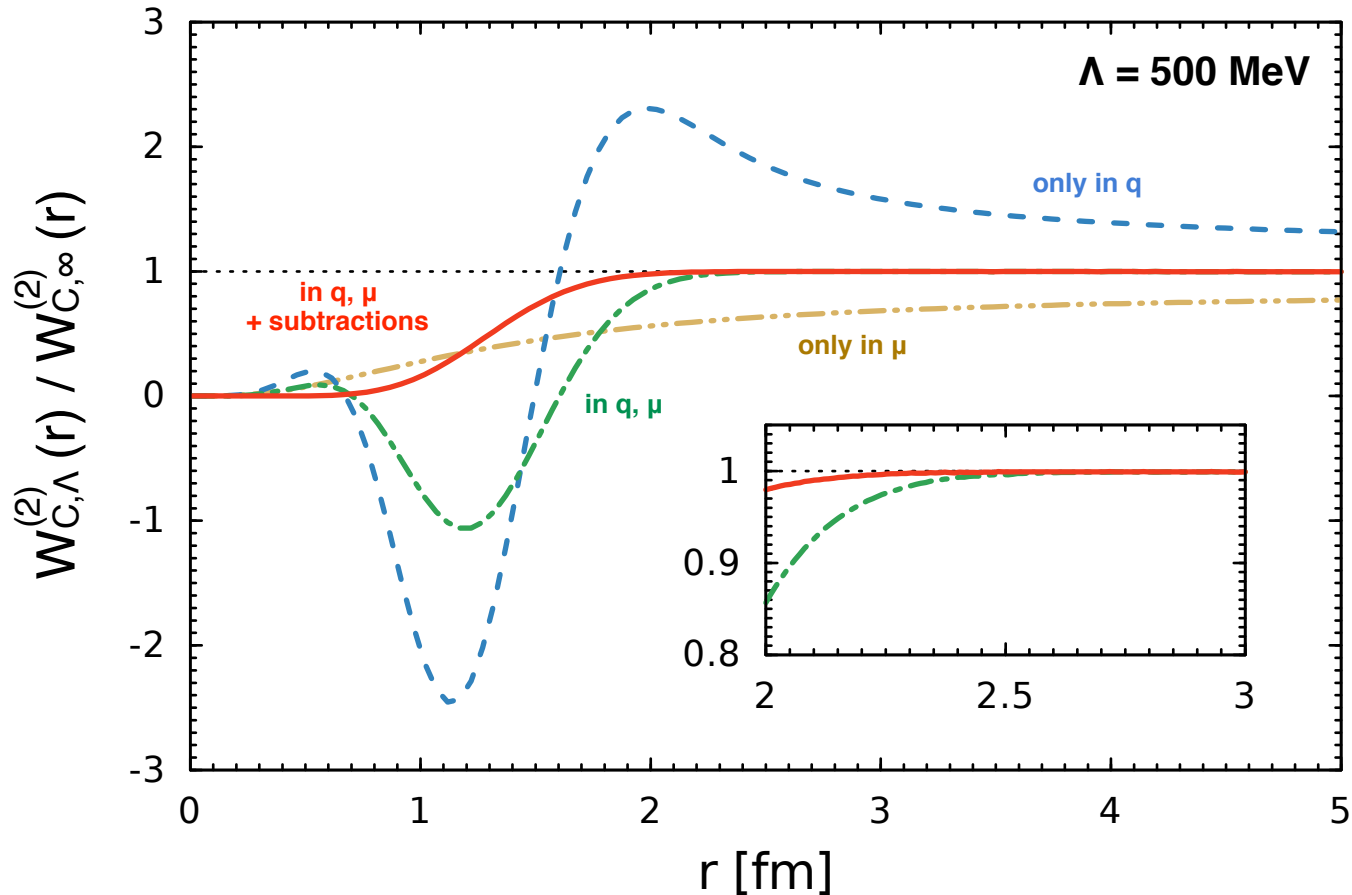
*polynomial in  $q^2, M_\pi$*

- Convention: choose polynomial terms such that  $\Delta^n V_{\Lambda, \text{long}}(\vec{r})|_{r=0} = 0$

# Regularization

Regularized  $2\pi$ -exchange potential: 
$$W_{C,\Lambda}(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{q^2 + \mu^2} e^{-\frac{\mu^2}{2\Lambda^2}}$$

## Various regularization approaches



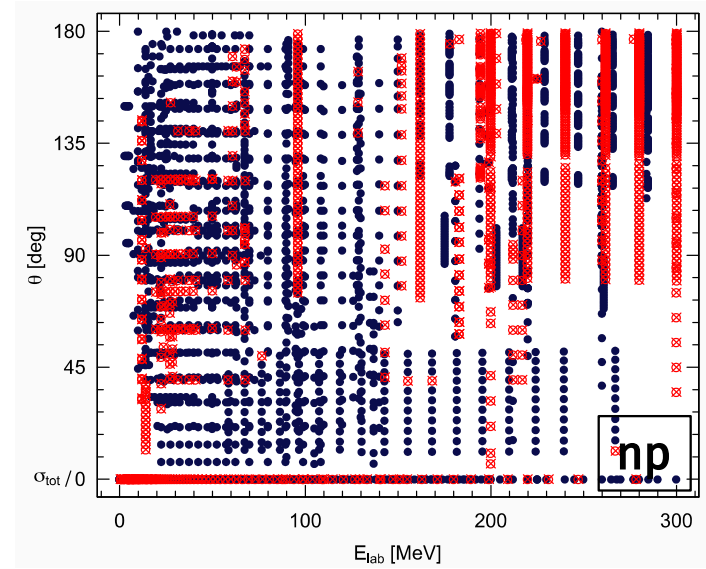
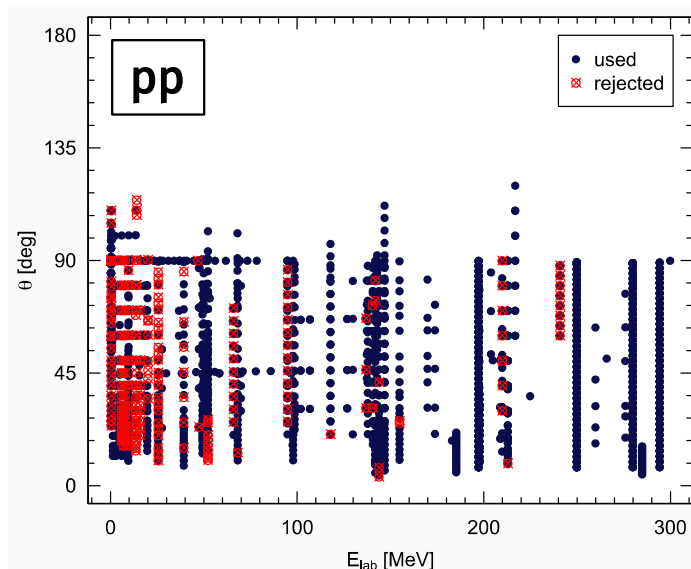
Does it matter in practice?



