XIIIth Quark Confinement and the Hadron Spectrum

BRST invariant d=2 condensates in Gribov-Zwanziger theory[a]

Caroline Felix

Collaborators: Prof. David Dudal, Prof. Leticia Palhares, François Rondeau and Prof. David Vercauteren

caroline.felix@kuleuven.be

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Outline

Introduction

- Quarks and gluon confinement.

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- To get a result in the infrared (IR), non-perturbative skills are needed. However, a good understanding about the Nature in this regime is hard to obtain.
- Our focus has been on the gluon propagator and also ghost propagator in the infrared region:
  - At large volume, the lattice results have showed that the gluon propagator is suppressed to a nonvanishing value at zero momentum and the ghost propagator is not enhanced anymore.
  - A good analytical attempt to explain this behavior is via the so-called Refined Gribov-Zwanziger (RGZ) framework (notice other approaches were successful as well).

Motivation

Some years ago, Dudal *et al.*[1] have proved that the Gribov-Zwanziger action in the presence of the condensates $\langle AA \rangle$, $\langle \bar{\varphi}\varphi \rangle$, $\langle \bar{\varphi}\bar{\varphi} \rangle$ and $\langle \varphi\varphi \rangle$ is renormalizable order by order.

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However, an estimate for the different condensates were not gotten due to the existence of unknown higher loop parameters. Nonetheless, strong indications have been provide that nonvanishing condensates are energetically favored.

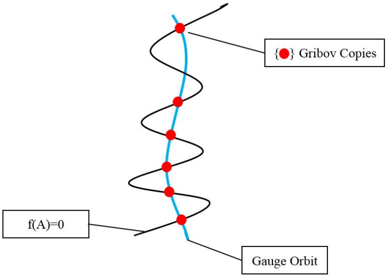
[1] D. Dudal, S. P. Sorella and N. Vandersickel, Phys. Rev. D **84** (2011) 065039.

Goal

- The effective action in the presence of the $\langle AA \rangle$ and $\langle \bar{\varphi}\varphi \rangle$ condensates at one-loop.
- A non-trivial minimum of the effective action leads us to dynamical transformation of the GZ action into the RGZ action which gives us the suppressed gluon propagator and non-enhanced ghost propagator in IR regime.

Gribov problem

- Gribov[2] showed that the Faddeev-Popov construction is not valid at the non-perturbative level.



[2]V. N. Gribov, Nucl. Phys. B **139** (1978) 1.

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Consequently, Gribov copies imply that:

- we are overcounting equivalent gauge configurations, since we have more than one configuration for each gauge orbit,
- the Faddeev-Popov measure is ill-defined, since there are zero-modes of the Faddeev-Popov operator when considering the infinitesimal copies ($det\ M = 0$).

Gribov region

The main idea of the Gribov method is to restrict the functional integral to a certain region Ω in field space, called the Gribov region, which is defined as

$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = i\alpha_g b^a, \qquad \mathcal{M}^{ab}(A^h) = -\partial_\mu D^{ab}_\mu(A^h) > 0\}. \qquad (1)$$

- Linear covariant gauge, $\partial_\mu A^a_\mu = i\alpha_g b^a$,
- Hermitian Faddeev-Popov operator,

$$\mathcal{M}^{ab}(A^h) = -\delta^{ab}\partial^2 + gf^{abc}(A^h)^c_\mu \partial_\mu, \qquad (2)$$

is positive. Inside the Gribov region, there are no infinitesimal copies, since $\mathcal{M}^{ab}(A^h) > 0$;

[3] G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326** (1989) 333.

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- it is convex, bounded and intersected by each gauge orbit[3].
- Its boundary, $\partial\Omega$, is called the first Gribov horizon and there, the first null eigenvalue of $\mathcal{M}^{ab}(A^h)$ (i.e. the first zero-mode of Faddeev-Popov operator) appears.

[3] G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326** (1989) 333.

The local gauge invariance of A^h_μ

The configuration A^h_μ is a non-local power series in the gauge field, obtained by minimizing the functional $f_A[u]$ along the gauge orbit of A_μ[4] , with

$$f_A[u] \equiv \min_{\{u\}} \mathrm{Tr} \int d^4x \, A^u_\mu A^u_\mu ,$$
$$A^u_\mu = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \tag{3}$$

One finds that a local minimum is given by

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \phi_\nu \, , \qquad \partial_\mu A^h_\mu = 0 \, ,$$
$$\phi_\nu = A_\nu - ig \left[\frac{1}{\partial^2} \partial A, A_\nu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\nu \frac{1}{\partial^2} \partial A \right] + O(A^3). \tag{4}$$

Notice that the A^h field collapses to the A field in Landau gauge.

