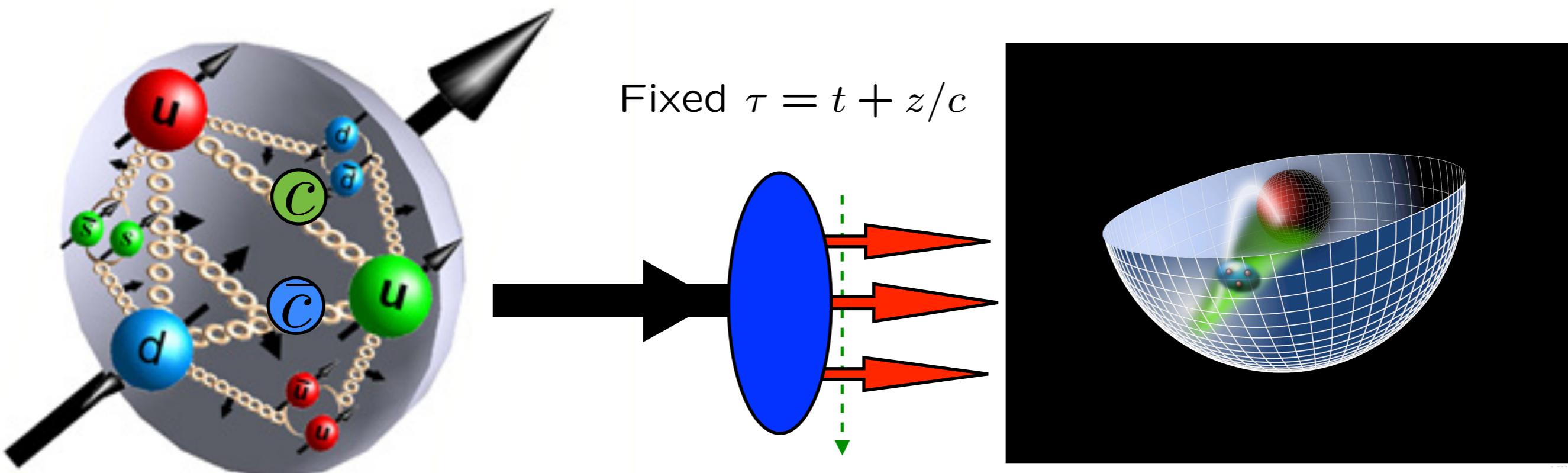


Color Confinement, Hadron Dynamics, and Hadron Spectroscopy from Light-Front Holography and Superconformal Algebra



8th Meeting on Quark Confinement and the Hadron Spectrum



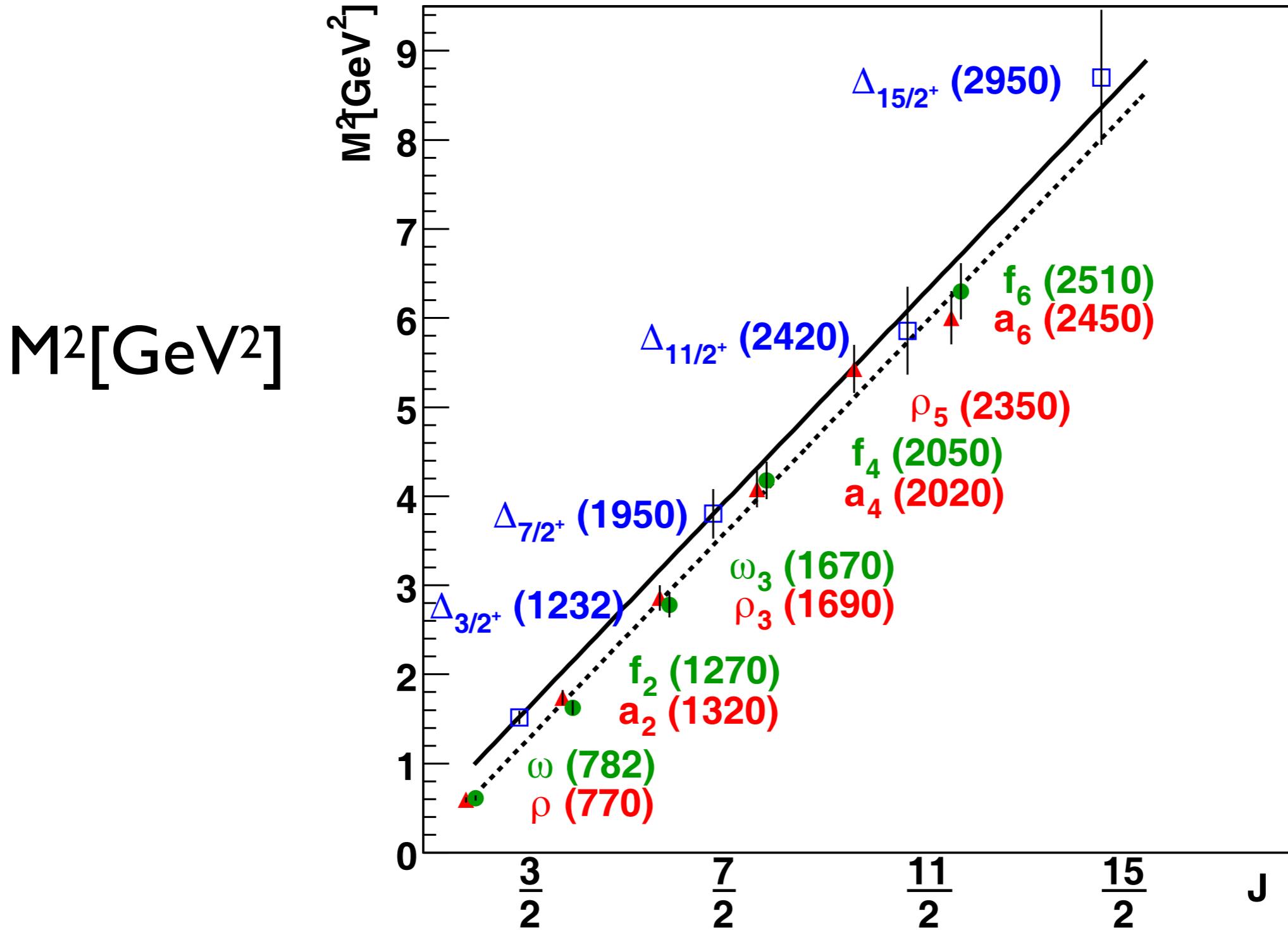
August 3, 2018

with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, Kelly Chiu, F. Navarra, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur, P. Lowdon

Profound Questions for Hadron Physics

- *Color Confinement*
- *Origin of QCD Mass Scale*
- *Spectroscopy: Tetraquarks, Pentaquarks, Gluonium, Exotic States*
- *Universal Regge Slopes: n , L , both Mesons and Baryons*
- *Massless Pion: Bound State*
- *Dynamics and Spectroscopy*
- *QCD Coupling at all Scales*
- *QCD Vacuum —Do Condensates Exist?*

Mesons and Baryons: Same Regge Slope $M^2 \propto J$!



The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with $J = L + S$.

M^2 (GeV 2)

$\rho - \Delta$ superpartner trajectories

0
1
2
3
4
5
6

MESONS
 $[q\bar{q}]$

ρ, ω

a_2, f_2

$\Delta \frac{3}{2}^+$

ρ_3, ω_3

$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$

a_4, f_4

$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$

BARYONS
 $[qqq]$

$L_M = L_B + 1$

0

1

2

3

4

5

bosons

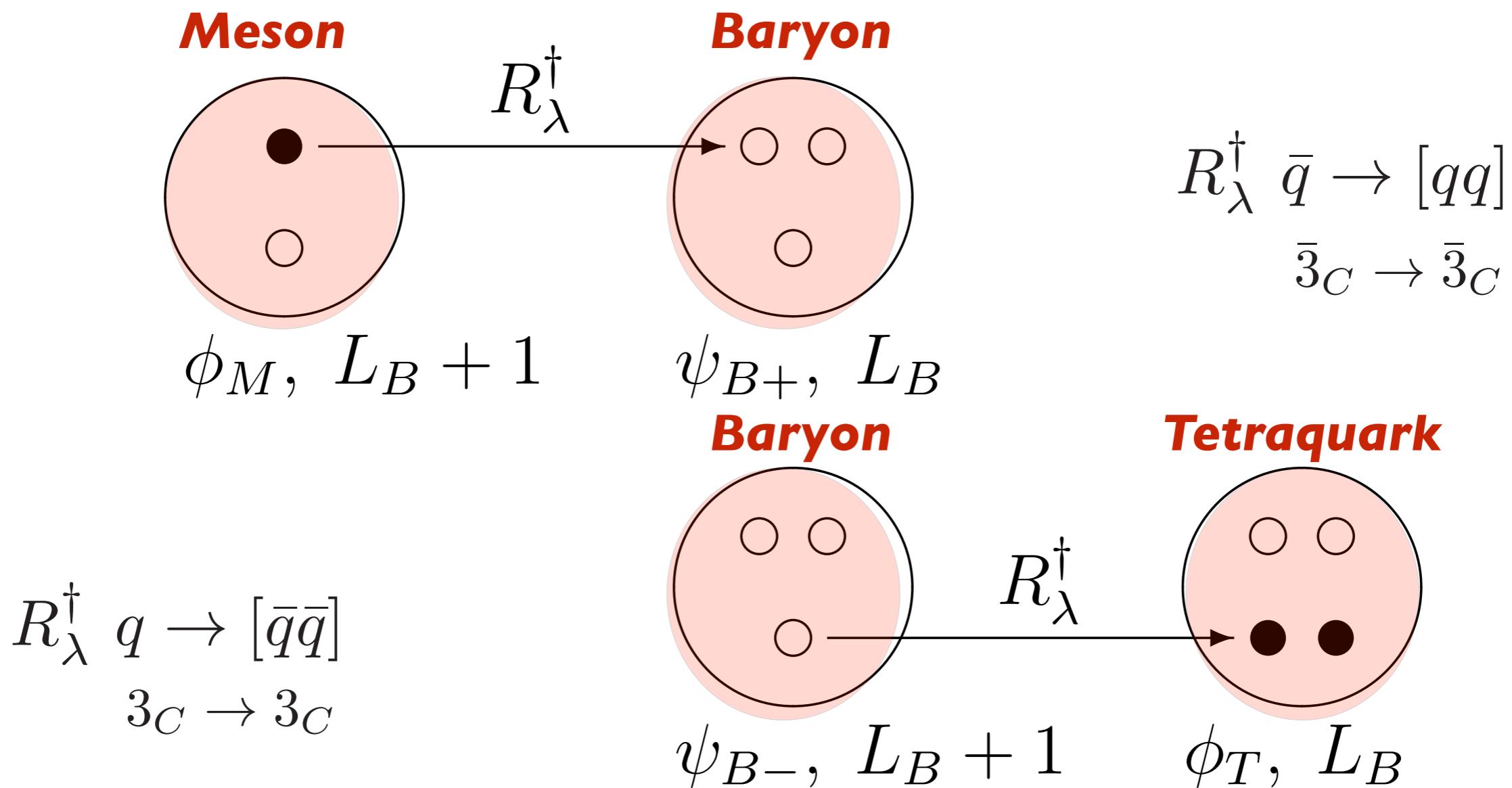
fermions

$\Delta \frac{11}{2}^+$

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

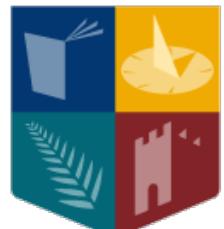


Proton: |u[ud]> Quark + Scalar Diquark

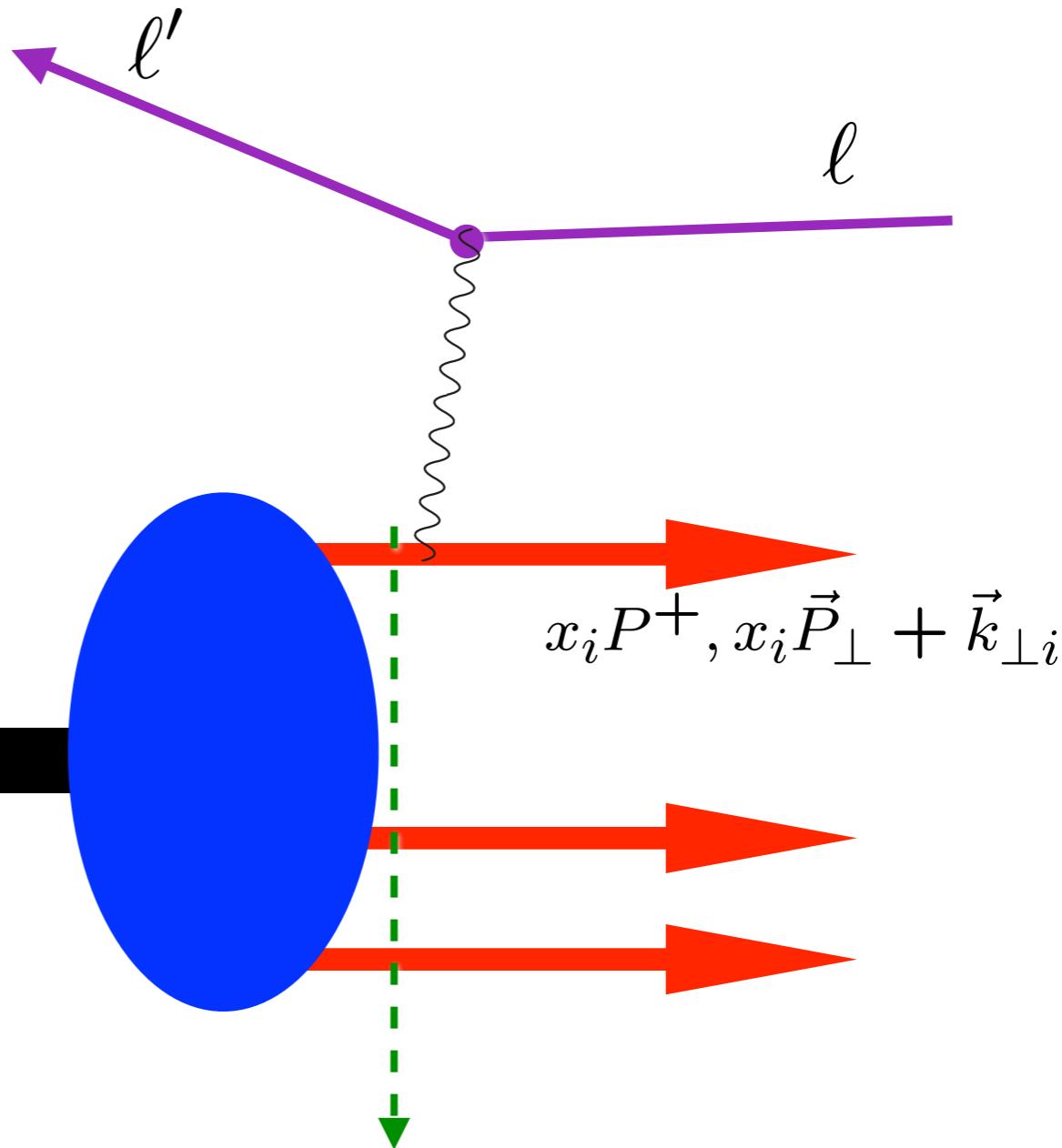
Equal Weight: L=0, L=1

Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit



$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac: Front Form

*Measurements of hadron LF
wavefunction are at fixed LF time*

Fixed $\tau = t + z/c$

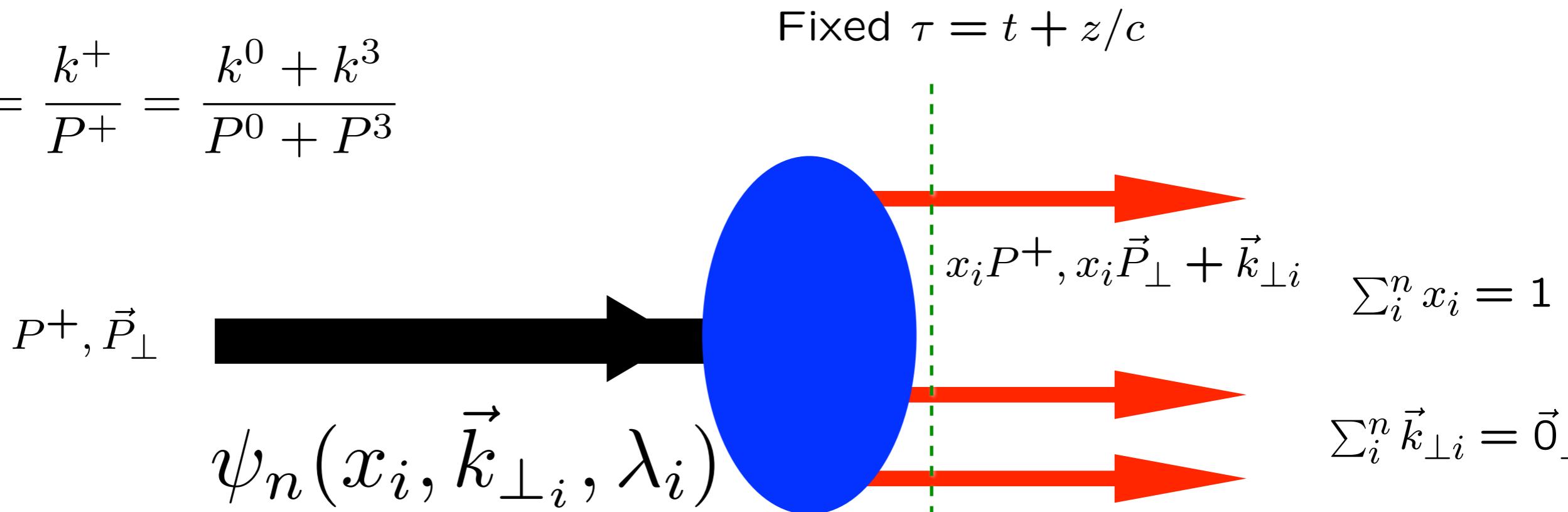
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Eigenstate of LF Hamiltonian

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

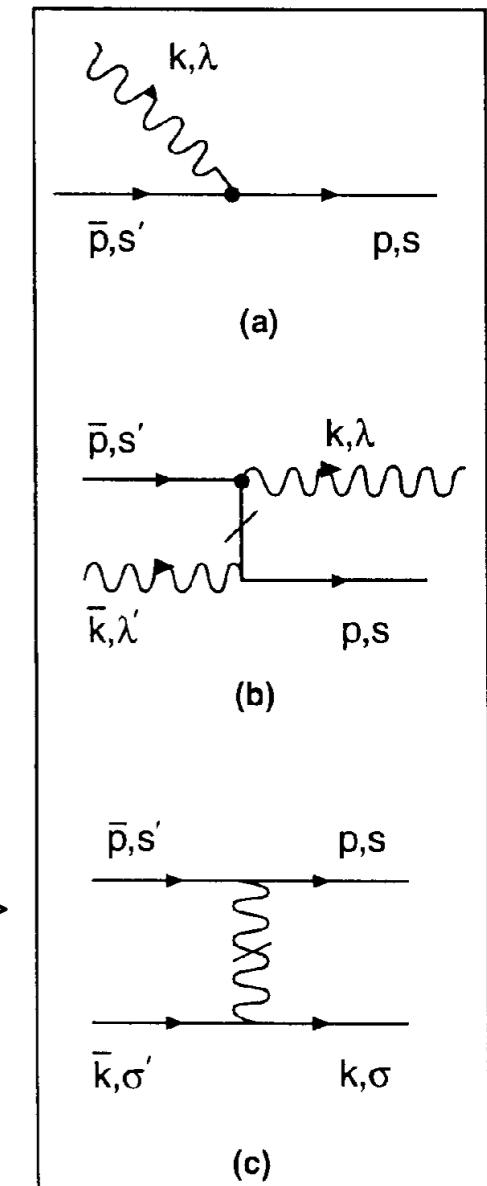
$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_\perp^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



LFWFs: Off-shell in P- and invariant mass

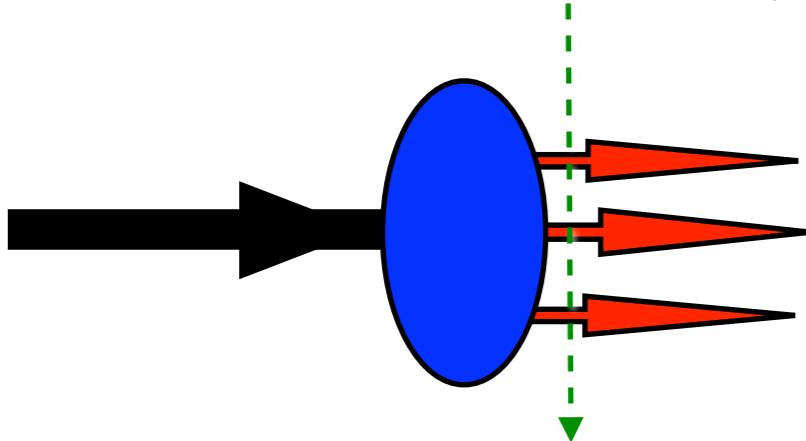
$$H_{LF}^{int}$$

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

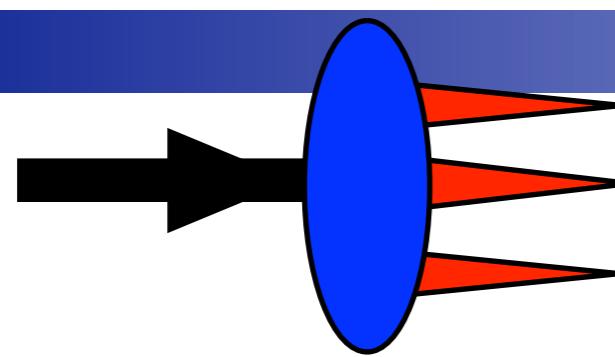
Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\Psi\rangle = M^2 |\Psi\rangle$$

Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space



$\Psi_n(x_i, \vec{k}_\perp i, \lambda_i)$

Transverse density in
momentum space

Light-Front Wavefunctions
underly hadronic observables

GTMDs

Momentum space

$$\begin{aligned} \vec{k}_\perp &\leftrightarrow \vec{z}_\perp \\ \vec{\Delta}_\perp &\leftrightarrow \vec{b}_\perp \end{aligned}$$