# NN data analysis

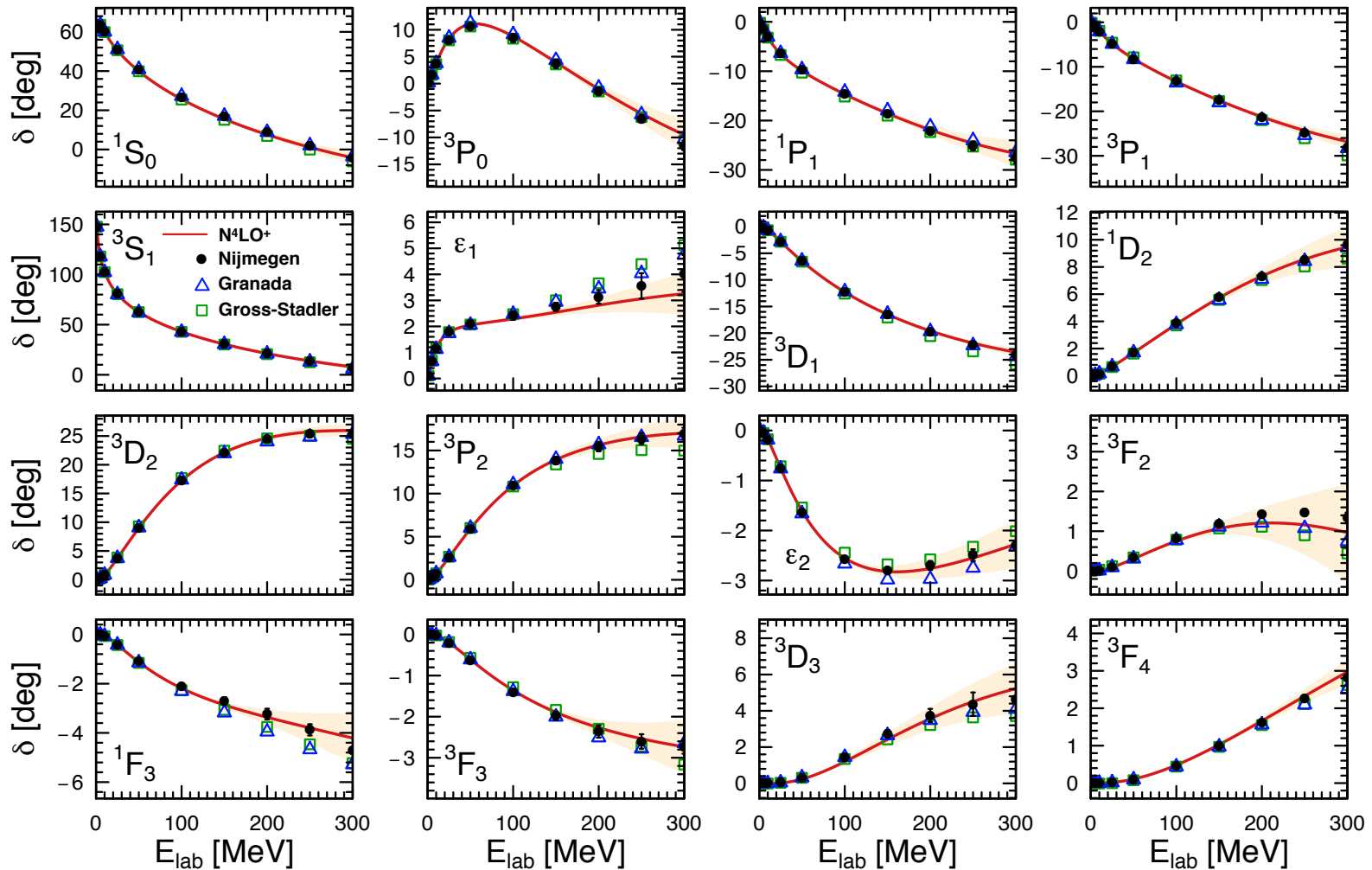
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

- Use local/nonlocal regulator for long-range/short-range contributions
- To fix NN contact interactions, use scattering data together with  $B_d = 2.224575(9)$  MeV and  $b_{np} = 3.7405(9)$  fm.
- Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured.
- However, certain data are mutually incompatible within errors and have to be rejected.  
2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:  
2158 proton-proton + 2697 neutron-proton data below  $E_{lab} = 300$  MeV



# State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

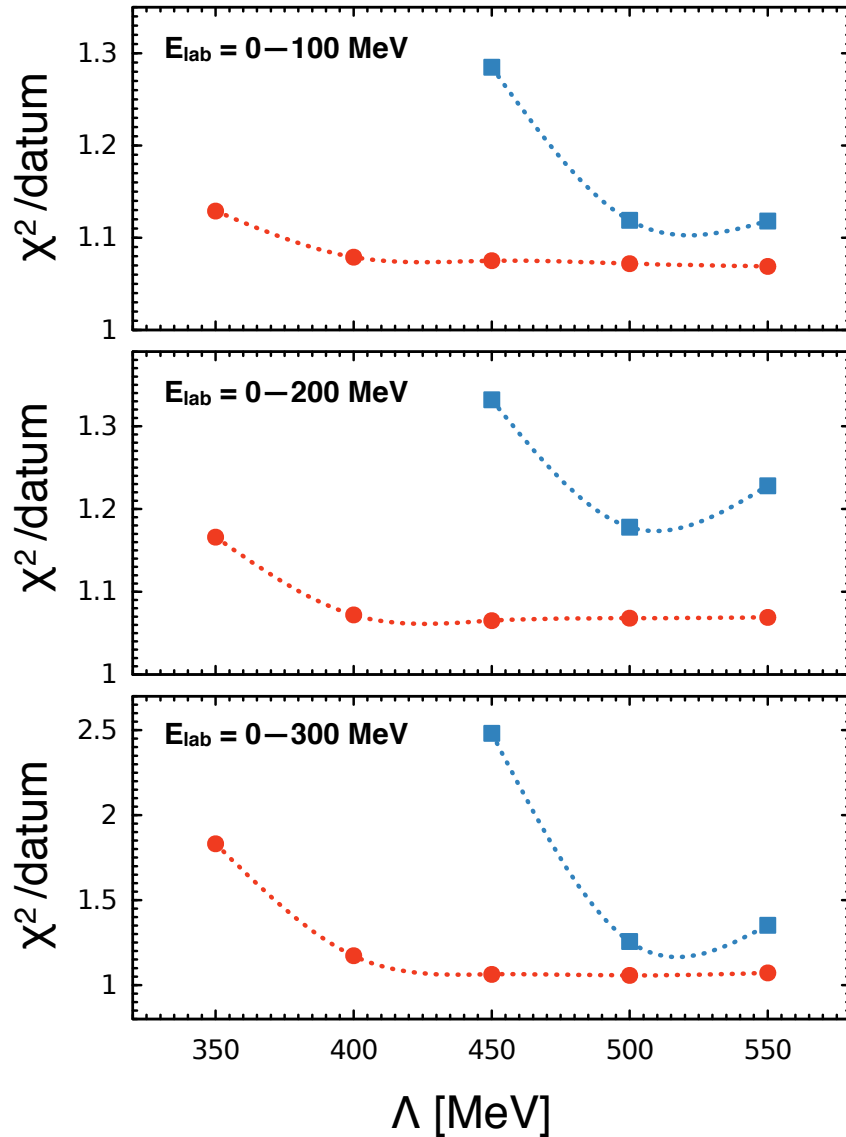


- $N^4\text{LO}^+$  yields currently the best description of the 2013 Granada database ( $E_{\text{lab}} < 300$  MeV)
- 40% less parameters (27+1) compared to high-precision potentials
- Clear evidence of the parameter-free chiral  $2\pi$  exchange

