FIELD STRENGTH DESCRIPITIONS

$E^a_{r,s,t}$ for a spherical system of $SU(2)$ charges subject to confining boundary conditions

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The Confinement Mechanism in QCD involves a domain wall of topological charge generated by color gradients between interior and exterior of hadrons.

This requirement can be illustrated in spherically-symmetris SU(2) by translating the "rink solution" of the Abelian-Higgs model into spherical coordinates.
OUTLINE

I. Bars-Witten ansatz
   spherical symmetry in SU(2) & SU(3)

II. Topological Charge
    and topological insulators

III. Type-0, Type-1 & Type-2
     solutions to static Yang-Mills eqns

IV. Arthur Jaffe's
    constructive field theory program
Bars-Witten ansatz for SU(2) with spherical symmetry

Ralston-Sivers field strength formulation

\[ J^a_{i\alpha} = \hat{J}^a_{i\alpha} \]

\[ \sigma^T_{ia} = \delta_{ia} - \sigma^T_{ia} = (\hat{\phi}_i \hat{\phi}_a + \hat{\phi}_a \hat{\phi}_i) \]

\[ \epsilon^T_{ia} = \epsilon_{ia} \hat{r}^a = (\hat{\phi}_i \hat{\phi}_a - \hat{\phi}_a \hat{\phi}_i) \]

\[ gA_0^a = A_0(r,t) \hat{r}^a \]

\[ gA_i^a = A_i(r,t) \sigma^T_{ia} \left( \frac{\alpha(r,t) \sin \omega(r,t)}{r} \delta^T_{ia} + \frac{\alpha(r,t) \cos \omega(r,t)}{r} - 1 \right) \epsilon^T_{ia} \]

2-dim + transverse gauge derivatives

\[ D^a_{i\alpha} \hat{r}^b = \frac{\alpha(r,t)}{r} \epsilon^T_{ia} (\omega(r,t)) = \frac{\alpha}{r} \left[ \delta^{T}_{ia} \cos \omega - \epsilon^T_{ia} \sin \omega \right] \]

\[ - \frac{1}{r} \left[ \hat{r}, D^a_i \hat{r}^a \right] = \frac{\alpha(r,t)}{r} \epsilon^T_{ia} (\omega(r,t)) = \frac{\alpha}{r} \left[ \delta^{T}_{ia} \sin \omega + \epsilon^T_{ia} \cos \omega \right] \]
quick note dimensional reduction as works in spherically symmetric SU(3)

The collapse to 1+1 dim Abelian-Higgs (AH) is important because QFT for AH model has a mass gap associated with kink solution implying confinement!!

Let's explore Yang-Mills Maxwell equations for a system of SU(2) charges with spherical symmetry and confining boundary conditions.
In terms of electric and magnetic field strengths

\[ g \mathbf{E}_i^a = E_L(r,t) \mathbf{a}_{ia} + E_s(r,t) A_{ia}^S(\omega r,t) + E_A(r,t) A_{ia}^A(\omega r,t) \]
\[ g \mathbf{B}_i^a = B_L(r,t) \mathbf{a}_{ia} + B_s(r,t) A_{ia}^S(\omega r,t) + B_A(r,t) A_{ia}^A(\omega r,t) \]

with

\[ E_L(r,t) = -\frac{\partial}{\partial t} A_1(r,t) + \frac{\partial}{\partial r} A_0(r,t) \]
\[ E_s(r,t) = \frac{acr(t)}{r} [-\frac{2}{r} \omega r(t) + A_0(t)] \]
\[ E_A(r,t) = -\frac{1}{r} \frac{\partial}{\partial r} acr(t) \]
\[ B_L(r,t) = \frac{a^2(r,t) - 1}{r^2} \]
\[ B_s(r,t) = -\frac{1}{r} \frac{\partial}{\partial r} acr(t) \]
\[ B_A(r,t) = \frac{acr(t)}{r} [A_1(r,t) + \omega r(t)] \]

Notice that any nonlinearity necessarily associated with with radially directed magnetic field \( B_L(r,t) = \frac{a^2(r,t) - 1}{r^2} \).
the tensors $\varepsilon^S_{ia}(\omega)$ and $\varepsilon^A_{ia}(\omega)$ define a representation of the complex plane connecting 3-space and color-space with complex conjugation $i \mapsto a$.