Position space

Transverse density in position
space

Weak transition
form factors

TMDs

$$x, \vec{k}_\perp$$

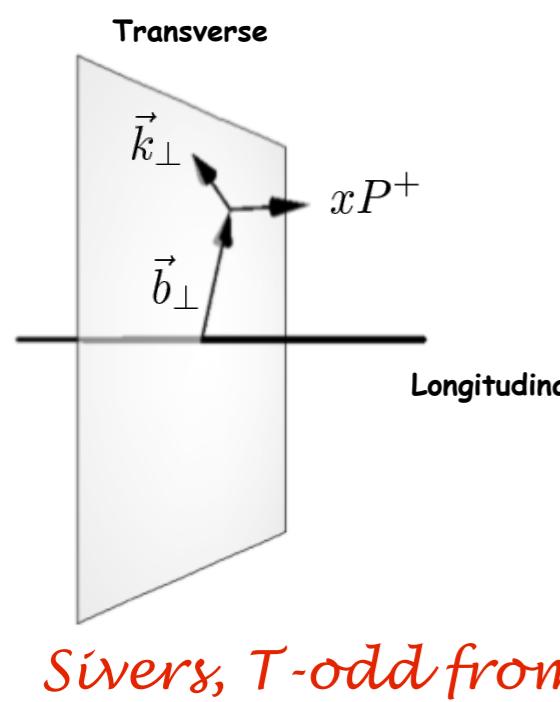
TMFFs

$$\vec{k}_\perp, \vec{b}_\perp$$

GPDs

$$x, \vec{b}_\perp$$

DGLAP, ERBL Evolution
Factorization Theorems



TMSDs

$$\vec{k}_\perp$$

PDFs

$$x,$$

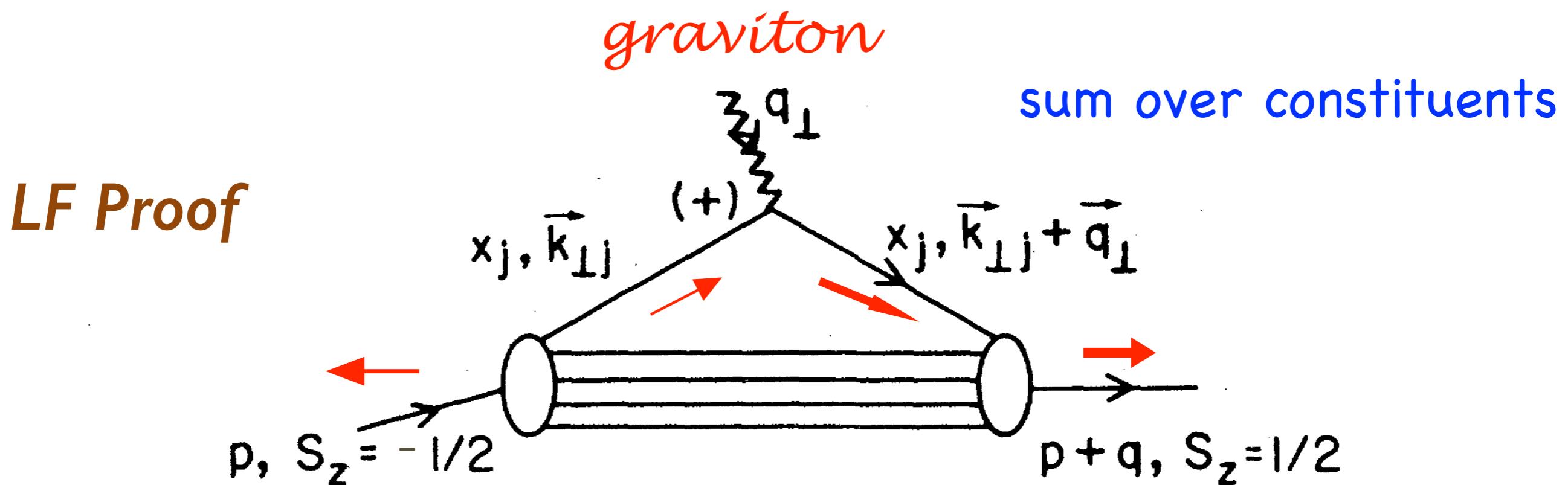
FFs

$$\vec{b}_\perp$$

Charges

- $\int d^2 b_\perp$
- $\int dx$
- $\int d^2 k_\perp$

Terayev, Okun: $B(0)$ Must vanish because of Equivalence Theorem

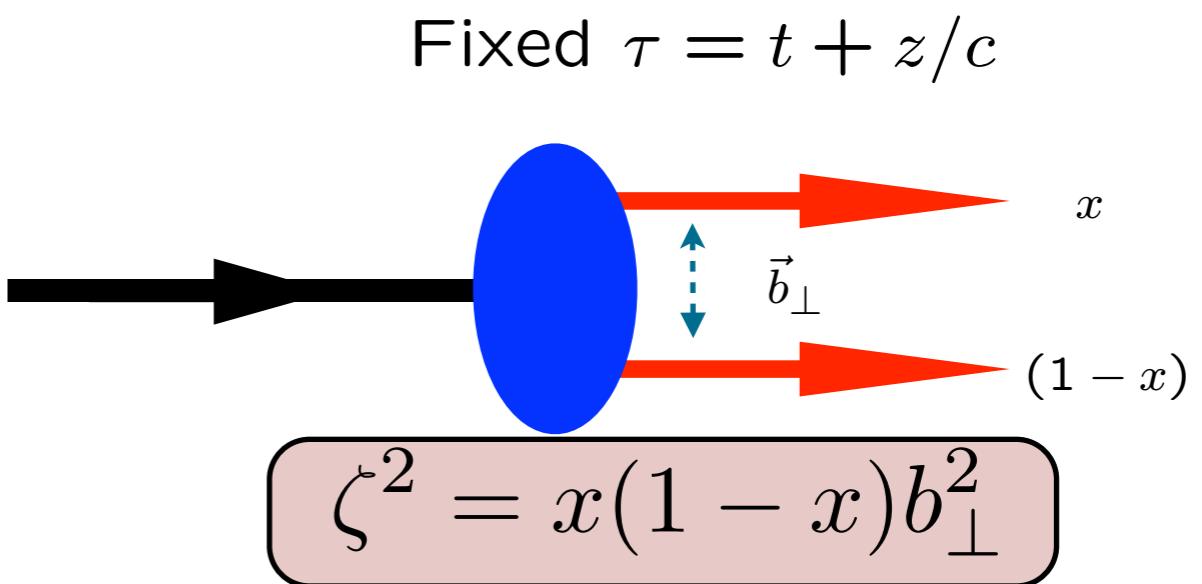
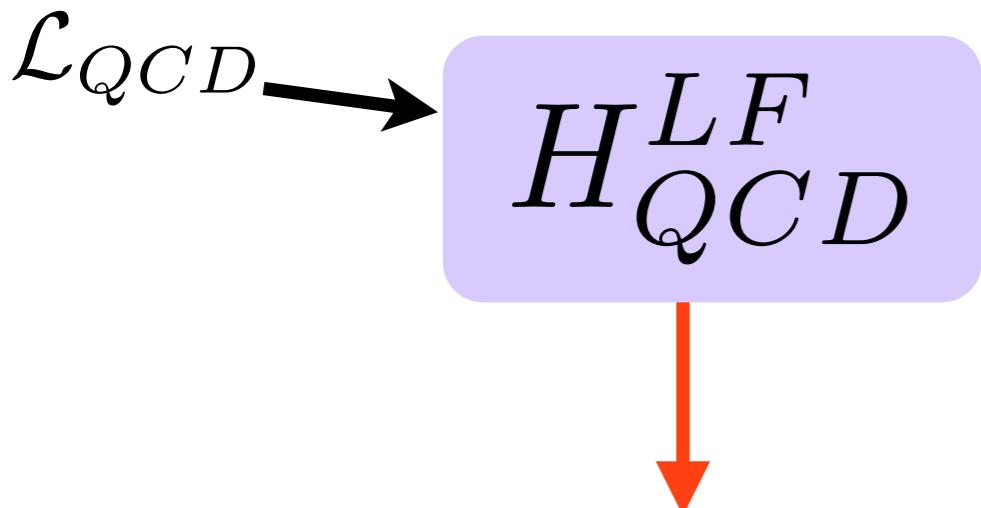


$B(0) = 0$

Each Fock State

Vanishing Anomalous gravitomagnetic moment $B(0)$

Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I)|\Psi\rangle = M^2|\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Eliminate higher Fock states
and retarded interactions

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$$m_q = 0$$

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

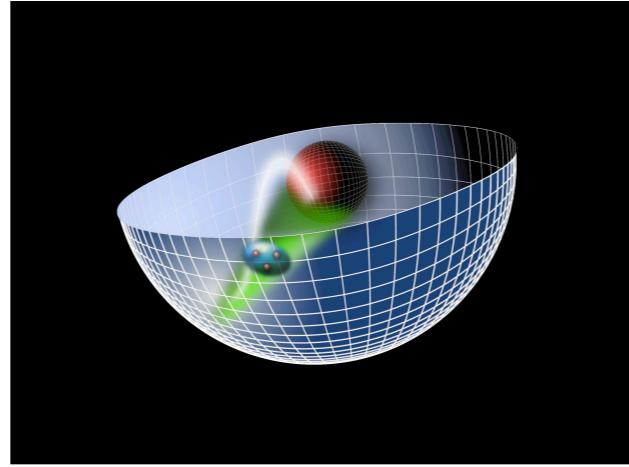
Confining AdS/QCD
potential!

Semiclassical first approximation to QCD

Sums an infinite # diagrams

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

$$\left[- \frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

Single variable ζ

Confinement scale: $\kappa \simeq 0.5 \text{ GeV}$

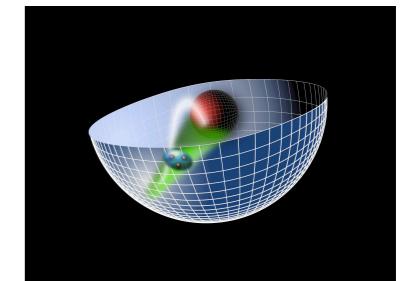
*Unique
Confinement Potential!
Conformal Symmetry
of the action*

- de Alfaro, Fubini, Furlan:
- Fubini, Rabinovici:

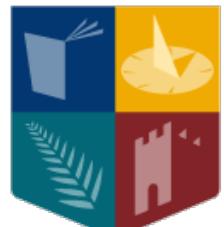
***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**



$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS₅

Identical to Single-Variable Light-Front Bound State Equation in ξ !

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp)$$

$$\longleftrightarrow$$

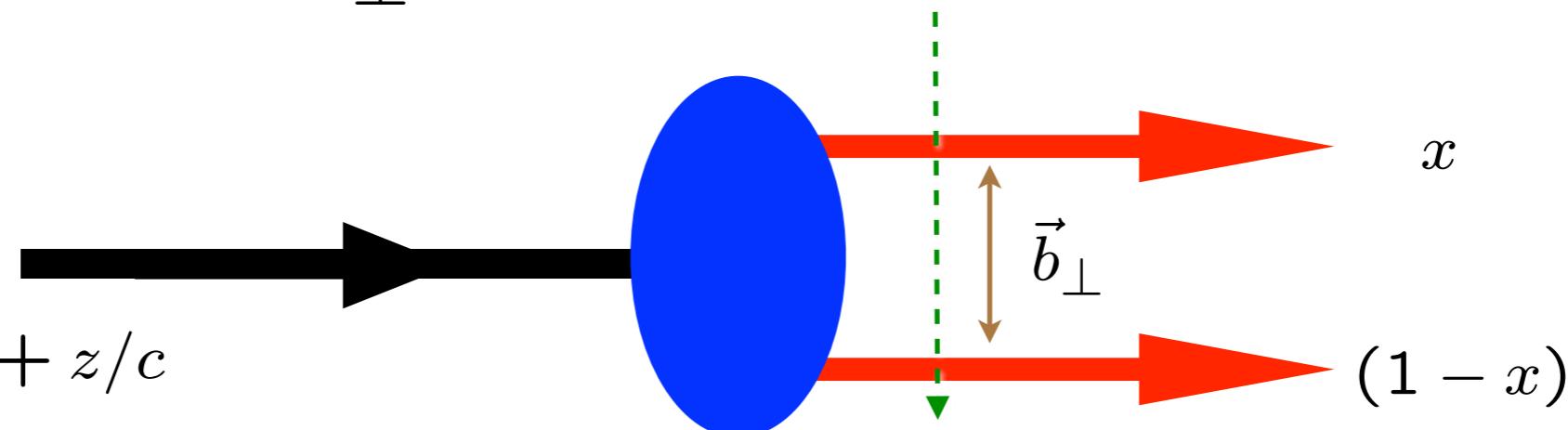
$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

$$\longleftrightarrow$$

$$z$$

Fixed $\tau = t + z/c$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

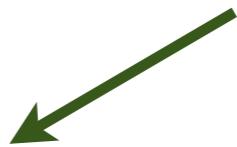
Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for $J=0$ cancels positive terms from LFKE and potential



- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J-1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

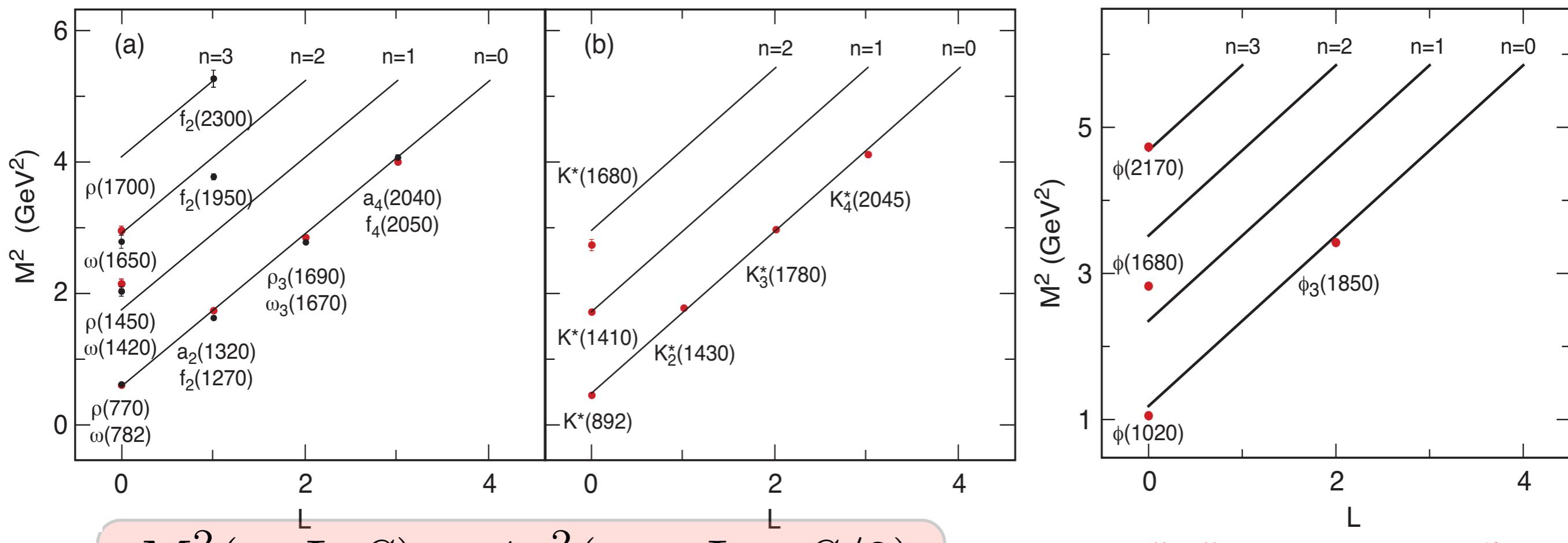
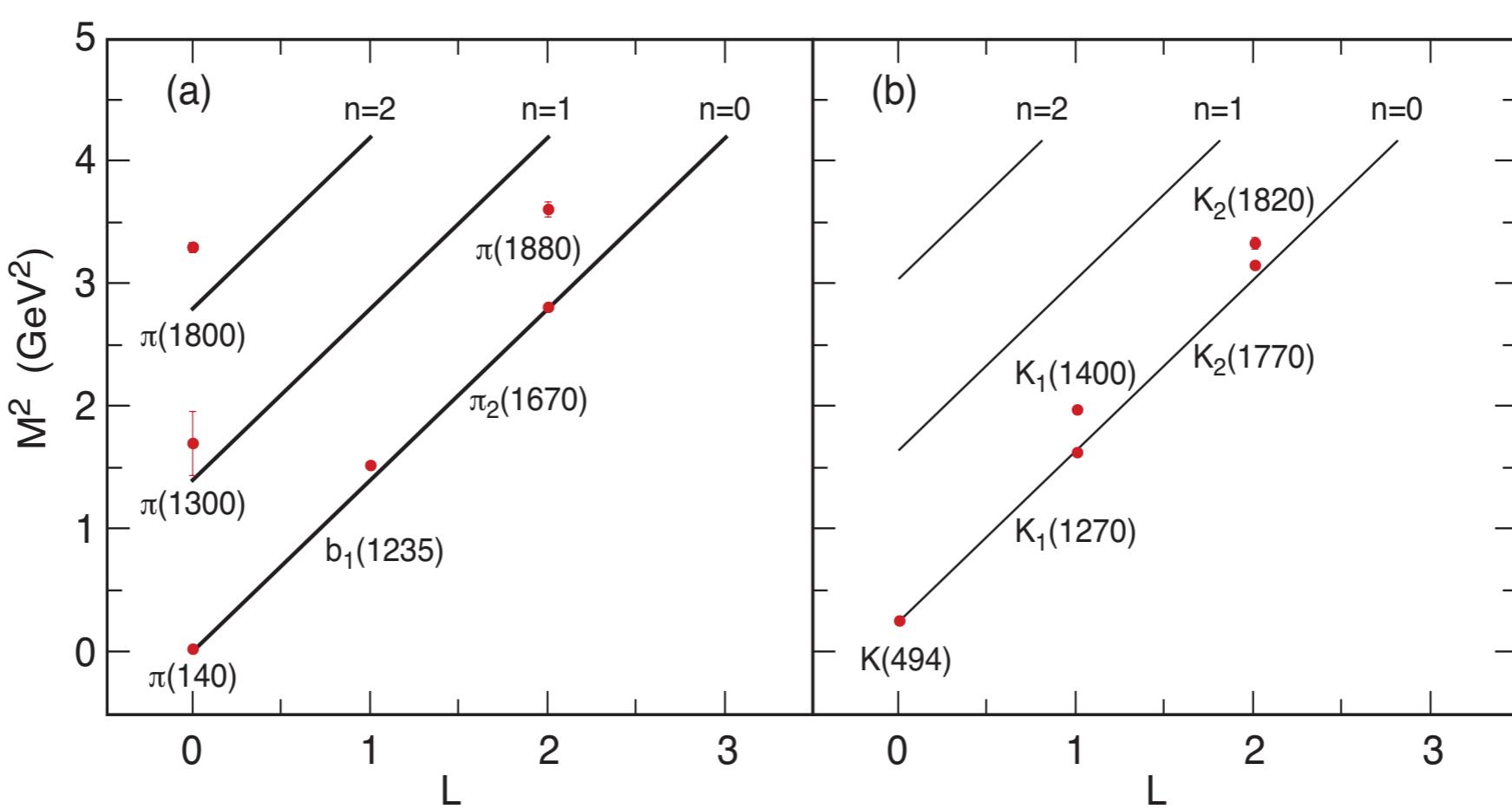
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

G. de Teramond, H. G. Dosch, sjb



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

Quark separation
increases with L

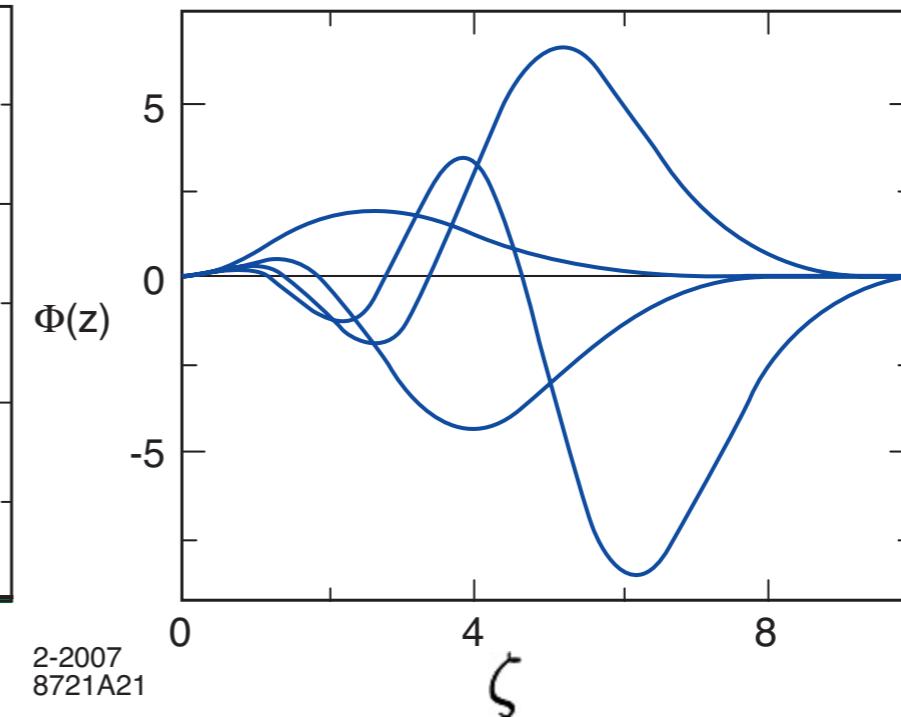
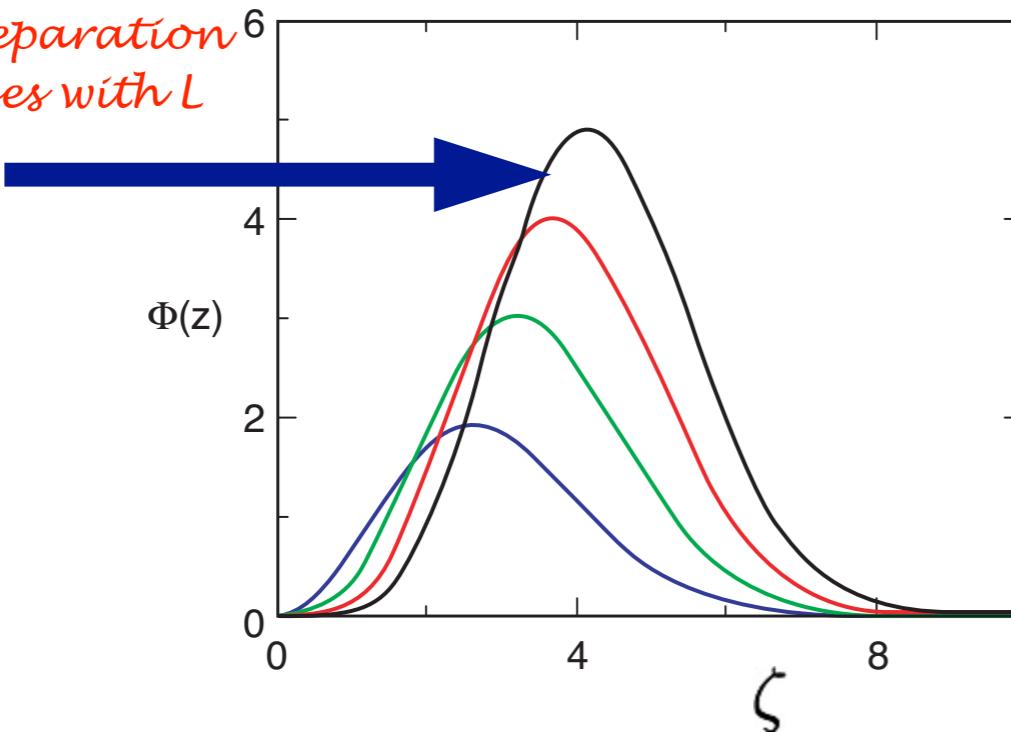
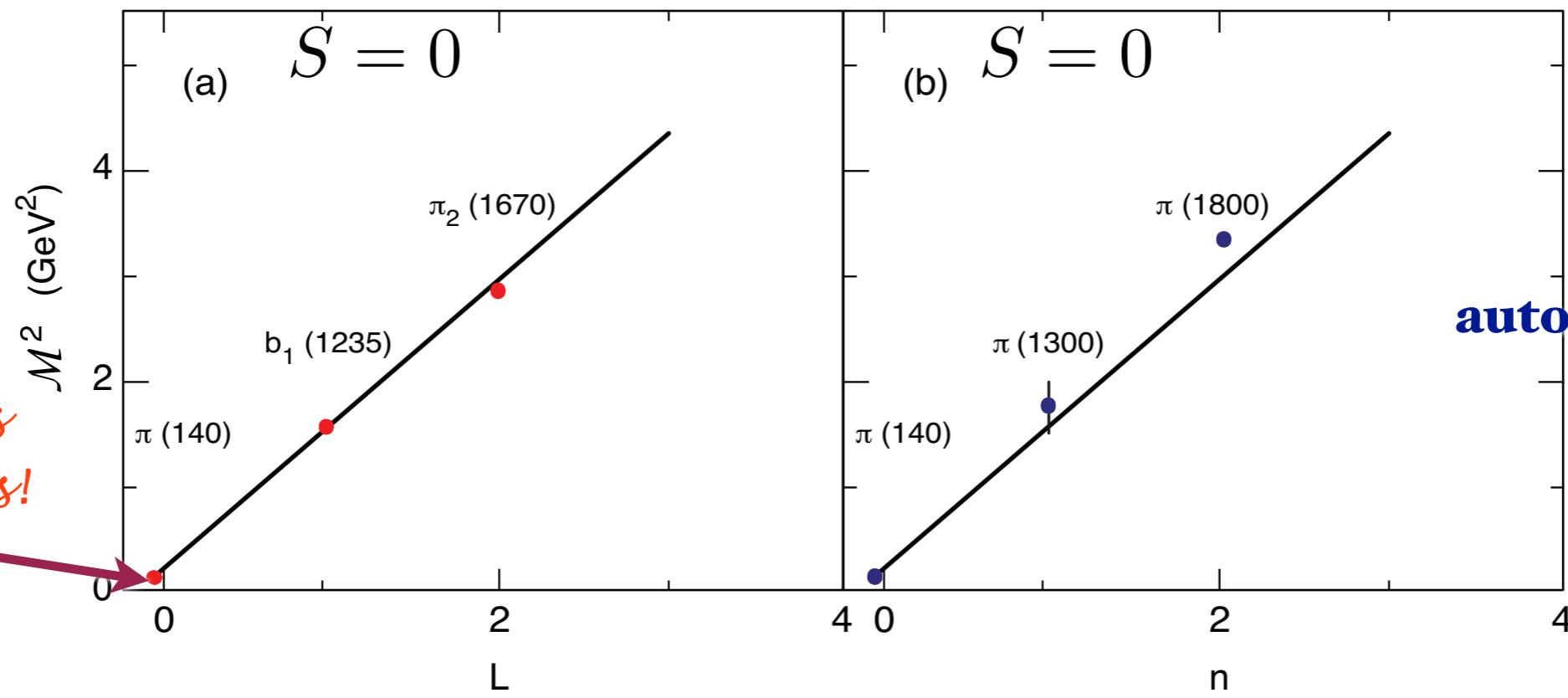


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Same slope in n and L !

**Soft Wall
Model**

Pion has
zero mass!