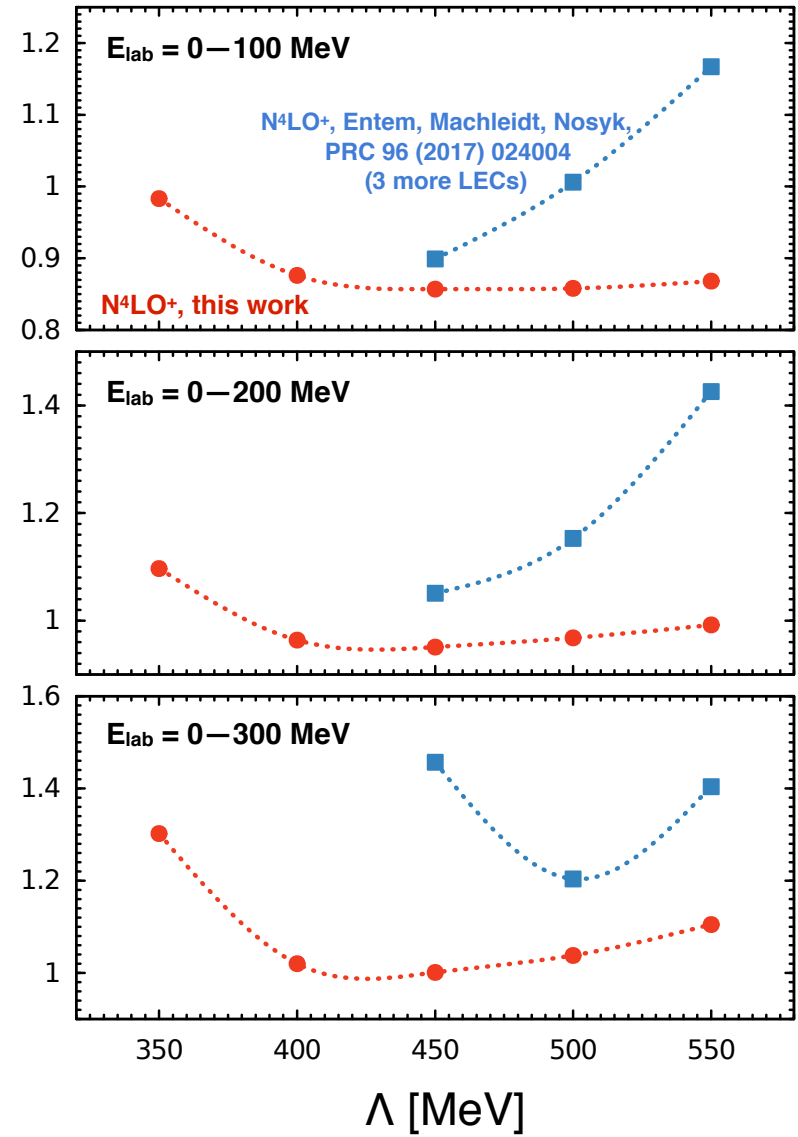
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## neutron-proton data



## proton-proton data



# Error analysis

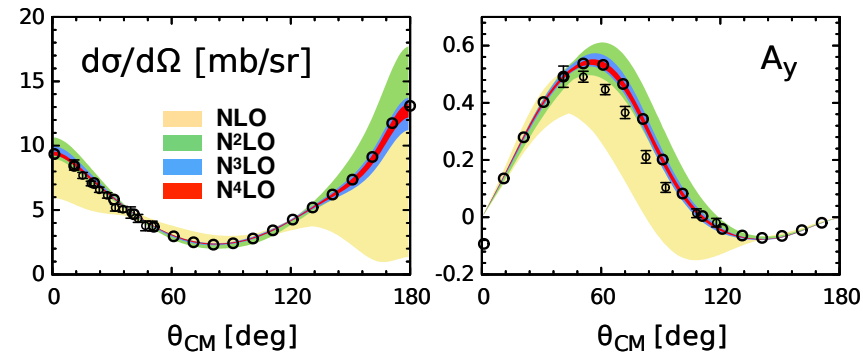
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## 1. Truncation error EE, Krebs, Meißner, EPJA 51 (2015) 53

$$X^{(i)}(p) = X^{(0)} + \underbrace{\Delta X^{(2)}}_{\sim Q^2 X^{(0)}} + \dots + \underbrace{\Delta X^{(i)}}_{\sim Q^i X^{(0)}}$$

$$\text{Expansion parameter: } Q = \max \left\{ \underbrace{\frac{p}{\Lambda_b}}_{\simeq 600 \text{ MeV}}, \frac{M_\pi}{\Lambda_b} \right\}$$

## proton-neutron scattering at $E_{\text{lab}}=143 \text{ MeV}$



Use the explicitly calculated  $\Delta X^{(i)}$  to estimate the uncertainty  $\delta X^{(i)}$  at order  $Q^i$ :

$$\left\{ \delta X^{(0)} = Q^2 |X^{(0)}|, \delta X^{(i)} = \max_{2 \leq j \leq i} (Q^{i+1} |X^{(0)}|, Q^{i+1-j} |\Delta X^{(j)}|) \right\} \wedge \delta X^{(i)} \geq \max_{j,k} (|X^{(j \geq i)} - X^{(k \geq i)}|)$$

Has been validated/extended within a Bayesian approach BUQEYE Collaboration, Furnstahl et al., '15 - '18

