CP violation in QCD

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Quark Confinement and the Hadron Spectrum XIII

Conventional QCD parameters α_s m_{q_i} Θ

Non-trivial connection to physical observables

- α_s tied to overall scale: m_p
- m_{q_i} determines pseudoscalar spectrum: m_π m_K $\ldots$
- Θ controls neutron electric dipole moment

Confinement: quarks are not free

- defining their masses non-trivial

Connection to scattering of physical particles subtle

- ambiguities can arise: "renormalons"

Related ambiguities in defining Θ

- non-differentiable fields in path integral

MC talk at QCHS II, Como, 1996 hep-ph/9608216

Connecting Θ with quark masses

Naive variable change $\psi \longrightarrow e^{i\gamma_5\theta}\psi$

- mass term $\overline{\psi}\psi \longrightarrow \cos(\theta)\ \overline{\psi}\psi + i\sin(\theta)\ \overline{\psi}\gamma_5\psi$

Study QCD dependence on m_1 and m_5

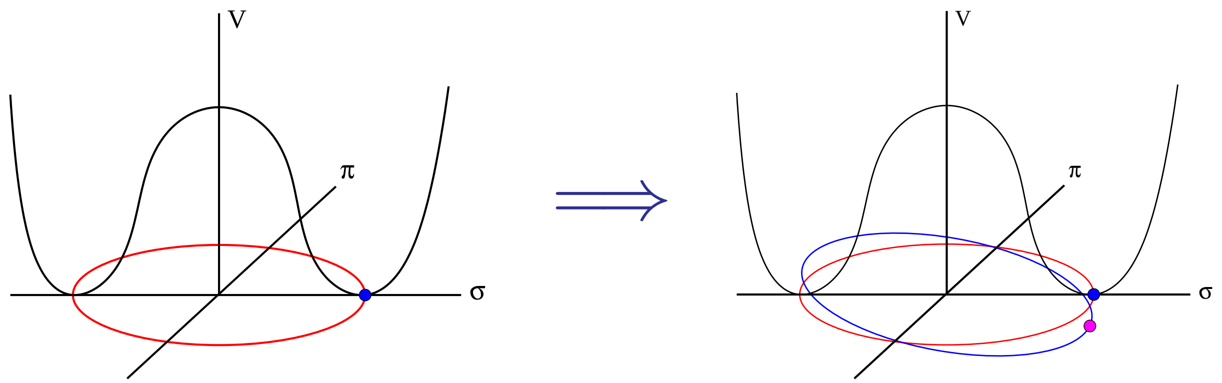
- $m\ \overline{\psi}\psi \to m_1\ \overline{\psi}\psi + im_5\ \overline{\psi}\gamma_5\psi$

Does physics depend only on $\sqrt{m_1^2 + m_5^2}$? X

Tool: effective chiral Lagrangian

Consider 2 flavors with effective potential

- $V = (\sigma^2 + \vec{\pi}^2 - v^2)^2 - m_1\sigma$
- mass term $m_1\overline{\psi}\psi \longrightarrow m_1\sigma$ tilts the sombrero



- pion becomes massive $M_\pi^2 \propto m_1$

What does m_5 do?

- $im_5\overline{\psi}\gamma_5\psi \longrightarrow m_5\eta$ not in above effective potential
- m_5 will give η an expectation value $\langle\eta\rangle \propto m_5/M_\eta^2$

Flavored chiral rotation $\psi \to e^{i\tau_3\gamma_5\theta}$

- mixes $i\overline{\psi}\gamma_5\tau_3\psi \sim \eta$ and $\overline{\psi}\vec{\tau}\psi \sim a_{03}$
- $(\sigma, \vec{\pi})$ and $(\eta, \vec{a}_0)$ independent chiral partners