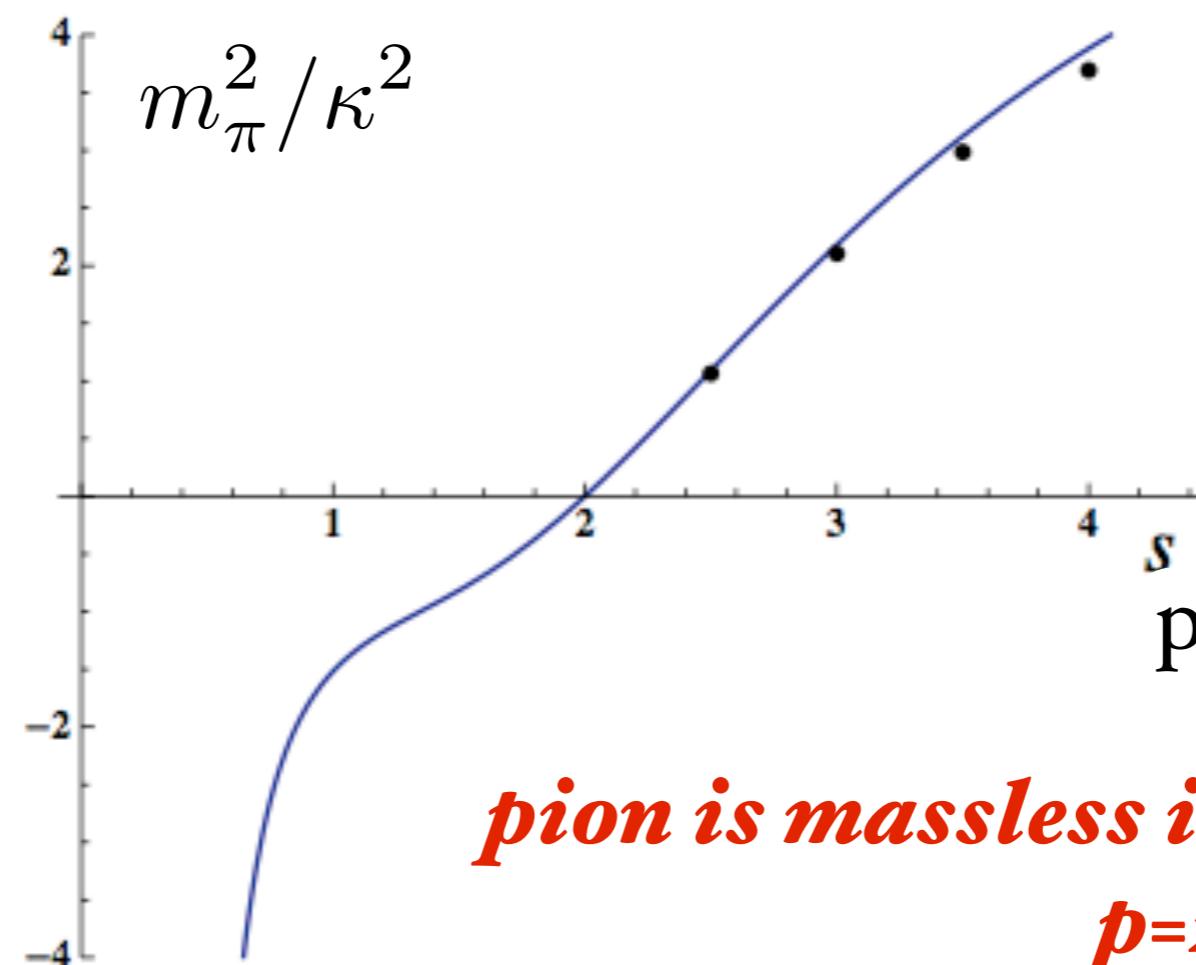
**Pion mass
automatically zero!**

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

● Dosch, de Tèramond, sjb

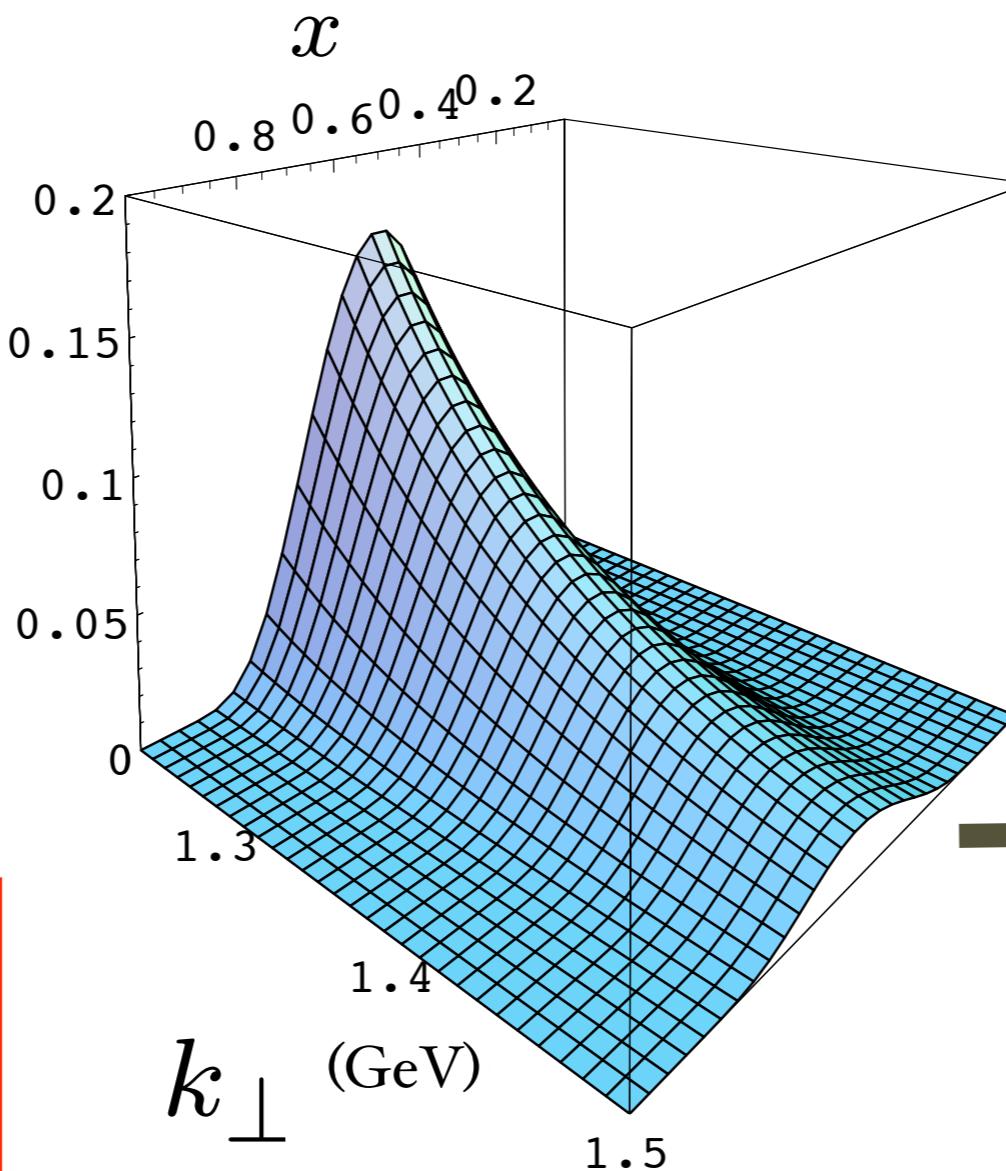
Prediction from AdS/QCD: Meson LFWF

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$

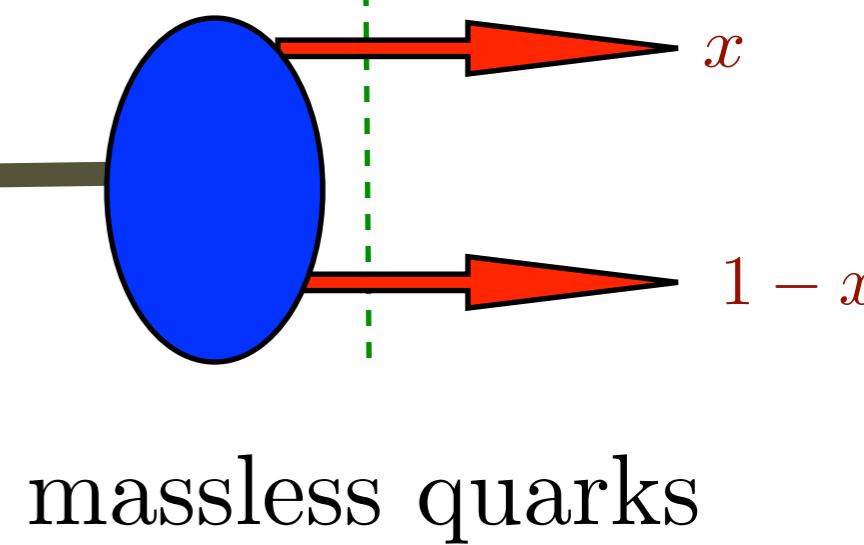
Note coupling

$$k_\perp^2, x$$



de Teramond,
Cao, sjb

**“Soft Wall”
model**



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

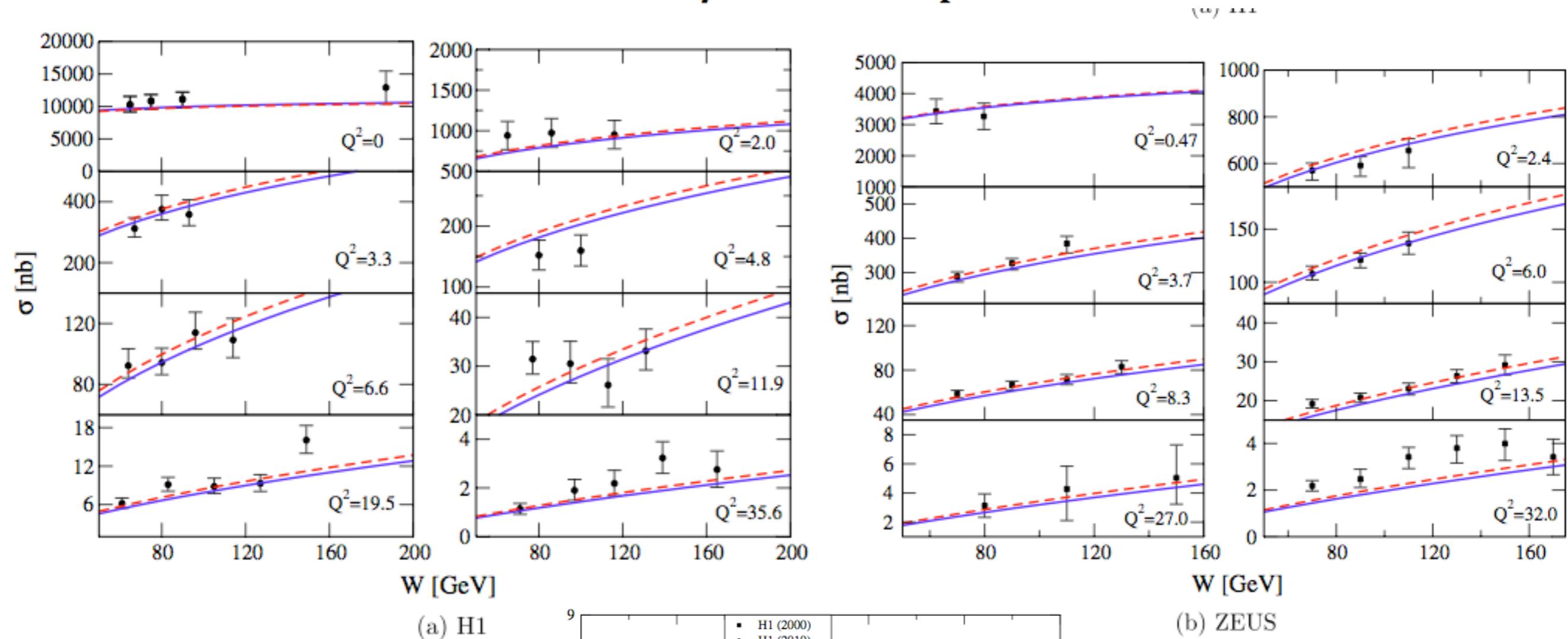
$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

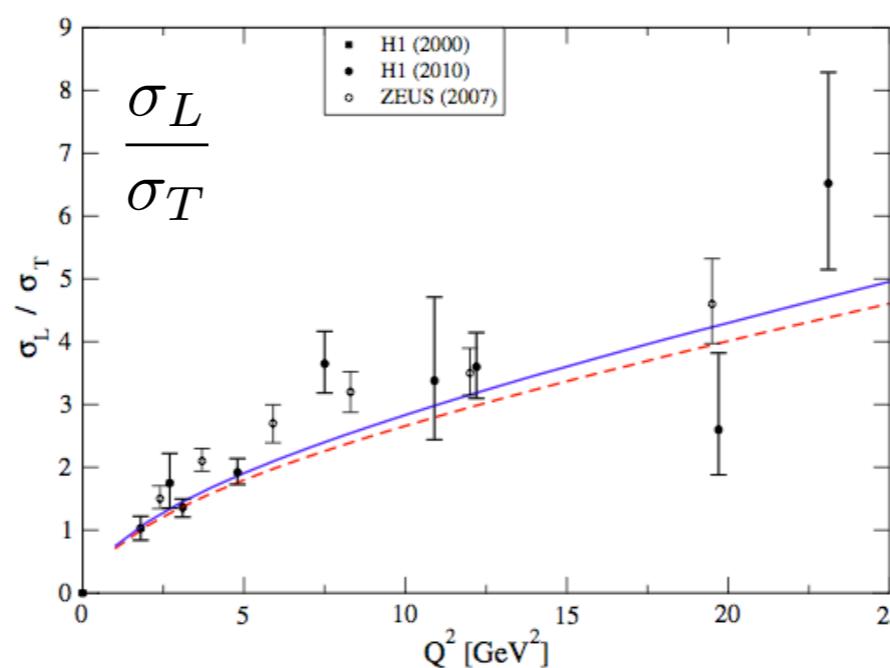
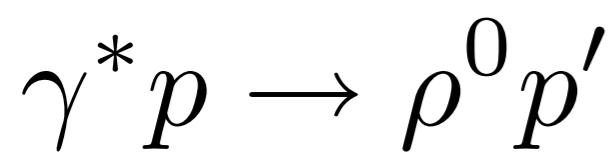
Same as DSE! C. D. Roberts et al.

Provides Connection of Confinement to Hadron Structure

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



**J. R. Forshaw,
R. Sandapen**



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Light-Front Perturbation Theory for p QCD

$$T = H_I + H_I \frac{1}{\mathcal{M}_{initial}^2 - \mathcal{M}_{intermediate}^2 + i\epsilon} H_I + \dots$$

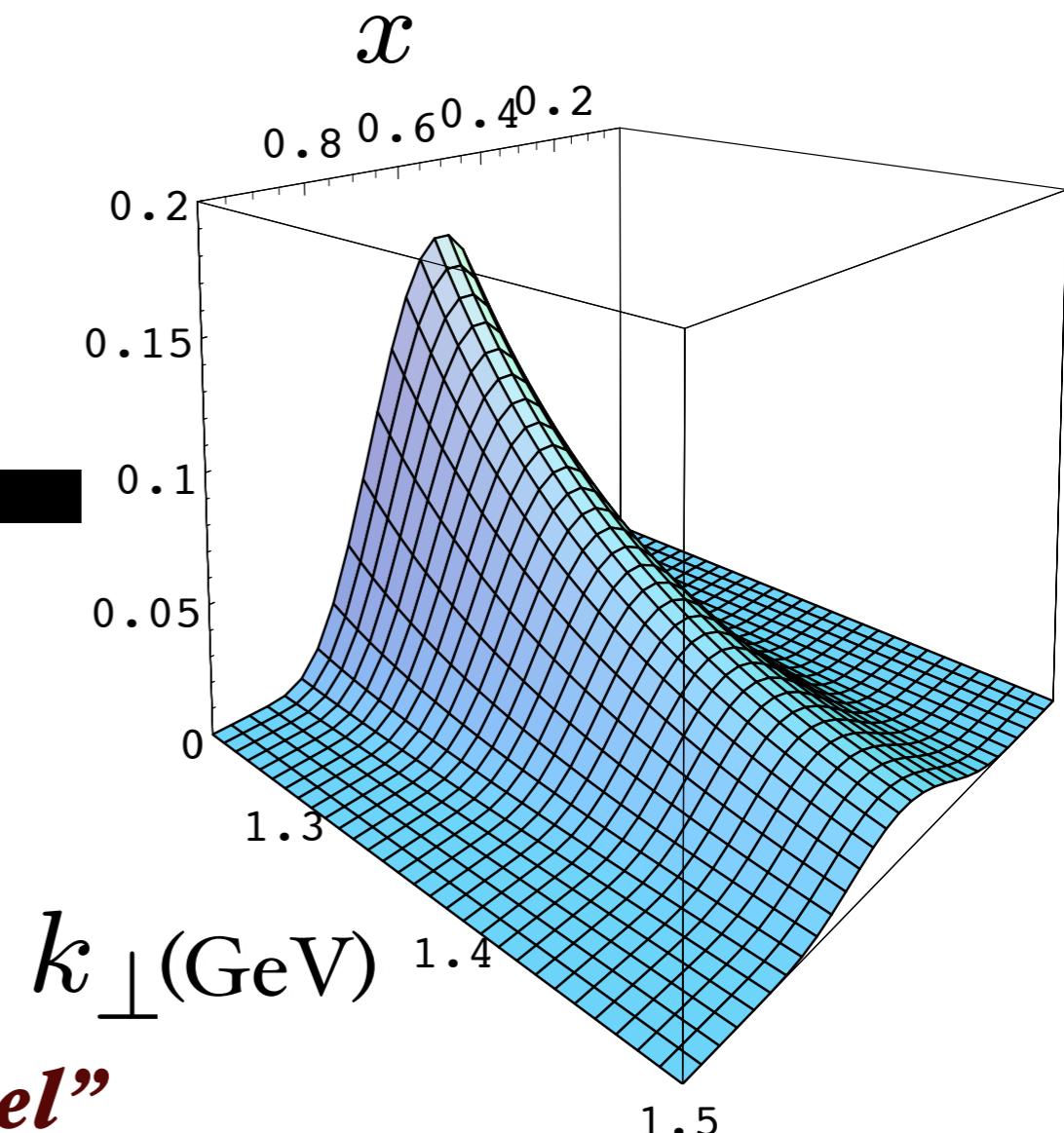
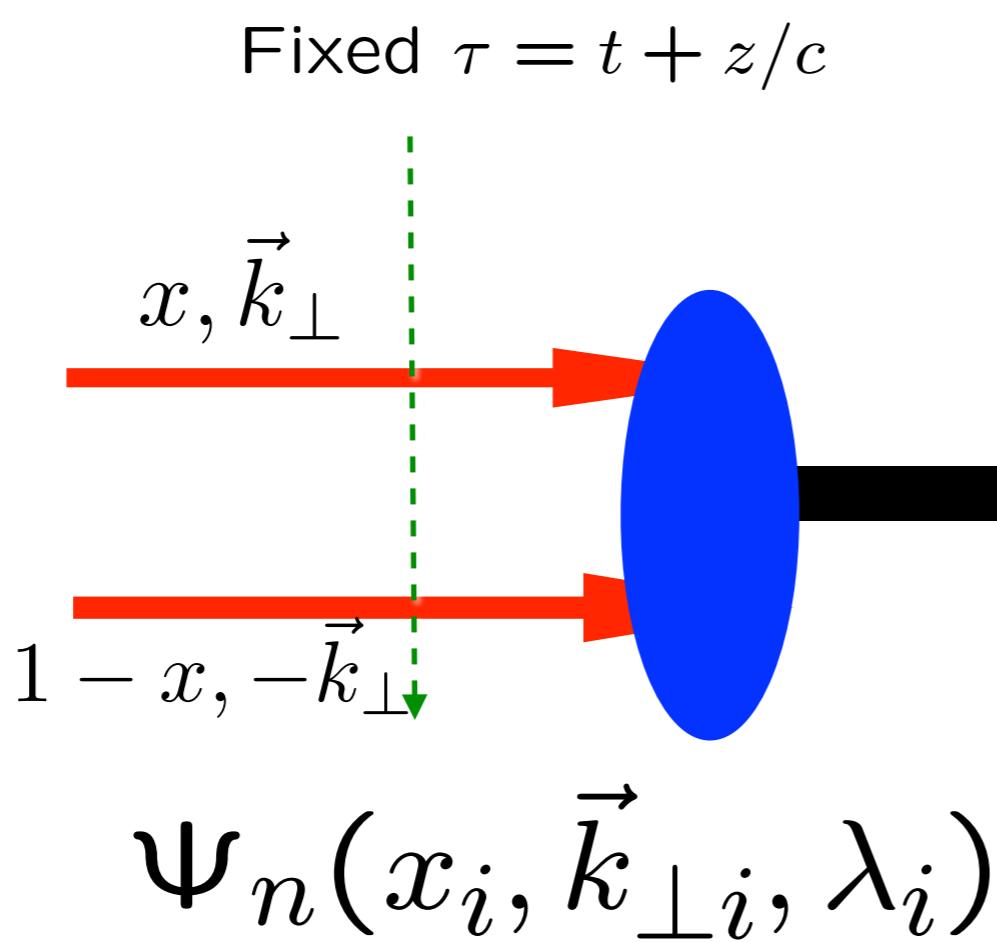
- “History”: Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!
- Wick Theorem applies, but few amplitudes since all $k^+ > 0$.
- J_z Conservation at every vertex $\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n$ at order g^n
- Unitarity is explicit
- Loop Integrals are 3-dimensional $\int_0^1 dx \int d^2 k_\perp$
- hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions $\Psi_n(x_i, \vec{k}_\perp i, \lambda_i)$

K. Chiu, sjb

- Light Front Wavefunctions:

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

off-shell in P^- and invariant mass $\mathcal{M}_{q\bar{q}}^2$



“Hadronization at the Amplitude Level”

Boost-invariant LFWF connects confined quarks and gluons to hadrons

Connection to the Linear Instant-Form Potential

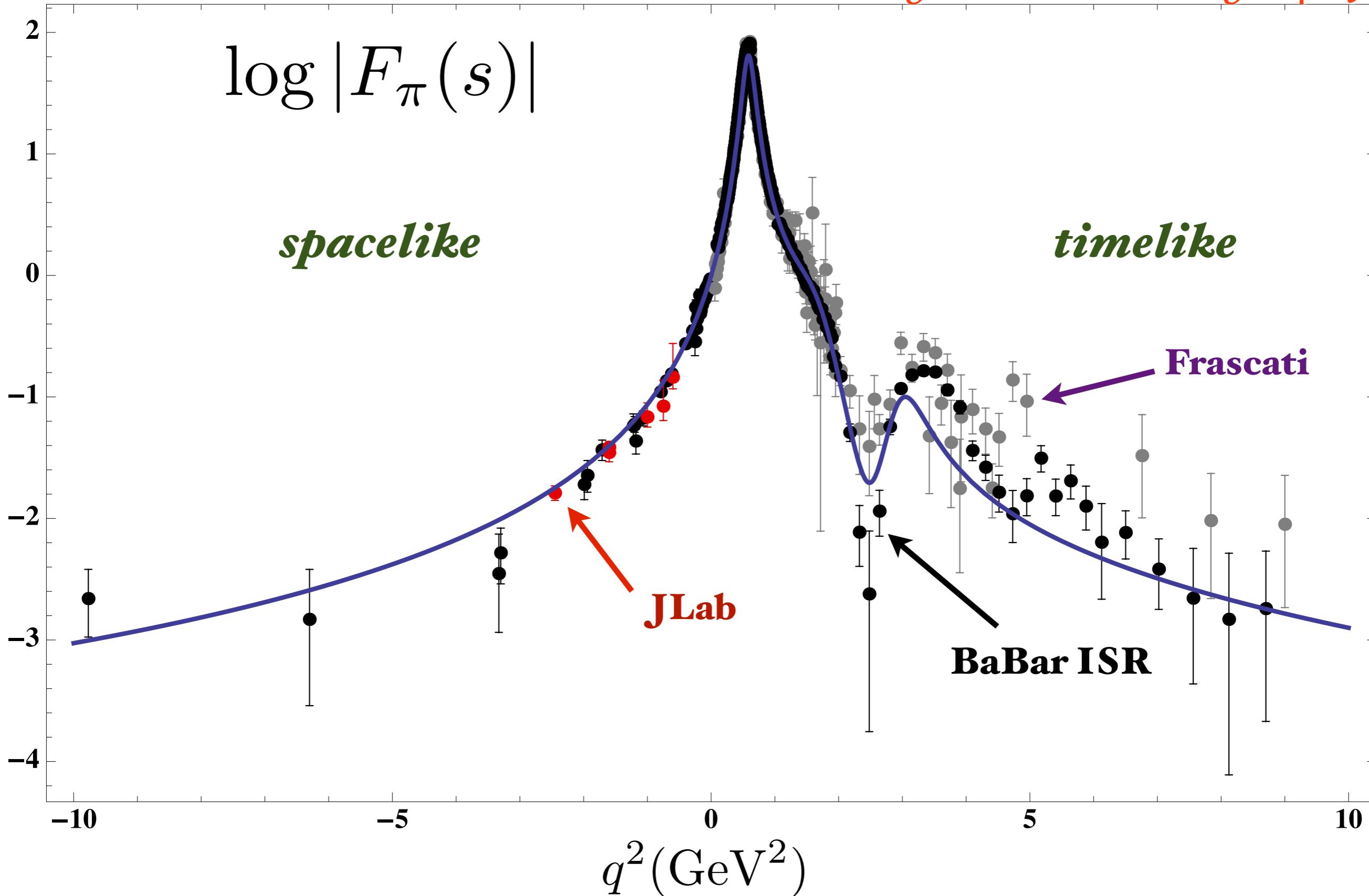
Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks



Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

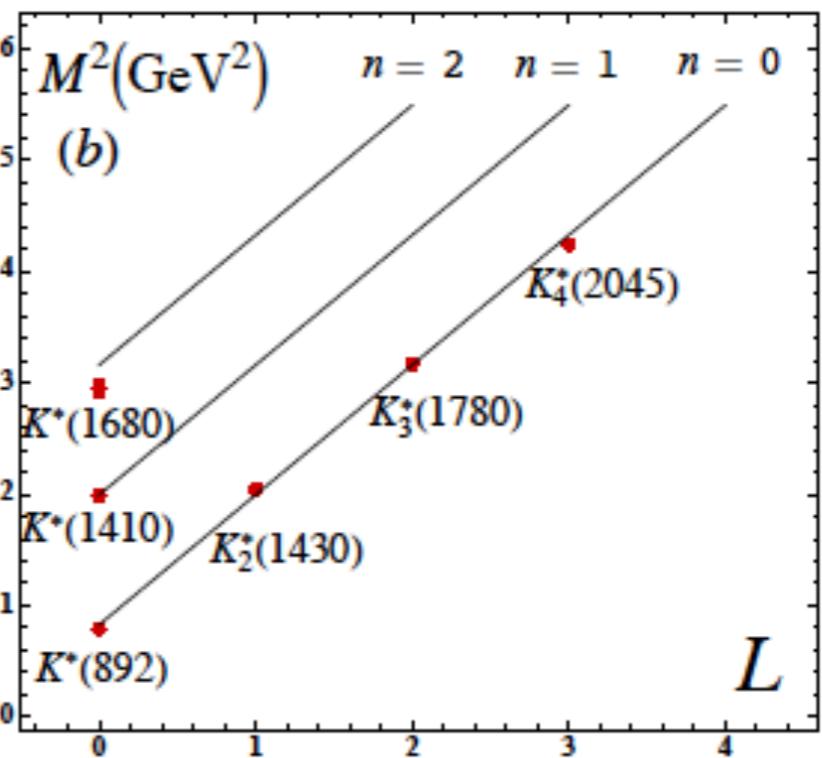
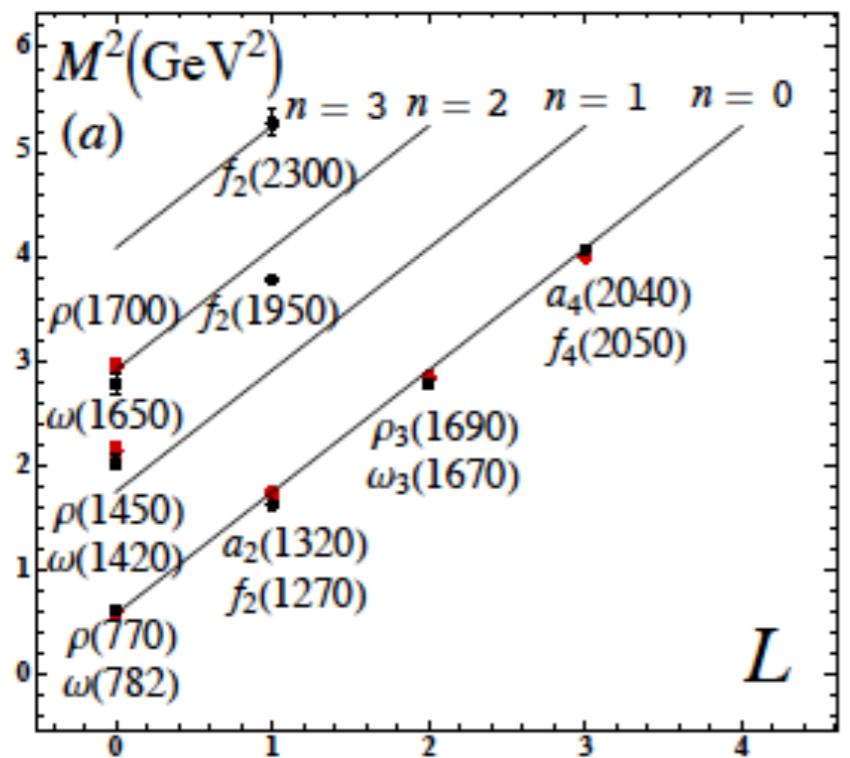
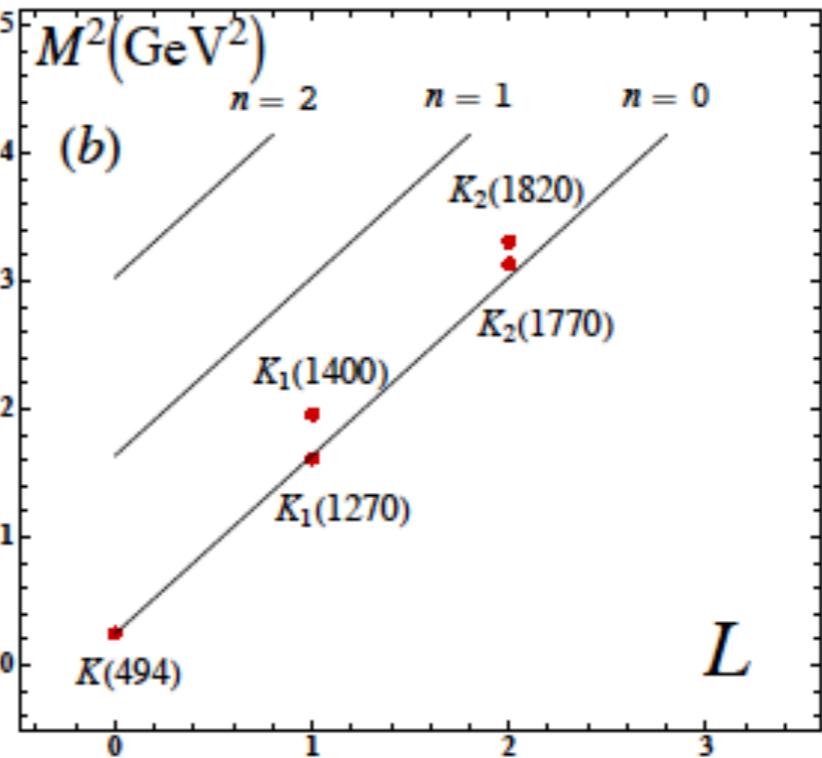
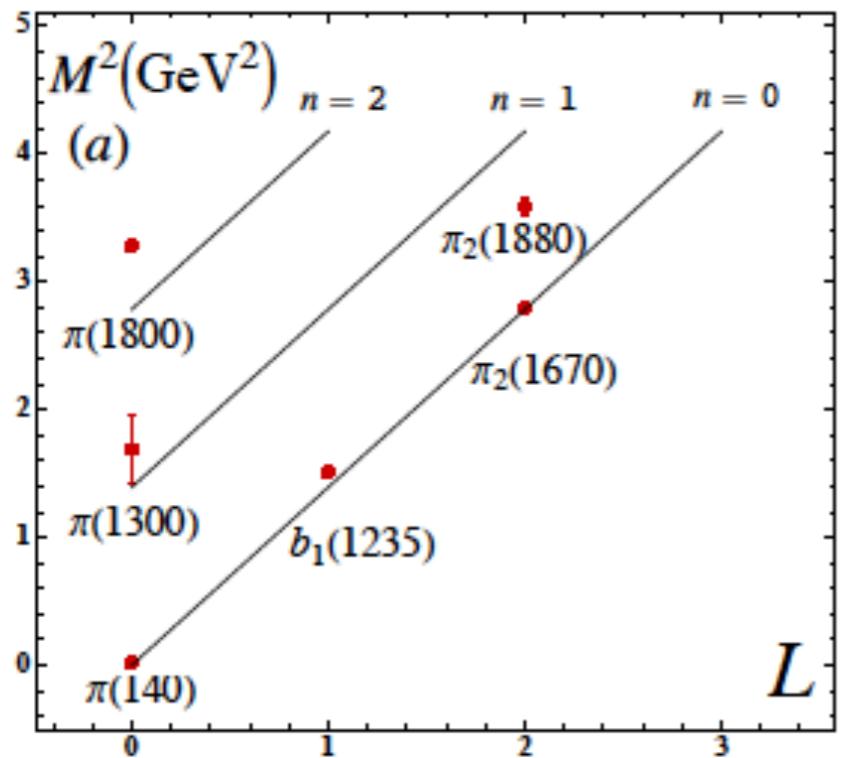
A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism

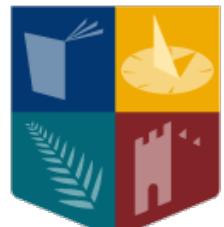


Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- Relativistic, frame-independent
- QCD scale appears - unique LF potential
- Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter
- Zero-mass pion for zero mass quarks!
- Regge slope same for n and L -- not usual HO
- Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry
- Phenomenology: LFWFs, Form factors, electroproduction
- Extension to heavy quarks

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$



QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

**QCD does not know what MeV units mean!
Only Ratios of Masses Determined**

- de Alfaro, Fubini, Furlan:

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

Unique confinement potential!

● de Alfaro, Fubini, Furlan (dAFF)

$$G|\psi(\tau)\rangle = i \frac{\partial}{\partial \tau} |\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2} \left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4} x^2 \right)$$

New term

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left(\frac{2tw + v}{\sqrt{4uw - v^2}} \right),$$

- Identify with difference of LF time $\Delta x^+/\mathbf{P}^+$ between constituents
- Finite range
- Measure in Double-Parton Processes

Retains conformal invariance of action despite mass scale!

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \quad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \quad S = \psi^+x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[H, D] = i H, \quad [H, K] = 2 i D, \quad [K, D] = -i K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$

w : dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \quad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B, \quad w = \kappa^2$

$$\lambda = \kappa^2$$

Eigenvalue of G : $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

$$\left(-\partial_\zeta^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

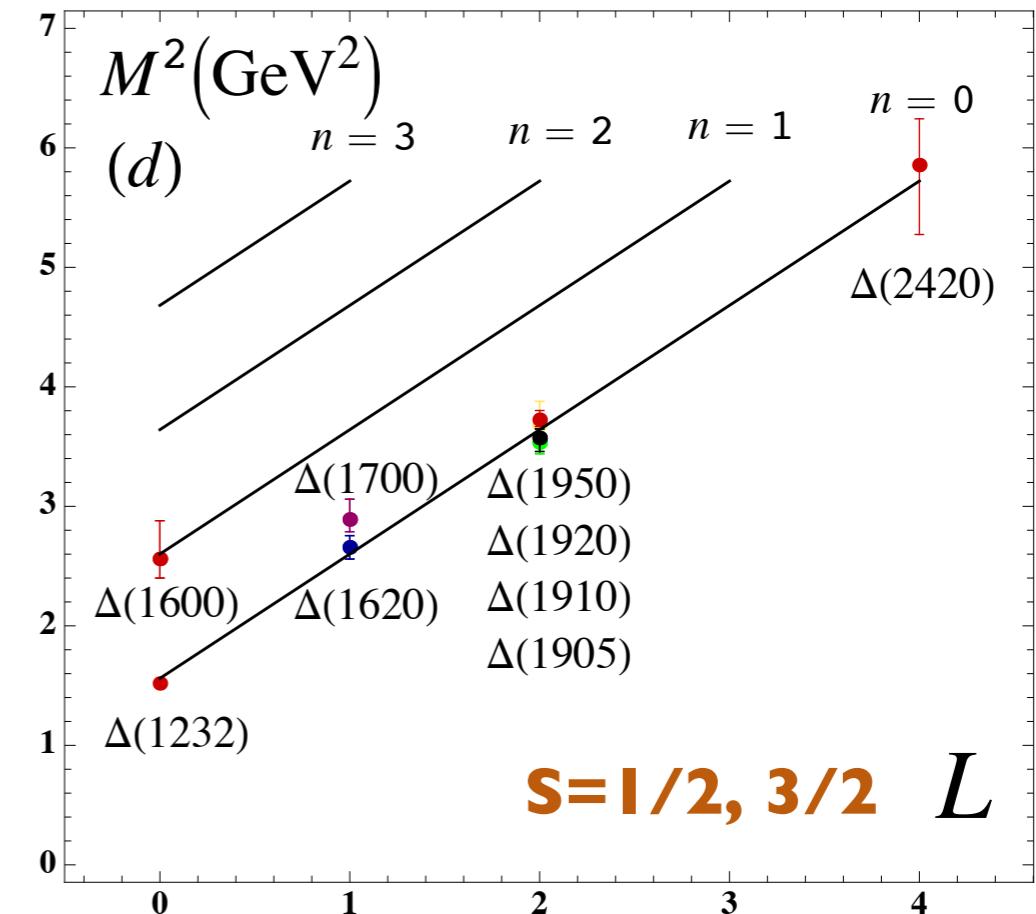
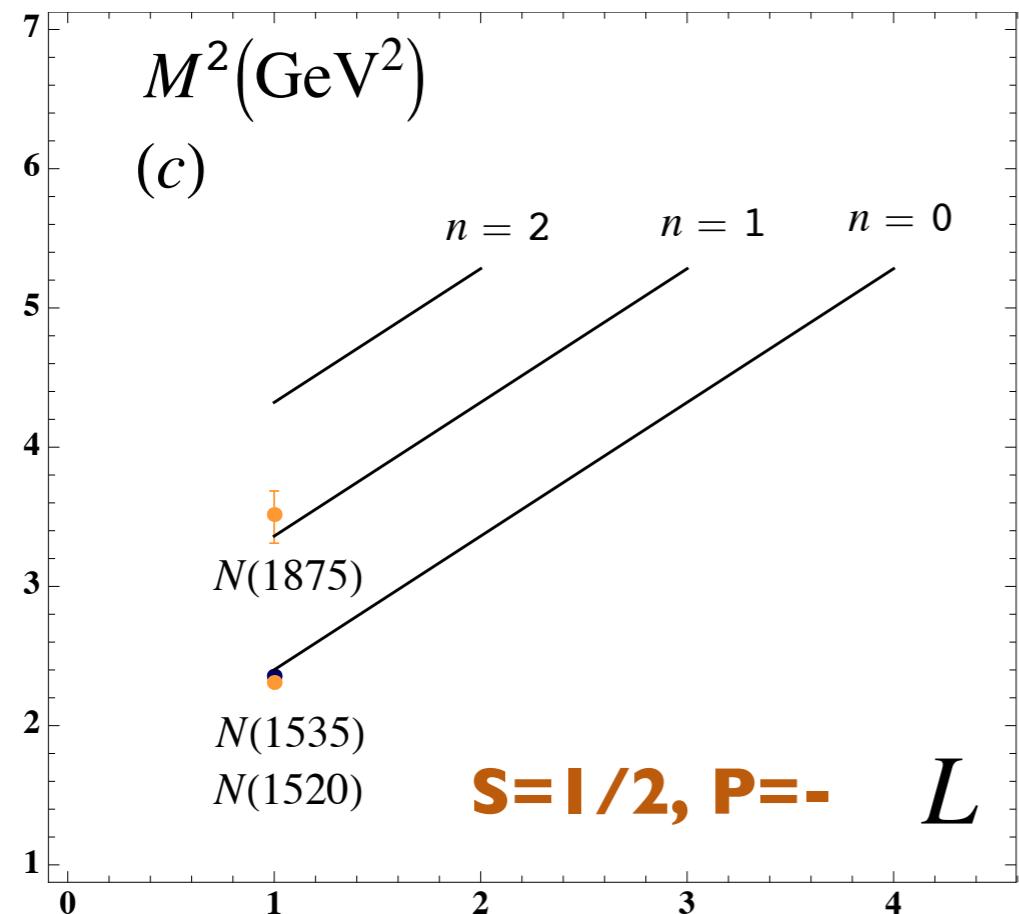
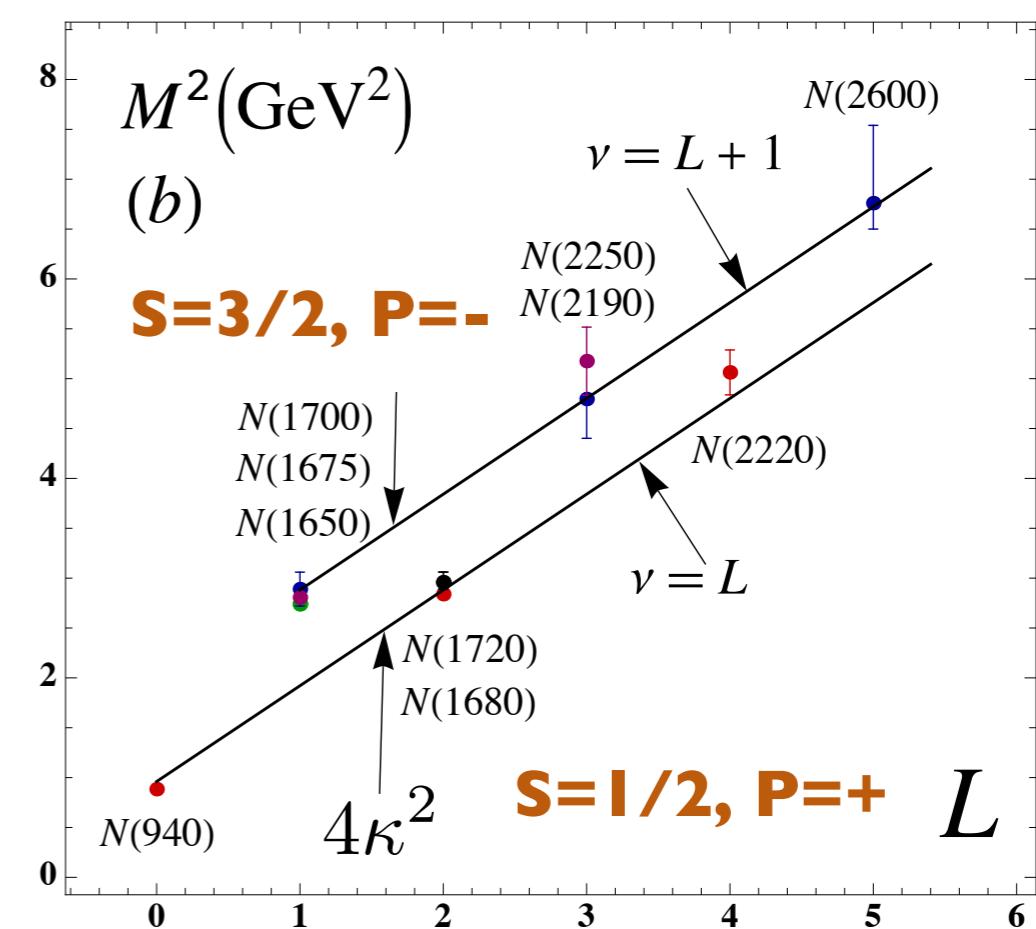
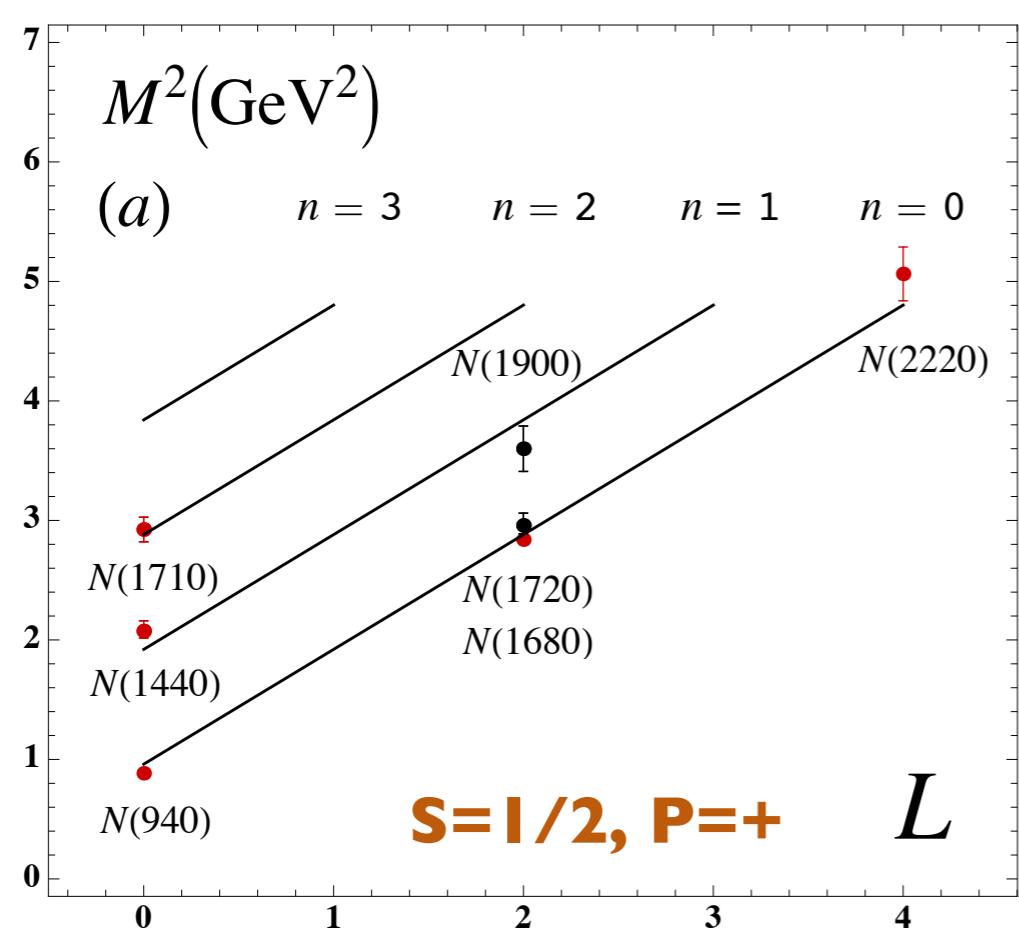
$$\left(-\partial_\zeta^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

S=0, I=I Meson is superpartner of S=1/2, I=I Baryon
Meson-Baryon Degeneracy for $L_M=L_B+1$



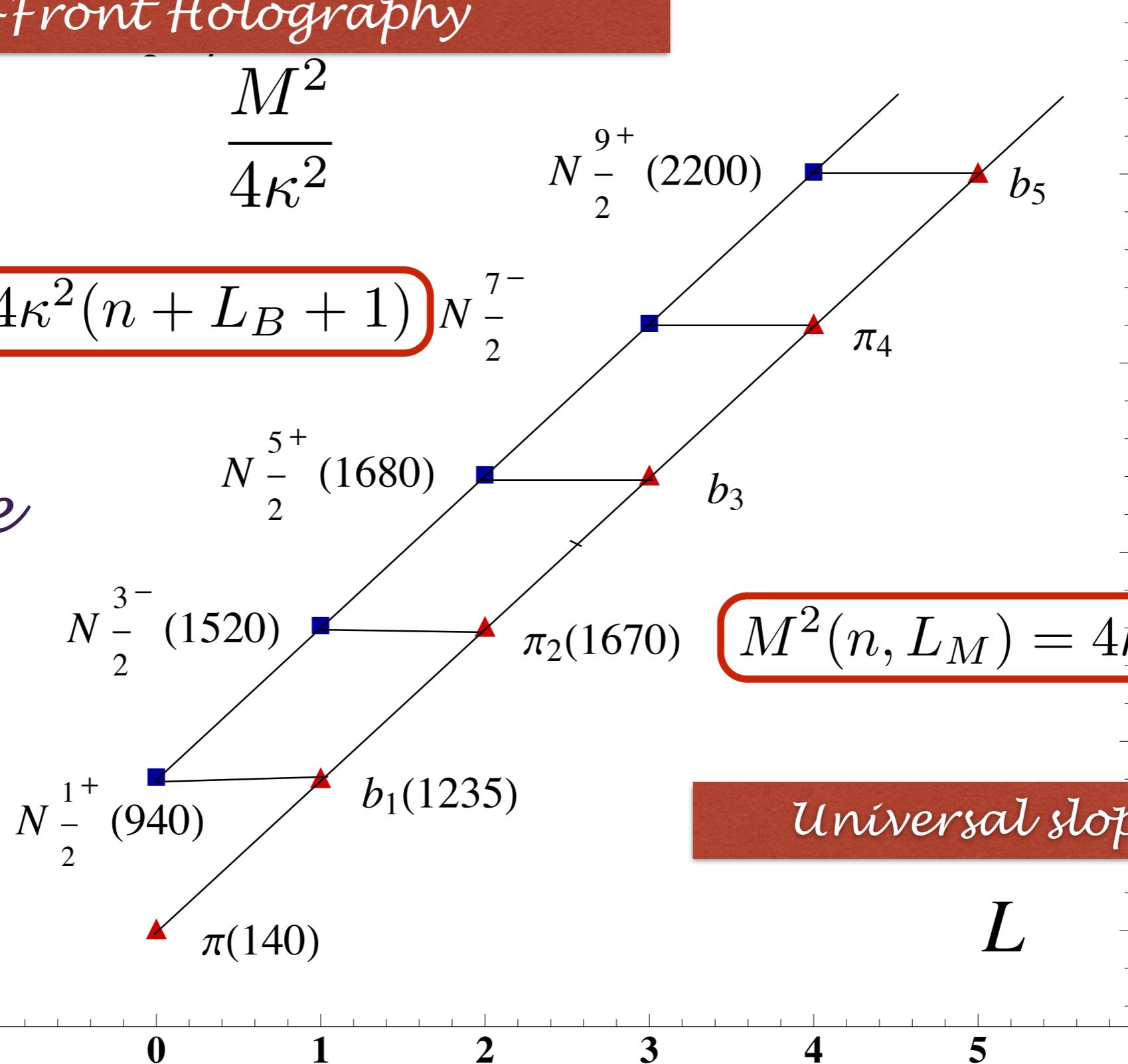
Superconformal Quantum Mechanics Light-Front Holography

de Tèramond, Dosch, Lorcè, sjb

$$\frac{M^2}{4\kappa^2}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon
Mass Degeneracy
for $L_M=L_B+1$**

