

FIELD STRENGTH DESCRIPTIONS

$E_i^a(r,t)$ for a spherical
system of charges
 $SU(2)$
subject to confining

$B_i^a(r,t)$

BOUNDARY CONDITIONS

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The Confinement Mechanism in QCD

involves a domain wall of topological charge generated by color gradients between interior and exterior of hadrons

This requirement can be illustrated in spherically-symmetric $SU(2)$ by translating the "kink solution" of the Abelian-Higgs model into spherical coordinates

OUTLINE

I. Bars-Witten ansätz

spherical symmetry in $SU(2)$ & $SU(3)$

II Topological Charge

and topological insulators

III Type-0, Type-1 & Type-2

solutions to static Yang-Mills eqn's

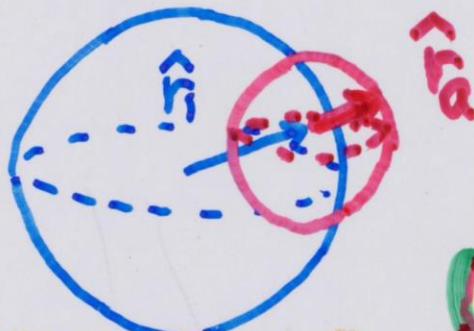
IV. Arthur Jaffe's

constructive field theory program

I. Bars-Witten ansatz for SU(2)

with spherical symmetry

Ralston-Sivers field strength formulation



spherical coordinates
for both spaces

2-dim + transv.

gauge derivatives

$$P_{ia} = \hat{r}_{ia} \quad i=1,2,3 \\ a=1,2,3$$

$$\delta_{ia}^T = \delta_{ia} - P_{ia} = (\hat{\theta}_i, \hat{\theta}_a + \hat{\phi}, \hat{\phi}_a)$$

$$\epsilon_{ia}^T = \epsilon_{ia} \hat{r}_i = (\hat{\phi}, \hat{\theta}_a - \hat{\theta}_i, \hat{\phi}_a)$$

$$gA_0^a = A_0(r,t) \hat{r}_a$$

$$gA_i^a = A_i(r,t) P_{ia} + \frac{a(r,t)}{r} \sin\omega(r,t) \delta_{ia}^T + \frac{a(r,t)}{r} \cos\omega(r,t) - 1 \epsilon_{ia}^T$$

$$D_i^{ab} \hat{r}_b = \frac{a(r,t)}{r} \epsilon_{ia}^S (\omega(r,t)) = \frac{a}{r} [\delta_{ia}^T \cos\omega - \epsilon_{ia}^T \sin\omega]$$

$$-i[\hat{r}, D_i \hat{r}] = \frac{a(r,t)}{r} \epsilon_{ia}^A (\omega(r,t)) = \frac{a}{r} [\delta_{ia}^T \sin\omega + \epsilon_{ia}^T \cos\omega]$$

quick note dimensional reduction also works in spherically-symmetric $SU(3)$

The collapse to 1+1 dim Abelian-Higgs (AH) is important because QFT for AH model has a mass gap associated with kink solution implying confinement !!

Let's explore Yang-Mills Maxwell equations for a system of $SU(2)$ charges with spherical symmetry and confining boundary conditions.

In terms of electric and magnetic field strengths

$$gE_i^a = E_L(r,t) \delta_{ia} + E_s(r,t) \epsilon_{ia}^S(\omega(r,t)) + E_A(r,t) \epsilon_{ia}^A(\omega(r,t))$$

$$gB_i^a = B_L(r,t) \delta_{ia} + B_s(r,t) \epsilon_{ia}^S(\omega(r,t)) + B_A(r,t) \epsilon_{ia}^A(\omega(r,t))$$

with

$$E_L(r,t) = -\frac{\partial}{\partial t} A_1(r,t) + \frac{\partial}{\partial r} A_0(r,t)$$

$$B_L(r,t) = \frac{a^2(r,t) - 1}{r^2}$$

$$E_s(r,t) = \frac{a(r,t)}{r} \left[-\frac{\partial}{\partial t} \omega(r,t) + A_0(r,t) \right]$$

$$B_s(r,t) = -\frac{1}{r} \frac{\partial a(r,t)}{\partial r}$$

$$E_A(r,t) = -\frac{1}{r} \frac{\partial a(r,t)}{\partial t}$$

$$B_A(r,t) = \frac{a(r,t)}{r} [A_1(r,t) - \frac{\partial}{\partial r} \omega(r,t)]$$