Chiral symmetry allows coupling

- $\sim \left(\begin{pmatrix} \sigma & \vec{\pi} \end{pmatrix} \cdot \begin{pmatrix} \eta \\ \vec{a}_0 \end{pmatrix} \right)^2$ (squared because of parity)

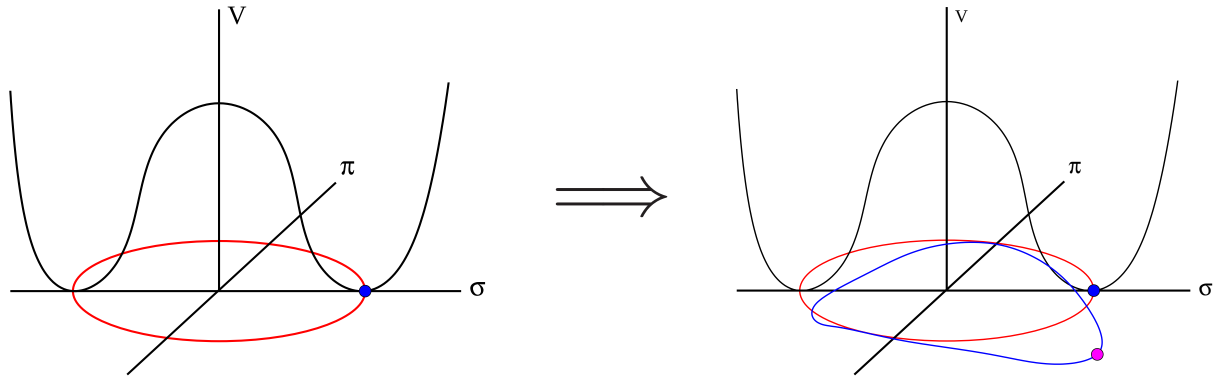
With an expectation for eta

- $(\sigma\eta + \vec{\pi} \cdot \vec{a}_0)^2 \to \langle \eta \rangle^2 \sigma^2$

Inducing in the effective potential

- $V \to V - \alpha m_5^2 \sigma^2$ (sign related to pi eta mixing)

Quadratic warping of the effective potential

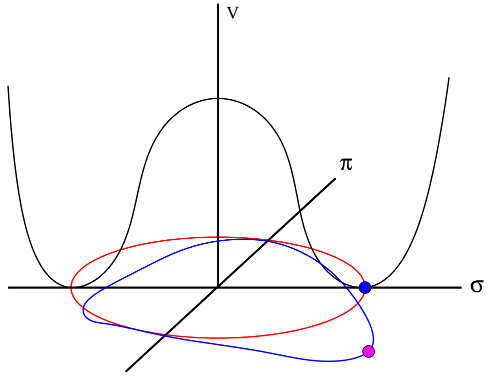


$$V \to V - \alpha m_5^2 \sigma^2$$

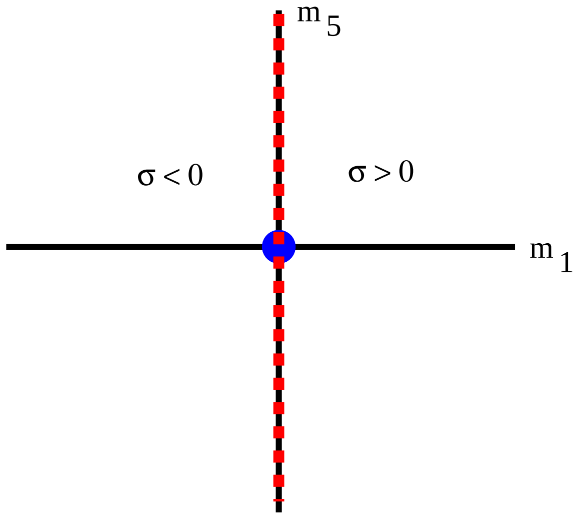
m_5 also gives pions a mass

- $M_\pi^2 \propto m_5^2$ not linear in m_5

m_5 induces a barrier between $\sigma > 0$ and $\sigma < 0$



Transition at $m_1 = 0$ becomes first order



Transition occurs at conventional $\Theta = \pi$

- $\frac{m_5}{m_1} = \tan(\Theta/2)$
- physics not only dependent on $\sqrt{m_1^2 + m_5^2}$
- variable change is anomalous

Physics depends non-trivially on Θ

- m_1 and m_5 are physically independent parameters

Why is $\psi \longrightarrow e^{i\gamma_5\theta}\psi$ not a symmetry?