[4] G. Dell'Antonio and D. Zwanziger, Nucl. Phys. B **326** (1989) 333. P. van Baal, Nucl. Phys. B **369** (1992) 259. M. Lavelle and D. McMullan, Phys. Rept. **279** (1997) 1.

The local gauge invariance of A_μ^h

The field A_μ^h can be localized adding an auxiliary Stueckelberg field ξ^a

$$A_\mu^h = (A^h)_\mu^a T^a = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h, \tag{5}$$

while

$$h = e^{ig\,\xi^a T^a}. \tag{6}$$

The local gauge invariance of A_μ^h under a gauge transformation $v \in SU(N)$ with

$$h \to v^\dagger h\,, \quad h^\dagger \to h^\dagger v\,, \quad A_\mu \to v^\dagger A_\mu v + \frac{i}{g} v^\dagger \partial_\mu v. \tag{7}$$

Action

Taking into account the BRST invariant the Grivov-Zwanziger action in the linear covariant gauges, the total action is given by

$$S = S_{YM} + S_{GF} + S_{GZ} + S_{\varepsilon}, \tag{8}$$

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S_{GF} denotes the Faddeev-Popov gauge-fixing in linear covariant gauges, i.e.

$$S_{GF} = \int d^4x \left(\frac{\alpha_g}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu (A) c^b \right), \tag{10}$$

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S_{GZ} is the Gribov-Zwanziger action in its local form, which stands by

$$S_{GZ} = \int d^4x \left[\bar{\varphi}^{ac}_{\mu}\partial_{\nu}(D^{ab}_{\nu}(A^h)\varphi^{bc}_{\mu}) - \bar{\omega}^{ac}_{\mu}\partial_{\nu}(D^{ab}_{\nu}(A^h)\omega^{bc}_{\mu})\right]$$
$$- \gamma^2 g \int d^4x \left[f^{abc}(A^h)^a_{\mu}\varphi^{bc}_{\mu} + f^{abc}(A^h)^a_{\mu}\bar{\varphi}^{bc}_{\mu} + \frac{d}{g}(N_c^2 - 1)\gamma^2\right], \tag{12}$$

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finally, the term S_{ε}

$$S_{\varepsilon} = \int d^4x \, \varepsilon^a \, \partial_{\mu}(A^h)^a_{\mu} \tag{13}$$

implements, through the Lagrange multiplier ε, the transversality of the composite operator $(A^h)^a_{\mu}$, namely $\partial_{\mu}(A^h)^a_{\mu} = 0$.

Gap Equation

The γ the Gribov parameter is dynamically fixed by gap equation:

$$\langle f^{abc}(A^h)^a_\mu(\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu)\rangle = 2d(N^2-1)\frac{\gamma^2}{g^2}, \tag{14}$$

which gives us the horizon function, and can also be rewritten as

$$\frac{\partial\Gamma}{\partial\gamma^2} = 0, \tag{15}$$

whereby the Γ is the quantum action defined by

$$\mathrm{e}^{-\Gamma} = \int [d\Phi]\mathrm{e}^{-S},$$

where

$$[d\Phi] \equiv [dA_\mu][dc][d\bar{c}][db][d\varphi][d\bar{\varphi}],$$

in our case.

BRST symmetry

The action $S = S_{YM} + S_{GF} + S_{GZ} + S_\varepsilon$ enjoys an exact nilpotent BRST invariance, $sS = 0$, if we define the following BRST transformation rules to all fields[5],

$$\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b \,, \quad sc^a = \frac{g}{2} f^{abc} c^b c^c \,, \\
s\bar{c}^a &= ib^a \,, \quad sb^a = 0 \,, \\
s\varphi^{ab}_\mu &= 0 \,, \quad s\omega^{ab}_\mu = 0 \,, \\
s\bar{\omega}^{ab}_\mu &= 0 \,, \quad s\bar{\varphi}^{ab}_\mu = 0 \,, \\
s(A^h)^a_\mu &= 0 \,, \quad s\varepsilon^a = 0 \,, \\
sh^{ij} &= -igc^a (T^a)^{ik} h^{kj} \,.
\end{aligned} \tag{16}$$

[5] M. A. L. Capri et al., Phys. Rev. D 92 (2015) 0450;
M. A. L. Capri et al., Phys. Rev. D 94 (2016) 0250.