Notice that any nonlinearity necessarily associated with with radially directed magnetic field $B_L(r,t) = \frac{a^2(r,t) - 1}{r^2}$.

the tensors $\epsilon_{ia}^s(\omega)$ and $\epsilon_{ia}^A(\omega)$ define a representation of the complex plane connecting 3-space and color-space with complex conjugation $i \leftrightarrow a$

Spherically symmetric SU(2) reduces to a 1+1 dim Abelian gauge theory

$$\int d^4x \mathcal{L}_g(x) = 4\pi \int dt dr (r^2 \mathcal{L}_g^{(2)})$$

$$\mathcal{L}_g^{(2)} = r^2 (F_{lm} F^{lm}) + 2 D^l \bar{\Phi} D_l \bar{\Phi}^* + \frac{1}{r^2} (|\bar{\Phi}|^2 - 1)^2$$

$$F_{lm} = (\partial_l A_m - \partial_m A_l)$$

$$l, m, = 0, 1$$

$$D_\varrho = \partial_\varrho - ie A_\varrho \quad \text{imbedded } e = 1$$

$$\bar{\Phi} = \alpha(r, t) e^{i\omega r, t}$$

$$g_{lm}^{(2)} = r^2 \eta_{lm}^{(2)}$$

$$\eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

curved 2-dim manifold

$$(\partial^\mu G_{\mu\nu})^a = J_\nu^a(r,t)$$

Yang-Mills

Maxwell

$$J_0^a(r,t) = \frac{1}{r^2} J_0(r,t) \hat{F}_a$$

$$J_i^a(r,t) = \frac{1}{r^2} J_i(r,t) \delta_{ia} + j_s(r,t) \epsilon_{ia}^s(\omega) + j_A(r,t) \epsilon_{ia}^A(\omega)$$

$$-\frac{\partial}{\partial r}(r^2 E_L) + 2arE_S = J_0(r,t)$$

$$-\frac{\partial}{\partial t}(r^2 E_L) + 2arB_A = J_i(r,t)$$

$$-\frac{\partial}{\partial r}(arE_S) + \frac{\partial}{\partial t}(arB_A) = arj_S(r,t)$$

$$a\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2}\right)a - r^2(E_S^2 - B_A^2) - \frac{a^2(a^2-1)}{r^2} = arj_A(r,t)$$

classical currents

Bianchi Constraints

$$\frac{\partial}{\partial r}(arE_A) - \frac{\partial}{\partial t}(arB_S) = 0$$

$$-E_L + \frac{\partial}{\partial r}(arE_S) + \frac{\partial}{\partial t}(arB_A) = g^2 r^2 E_i^a B_i^a$$

Extension to SU(3)

$a=1-3 \text{ SU}(2) \Rightarrow a=1-8 \text{ SU}(3)$

$$\hat{\Sigma}_a \Rightarrow \hat{\Sigma} |h\rangle_a \quad |h\rangle_a = (h_3, h_8)$$

t_3, t_8 diagonal

$t_1-t_2, t_4-t_3 \Rightarrow 3 \text{ SU}(2)$ subgroups

O(r y) G(y b) P(br) with \pm charge

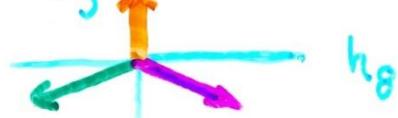
$$[t_3, Q_O^\pm] = \pm Q_O^\pm \quad [t_3, Q_G^\pm] = \mp \frac{1}{2} Q_G^\pm \quad [t_3, Q_P^\pm] = \mp \frac{1}{2} Q_P^\pm$$

$$[t_8, Q_O] = 0$$

$a = (1, 2)$

$$[t_8, Q_G^\pm] = \mp \frac{\sqrt{3}}{2} Q_G^\pm \quad [t_8, Q_P^\pm] = \pm \frac{\sqrt{3}}{2} Q_P^\pm$$

$$\xi_{ia} \Rightarrow \xi_{i\bar{a}} \hat{\Sigma}_i \hat{\Sigma} |h\rangle_a$$



$$\sum_c |w_c\rangle = (0, 0)$$

$$|w_O\rangle = (1, 0) \quad |w_G\rangle = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$|w_P\rangle = \left(-\frac{1}{2}, +\frac{\sqrt{3}}{2}\right)$$