M^2 (GeV 2)

$\rho - \Delta$ superpartner trajectories

0
1
2
3
4
5
6

MESONS
 $[q\bar{q}]$

ρ, ω

a_2, f_2

$\Delta \frac{3}{2}^+$

ρ_3, ω_3

$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$

a_4, f_4

$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$

BARYONS
 $[qqq]$

$L_M = L_B + 1$

0

1

2

3

4

5

bosons

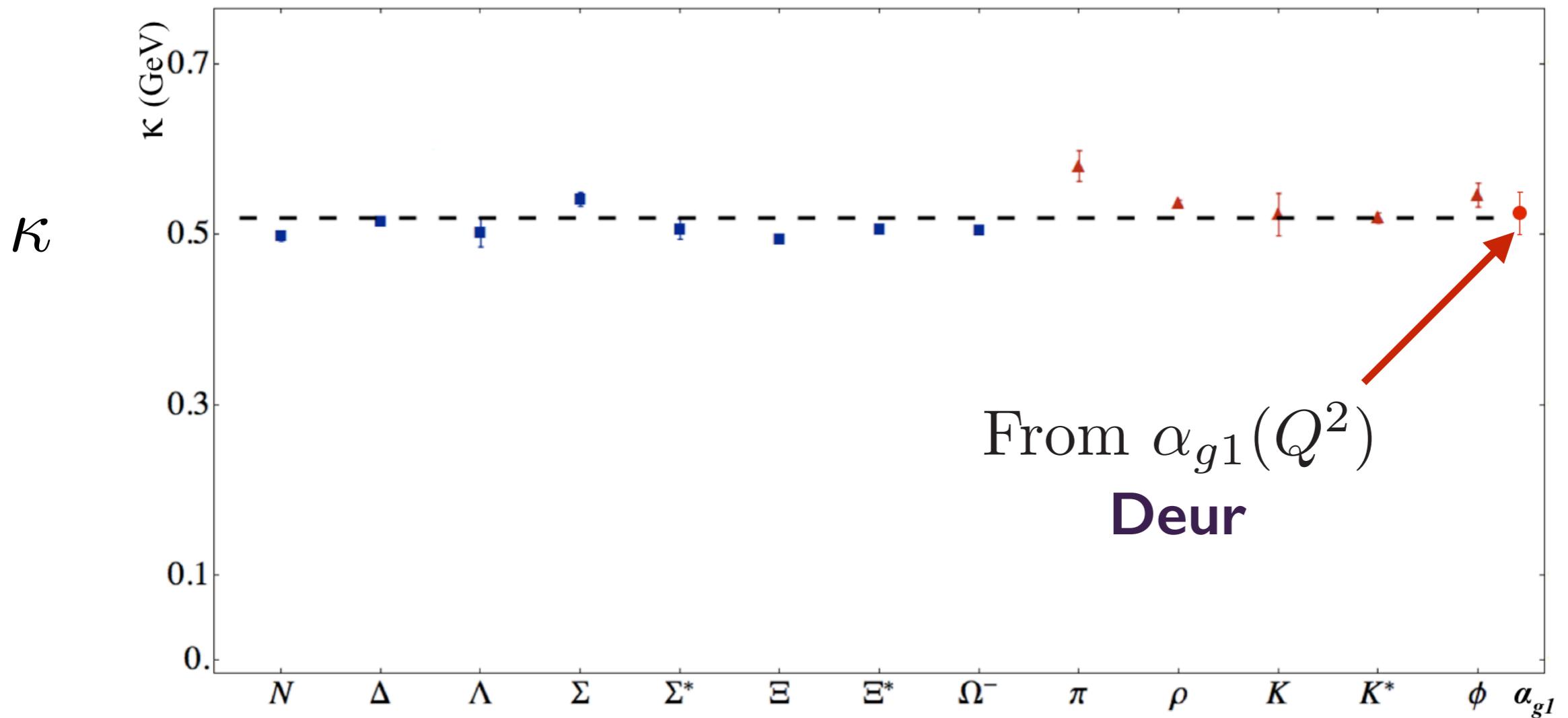
fermions

$\Delta \frac{11}{2}^+$

$$\lambda = \kappa^2$$

de Tèramond, Dosch, Lorce', sjb

$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



***Fit to the slope of Regge trajectories,
including radial excitations***

***Same Regge Slope for Meson, Baryons:
Supersymmetric feature of hadron physics***

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}(\kappa^2 z^2) e^{-\kappa^2 z^2/2}$$

- Normalization $(F_1^p(0) = 1, V(Q=0, z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

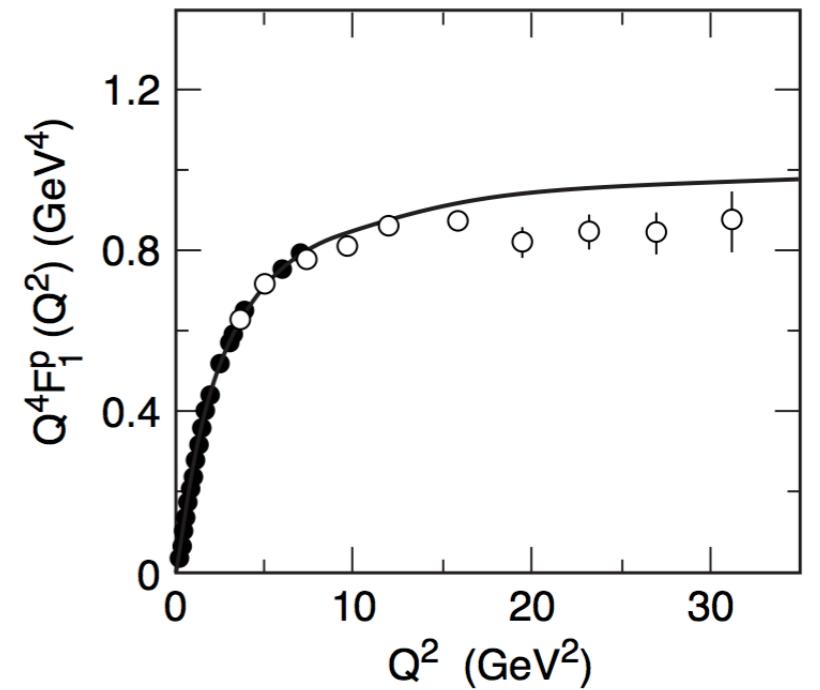
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_\rho^2 \rightarrow 4\kappa^2(n + 1/2)$



Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

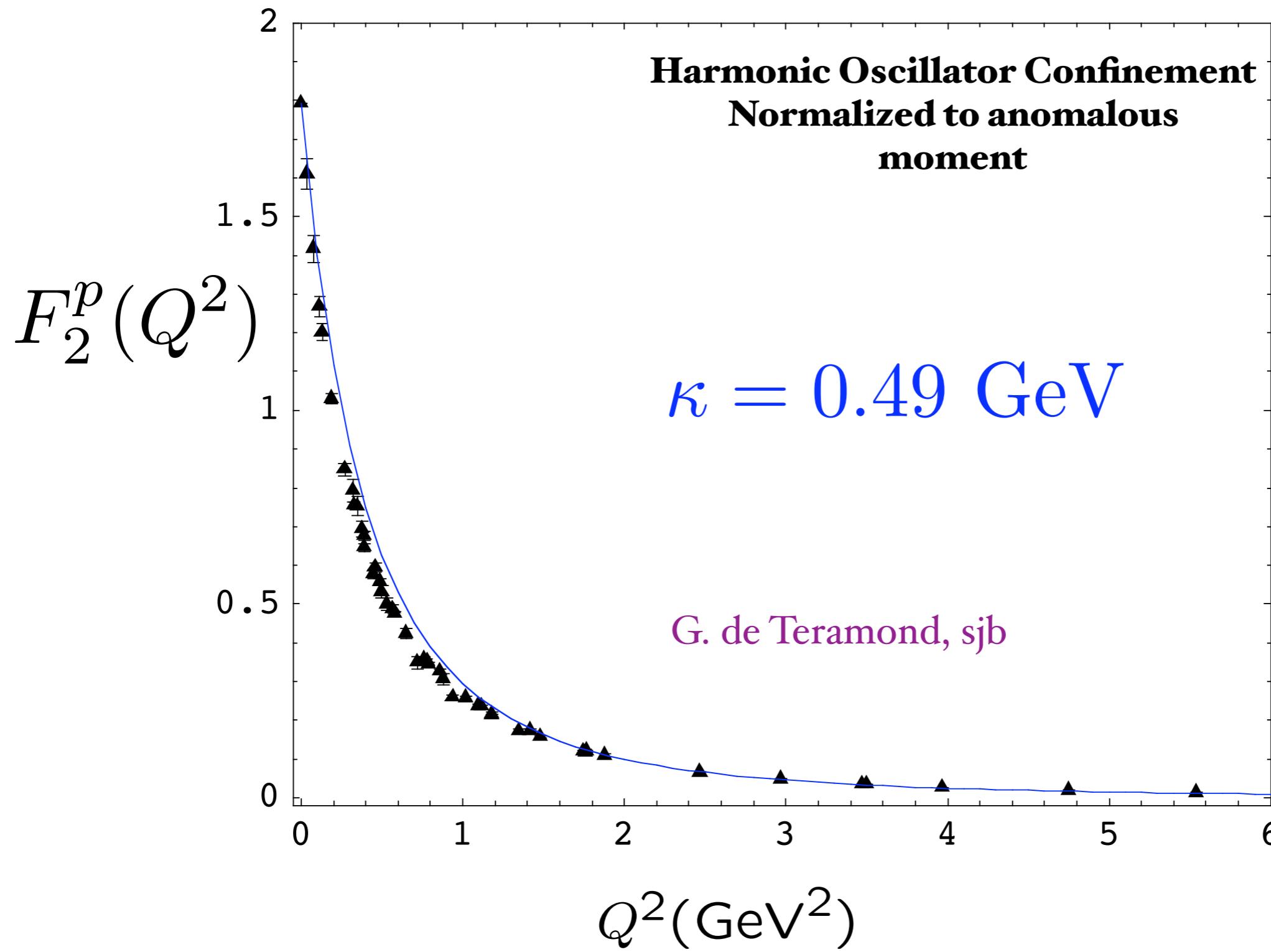
$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

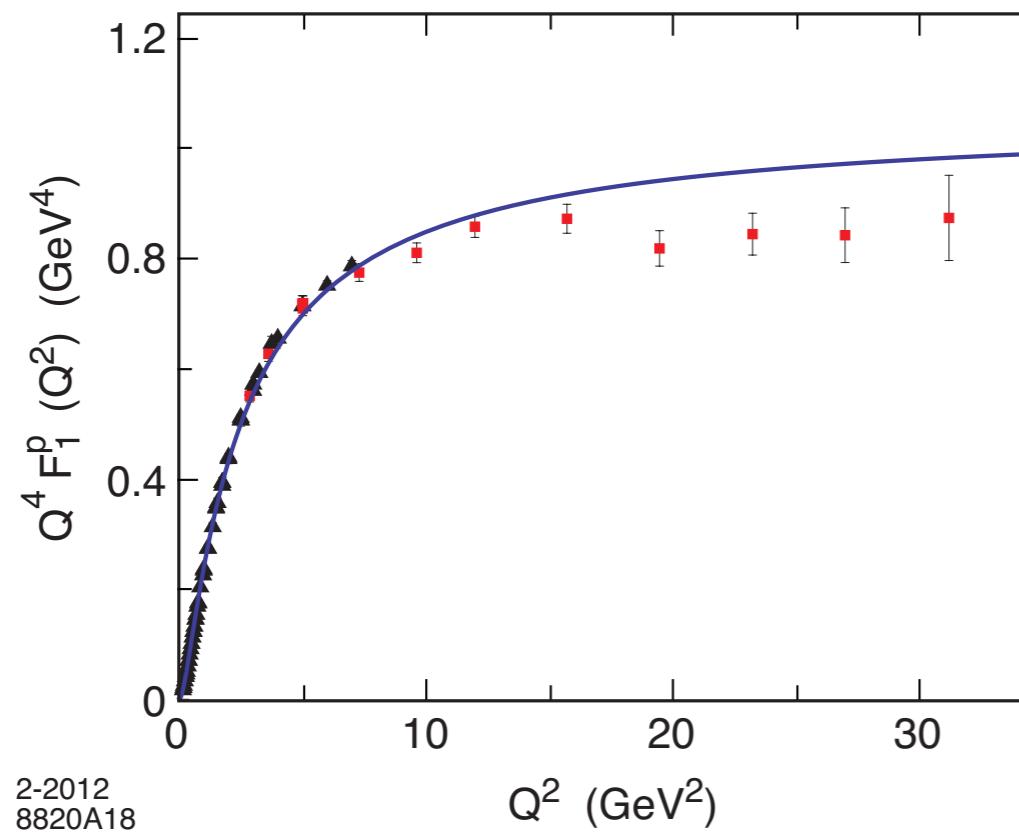
where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

Spacelike Pauli Form Factor

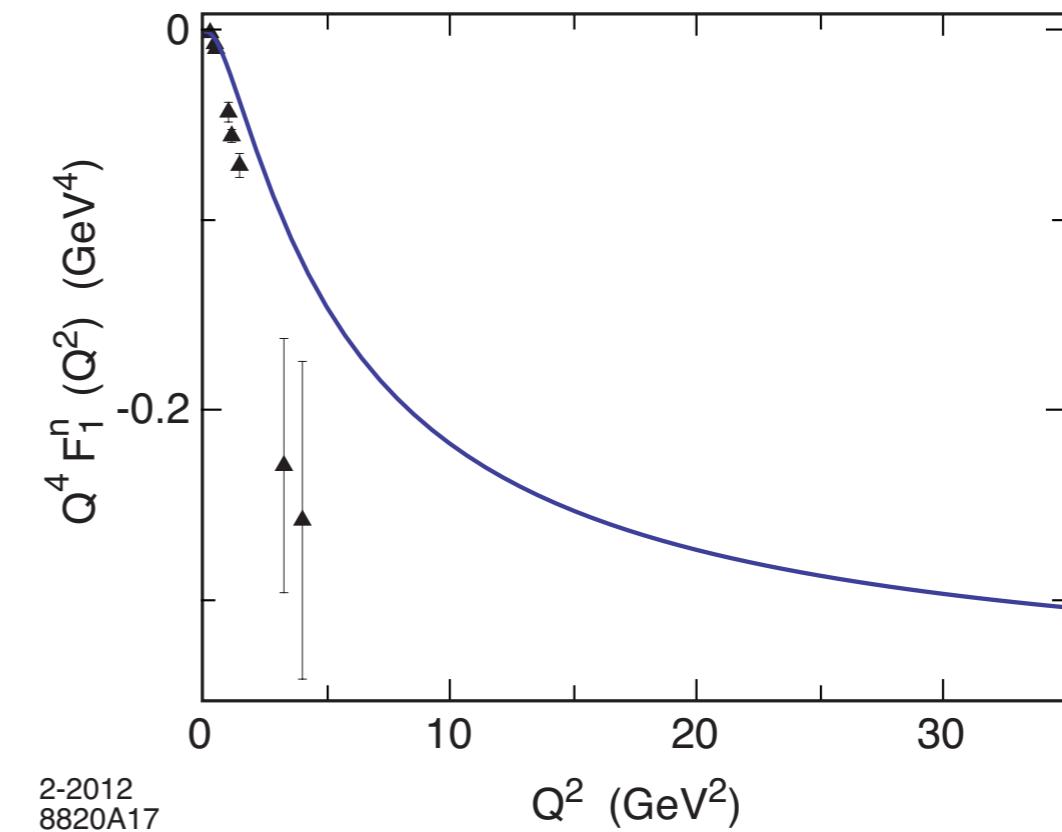
From overlap of $L = 1$ and $L = 0$ LFWFs



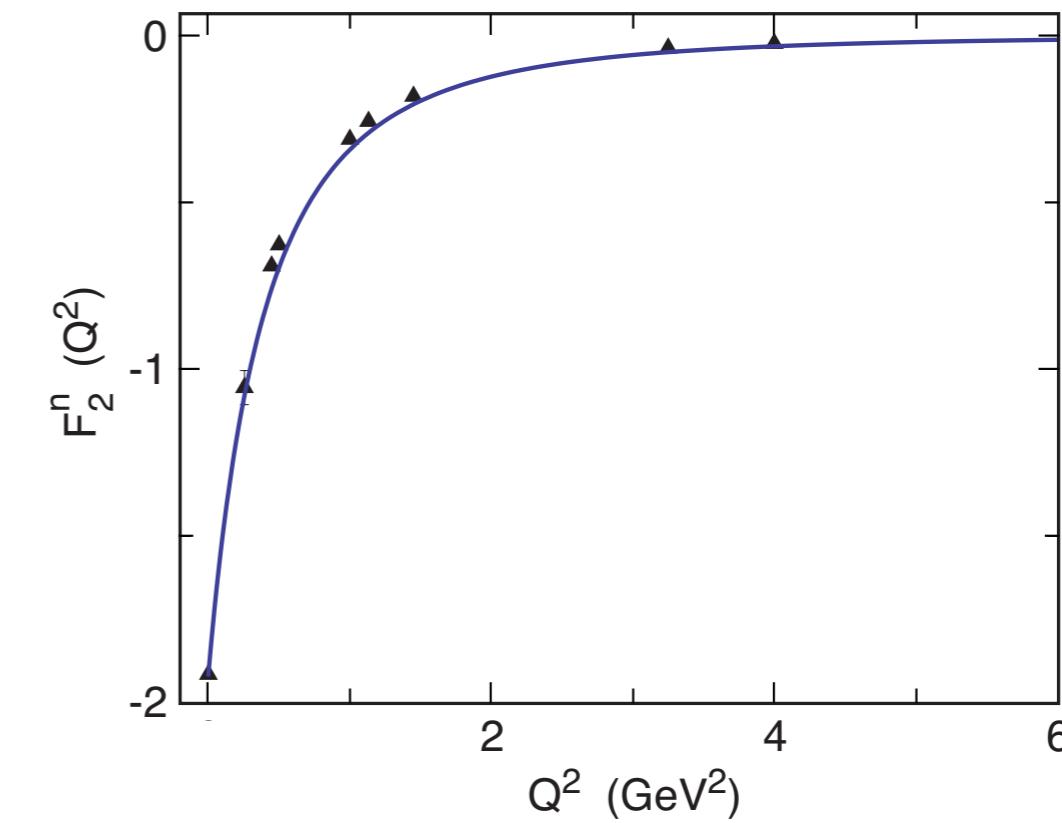
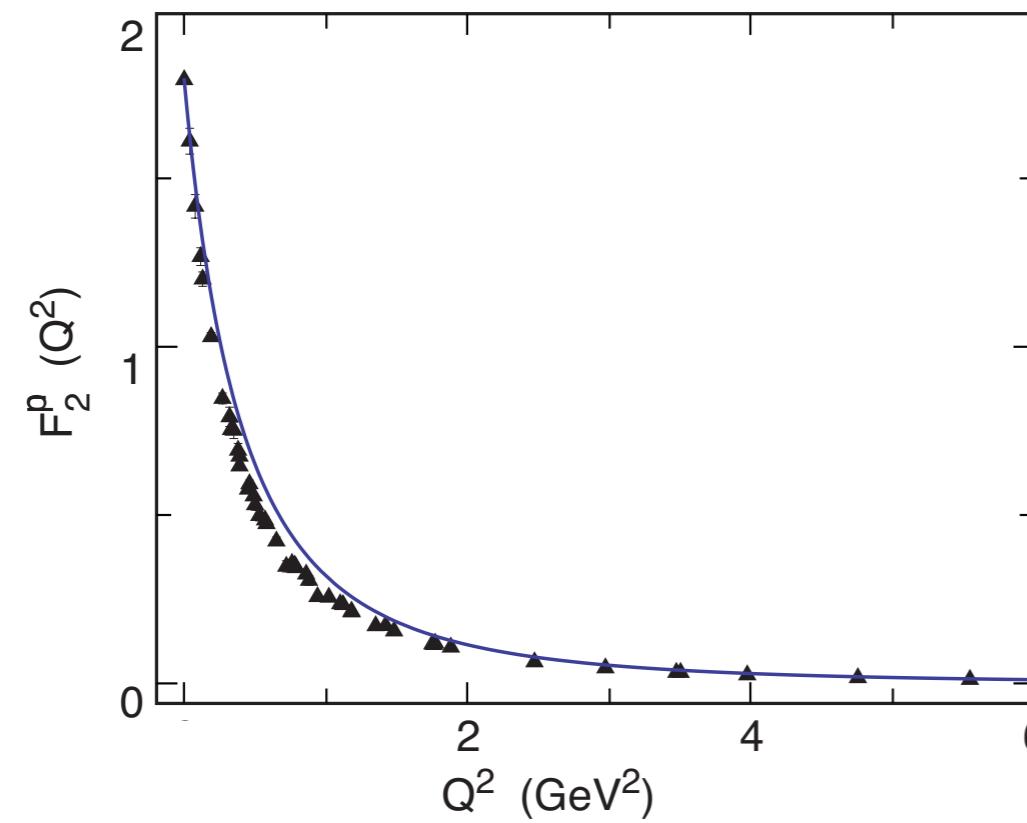
Using $SU(6)$ flavor symmetry and normalization to static quantities



2-2012
8820A18



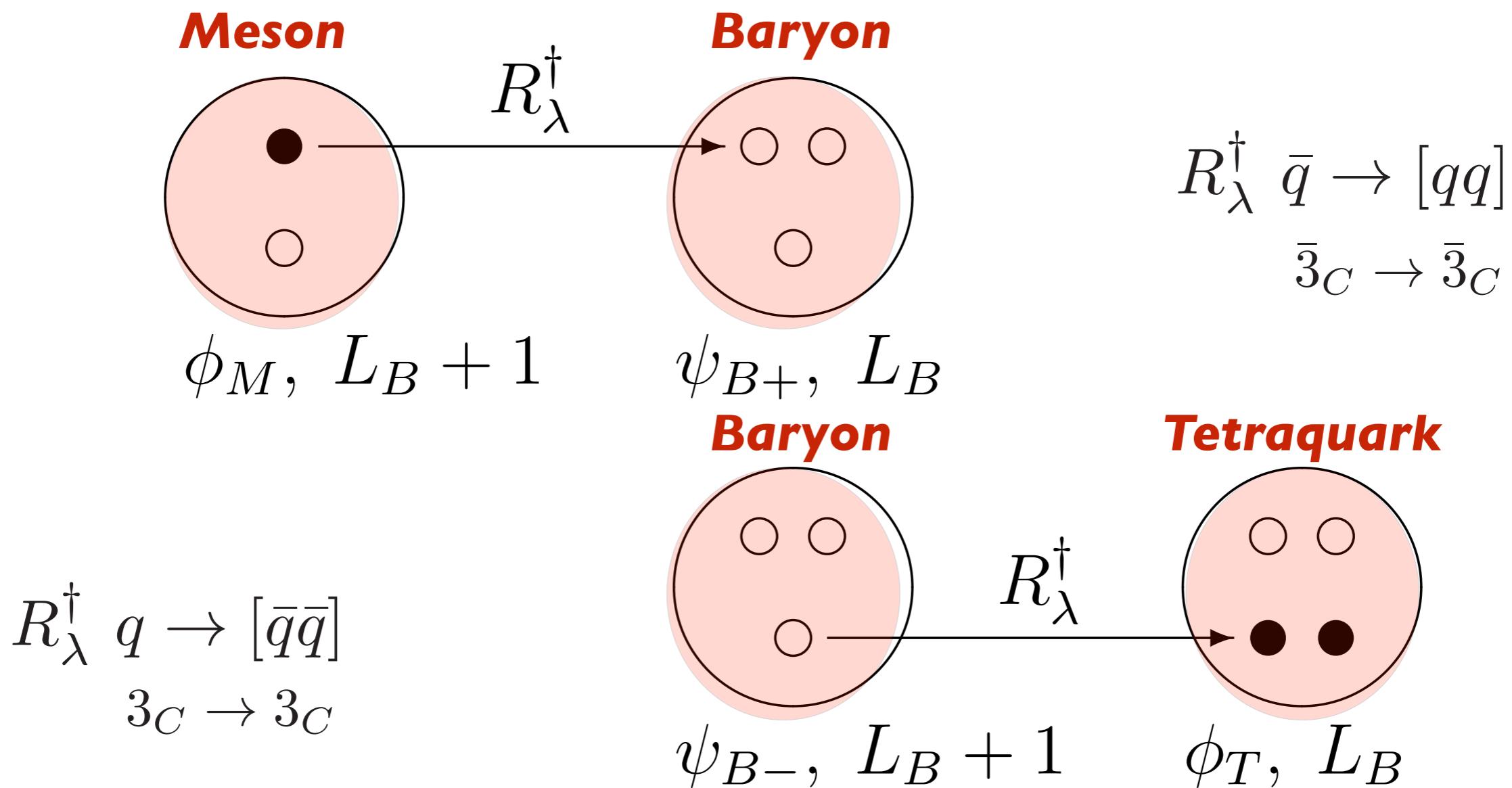
2-2012
8820A17



Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



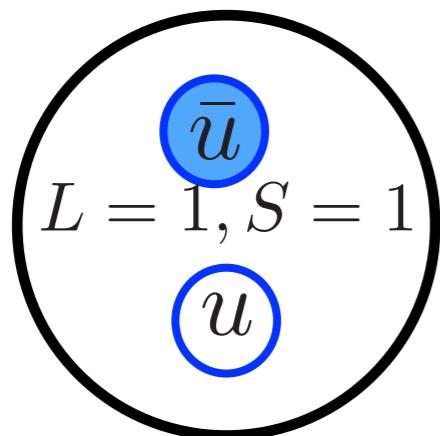
Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Superconformal Algebra 4-Plet