Refined Gribov-Zwanziger (RGZ) action

The BRST invariant $d = 2$ condensates, $\langle A_\mu^h A_\mu^h \rangle$ and $\langle \bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} \rangle$, cause non-perturbative dynamical instabilities disturbing the Gribov-Zwanziger formalism.

- $\langle \bar{\varphi} \varphi \rangle$ guarantees that the gluon propagator is non-vanishing at zero momentum;
- $\langle AA \rangle$ assures to fit the result with the lattice data.
- Adding these condensates at the GZ action via the local composite operator (LCO) formalism, the refined Gribov-Zwanziger action (RGZ) is obtained.
- From here, we work with Landau gauge: $\partial A = 0$, $A^h \Longrightarrow A$.

Refined Gribov-Zwanziger (RGZ) action

Then, the action with these LCOs is written as

$$\Sigma = S + S_{A^2} + S_{\varphi\bar{\varphi}} + S_{\mathrm{vac}}, \tag{17}$$

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whereby S is action given by (11) and we also have

$$S_{A^2} = \int d^d x \frac{\tau}{2} A^a_\mu A^a_\mu,$$
$$S_{\varphi\bar{\varphi}} = \int d^d x Q \bar{\varphi}^{ac}_\mu \varphi^{ac}_\mu. \qquad (18)$$

LCO formalism

To add the operators AA and $\bar{\varphi}\varphi$, we have introduced two BRST invariant bosonic sources τ and Q,

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- new divergences proportional to τ^2, Q^2 and τQ come out.
- Due to the divergences from $\langle \mathcal{O}_j(k)\mathcal{O}_j(-k)\rangle_{k\to 0}$, with $\mathcal{O}_i$ one of the $d = 2$ operators in the RGZ action given by

$$\mathcal{O}_j = \{A_\mu A_\mu,\, \varphi_i^a \bar{\varphi}_i^a\}, \tag{20}$$

where the indices (a, i) are fully contracted, e.g. $\varphi_i^a \bar{\varphi}_i^a = \varphi_\mu^{ac} \bar{\varphi}_\mu^{ac}$.

LCO formalism

For this reason, the term S_{vac},

$$S_{\text{vac}} = -\int d^d x \left(\frac{\zeta}{2}\tau^2 + \alpha QQ + \chi Q\tau \right), \tag{21}$$

is necessary in $\Sigma = S + S_{A^2} + S_{\varphi\bar{\varphi}} + S_{\text{vac}}$.

The parameters α, ζ and χ (LCO parameters) absorb the divergence in τ^2, Q^2 and $Q\tau$, i.e. $\delta\zeta\tau^2$, $\delta\alpha Q^2$ and $\delta\chi Q\tau$.

Hubbard-Stratonovich transformation

In order to get the effective action, we have written the energy functional as

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The easiest way to get rid of these quadratic terms in sources is introducing two auxiliary fields σ_1 and σ_2 through two identities:

$$\begin{aligned} 1 &= \int [\mathcal{D}\sigma_1]\, e^{-\frac{1}{2}\int d^dx \left(\sigma_1 + \frac{\bar{a}}{2}A^2 + \bar{b}Q + \bar{c}\tau\right)^2}, \\ 1 &= \int [\mathcal{D}\sigma_2]\, e^{+\frac{1}{2}\int d^dx \left(\sigma_2 + \bar{d}\bar{\varphi}\varphi + \bar{e}Q + \frac{\bar{f}}{2}A^2\right)^2}, \end{aligned} \qquad (22)$$

choosing the coefficients:

$$\bar{a} = -\frac{Z_A}{\sqrt{Z_{\zeta\zeta}\zeta}}\mu^{\epsilon/2}, \qquad \bar{b} = \frac{Z_{QQ}Z_{\chi\chi}\chi}{\sqrt{Z_{\zeta\zeta}\zeta}}\mu^{-\epsilon/2}, \qquad \bar{c} = Z_{\tau\tau}\sqrt{Z_{\zeta\zeta}\zeta}\mu^{-\epsilon/2},$$