"Hawser Laid"

off-diagonal gluons carry charge

II TOPOLOGICAL CHARGE in Spherical Symmetric SU(2)

topological current $\partial^i K_0(r,t) = g^2 r^2 \epsilon_i^a B^a$

$$K_0(r,t) = (\alpha^2 - 1) A_1 - \alpha^2 \frac{\partial}{\partial r} \omega$$

$$K_1(r,t) = -(\alpha^2 - 1) A_0 + \alpha^2 \frac{\partial}{\partial t} \omega$$

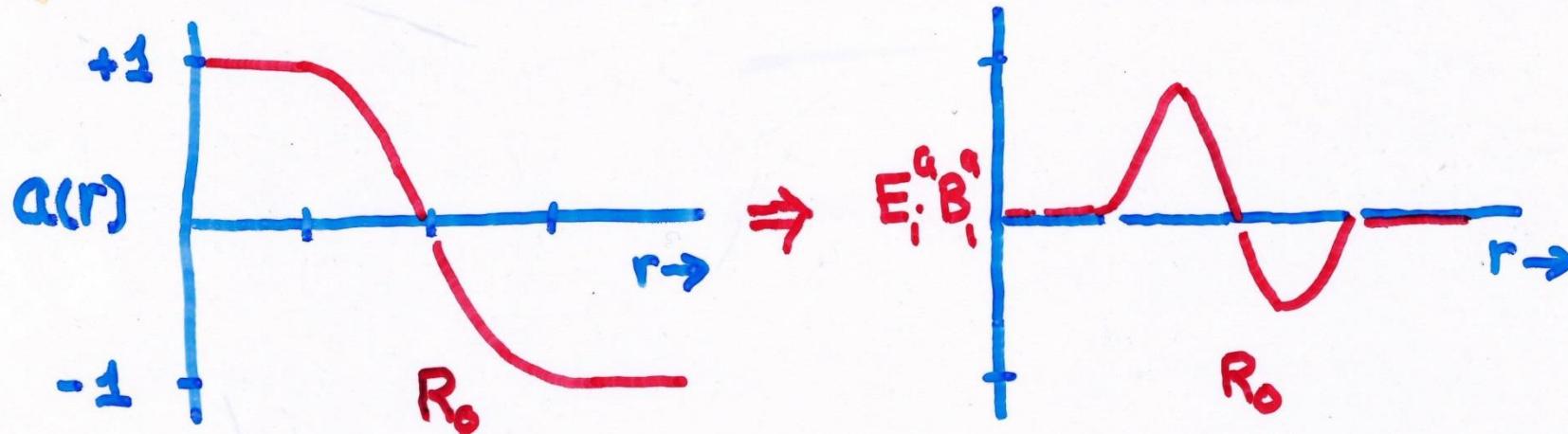
when $\alpha(r,t) = \pm 1$

$$K_0(r,t) = -\frac{\partial}{\partial r} \omega(r,t)$$

$$\partial^i K_0 = 0$$

$$K_1(r,t) = +\frac{\partial}{\partial t} \omega(r,t)$$

A "kink" solution to SU(2) eq'ns generates
3-dim domain wall of CP-odd topological charge



Providing a non-trivial emergent dynamical structure with significant chiral (Adler-Bell-Jackiw) properties and a mass gap

Is this sufficient to give confinement?

DUAL TOPOLOGICAL INSULATOR

A kink solution to spherical-symmetric $SU(2)$ creates a magnetic dual to the topological insulators being studied in solid-state physics

- C.P.-ODD (O^{+-}) domain wall
- geometric effective fermionic masses
- induced dyonic charge densities
- field theoretical mass gap

field strength descriptions
for a system of $SU(2)$ charges with
spherical symmetry
and confining boundary conditions