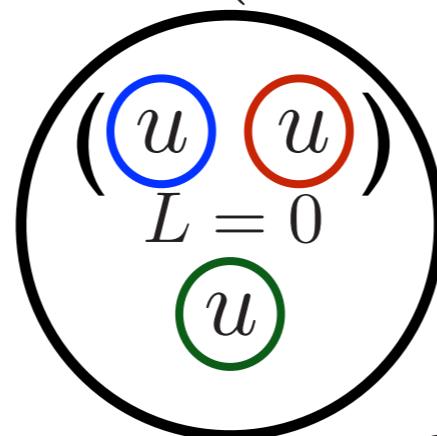
$$R_\lambda^\dagger \quad \bar{q} \rightarrow (qq) \quad S = 1 \\ \bar{3}_C \rightarrow \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$



$\Delta^+(1232)$

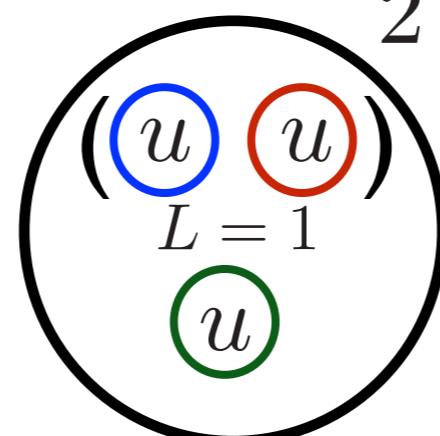


Tetraquark

$J^{PC} = 1^{++}$

$a_1(1260)$

$J^{PC} = 2^{++}$



$S = 0$
 $L = 0$

Meson

$$R_\lambda^\dagger \quad q \rightarrow [\bar{q}\bar{q}] \\ 3_C \rightarrow 3_C$$

Baryon

Meson			Baryon			Tetraquark		
q -cont	$J^{P(C)}$	Name	q -cont	J^P	Name	q -cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$h_1(1170)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$\sigma(500)$
$\bar{q}q$	2^{-+}	$\eta_2(1645)$	$[ud]q$	$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	—
$\bar{q}q$	1^{--}	$\rho(770), \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320), f_2(1270)$	$(qq)q$	$(3/2)^+$	$\Delta(1232)$	$(qq)[\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^-	$\rho_3(1690), \omega_3(1670)$	$(qq)q$	$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$	$(qq)[\bar{u}\bar{d}]$	1^-	$\pi_1(1600)$
$\bar{q}q$	4^{++}	$a_4(2040), f_4(2050)$	$(qq)q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$(qq)[\bar{u}\bar{d}]$	—	—
$\bar{q}s$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	1^+	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	0^+	$K_0^*(1430)$
$\bar{q}s$	2^-	$K_2(1770)$	$[ud]s$	$(3/2)^-$	$\Lambda(1520)$	$[ud][\bar{s}\bar{q}]$	1^-	—
$\bar{s}q$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	1^+	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
$\bar{s}q$	2^+	$K_2^*(890)$	—	—	—	—	—	—
$\bar{s}q$	3^-	$K_2^*(1430)$	$(sq)q$	$(3/2)^+$	$\Sigma(1385)$	$(sq)[\bar{u}\bar{d}]$	1^+	$K_1(1400)$
$\bar{s}q$	4^+	$K_3^*(1780)$	$(sq)q$	$(3/2)^-$	$\Sigma(1670)$	$(sq)[\bar{u}\bar{d}]$	2^-	$K_2(1820)$
$\bar{s}q$	5^+	$K_4^*(2045)$	$(sq)q$	$(7/2)^+$	$\Sigma(2030)$	$(sq)[\bar{u}\bar{d}]$	—	—
$\bar{s}s$	0^{-+}	$\eta'(958)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1380)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
$\bar{s}s$	2^{-+}	$\eta_2(1870)$	$[sq]s$	$(3/2)^-$	$\Xi(1620)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$a_0(1450)$
$\bar{s}s$	3^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	4^{++}	$f_2'(1525)$	$(sq)s$	$(3/2)^+$	$\Xi^*(1530)$	$(sq)[\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	5^{+-}	$\Phi_3(1850)$	$(sq)s$	$(3/2)^-$	$\Xi(1820)$	$(sq)[\bar{s}\bar{q}]$	—	$a_1(1420)$
$\bar{s}s$	6^{++}	$f_2(1640)$	$(ss)s$	$(3/2)^+$	$\Omega(1672)$	$(ss)[\bar{s}\bar{q}]$	1^+	$K_1(1650)$

Meson

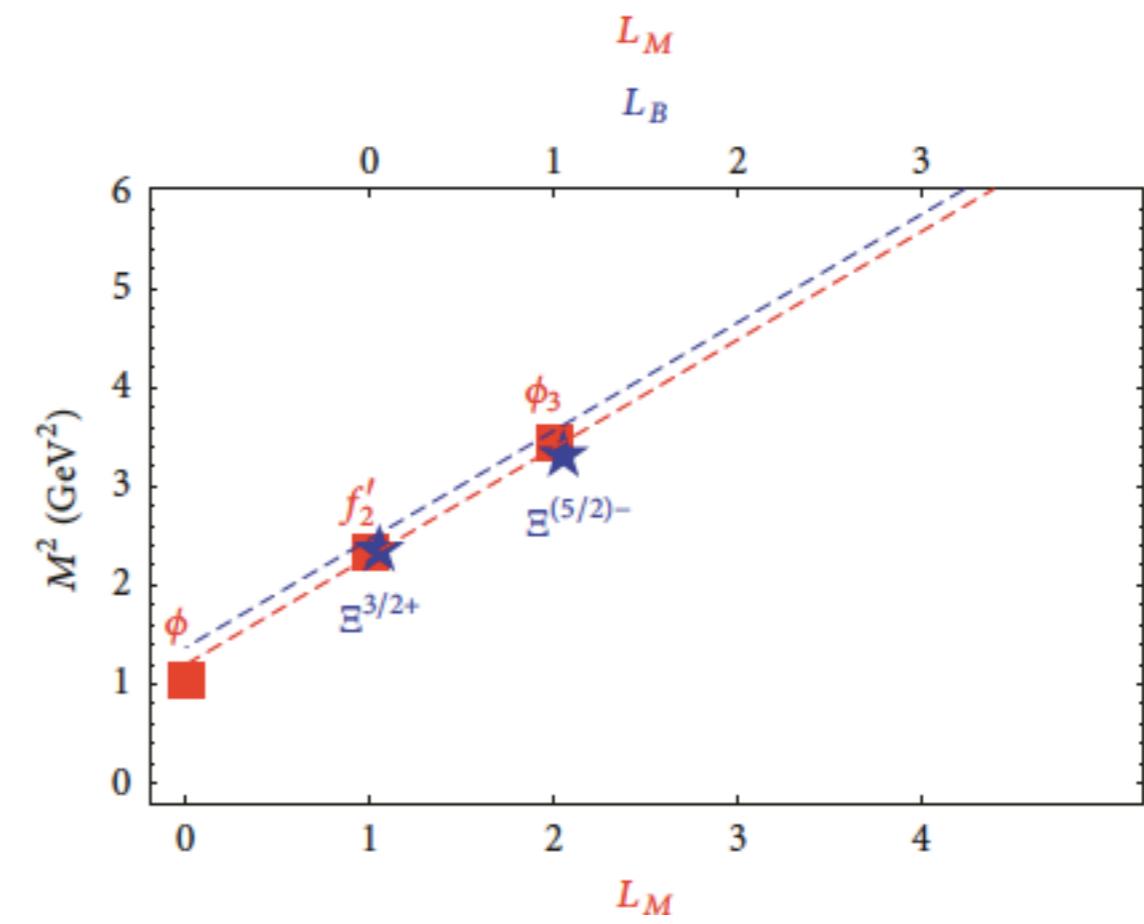
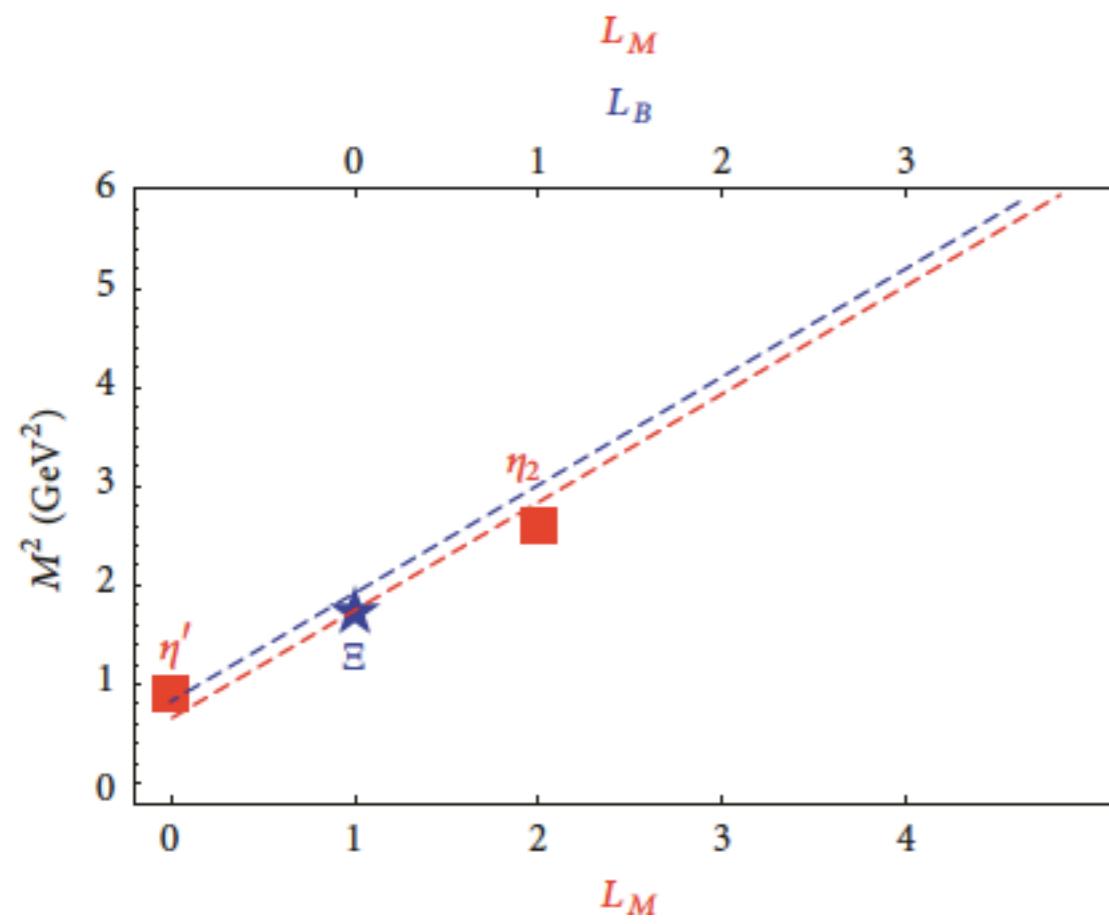
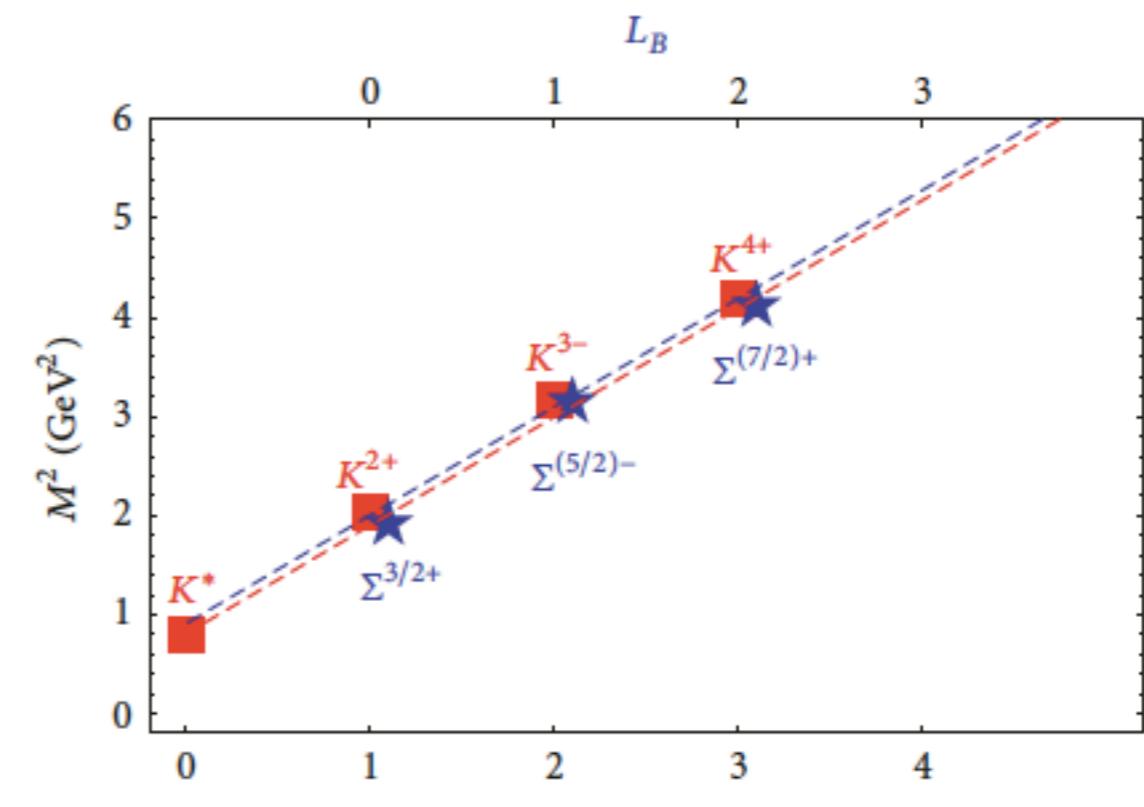
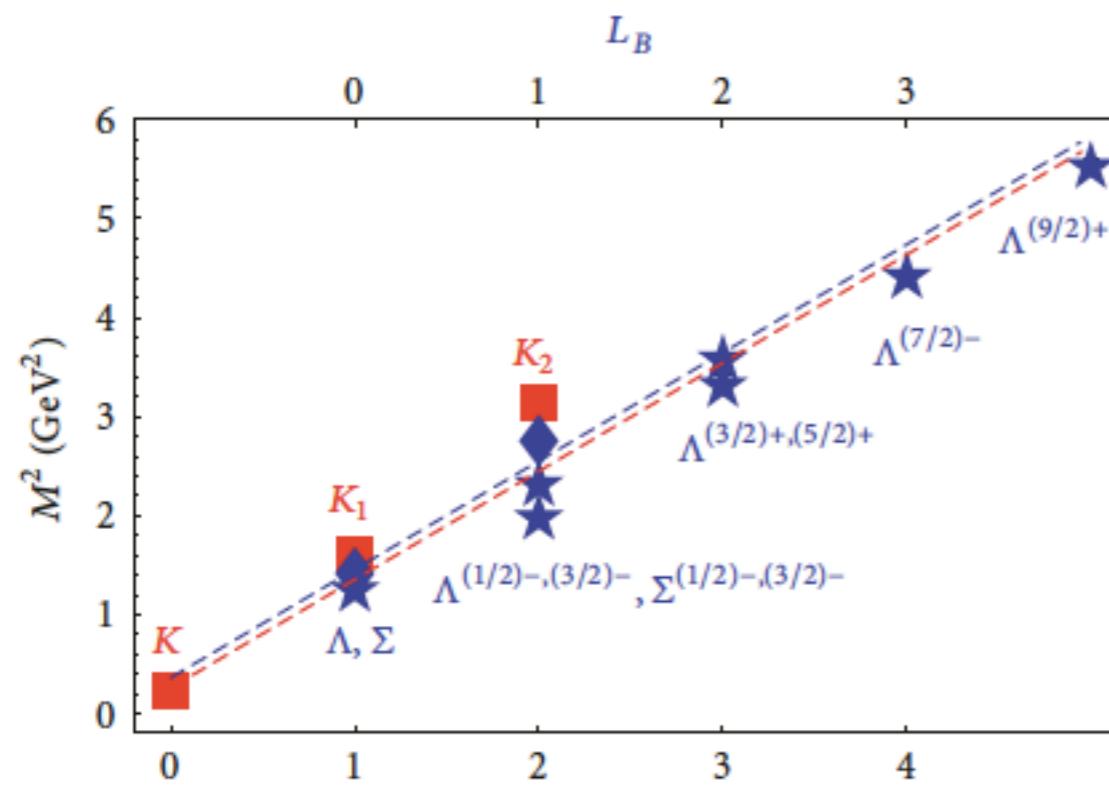
Baryon

Tetraquark

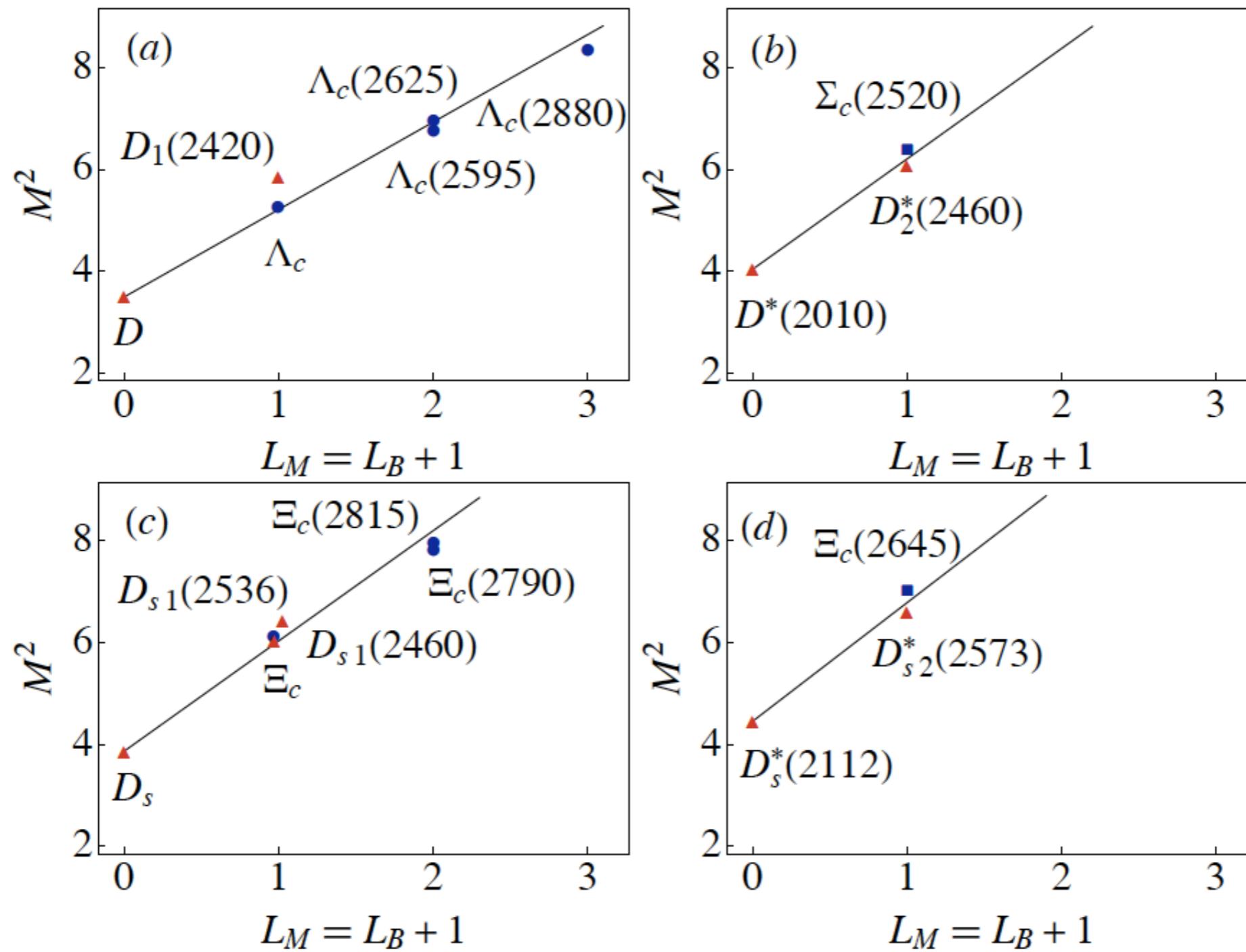
New Organization of the Hadron Spectrum

M. Nielsen,
sjb

Supersymmetry across the light and heavy-light spectrum



Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

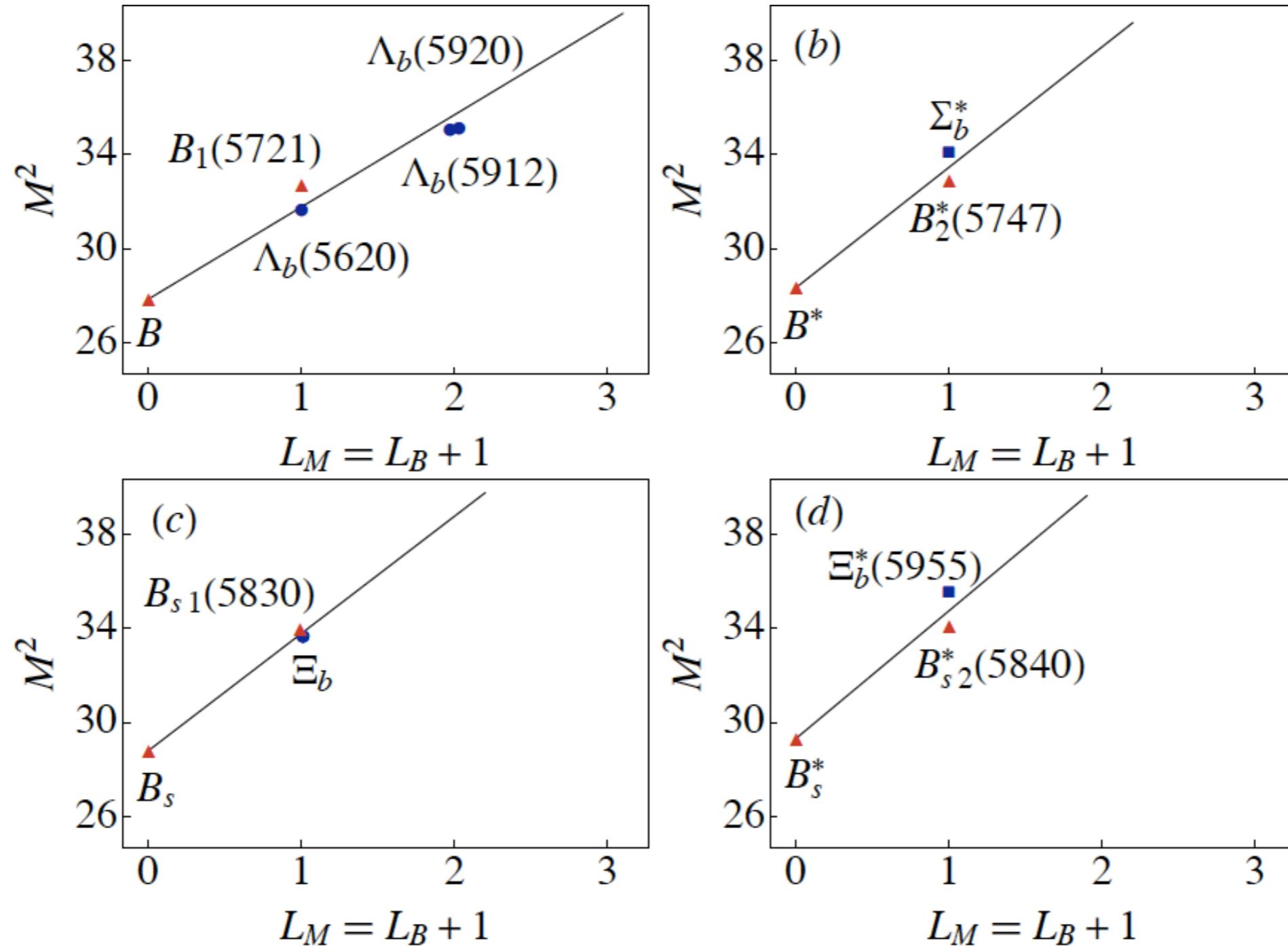
Meson			Baryon			Tetraquark		
q -cont	$J^{P(C)}$	Name	q -cont	J^P	Name	q -cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$D_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