Spherically symmetric $SU(2)$ reduces to a $1+1$-dim Abelian gauge theory:

$$\int d^4x \mathcal{L}_g(x) = 4\pi \int dt dr (r^2 \rho_{g}^{(2)})$$

$$\rho_{g}^{(2)} = r^2 \left( F_{lm} F_{lm}^{(1)} + 2 D^l \Phi \Phi^* + \frac{1}{r^2} (\Phi^* \Phi - 1)^2 \right)$$

$$F_{lm} = (\partial_l A_m - \partial_m A_l)$$  

$l, m = 0, 1$

$$D_l = \partial_l - ie A_l$$  

embedded $e = 1$

$$\Phi = a(r, t) e^{i\omega r, t}$$

$$g_{lm}^{(2)} = r^2 \eta_{lm}$$

$$\eta = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right)$$  

curved 2-dim manifold
\[
(D^\mu G_{\mu \nu})^a = J^a_{\nu}(r, t) \\
J^a_{0}(r, t) = \frac{1}{r^2} J^a_0(r, t) F^a_0 \\
J^a_1(r, t) = \frac{1}{r^2} J^a_1(r, t) A^a + J^a_1(r, t) E^a(t) + J^a_1(r, t) E^a_{1a}(w) \\
J^a_{\text{classical currents}}(r, t) = J^a_0(r, t) + J^a_1(r, t)
\]

Bianchi Constraints

\[
\frac{\partial}{\partial r} (arE_a) - \frac{\partial}{\partial t} (arB_a) = 0 \\
-E^a_L + \frac{\partial}{\partial r} (arE_a) + \frac{\partial}{\partial t} (arB_a) = g^2 r^2 E_i B_i^a
\]
Extension to SU(3)

\[ a = 1 - 3 \quad \text{SU}(2) \Rightarrow a = 1 - 8 \quad \text{SU}(3) \]

\[ \hat{e}_a \Rightarrow \hat{e}_a \hat{1}_h \]

\[ \hat{1}_h_a = (h_3, h_8) \]

\[ t_3, t_8 \text{ diagonal} \]

\[ t_2, t_4, t_3 \Rightarrow 3 \quad \text{SU}(2) \text{ subgroups} \]

\[ O(ry) \quad G(yb) \quad P(br) \quad \text{with } \pm \text{ charge} \]

\[ [t_3, Q^\pm_{Q_0}] = \pm Q^\pm_{Q_0} \]

\[ [t_8, Q_{Q_0}] = 0 \]

\[ a = (1, 2) \]

\[ \hat{e}_a \hat{1}_h \]

\[ \hat{e}^{\pm} a (\omega) \Rightarrow e^{\pm} (\omega) \]

\[ 1 w_0 = (1, 0) \]

\[ 1 w_e = (-\frac{1}{2}, -\frac{5}{2}) \]

\[ 1 w_e = (-\frac{1}{2}, +\frac{5}{2}) \]

\[ \text{"Hawser Laid"} \]
II TOPOLOGICAL CHARGE
in Spherical Symmetric SU(2)

topological current \( \partial^0 K_\phi(r,t) = g^2 r^2 E_\phi B_\phi \)

\[
K_0(r,t) = (a^2 - 1)A_1 - a^2 \frac{\partial}{\partial r} \omega \\
K_1(r,t) = -(a^2 - 1)A_\phi + a^2 \frac{\partial}{\partial t} \omega
\]

when \( a(r,t) = \pm 1 \)

\[
K_0(r,t) = -\frac{\partial}{\partial r} \omega(r,t) \\
K_1(r,t) = +\frac{\partial}{\partial t} \omega(r,t)
\]

\( \partial^0 K_\phi = 0 \)
A "kink" solution to SU(2) eq's generates 3-dim domain wall of CP-odd topological charge

Providing a non-trivial emergent dynamical structure with significant chiral (Adler-Bell-Jackiw) properties and a mass gap

Is this sufficient to give confinement?
DUAL TOPOLOGICAL INSULATOR

A kink solution to spherical-symmetric SU(2) creates a magnetic dual to the topological insulators being studied in solid-state physics.

- CP-odd (O^+-) domain wall
- Geometric effective fermionic masses
- Induced dyonic charge densities
- Field theoretical mass gap
field strength descriptions for a system of $su(2)$ charges with spherical symmetry and confining boundary conditions (draft manuscript) topological charge and induced dyonic charge


II. SOLUTIONS to STATIC YANG-MILLS MAXWELL eqns

type-0  type-1  type-2
confining boundary conditions

0. \( \Omega(\infty) = 1 \)
1. \( \Omega(\infty) = 0 \)
2. \( \Omega(\infty) = \infty \)