(draft manuscript)

topological charge and induced
dyonic charge

III. SOLUTIONS to STATIC YANG-MILLS MAXWELL eqns

type-0 type-1 type-2
confining boundary conditions

- 0. $A(\infty) = 1$
- 1. $A(\infty) = 0$
- 2. $A(\infty) = -1$

$$\text{Let } \frac{\partial f(r)}{\partial r} = f' - (r^2 E_L)' + 2ra E_S = J_0(r)$$

$$2ra B_A = J_1(r)$$

$$(ra B_A)' = ra j_S(r)$$

$$\alpha\alpha'' + r^2(E_S^2 - B_A)^2 + \frac{\alpha^2(\alpha^2 - 1)}{r^2} = ra j_A(r)$$

$$\text{with } K_1' = -E_L + (ra E_S)'$$

Interior Volume

$$E_L = C_E \quad E_S = C_E \quad E_A = 0 \\ B_L = 0 \quad B_S = 0 \quad B_A = C_M$$

(all solutions)

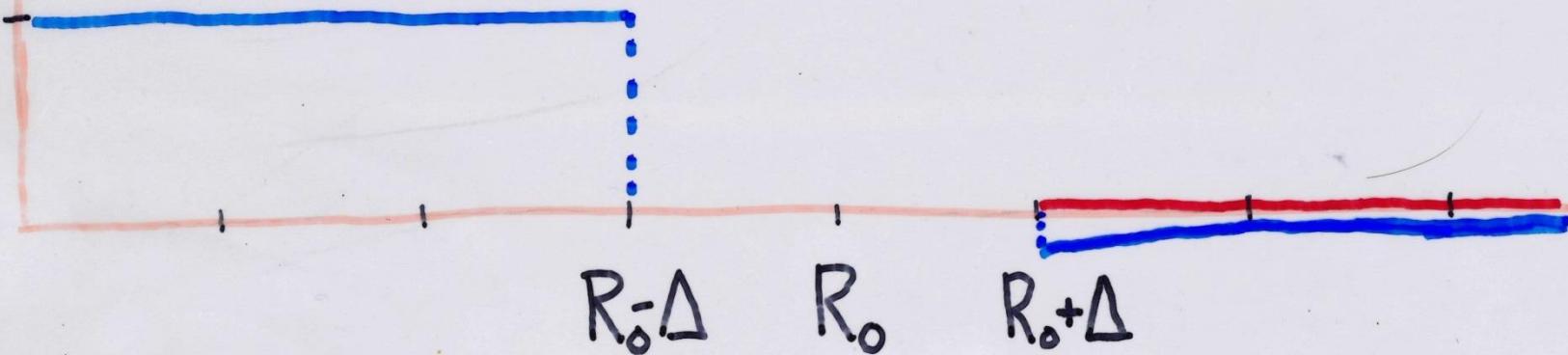
Exterior Volume

type-0 type-2

$$E_L = E_S = E_A = 0 \\ B_L = B_S = B_A = 0$$

type-1

$$B_L = -\frac{1}{r^2}$$



[Interpolations in transition]

type - 0

$$\alpha(r) = 1 \quad C_E = C_E \beta_K(r) \quad C_M = C_M \beta_K(r)$$

$$\omega(r) = 0$$

type - 1

$$\alpha(r) = 1 \cdot \beta_K(r) \quad C_E = C_E \beta_K(r) \quad C_M = C_M \beta_K(r)$$

$$\omega(r) = 0$$

type - 2

$$\alpha(r) = 1 \cdot \alpha_K(r) \quad C_E = C_E \beta_K(r) \quad C_M = C_M \beta_K(r)$$