$$\bar{d} = \frac{Z_\varphi}{\sqrt{\frac{Z_{\chi\chi}^2\chi^2}{Z_{\zeta\zeta}\zeta} - 2Z_{\alpha\alpha}\alpha}}\mu^{\epsilon/2}, \qquad \bar{e} = Z_{QQ}\sqrt{\frac{Z_{\chi\chi}^2\chi^2}{Z_{\zeta\zeta}\zeta} - 2Z_{\alpha\alpha}\alpha}\,\mu^{-\epsilon/2},$$

$$\bar{f} = \frac{Z_A}{\sqrt{Z_{\zeta\zeta}\zeta}}\left(\frac{Z_{\tau Q}Z_{\zeta\zeta}\zeta - Z_{QQ}Z_{\chi\chi}\chi}{Z_{QQ}\sqrt{Z_{\chi\chi}^2\chi^2 - 2Z_{\alpha\alpha}Z_{\zeta\zeta}\zeta\alpha}}\right)\mu^{\epsilon/2}.$$

Z factors

In $\overline{\mathrm{MS}}$ scheme and at one loop, these Z factors are given by

$$
\begin{aligned}
Z_A &= 1 + \frac{13}{3}\frac{Ng^2}{16\pi^2\epsilon}, & \tilde{Z}_\zeta = Z_{\zeta\zeta}Z_{\tau\tau}^2 = 1 - \frac{13}{3}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_{\zeta\zeta} &= 1 + \frac{22}{3}\frac{Ng^2}{16\pi^2\epsilon}, & Z_g = 1 - \frac{11}{3}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_\tau &= 1 - \frac{35}{6}\frac{Ng^2}{16\pi^2\epsilon}, & Z_{QQ} = Z_g Z_A^{1/2} = 1 - \frac{3}{2}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_{\chi\chi} &= 1, & Z_{\tau Q} = 0, \\
\tilde{Z}_\alpha &= Z_{\alpha\alpha}Z_{QQ}^2 = 1 + \frac{35}{6}\frac{Ng^2}{16\pi^2\epsilon}, & Z_{\alpha\alpha} = 1 + \frac{53}{6}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_\varphi &= Z_{\bar{\varphi}} = Z_g^{-1}Z_A^{-1/2} = 1 + \frac{3}{2}\frac{Ng^2}{16\pi^2\epsilon} & Z_{\gamma^2}^2 = 1 + \frac{3}{2}\frac{g^2N}{16\pi^2\epsilon}.
\end{aligned}
$$

Energy functional

The energy functional can be written as

$$e^{-W(Q,\tau)} = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_{1,3}] \exp\left[-S_{GZ} - \frac{1}{2}\int d^d x \left(2\bar{c}\sigma_1\tau + 2\sigma_3 Q \left(1 - \frac{\bar{b}^2}{\bar{e}^2}\right)\sigma_1^2 \right.\right.$$
$$\left.\left. - \frac{1}{\bar{e}^2}(\sigma_3^2 - 2\bar{b}\sigma_1\sigma_3) + \left(\left(\bar{a} - \frac{\bar{f}\bar{b}}{\bar{e}}\right)\langle\sigma_1\rangle + \frac{\bar{f}}{\bar{e}}\langle\sigma_3\rangle\right) A^2 - 2\frac{\bar{d}}{\bar{e}}(\bar{b}\langle\sigma_1\rangle - \langle\sigma_3\rangle)\overline{\varphi}\varphi\right)\right],$$

where

$$\sigma_3 = \sigma_1\bar{b} - \sigma_2\bar{e}.$$

- The term $\bar{c}\sigma_1\tau = Z_\tau\sqrt{Z_\zeta\zeta}\mu^{-\epsilon/2}\sigma_1\tau$ looks ∞, but we need a finite quantity multiplying the finite source τ.
- It is then natural to define a renormalized finite field σ_1' by $\sigma_1' \equiv Z_{\tau\tau}\sqrt{Z_{\zeta\zeta}}\sigma_1 \equiv \sqrt{\widetilde{Z}_\zeta}\sigma_1$.