# Error analysis

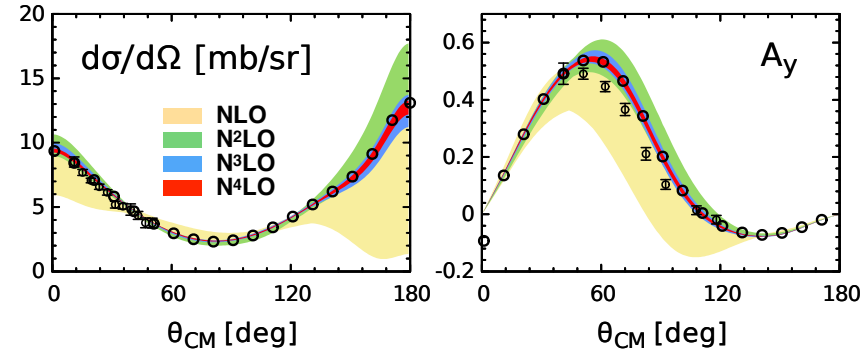
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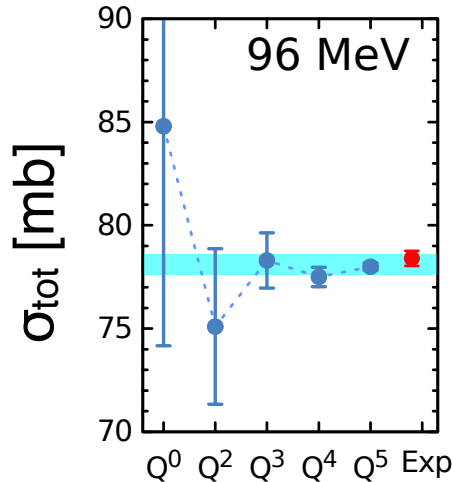
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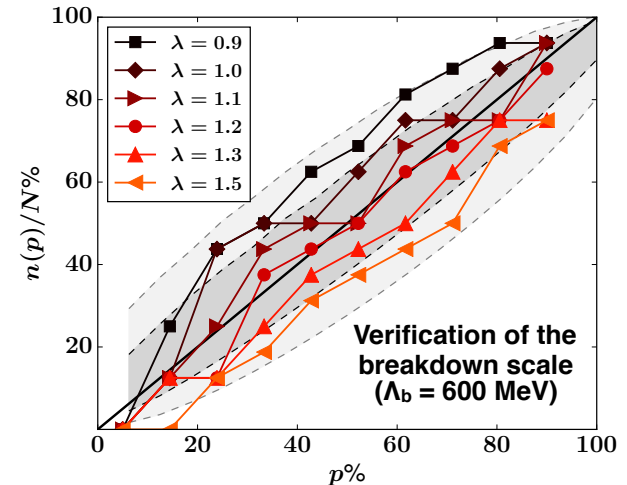
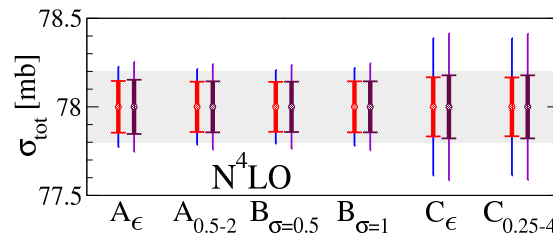
$$\left\{ \delta X^{(0)} = Q^2 |X^{(0)}|, \delta X^{(i)} = \max_{2 \leq j \leq i} (Q^{i+1} |X^{(0)}|, Q^{i+1-j} |\Delta X^{(j)}|) \right\} \wedge \delta X^{(i)} \geq \max_{j,k} (|X^{(j \geq i)} - X^{(k \geq i)}|)$$

## Example: np total cross section at 96 MeV



— calculations based on the  
EE et al., PRL 115 (2015) 122301

— Bayesian analysis [BUQEYE],  
Furnstahl et al., PRC92 (15) 024005

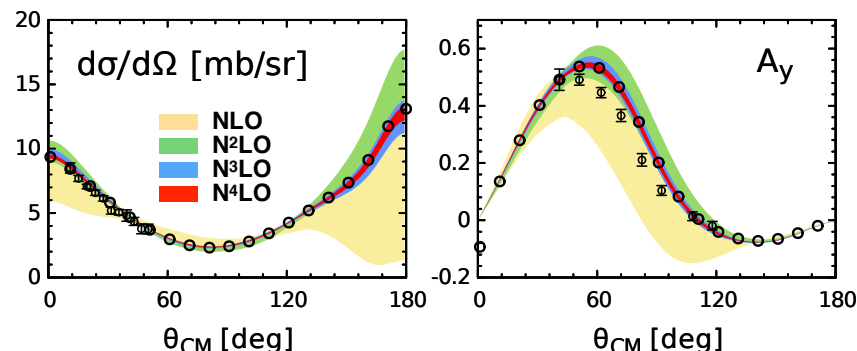


## 1. Truncation error EE, Krebs, Meißner, EPJA 51 (2015) 53

$$X^{(i)}(p) = X^{(0)} + \underbrace{\Delta X^{(2)}}_{\sim Q^2 X^{(0)}} + \dots + \underbrace{\Delta X^{(i)}}_{\sim Q^i X^{(0)}}$$

$$\text{Expansion parameter: } Q = \max \left\{ \underbrace{\frac{p}{\Lambda_b}}_{\simeq 600 \text{ MeV}}, \frac{M_\pi}{\Lambda_b} \right\}$$

## proton-neutron scattering at $E_{\text{lab}}=143 \text{ MeV}$



Use the explicitly calculated  $\Delta X^{(i)}$  to estimate the uncertainty  $\delta X^{(i)}$  at order  $Q^i$ :

$$\left\{ \delta X^{(0)} = Q^2 |X^{(0)}|, \delta X^{(i)} = \max_{2 \leq j \leq i} \left( Q^{i+1} |X^{(0)}|, Q^{i+1-j} |\Delta X^{(j)}| \right) \right\} \wedge \delta X^{(i)} \geq \max_{j,k} \left( |X^{(j \geq i)} - X^{(k \geq i)}| \right)$$

Has been validated/extended within a Bayesian approach BUQEYE Collaboration, Furnstahl et al., '15 - '18

## 2. Statistical uncertainties

Estimated in the standard way using the covariance matrix (quadratic approximation)

## 3. Uncertainties due to $\pi N$ LECs $\mathbf{c}_{1,2,3,4}$ , $\mathbf{d}_{1,2,3,5,14,15}$ and $\mathbf{e}_{14,17}$

Estimated using 2 sets of  $\pi N$  LECs (Roy-Steiner equation analysis & KH PWA)

## 4. Choice of $E_{\text{max}}$ in the fits

Uncertainty estimated at N<sup>4</sup>LO/N<sup>4</sup>LO+ by performing fits with  $E_{\text{max}} = 220 \dots 300 \text{ MeV}$

# Error analysis

In most cases, **the uncertainty is dominated by truncation errors**. At N<sup>4</sup>LO and at very low energies, other sources of errors become comparable (especially  $\pi$ N LECs...).

**Example: deuteron asymptotic normalizations** (relevant for nuclear astrophysics)

Our determination:

$$A_S = 0.8847_{(-3)}^{(+3)} (3)(5)(1) \text{ fm}^{-1/2}$$

truncation error ———> (3)(5)(1)  
 statistical error ———> (3)(5)(1)  
 $\pi$ N LECs ———> (3)(5)(1)  
 variation of  $E_{\text{max}}$  ———> (3)(5)(1)