$$\omega(r) = \pi \beta_K(-r)$$

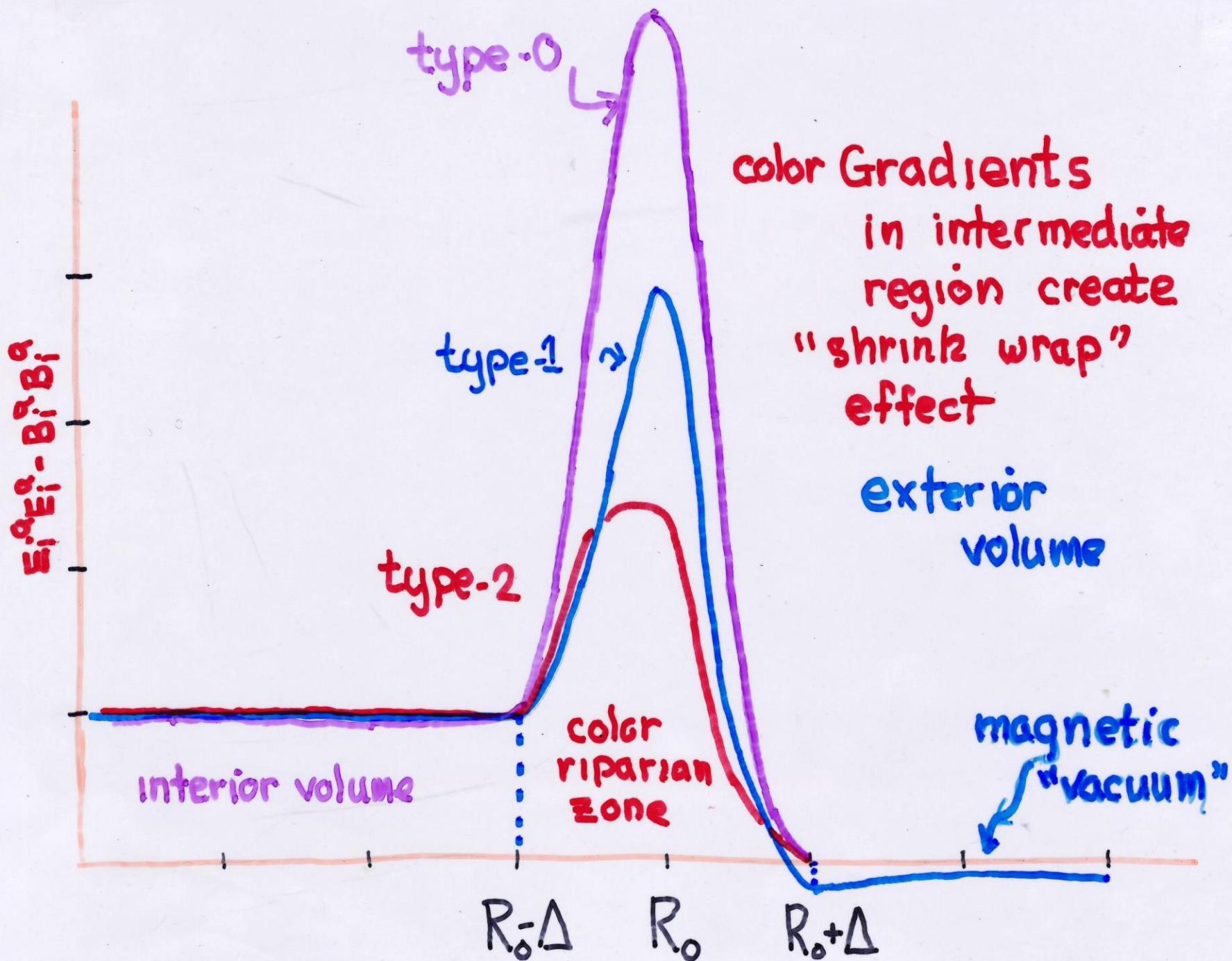
$$(1 \rightarrow 0) \quad \beta_K(r) = \frac{1}{2} [1 - \tanh(\kappa z)]$$