Fugikawa: fermion measure changes

- $d\psi \to |e^{i\gamma_5\theta}|\ d\psi = e^{i\theta \mathrm{Tr}\gamma_5}\ d\psi$
- index theorem: $\not{D}$ has chiral zero modes
  - $n_+ - n_- = \nu =$ gauge field winding number
  - $\mathrm{Tr}\gamma_5 \equiv \sum_i \langle\psi_i|\gamma_5|\psi_i\rangle = \nu$

$$d\psi \to e^{iN_f\nu\theta}\ d\psi$$

$\psi \longrightarrow e^{i\gamma_5\theta}\psi$ equivalent to inserting $e^{i\nu\theta}$ in path integral

$$Z = \int (dA)(d\psi)(d\overline{\psi})\ e^{-\beta S} \longrightarrow \int (dA)(d\psi)(d\overline{\psi})\ e^{i\nu\Theta}\ e^{-\beta S}$$

- a physically different theory
- CP violating

m_1 and m_5 are inequivalent

Is there a m_5 for each flavor?

- no, $\psi \longrightarrow e^{i\gamma_5 \lambda_\alpha \theta}\psi$ is a valid symmetry
- λ_α a traceless generator of $SU(N_f)$

Can rotate θ into any flavor

- including the top quark! decoupling?

The strong CP problem:

- m_5 is CP violating
- why is this parameter so small? $O(10^{-10})$

Only a problem for unification

- weak interactions violate CP
- absence in QCD at low energies is not "natural"

Axions

- make Θ dynamical
- add $(\partial_\mu \Theta)^2$ to the action
- Θ relaxes to zero

Ad hoc, introduces a new particle

- why no linear term in Θ?
- "natural?"

What about $m_u = 0$?

- $M_{\pi_0} \neq 0$ means m_u and m_d cannot both be zero

Introduce an up-down mass difference

- $m\ \overline{\psi}\psi \rightarrow m_1\ \overline{\psi}\psi + m_2\ \overline{\psi}\tau_3\psi$
- $\overline{\psi}\vec{\tau}\psi \sim \vec{a}_0$ isovector scalar

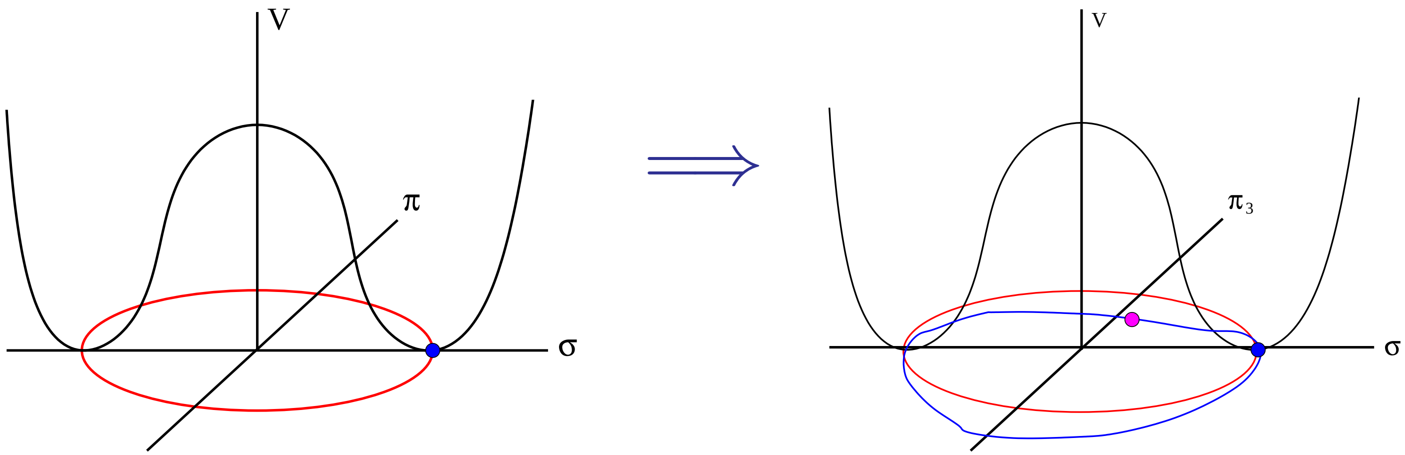
m_2 also not in starting effective potential