$$\eta \equiv \frac{A_D}{A_S} = 0.0255_{(-1)}^{(+1)} (1)(4)(1)$$

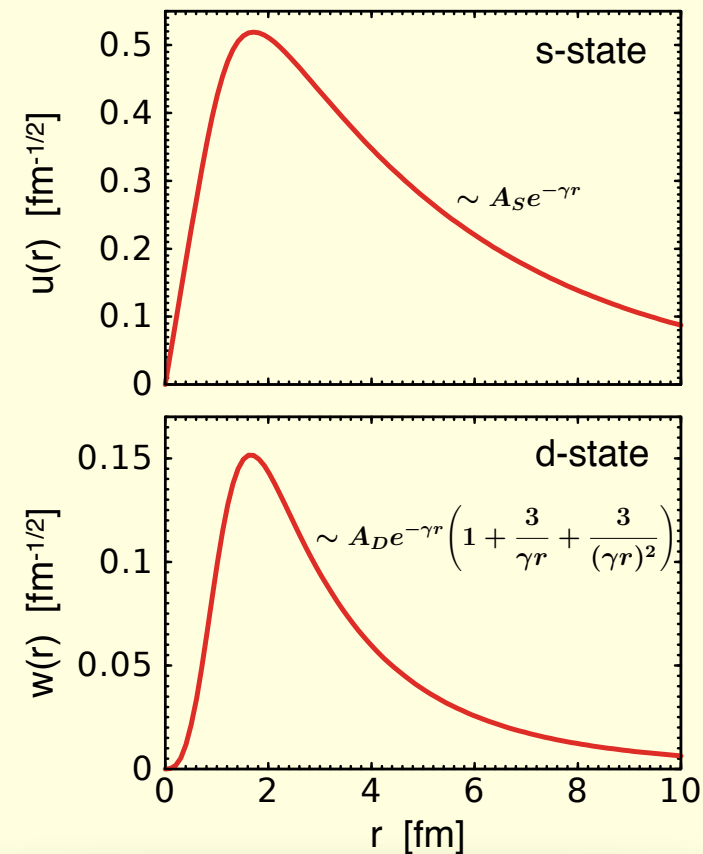
Exp:  $A_S = 0.8781(44) \text{ fm}^{-1/2}$ ,  $\eta = 0.0256(4)$   
Borbely et al. '85 Rodning, Knutson '90

Nijmegen PWA [errors are „educated guesses“] Stoks et al. '95

$$A_S = 0.8845(8) \text{ fm}^{-1/2}, \quad \eta = 0.0256(4)$$

Granada PWA [errors purely statistical] Navarro Perez et al. '13

$$A_S = 0.8829(4) \text{ fm}^{-1/2}, \quad \eta = 0.0249(1)$$





# Applications 2: Beyond the 2N system

— LENPIC Collaboration —

**Goal: precision tests of chiral nuclear forces & currents in light nuclei**

**Strategy: go to high orders, do not compromise the  $\pi$ N LECs, no fine tuning to heavy nuclei, careful error analysis**



**LENPIC: Low Energy Nuclear Physics International Collaboration**

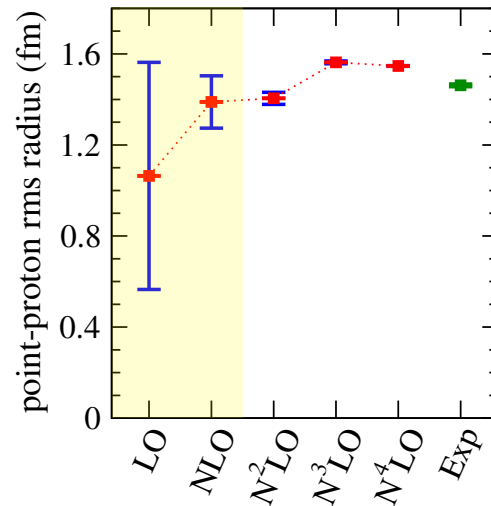
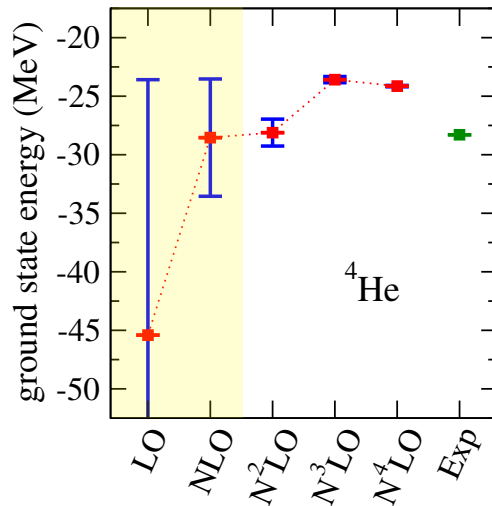


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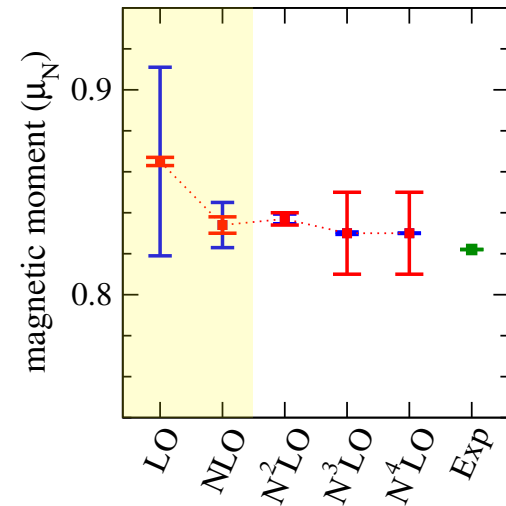
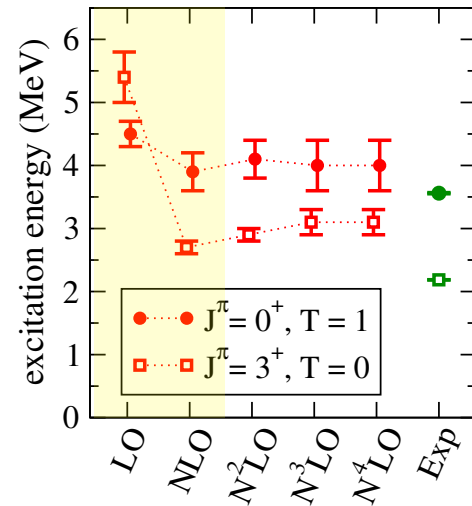
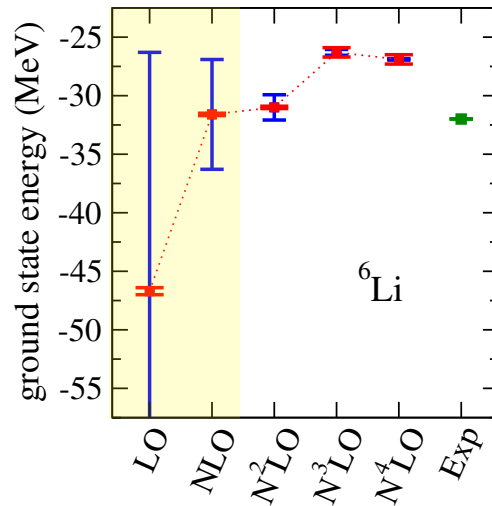


# Light nuclei based on 2NF alone

LENPIC Collaboration (Maris et al.), EPJ Web of Conf. 113 (2016) 04015



Is there any evidence of the missing 3NF?