$$z = \frac{r - R_0}{\Delta}$$

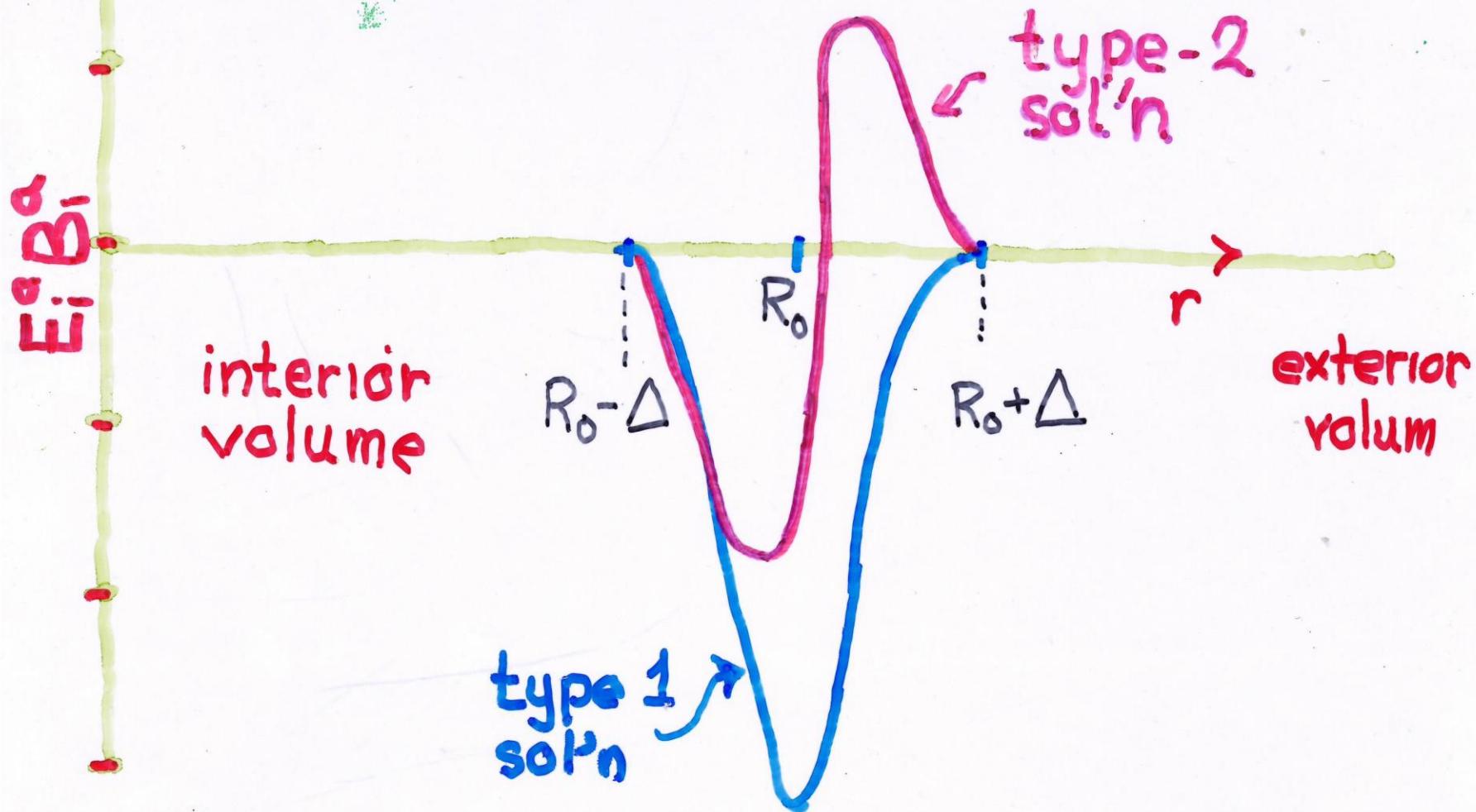
$$(1 \rightarrow -1) \quad \alpha_K(r) = -\tanh(\kappa z)$$

$$\beta' = -\frac{1}{2} \frac{\kappa}{\Delta \cosh^2(\kappa z)} \quad \beta'' = \frac{\kappa^2 \tanh(\kappa z)}{\Delta^2 \cosh^2(\kappa z)}$$

$$\frac{\kappa}{R_0} \geq C_E \quad \frac{R_0}{\Delta} \geq C_E \quad \frac{\kappa}{\Delta} \geq C_E^2$$



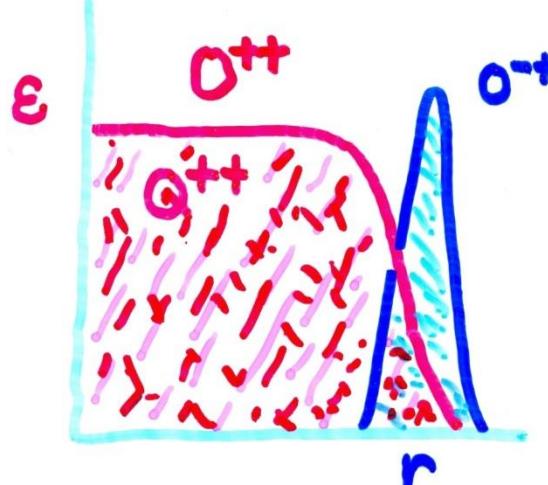
Topological Charge Densities in transition zone