m_2 will give a_{03} an expectation value

- $\langle a_{03} \rangle \propto m_2 / M_{a_0}^2$

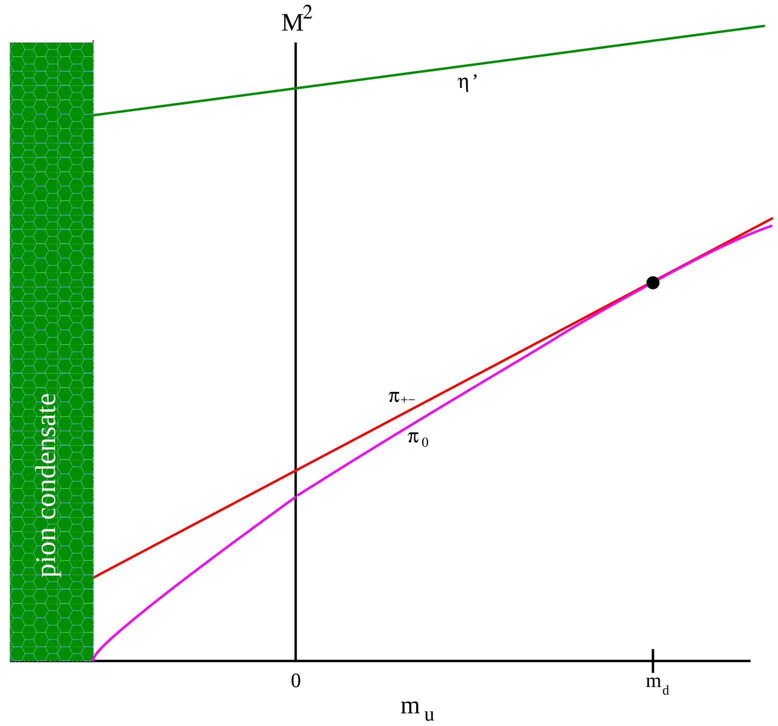
Effective coupling $(\vec{\pi} \cdot \vec{a}_0 + \sigma\eta)^2$

- $V \to V - \alpha m_2^2 \pi_3^2$



Without tilt from m_1,

- π_3 gains an expectation value!
- the CP violating "Dashen phase" (Dashen 1971)



Mass gap persists at $m_u \sim 0$

- no singularity!