Energy functional

The energy functional

$$e^{-W(Q,\tau)} = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_{1,3}] \exp\Bigg[-S_{GZ} - \frac{1}{2}\int d^d x \Bigg(2\bar{c}\sigma_1\tau + 2\sigma_3 Q\left(1 - \frac{\bar{b}^2}{\bar{e}^2}\right)\sigma_1^2 - \frac{1}{\bar{e}^2}(\sigma_3^2 - 2\bar{b}\sigma_1\sigma_3) + \left(\left(\bar{a} - \frac{\bar{f}\bar{b}}{\bar{e}}\right)\langle\sigma_1\rangle + \frac{\bar{f}}{\bar{e}}\langle\sigma_3\rangle\right)A^2 - 2\frac{\bar{d}}{\bar{e}}(\bar{b}\langle\sigma_1\rangle - \langle\sigma_3\rangle)\overline{\varphi}\varphi\Bigg)\Bigg].$$

At one loop:

- $\chi = 0$, $Z_{\tau Q} = 0$, $\bar{b} = \bar{f} = 0$ and $\sigma_3 = -\bar{e}\sigma_2$.
- Now, $\sigma_3 Q = -\bar{e}\sigma_2 Q$ looks ∞. It is the same case of σ_1.
- $\sigma_2' \equiv Z_{QQ}\sqrt{Z_{\alpha\alpha}}\sigma_2 \equiv \sqrt{\widetilde{Z}_\alpha}\sigma_2$.

Energy functional

At one-loop and in terms of the *finite* fields σ_1' and σ_2' the energy functional is given by

$$
\begin{aligned}
e^{-W(Q,\tau)} &= \int [\mathcal{D}\Phi][\mathcal{D}\sigma_{1,2}'] \exp\left[-S_{GZ} - \frac{1}{2}\int d^d x \left(\frac{{\sigma_1'}^2}{\widetilde{Z}_\zeta} - \frac{{\sigma_2'}^2}{\widetilde{Z}_\alpha} \right.\right. \\
&\left.\left. + \bar{a}\frac{\langle\sigma_1'\rangle}{\sqrt{\widetilde{Z}_\zeta}} A^2 - 2\bar{d}\frac{\langle\sigma_2'\rangle}{\sqrt{\widetilde{Z}_\alpha}}\overline{\varphi}\varphi - 2\sqrt{\zeta}\sigma_1'\tau + 2\sqrt{-2\alpha}\sigma_2' Q \right)\right]
\end{aligned}
$$

Energy functional in function of the condensate masses m^2 and M^2

If we define in (finite) tree level:

$$m^2 \equiv \frac{\bar{a}}{\sqrt{\widetilde{Z_\zeta}}} \langle \sigma_1' \rangle = \sqrt{\frac{13Ng^2}{9(N^2-1)}} \langle \sigma_1' \rangle \Rightarrow \langle AA \rangle, \quad M^2 \equiv \frac{\bar{d}}{\sqrt{\widetilde{Z_\alpha}}} \langle \sigma_2' \rangle = -\sqrt{\frac{35Ng^2}{48(N^2-1)^2}} \langle \sigma_2' \rangle \Rightarrow \langle \bar{\varphi}\varphi \rangle .$$

and $\lambda^4 \equiv 2Ng^2\gamma^4$.

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and $\lambda^4 \equiv 2Ng^2\gamma^4$. The renormalized effective potential becomes

$$\begin{aligned}
\Gamma(m^2, M^2, \lambda^4) &= \frac{9(N^2-1)}{13Ng^2}\frac{m^4}{2} - \frac{48(N^2-1)^2}{35Ng^2}\frac{M^4}{2} - \frac{2\lambda^4(N^2-1)}{Ng^2} \\
&- \frac{N^2-1}{16\pi^2}\left\{-2\lambda^4 + \frac{5}{8}m^4 + 2(N^2-1)M^4 - \left(2(N^2-1) - \frac{3}{4}\right)\ln\left(\frac{M^2}{\bar{\mu}^2}\right)M^4\right\} \\
&+ \frac{3}{8}\frac{N^2-1}{16\pi^2}\left\{m^4 + M^4 - 2\lambda^4 + \left(m^2 + M^2\right)\sqrt{(m^2-M^2)^2 - 4\lambda^4}\right\} \\
&\times \ln\left[\frac{1}{2\bar{\mu}^2}\left(m^2 + M^2 + \sqrt{(m^2-M^2)^2 - 4\lambda^4}\right)\right] \\
&+ \frac{3}{8}\frac{N^2-1}{16\pi^2}\left\{m^4 + M^4 - 2\lambda^4 - \left(m^2 + M^2\right)\sqrt{(m^2-M^2)^2 - 4\lambda^4}\right\} \\
&\times \ln\left[\frac{1}{2\bar{\mu}^2}\left(m^2 + M^2 - \sqrt{(m^2-M^2)^2 - 4\lambda^4}\right)\right]. \qquad (23)
\end{aligned}$$