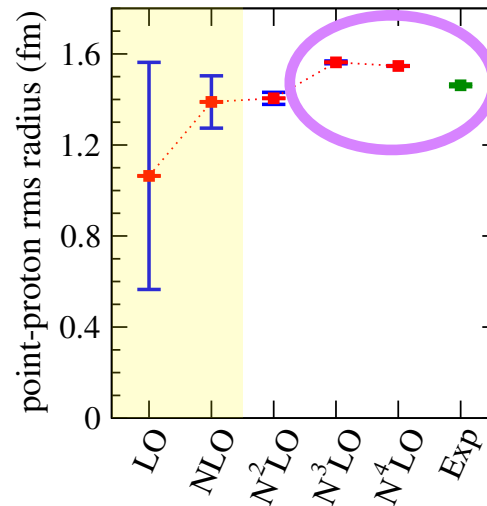
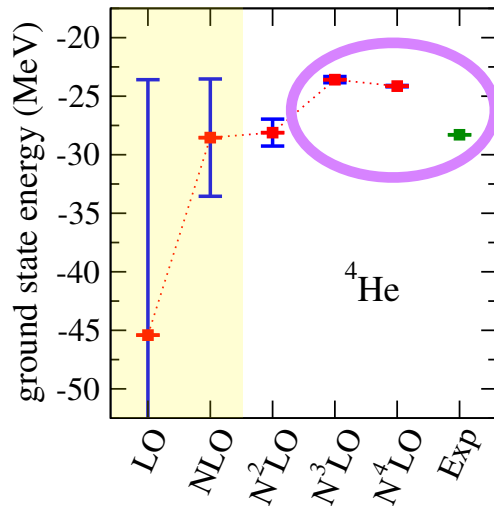


LENPIC: Low Energy Nuclear Physics International Collaboration



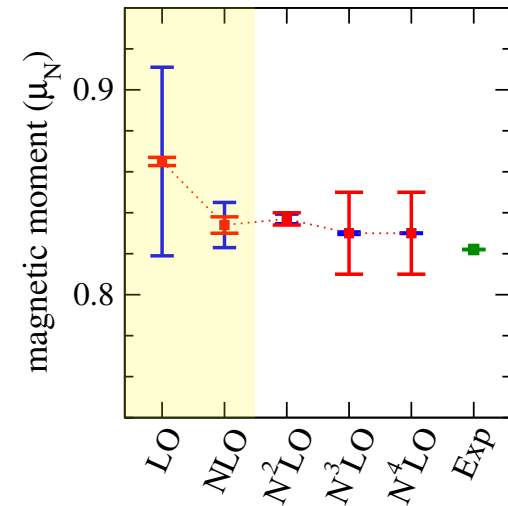
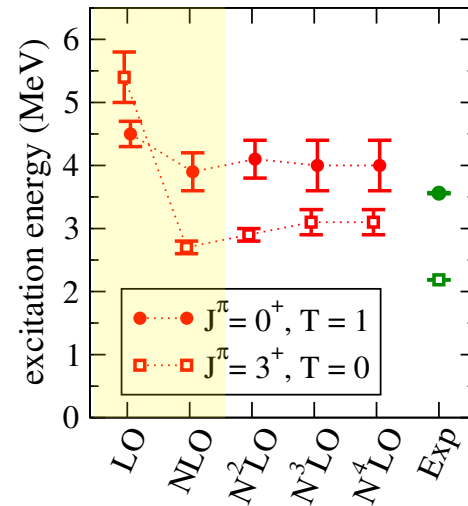
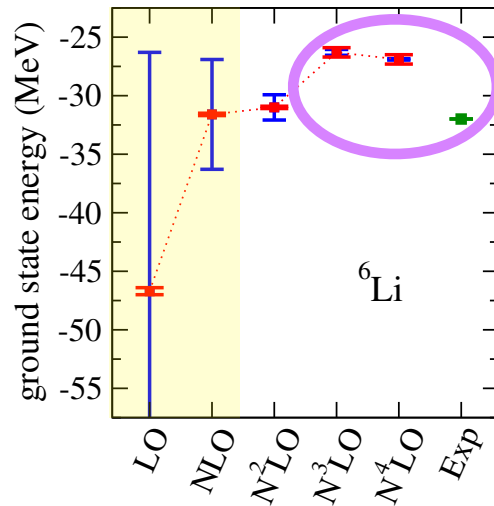
# Light nuclei based on 2NF alone

LENPIC Collaboration (Maris et al.), EPJ Web of Conf. 113 (2016) 04015



Is there any evidence of the missing 3NF?

Deviations from the data are consistent with the estimated size of the 3N forces



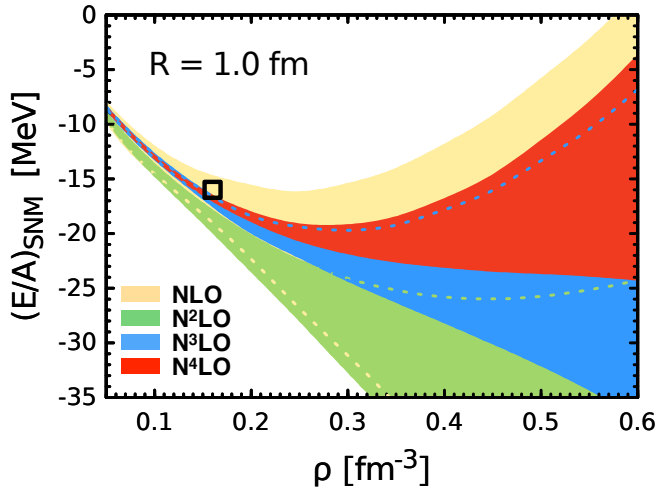
LENPIC: Low Energy Nuclear Physics International Collaboration



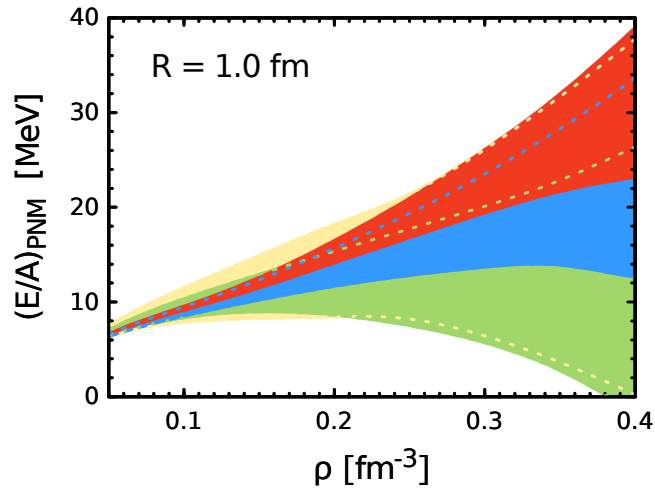
# Brueckner-Hartree-Fock based on 2NF alone

Jinniu Hu, Ying Zhang, EE, Ulf-G. Meißner, Jie Meng, PRC 96 (2017) 034307

## Symmetric nuclear matter



## Pure neutron matter

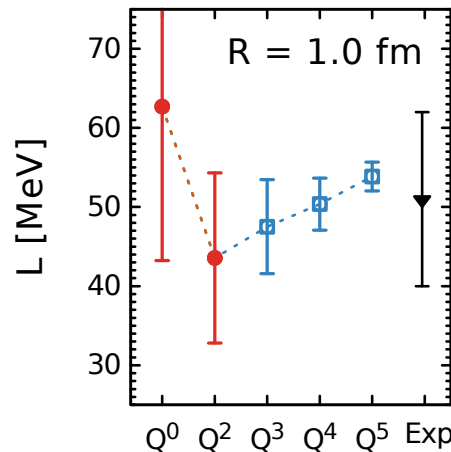
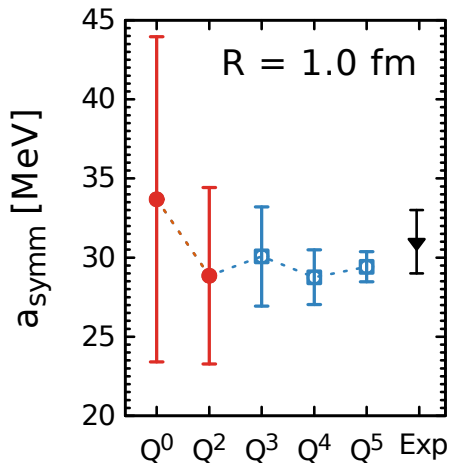