M. Nielsen, sjb

predictions

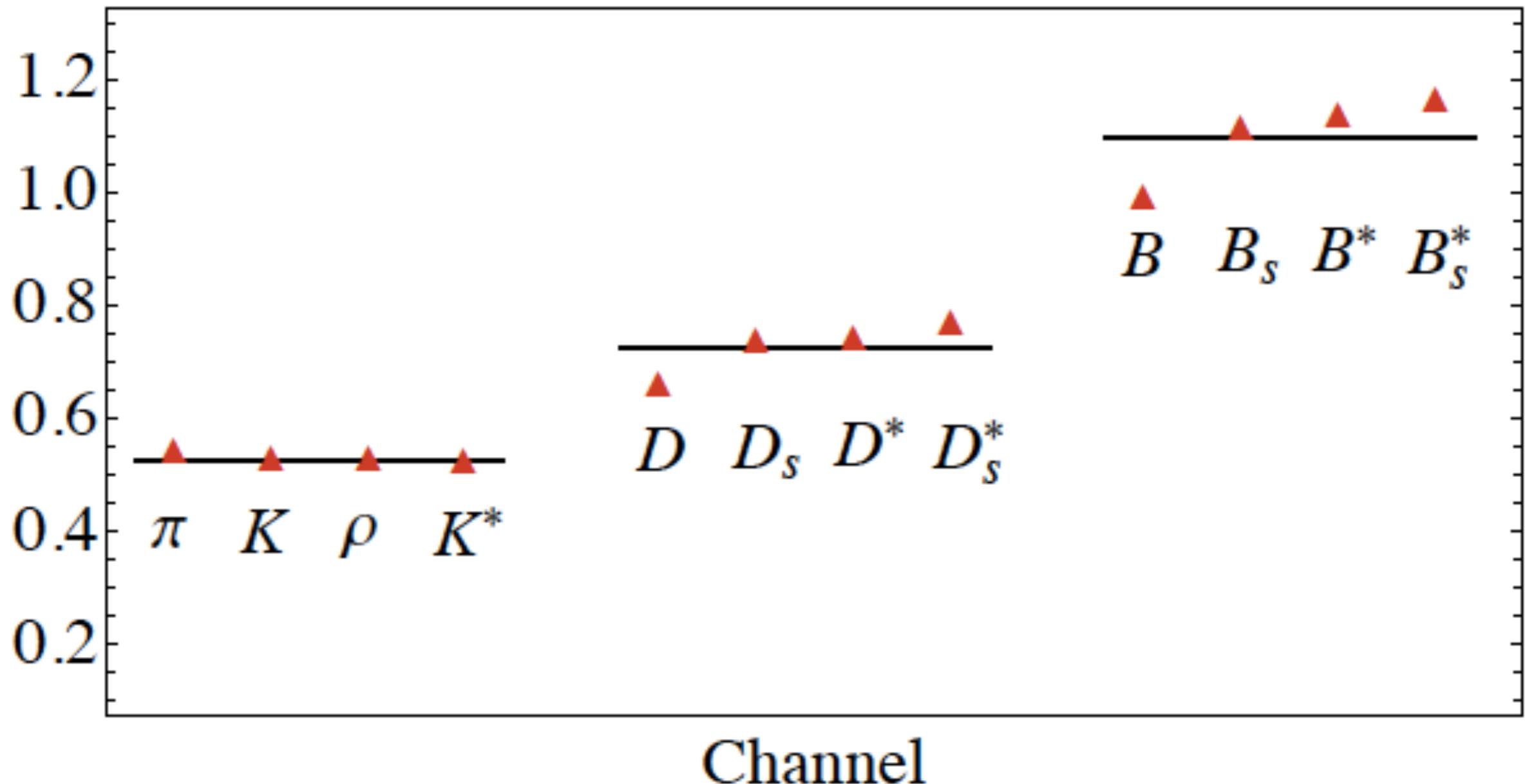
beautiful agreement!

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

$\kappa_R(\text{GeV})$



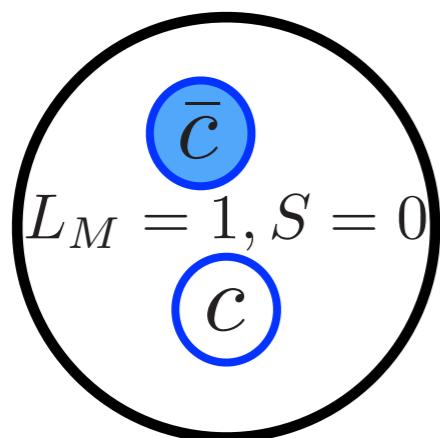
*Regge slope for heavy-light mesons, baryons:
increases with heavy quark mass*

Double-Charm Baryon (SELEX)

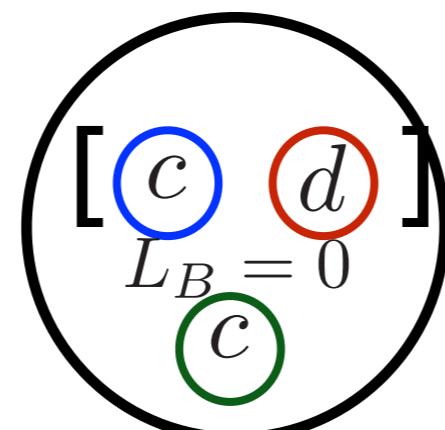
$$R_\lambda^\dagger \bar{q} \rightarrow [qq]$$

$$\bar{3}_C \rightarrow \bar{3}_C$$

$h_c(3525)$



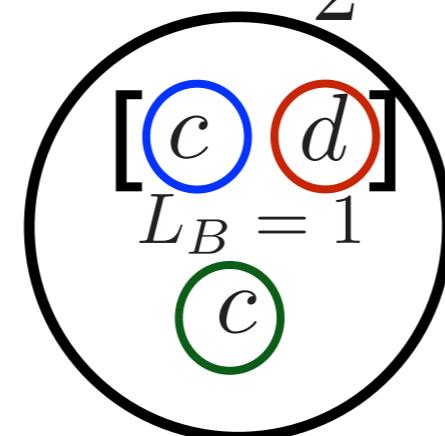
$\Xi_{CC}^+(3520)$



η'_c

$J^{PC} = 1^{+-}$

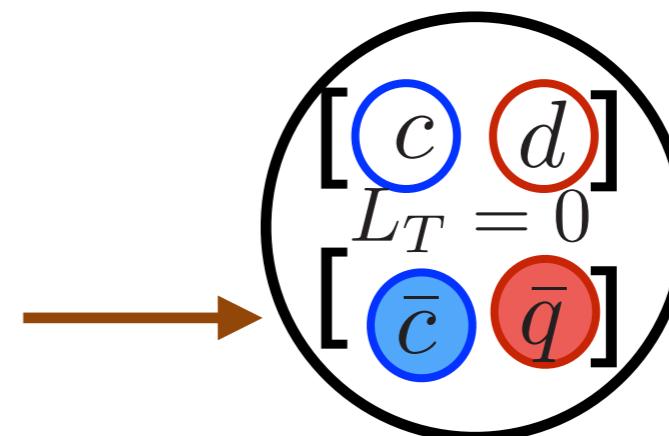
$$J^P = \frac{1}{2}^+$$



Predict Tetraquark $T_{c\bar{c}q\bar{q}}$
 $M_T \sim 3520 \text{ MeV}$

Scalar Diquarks

$$J^{PC} = 0^{++}$$



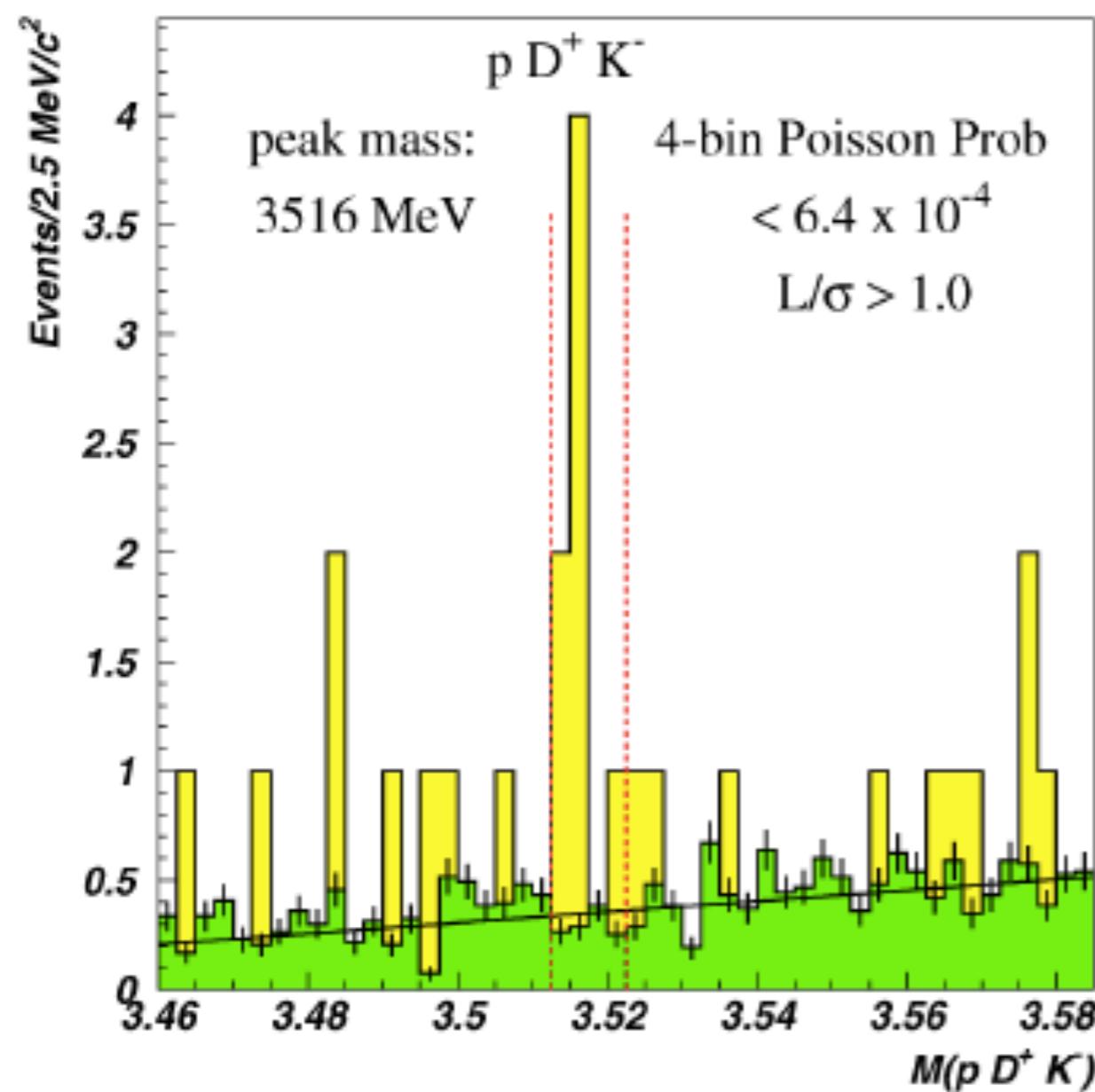
$$R_\lambda^\dagger q \rightarrow [\bar{q}\bar{q}] \quad S = 0$$

$$3_C \rightarrow 3_C$$

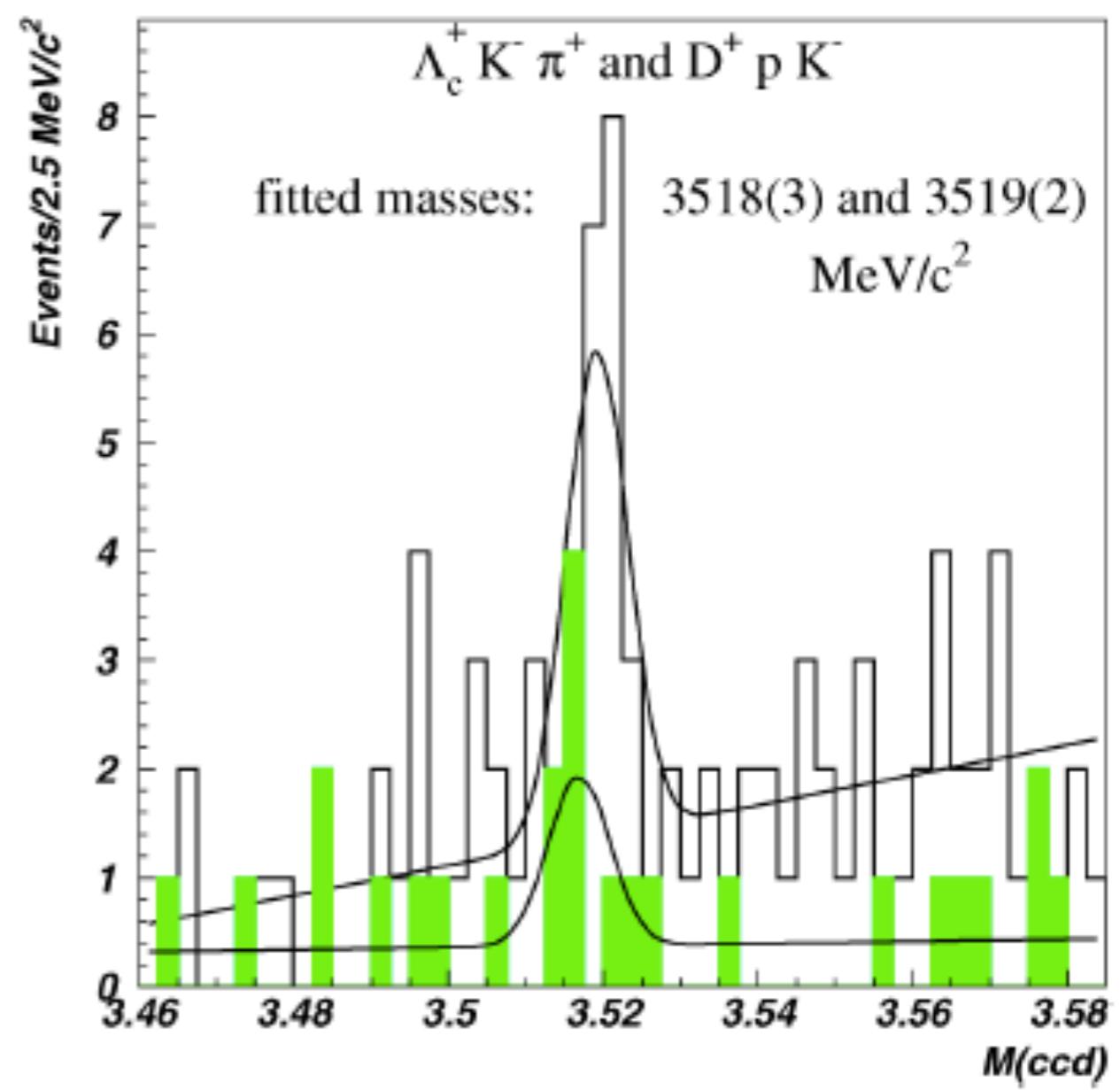
SELEX (3520 ± 1 MeV) $J^P = \frac{1}{2}^- | [cd]c >$

Two decay channels: $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+, p D^+ K^-$

SELEX Collaboration / Physics Letters B 628 (2005) 18–24

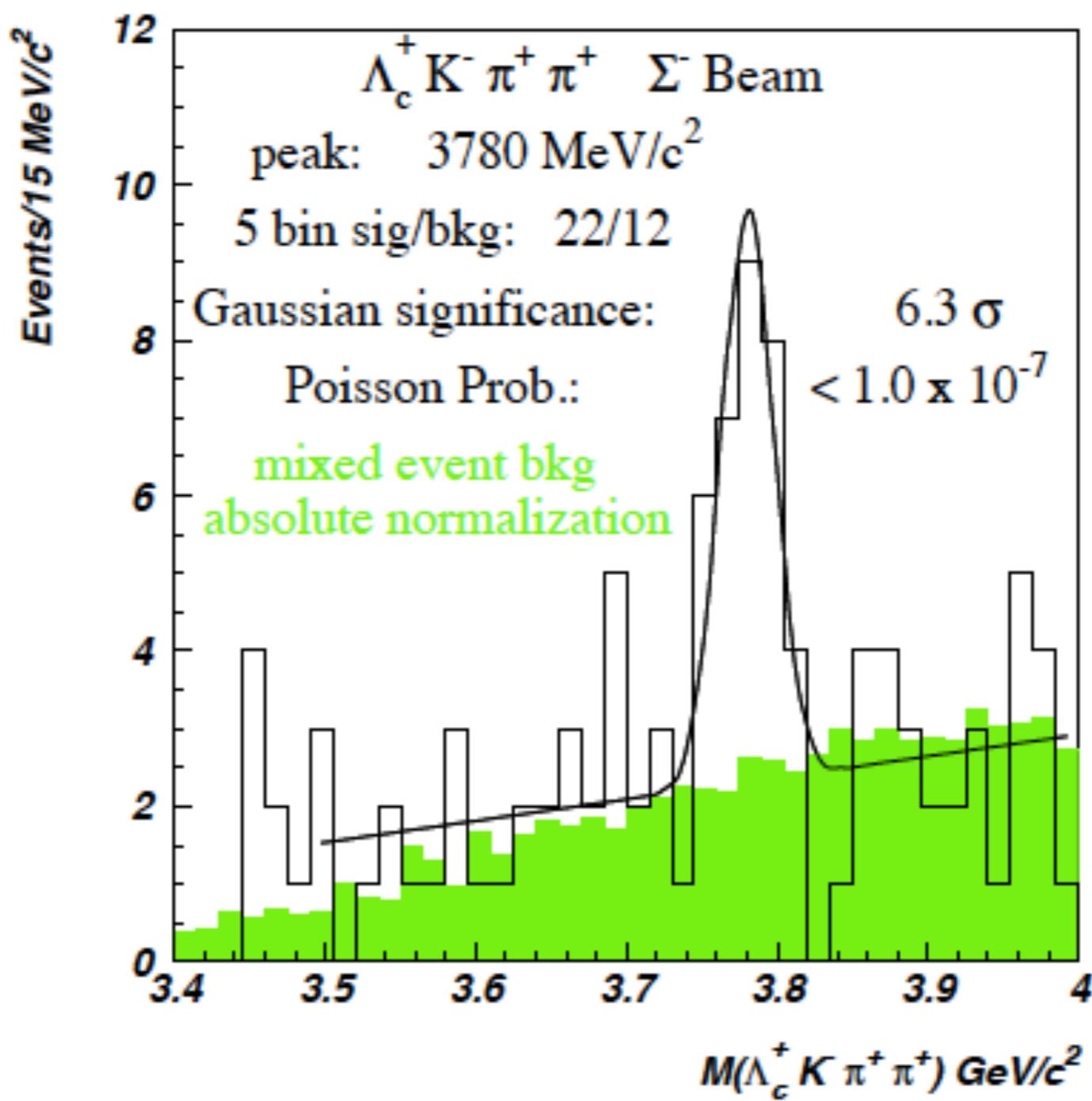


$\Xi_{cc}^+ \rightarrow p D^+ K^-$ mass distribution from Fig. 2(a) with high-statistics measurement of random combinatoric background computed from event-mixing.

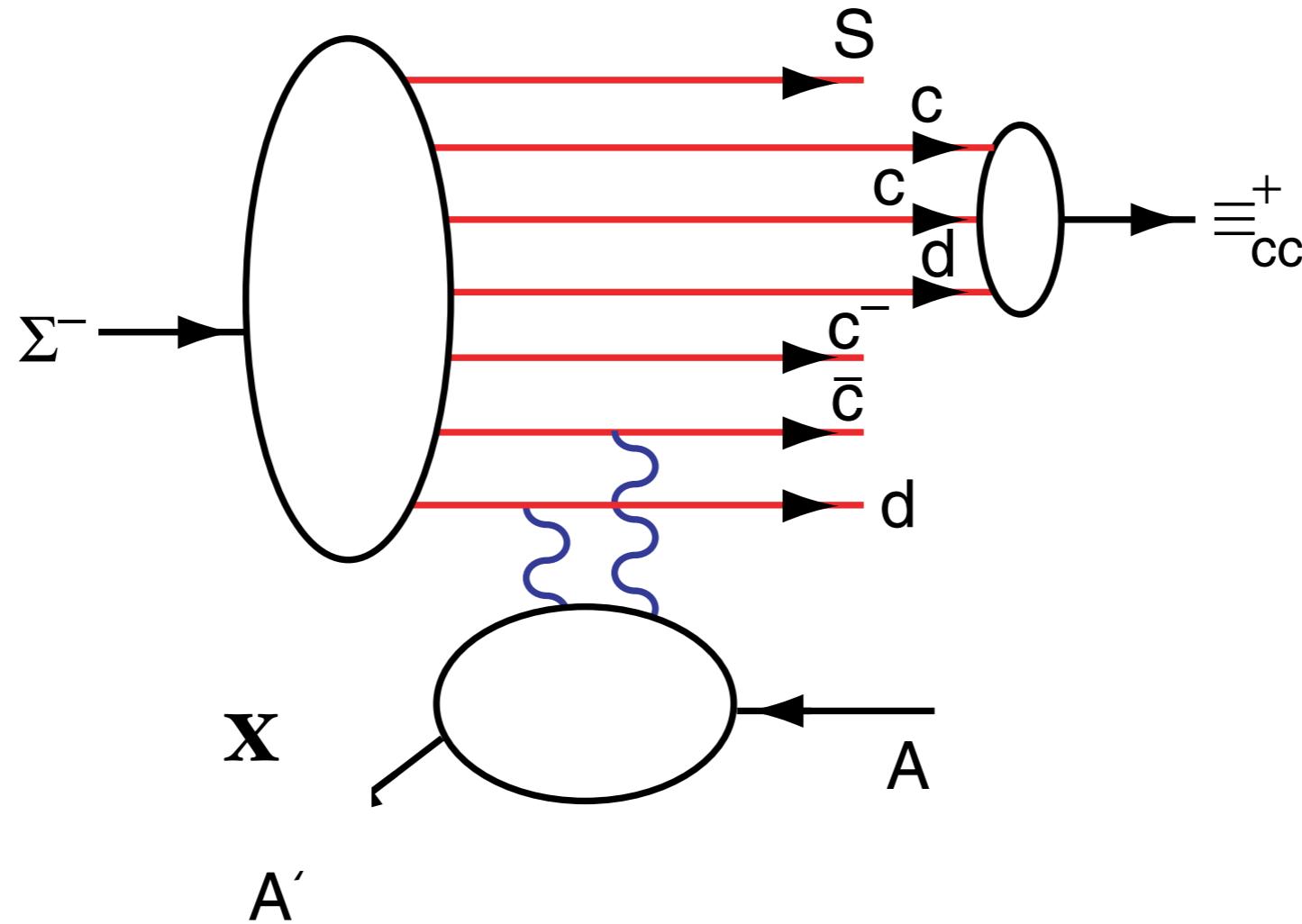


Gaussian fits for $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+ \rightarrow p D^+ K^-$ (shaded data) on same plot.

SELEX: Recent Progress in the Analysis of Charm-Strange and Double-Charm Baryons

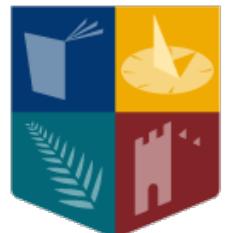


The $\Lambda_c^+ K^- \pi^+ \pi^+$ invariant mass distribution, for Σ^- beam only.



Production of a Double-Charm Baryon

SELEX high x_F $\langle x_F \rangle = 0.33$



SELEX (3520 ± 1 MeV) $J^P = \frac{1}{2}^- |[cd]c >$

Two decay channels: $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+, p D^+ K^-$

LHCb (3621 ± 1 MeV) $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^- |(cu)c >$

$\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$

Groote, Koshkarev, sjb: SELEX& LHCb could both be correct!