Solving the gap equations

$$\frac{\partial \Gamma}{\partial M^2} = 0, \qquad \frac{\partial \Gamma}{\partial m^2} = 0, \qquad \frac{\partial \Gamma}{\partial \lambda^4} = 0. \tag{24}$$

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Unfortunately, in $\overline{\mathrm{MS}}$ and MOM schemes, these equations do not have a numerically trustworthy solution.

Effective potential in general scheme

The effective potential, in general renormalization scheme, becomes[6]

$$\begin{aligned}\Gamma_{gen}(m^2, M^2, \lambda^4, b_0) = {} & \frac{9(N^2-1)}{26Ng^2} m^4 - \frac{24(N^2-1)^2}{35Ng^2} M^4 - \frac{2(N^2-1)^2 M^4}{16\pi^2}\left(1 - \ln\left(\frac{M^2}{\bar{\mu}^2}\right)\right) \\ & - 2\lambda^4 \frac{N^2-1}{Ng^2} - 2\lambda^4 \frac{N^2-1}{16\pi^2}(b_0 - 1) + \frac{3}{4}\frac{N^2-1}{16\pi^2}\left\{\frac{5}{4}(m^4 + M^4 - 2\lambda^4)\right. \\ & - \frac{m^2 + M^2 - 2\lambda^4}{2} \ln\left[\frac{m^2 M^2 + \lambda^4}{\bar{\mu}^4}\right] + \ln\left[\frac{M^2}{\bar{\mu}^2}\right] M^4 \\ & \left. + (m^2 + M^2)\sqrt{4\lambda^4 - (m^2 - M^2)^2} \arctan\left[\frac{\sqrt{4\lambda^4 - (m^2 - M^2)^2}}{m^2 + M^2}\right]\right\}, \qquad (25)\end{aligned}$$

where b_0 is parametrizing at leading order the most general renormalization scheme.

[6] D. Dudal, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D **72** (2005) 014016.

Minimum of effective potential

Now, the three gap equations are

$$\frac{\partial\Gamma_{gen}}{\partial M^2} = 0 \qquad \frac{\partial\Gamma_{gen}}{\partial m^2} = 0 \qquad \frac{\partial\Gamma_{gen}}{\partial\lambda^4} = 0. \tag{26}$$

[7] D. Dudal et al, arXiv:1803.02281.

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Solving...

- $m^2(b_0, \bar{\mu})$, $M^2(b_0, \bar{\mu})$ and $\gamma^2(b_0, \bar{\mu})$ in function of b_0 and $\bar{\mu}$ with $N = 3$ and units $\Lambda_{\overline{\mathrm{MS}}} = 1$;

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- the parameters b_0 and $\bar{\mu}$ were fixed comparing the masses with the lattice estimates of the RGZ complex conjugate gluon poles[7]. One can show that these poles are both RG and scheme independent (as well as gauge parameter independent). So we selected that scheme that brings us as close as possible to a priori scheme independent quantities;

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- $\alpha_{coupling} = 0.382$ that is small enough and the perturbative tools can be used;

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- $b_0 = -3.643$ and $\bar{\mu} = 1.429$.
- $\alpha_{coupling} = 0.382$ that is small enough and the perturbative tools can be used;
- $\gamma^2 = 0.637$ and
- the vacuum energy is -26.955 that is the minimum of potential in the presence of two dimensional condensate $\langle AA \rangle$ and $\langle \bar{\varphi}\varphi \rangle$ with all parameters determined.
- The Hessian determinant is positive and

$$\left.\frac{\partial^2 \Gamma_{gen}}{\partial {M^2}^2}\right|_{solved} = 1.668, \qquad \left.\frac{\partial^2 \Gamma_{gen}}{\partial {m^2}^2}\right|_{solved} = 0.216, \qquad \left.\frac{\partial^2 \Gamma_{gen}}{\partial M^2 \partial m^2}\right|_{solved} = 0.011.$$

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thank YOU