Achievable accuracy at N<sup>4</sup>LO at  $\rho_0$ :

$\pm 0.3 \text{ MeV}$  for SNM,  
 $\pm 0.7 \text{ MeV}$  for PNM,

semi-quantitative up to  $\sim 2\rho_0 \dots$

## Symmetry energy and the slope parameter at the saturation density

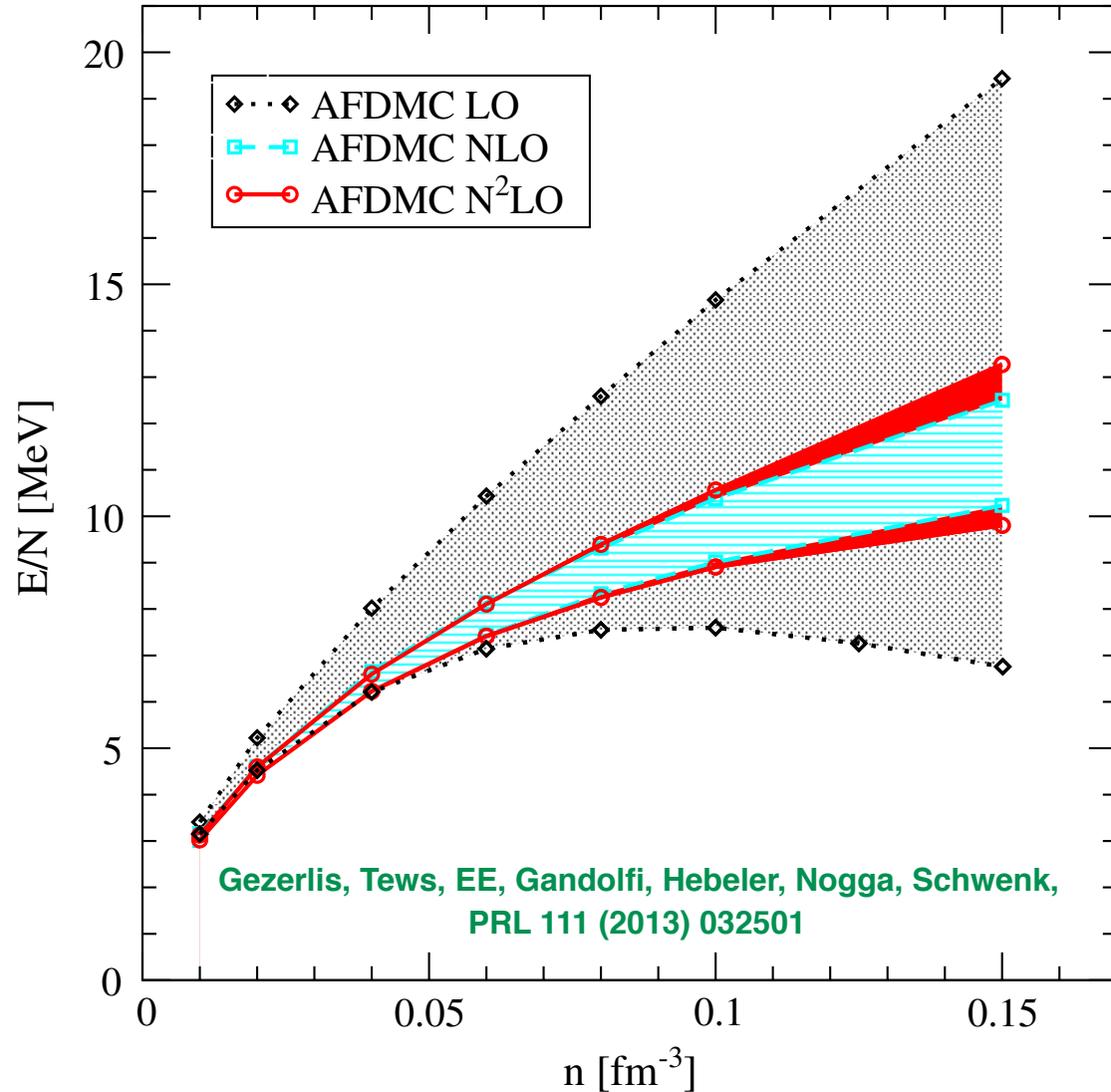


$$a_{\text{symm}}(\rho) = \left( \frac{E}{A} \right)_{\text{PNM}} - \left( \frac{E}{A} \right)_{\text{SNM}}$$

$$L = 3\rho \frac{\partial (E/A)_{\text{SNM}}}{\partial \rho}$$

# Brueckner-Hartree-Fock based on 2NF alone

Jinniu Hu, Ying Zhang, EE, Ulf-G. Meißner, Jie Meng, PRC 96 (2017) 034307

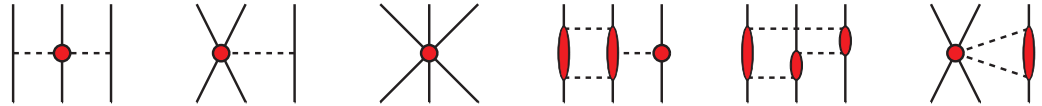


Estimated truncation error at  $N^4$ LO

Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk, PRL 111 (2013) 032501

# Three-nucleon forces

**N<sup>2</sup>LO:** tree-level graphs, 2 new LECs  
van Kolck '94; EE et al '02



**N<sup>3</sup>LO:** leading 1 loop, **parameter-free**

Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08, '11

**N<sup>4</sup>LO:** full 1 loop, almost completely worked out, several new LECs

Girlanda, Kievski, Viviani '11; Krebs, Gasparyan, EE '12, '13; EE, Gasparyan, Krebs, Schat '14