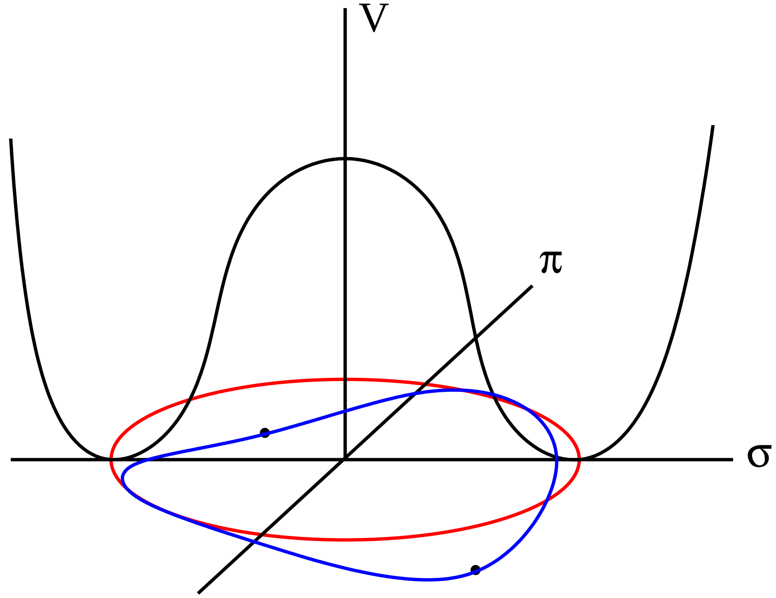
$M^2_{\pi_0}$ can go negative at negative m_u

- pion condensate $\langle \pi_0 \rangle \neq 0$
- $\prod_q m_q < 0$ formally at $\Theta = \pi$

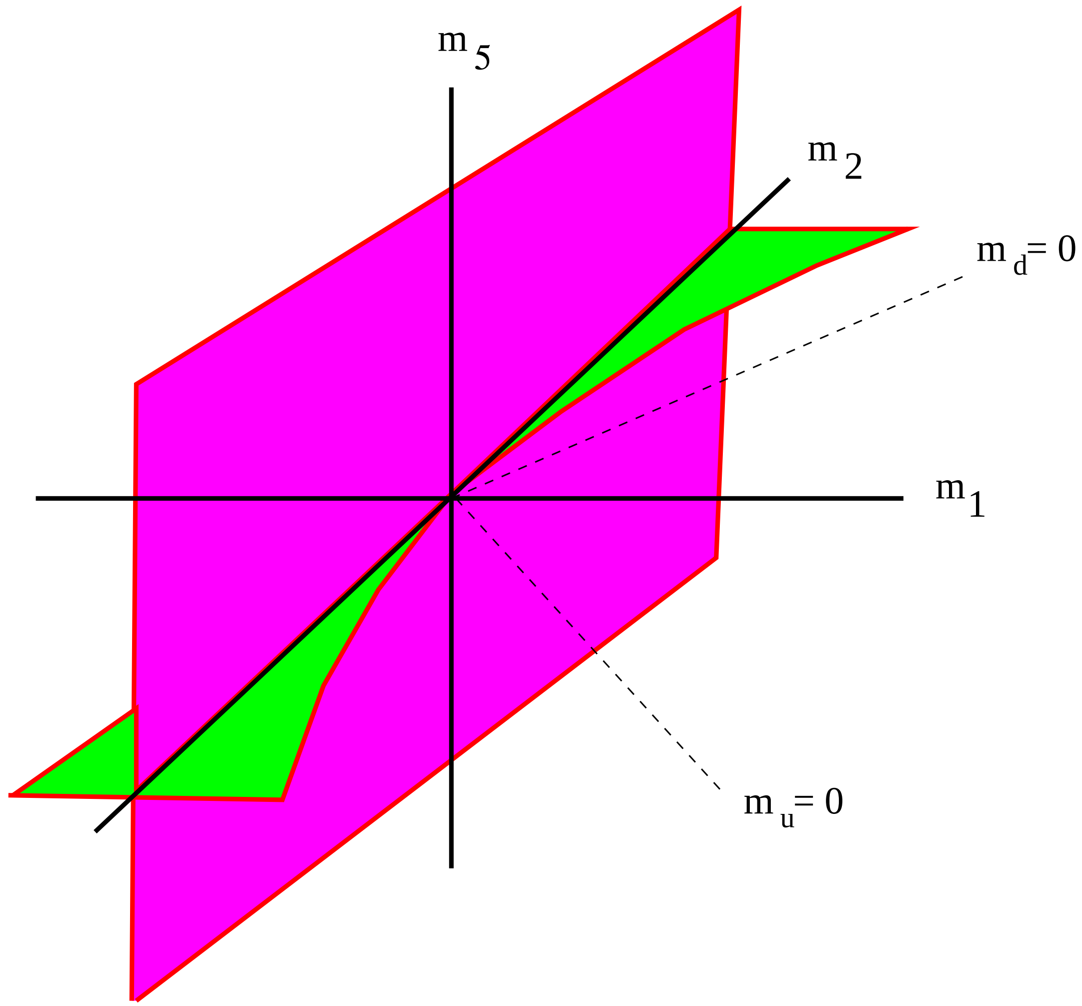
In chiral Lagrangian language:

α_s m_u m_d Θ map onto effective potential

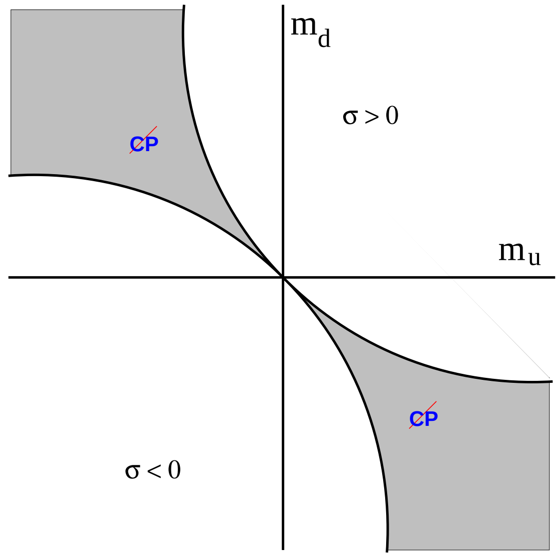
- overall scale
- tilt
- warp
- angle between tilt and warp



Full 2 flavor phase diagram



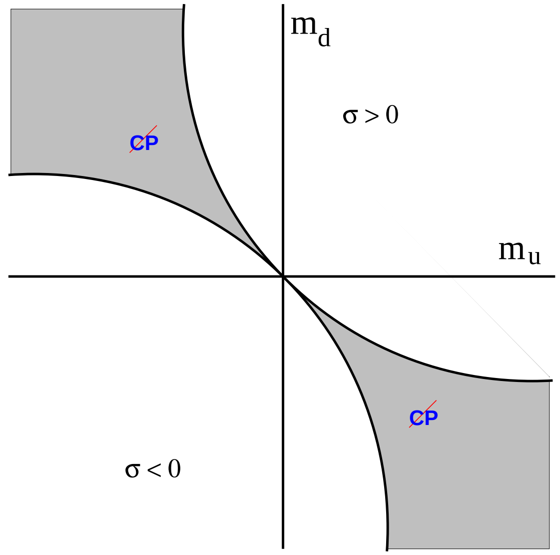
Concentrate on $m_5 = 0$ plane



Second order transition at $m_u m_d < 0$; i.e. $\Theta = \pi$

- order parameter $\langle \pi_0 \rangle$
- massless neutral pion along transition line