Let \( \frac{df(r)}{dr} = f' \quad - \left( r^2 E_L \right)' + 2ra E_s = J_0(r) \)
\( 2ra B_A = J_1(r) \)
\( (ra B_A)' = ra j_s(r) \)
\( qa'' + r^2(E_s^2 - B_A)^2 + \frac{a^2(a^2 - 1)}{r^2} = ra j_A(r) \)

with \( K'_s = -E_L + (ra E_s)' \)
Interior Volume

\[ E_L = C_E \quad E_S = C_E \quad E_A = 0 \]
\[ B_L = 0 \quad B_S = 0 \quad B_A = C_M \]
(all solutions)

Exterior Volume

type-0 type-2

\[ E_L = E_S = E_A = 0 \]
\[ B_L = B_S = B_A = 0 \]

type-1
\[ B_L = -\frac{1}{r^2} \]
Interpolations in transition

**type-0**
\[ \alpha(r) = 1 \quad C_E = C_E \beta_k(r) \quad C_m = C_m \beta_k(r) \]
\[ \omega(r) = 0 \]

**type-1**
\[ \alpha(r) = 1 \cdot \beta_k(r) \quad C_E = C_E \beta_k(r) \quad C_m = C_m \beta_k(r) \]
\[ \omega(r) = 0 \]

**type-2**
\[ \alpha_k(r) = 1 - \alpha_k(r) \quad C_E = C_E \beta_k(r) \quad C_m = C_m \beta_k(r) \]
\[ \omega(r) = \pi \beta_k(-r) \]

\[(1 \to 0)\]
\[ \beta_k(r) = \frac{1}{2} \left[ 1 - \tanh(Kz) \right] \quad z = \frac{r - R_0}{\Delta} \]

\[(1 \to -1)\]
\[ \alpha_k(r) = -\tanh(Kz) \]

\[ \beta' = -\frac{1}{2} \frac{K}{\Delta \cosh^2(Kz)} \quad \beta'' = \frac{K^2 \tanh(Kz)}{\Delta^2 \cosh^2(Kz)} \]

\[ \frac{K}{R_0} \geq C_E \quad \frac{R_0}{\Delta} \geq C_E \quad \frac{K}{\Delta} \geq C_E^2 \]
color Gradients in intermediate region create "shrink wrap" effect

exterior volume

magnetic "vacuum"
Topological Charge Densities in transition zone

- Type 1 solution
  - Interior volume
  - \( R_0 - \Delta \)
  - \( R_0 + \Delta \)

- Type 2 solution
  - Exterior volume
natural parity hadron


"pion"

energy density

non-Abelian field strengths

virtual
IV. Constructive Field Theory

A topological stable solution to the classical (Yang-Mills Maxwell) eqns provides robust scaffolding for understanding hadron structure.
the type-2 layer of topological charge provides a chiral version of MIT bag

non-abelian Dirac equations show \(<E^{a}_i B^{i}_a> \neq 0\) region allows helicity-flip required by surface reflection

Bridges, Fröhlich & Seiler construction for [Landau-Ginsberg] 1+1 dimensional abelian-Higgs model translates to Bars-Witten spherical-symmetric SU(2)
A. The Yang-Mills Millenium Prize
(* Clay Mathematics Institute ... $1G each)
posed by A. Jaffe & E. Witten (2000)

\[ S_{\text{ym}} = \frac{1}{4g^2} \int \text{Tr} \ F \wedge *F \]
  * Hodge Dual
  G compact group

F curvature, F = \text{d}A + A \wedge A, of G-bundle connection A.

Prove \exists QFT in 4-dim. Space time

1.) "mass gap", \( \Delta > 0 \)
2.) confinement (quarks & gluons)
3.) chiral symmetry breaking
These 3 requirements for solution recognize that "hadronic" sector different from perturbative sector.

\[ \text{SU}(3) \times G \]

\[ \text{QCD} \neq \text{PQCD} \]

Infrared "Slavery"

While the mathematics is hard (M.Douglas - Clay.org) nature has solved the physics problem.

Transverse spin observables (everything at Transversity 2011) pierce to the meat of the question "What distinguishes QCD from PQCD?"
ORBITAL CHROMODYNAMICS
and the PION TORNADO

\[ J = L + S \]
\[ J \left( \frac{J}{2} + 1 \right) = L \left( L+1 \right) + s \left( s+1 \right) + 2L \cdot S \]

den sivers
"Constituents" in QCD (degrees of freedom in hadrons & jets) forged from nonlinear dynamics from the "partons" of PQCD (quarks & gluons)

**EMERGENT STRUCTURES**

- Constituent Quarks
- Diquarks
- Field-Strength Densities
- Topologically Structured condensates (dyons, merons, instantons, ...?)
- Virtual baryons, mesons
  - (\(\Upsilon, \Omega, \Lambda\) ... Chiral Pert Thy)