Very different production kinematics:

LHCb (central region)

SELEX (Forward, High x_F) where Λ_c, Λ_b were discovered

NA3: Double J/ ψ Hadroproduction measured at High x_F

Radiative Decay:

$\text{LHCb}(3621) \rightarrow \text{SELEX}(3520) + \gamma$

strongly suppressed: $\left[\frac{100 \text{ MeV}}{M_c}\right]^7$

Also: Different diquark structure possible for LHCb: $|(cc)u >$

Underlying Principles

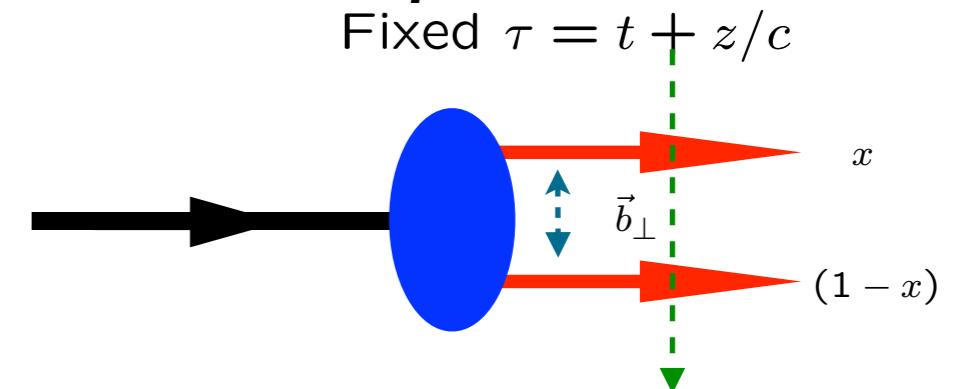
- **Poincarè Invariance: Independent of the observer's Lorentz frame**

- **Quantization at Fixed Light-Front Time τ**

- **Causality: Information within causal horizon**

- **Light-Front Holography: $AdS_5 = LF(3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



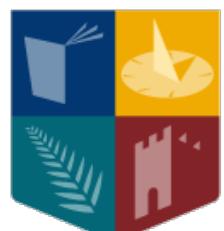
$$\zeta^2 = x(1-x)b_\perp^2$$

- **Single fundamental hadronic mass scale κ : but retains the Conformal Invariance of the Action (dAFF)!**

- **Unique color-confining LF Potential!** $U(\zeta^2) = \kappa^4 \zeta^2$

- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson $q\bar{q} \leftrightarrow$ Baryon $q[qq] \leftrightarrow$ Tetraquark $[qq][\bar{q}\bar{q}]$



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \quad S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

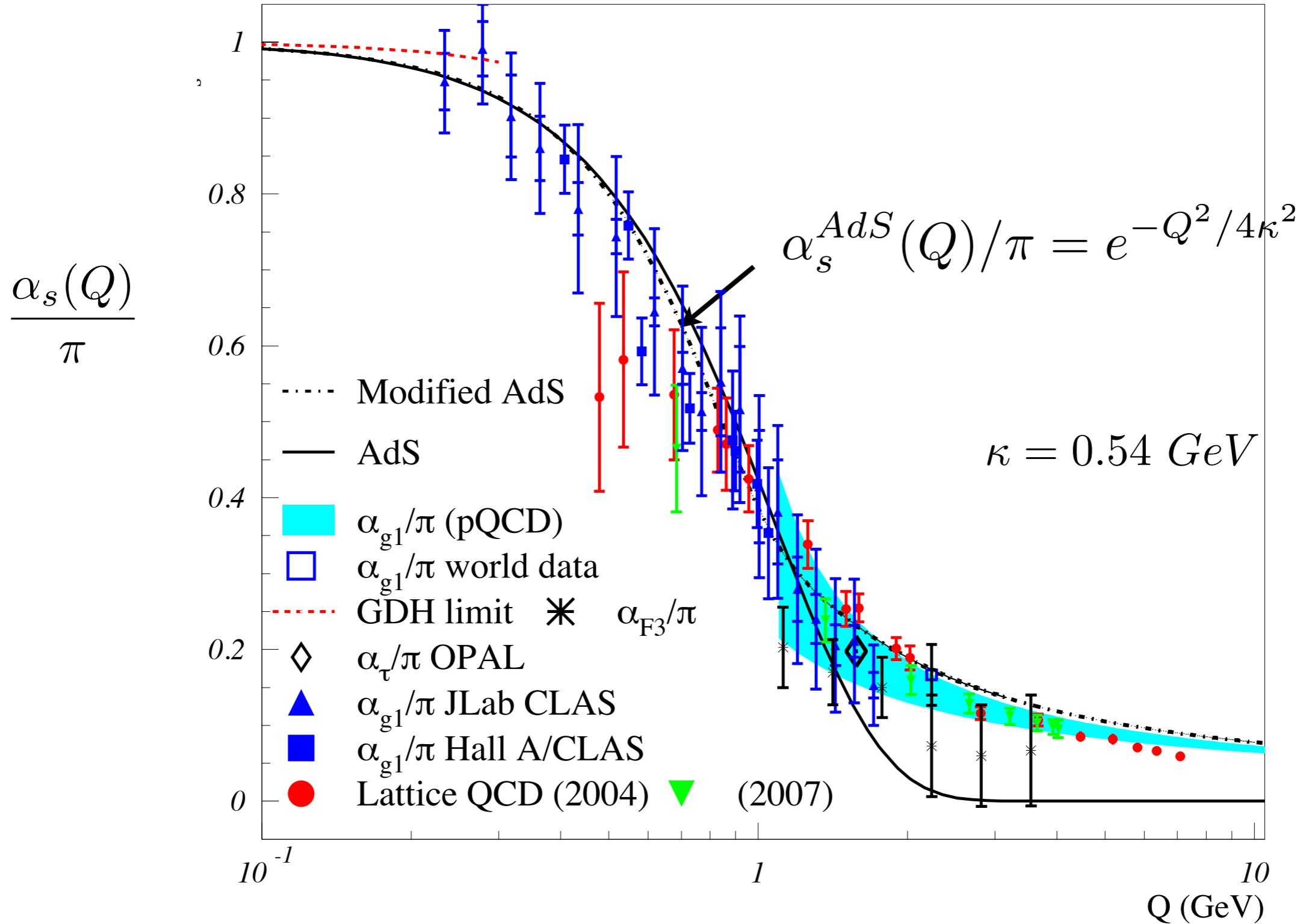
Bjorken sum rule defines effective charge

$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal $\beta_0, \beta,$***

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1 \text{ GeV}$

$$e^\varphi = e^{+\kappa^2 z^2}$$

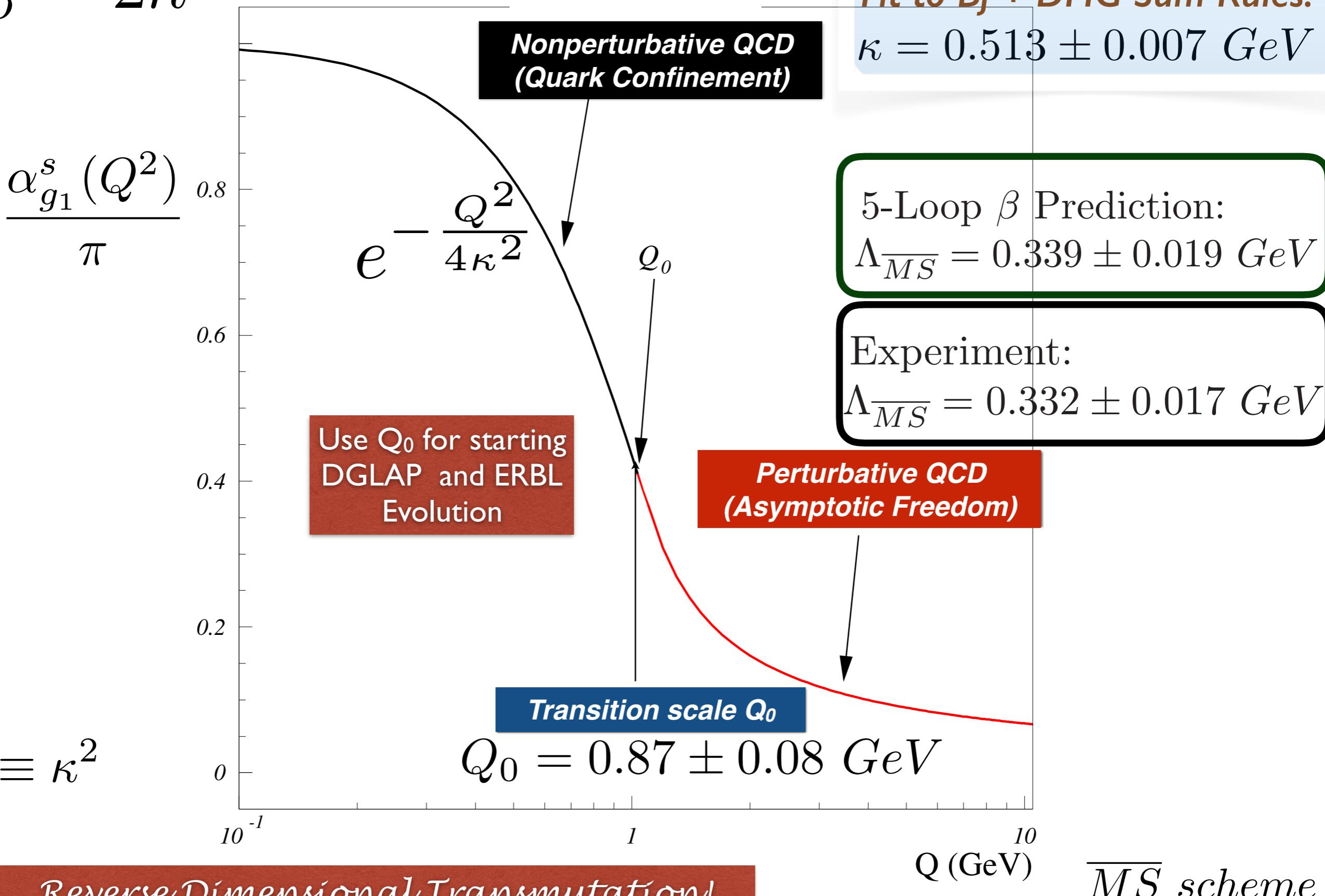
Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

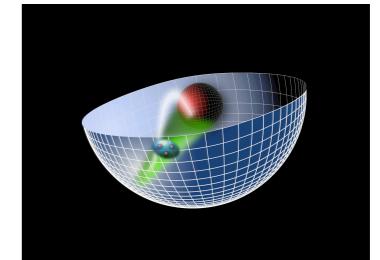
All-Scale QCD Coupling



Underlying Principles

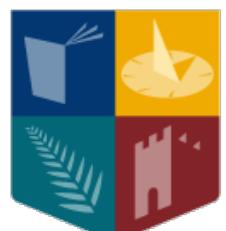
- **Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time \mathbf{T}**
- **Causality: Information within causal horizon: Light-Front**
- **Light-Front Holography: $AdS_5 = LF(3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



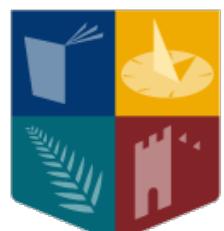
- **Introduce Mass Scale κ while retaining the Conformal Invariance of the Action (dAFF)**
- **Unique Dilaton in AdS_5 : $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson $q\bar{q} \leftrightarrow$ Baryon $q[qq] \leftrightarrow$ Tetraquark $[qq][\bar{q}\bar{q}]$



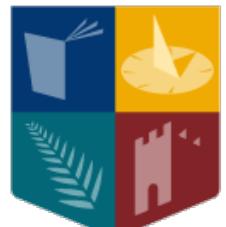
Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale
(DeAlfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincarè Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests
- Systematically improvable: Basis LF Quantization (BLFQ)

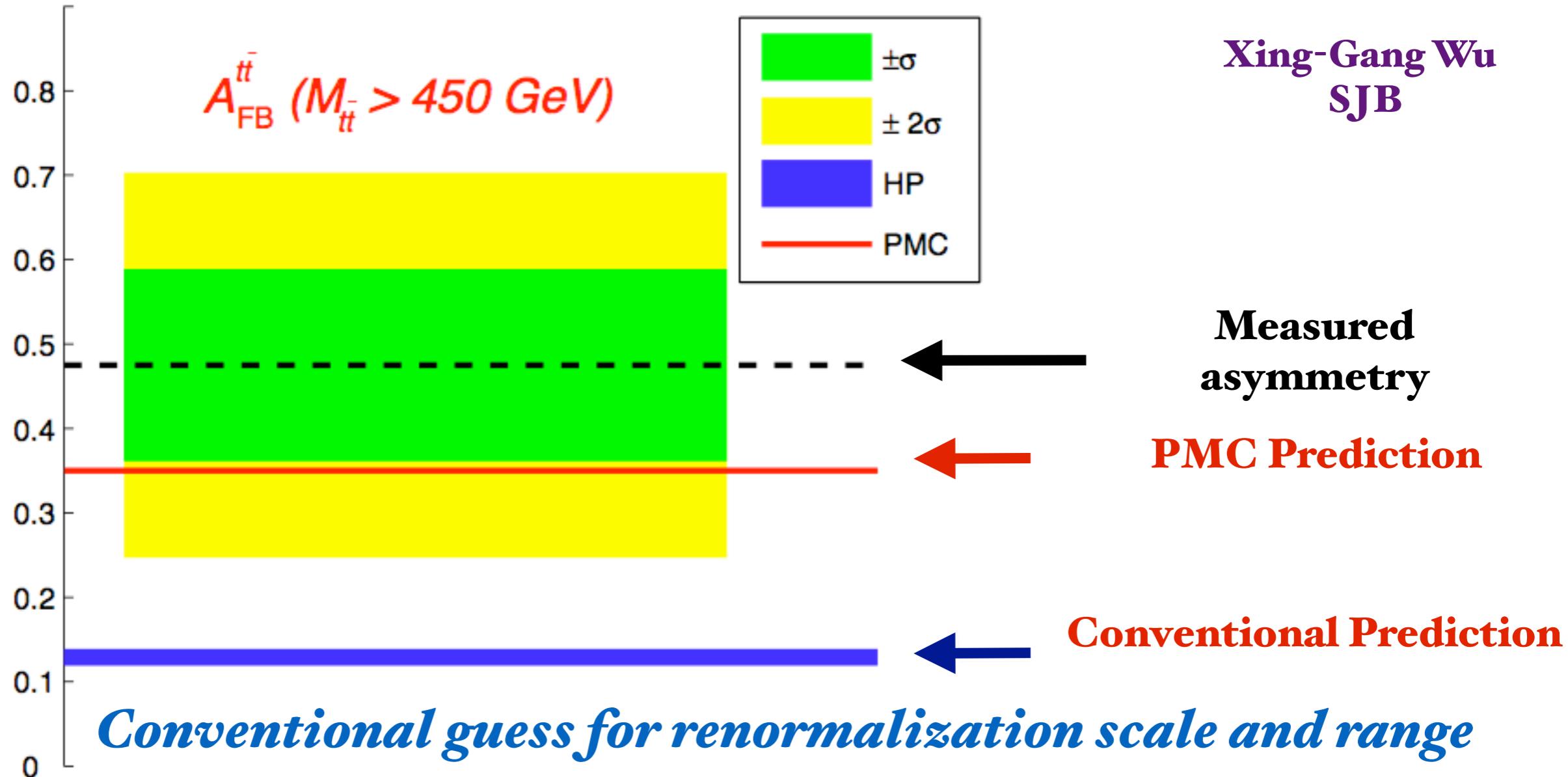


Invariance Principles of Quantum Field Theory

- **Polcarè Invariance:** *Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- **Causality:** *Information within causal horizon: Front Form*
- **Gauge Invariance:** *Physical predictions of gauge theories must be independent of the choice of gauge*
- **Scheme-Independence:** *Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*
- **Mass-Scale Invariance:**
Conformal Invariance of the Action (DAFF)

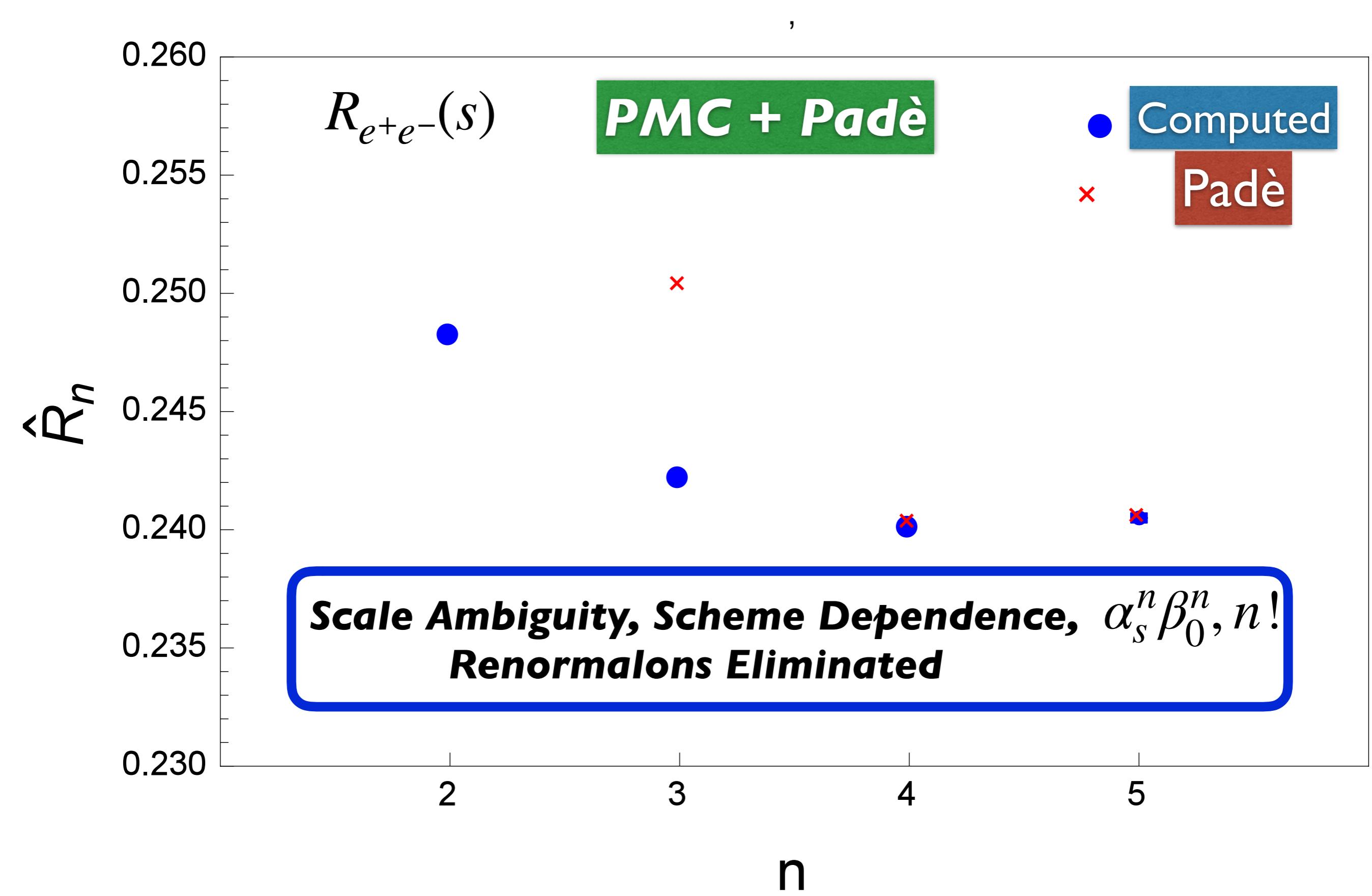


The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the ‘Principle of Maximum Conformality’ (PMC)



BLM/PMC: Scheme-Independent, same as Gell-Mann-Low in pQED

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1σ of CDF/D0 measurements using PMC/BLM scale setting



Extending the Predictive Power of pQCD

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA

*Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(\text{expt})$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:

- (A) Light-Front Quantization: causal, frame-independent vacuum
- (B) New understanding of QCD “Condensates”
- (C) Higgs Light-Front Zero Mode

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0\rangle = E_0|\psi_0\rangle, E_0 = \min\{E_i\}$$

*Eigenstate defined at one time t over all space;
Acausal! Frame-Dependent*

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0\rangle_{LF} = M_0^2|\psi_0\rangle_{LF}, M_0^2 = 0.$$

*Frame-independent eigenstate at fixed LF time $\tau = t+z/c$
within causal horizon*

Frame-independent description of the causal physical universe!

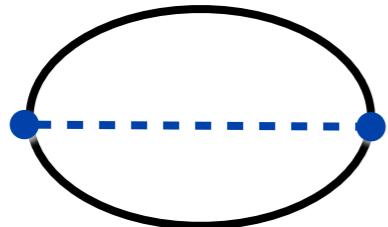
Front-Form Vacuum

All LF propagators have positive k^+

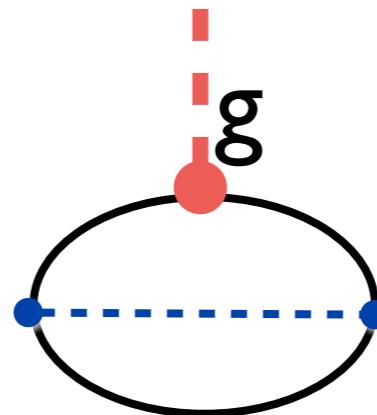
$$k^+ = k^0 + k^3 \geq 0 \text{ since } |\vec{k}| \leq k^0$$

P^+ Momentum Conserved

$$P^+ = 0$$



zero !!



zero !!

$$\langle 0 | T^{\mu\nu} | 0 \rangle = 0$$

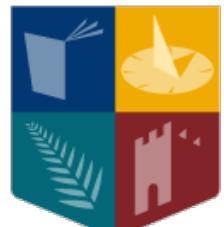
Graviton does not couple to LF vacuum!

Vanishing gravitational coupling even in presence of zero modes

Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state $M=0$.
- Trivial up to $k^+=0$ zero modes -- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (**Srivastava, sjb**)
- QCD and AdS/QCD: “In-hadron” condensates (**Maris, Tandy Roberts**) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD, EW



Advantages of the Dirac's Front Form for Hadron Physics Poincare' Invariant

Physics Independent of Observer's Motion

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!

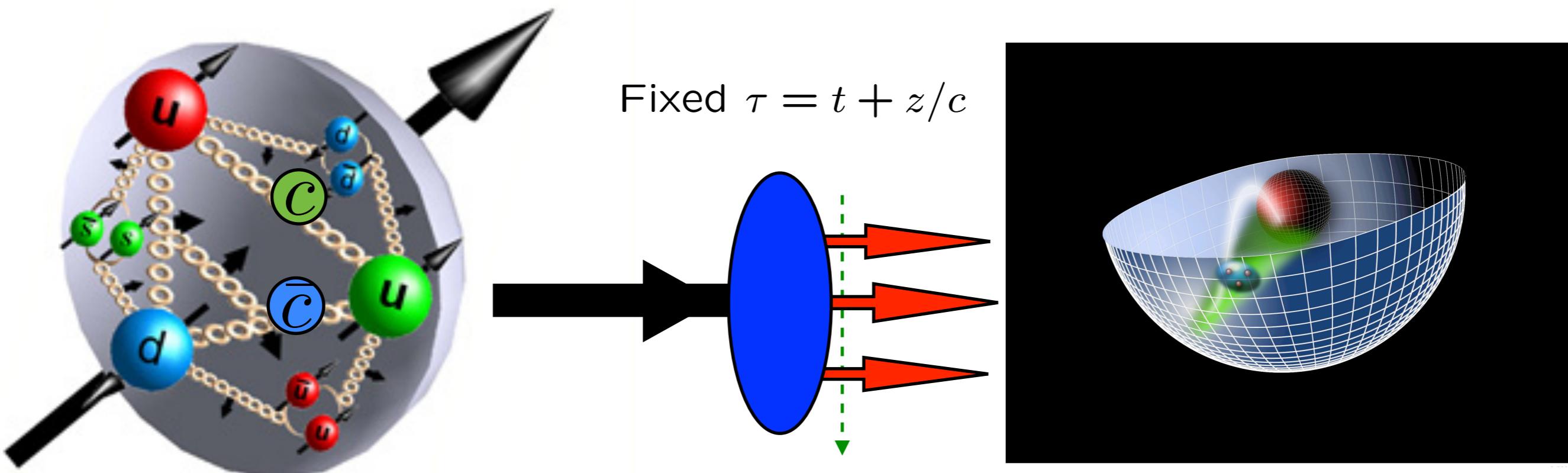


Penrose, Terrell, Weisskopf

- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- J_z Conservation, bounds on ΔL_z **Chiu, sjb**
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no vacuum condensates!

Roberts, Shrock, Tandy, sjb

Color Confinement, Hadron Dynamics, and Hadron Spectroscopy from Light-Front Holography and Superconformal Algebra



8th Meeting on Quark Confinement and the Hadron Spectrum



with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, Kelly Chiu, F. Navarra,
Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur, P. Lowdon