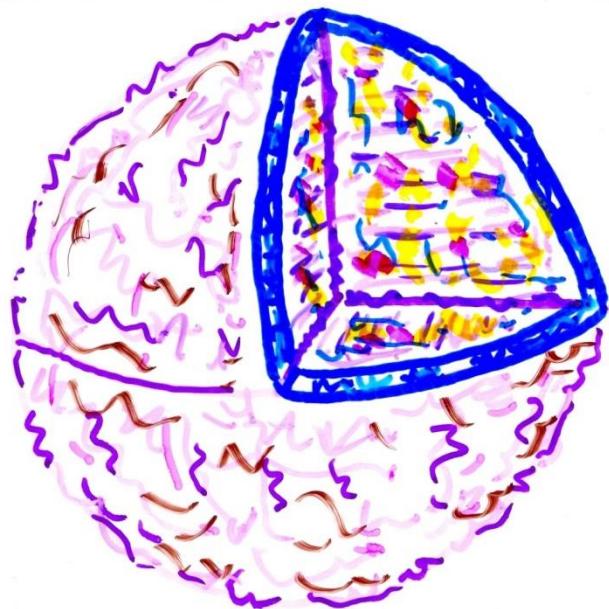
natural
parity
hadron



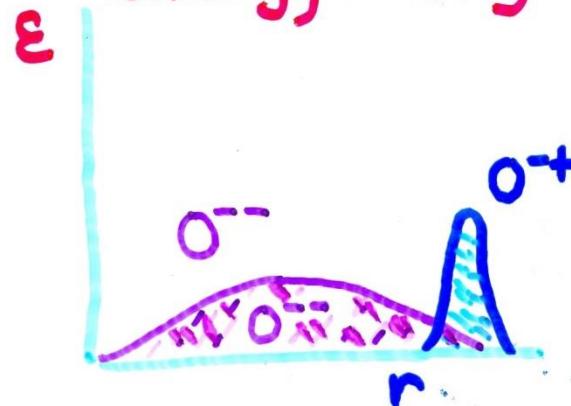
energy density



"pion"



energy density



virtual
non-Abelian field strengths

IV. Constructive Field Theory

a. jaffe

A topological stable solution to
the classical (Yang-Mills Maxwell) eq.'ns
provides robust scaffolding



for understanding hadron structure

the type-2 layer of topological charge provides a chiral version of MIT bag

non-abelian Dirac equations show $\langle E_i^a B_j^a \rangle \neq 0$ region allows helicity-flip required by surface reflection

Bridges, Fröhlich & Seiler construction
for [Landau-Ginsberg] 1+1 dimensions
abelian-Higgs model translates to
Bars-Witten spherical-symmetric SU(2)

A. The Yang-Mills Millennium Prize*

(* Clay Mathematics Institute .. \$1G each)

posed by A. Jaffe & E. Witten (2000)

$$S_{YM}^G = \frac{1}{4g^2} \int Tr F \wedge {}^*F$$

* Hodge Dual
G compact group

F curvature, $F = dA + A \wedge A$, of G-bundle connection A

Prove \nexists QFT in 4-dim. Space time

- 1.) "mass gap", $\Delta > 0$
- 2.) confinement (quarks & gluons)
- 3.) chiral symmetry breaking

These 3 requirements for solution recognize
that "hadronic" sector different from perturbative
sector

$SU(3) = G$ $QCD \neq PQCD$

Infrared
"Slavery"

While the mathematics is hard (M.Douglas - Clay.org)
nature has solved the physics problem.

Transverse spin observables (everything at
Transversity 2011) pierce to the meat of the question

"What distinguishes QCD from PQCD?"

ORBITAL CHROMODYNAMICS

and the

PION TORNADO

$$\begin{aligned} L(T \rightarrow d \downarrow T^*) & \\ P(T \rightarrow n \downarrow T^*) & \\ \langle L \rangle = 1 & \end{aligned}$$

$$J(J+1) = L(L+1) + S(S+1) + 2L \cdot S$$

den sivers

"Constituents" in QCD (degrees of freedom in hadrons & jets) forged from nonlinear dynamics from the "partons" of PQCD (quarks & gluons)

EMERGENT STRUCTURES

- > Constituent Quarks
- > Diquarks
- > Field · Strength Densities
- > Topologically Structured condensates
(dyons, mesons, instantons, ...?)
- > Virtual baryons, mesons
(π, σ, N ... chiral Pert Thy)