**LENPIC: Low Energy Nuclear Physics International Collaboration**

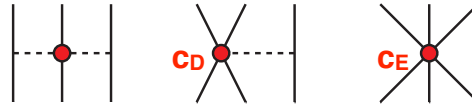


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# Three-nucleon forces

**N<sup>2</sup>LO:** tree-level graphs, 2 new LECs  
van Kolck '94; EE et al '02



**Determination of the LECs  $C_D$ ,  $C_E$ :** Triton BE & pd elastic cross section minimum @70 MeV



**LENPIC: Low Energy Nuclear Physics International Collaboration**



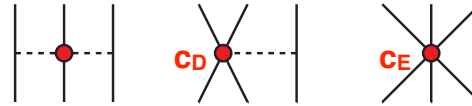
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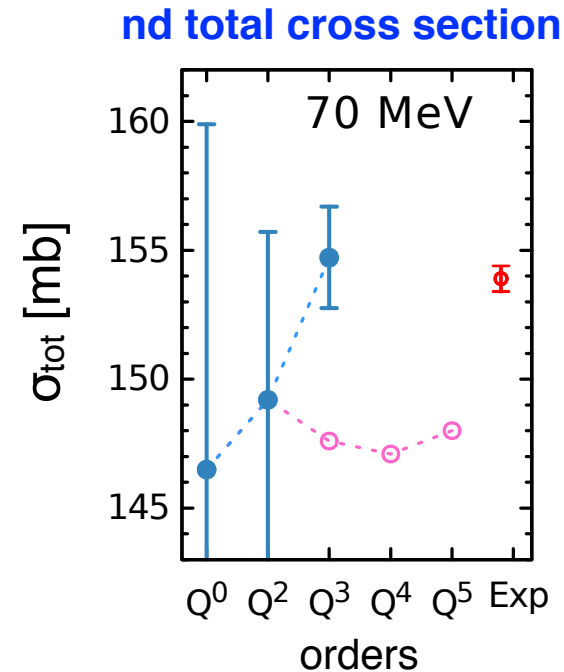
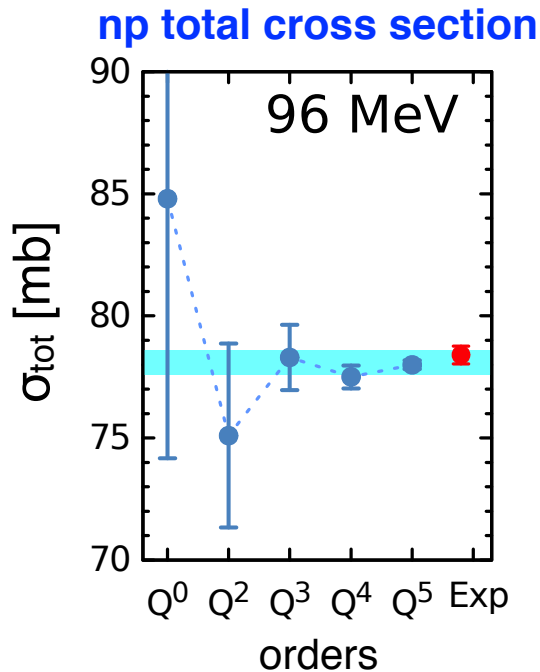


# Three-nucleon forces

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LENPIC, preliminary



LENPIC: Low Energy Nuclear Physics International Collaboration

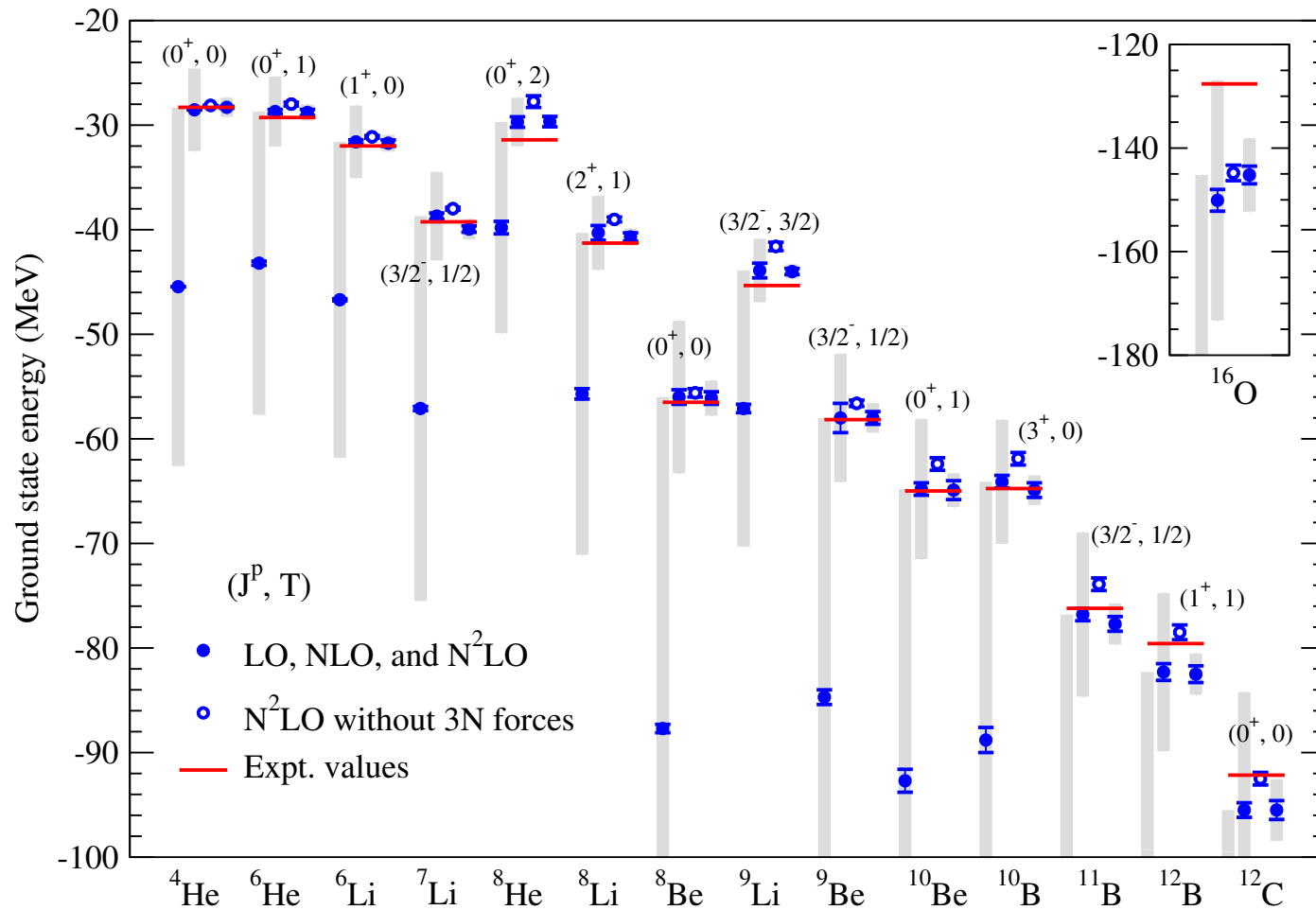


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# Light nuclei

EE et al. (LENPIC), arXiv:1807.02848



[based on EKM potential,  $R = 1.0$  fm]



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# Summary and outlook

## Nuclear Hamiltonian:

- derivation of contributions up to N<sup>3</sup>LO completed already in 2011; derivation of N<sup>4</sup>LO corrections done for V<sub>2N</sub> and almost done for V<sub>3N</sub> (new LECs...) and V<sub>4N</sub>
- accurate & precise 2N potentials at N<sup>4</sup>LO+ are available,
- promising results for few-N systems based on 2NF + 3NF@N<sup>2</sup>LO [LENPIC]

## Work in progress:

- regularization of 3NF & currents beyond N<sup>2</sup>LO (nontrivial to maintain  $\chi$ -symm!)

## Next steps:

- Precision tests of the theory for <sup>3</sup>H  $\beta$  decay &  $\mu$  capture (validation)
- Extension to other processes, heavier nuclei, N<sup>4</sup>LO, explicit  $\Delta$ 's, ...