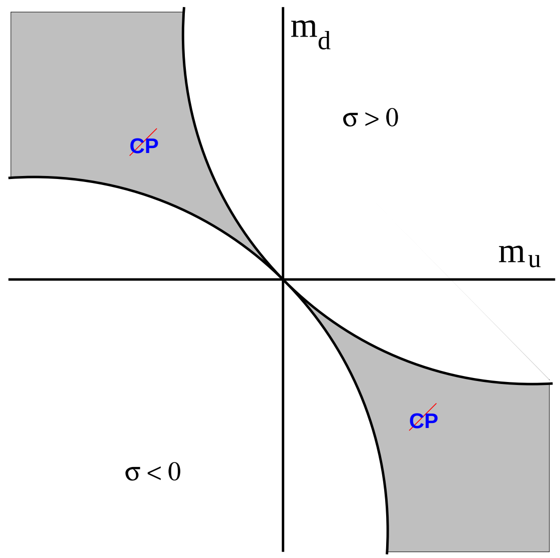
Symmetries



$m_u \leftrightarrow m_d$

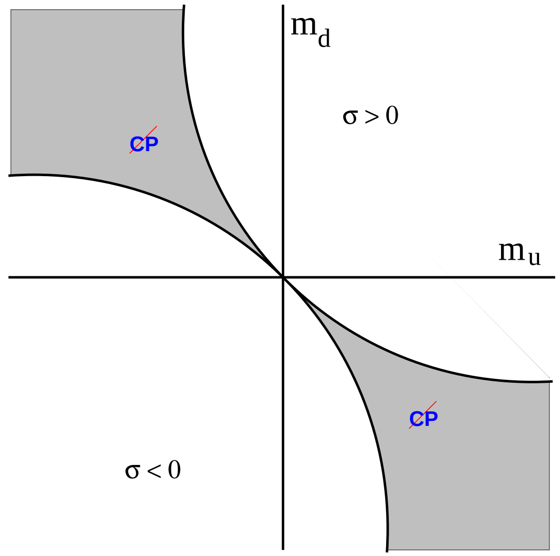
- if $m_u = m_d$ isospin is exact
- $m_2 = 0$ protected from additive renormalization

Symmetries



$m_u \leftrightarrow -m_d$

- $m_u = -m_d$ isospin symmetry at $\Theta = \pi$
- $m_1 = 0$ also protected: $m_u + m_d$



NO symmetry under $m_u \leftrightarrow -m_u$

- $m_u = 0$ not protected by any symmetry!

Symmetries protect m_1, m_2, m_5 separately

- renormalizations not in general equal
- no symmetry to protect $m_u \sim m_1 + m_2$

m_1, m_2, m_5 physically distinct parameters

- independent renormalizations
- $``m_u" = \frac{m_1+m_2}{2} + im_5$ an artificial construct

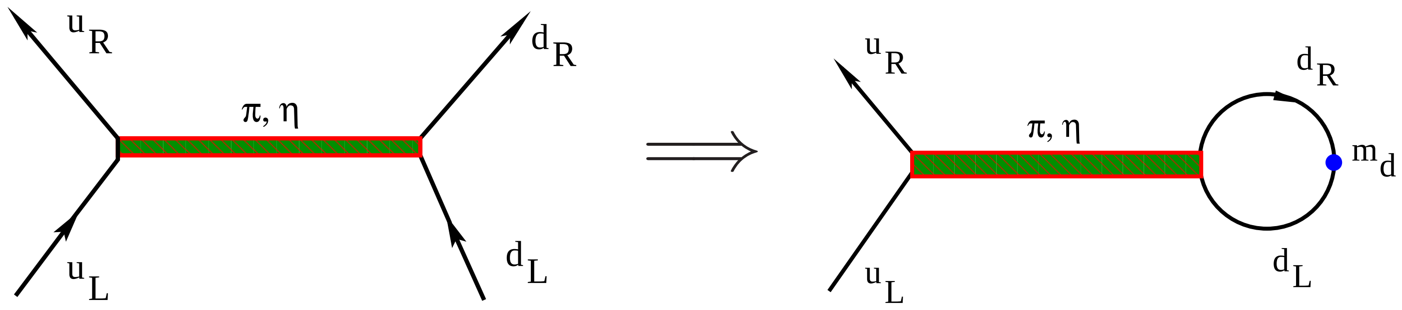
Question: Can any experiment tell if $m_u = 0$?

- is $m_u = 0$ well-defined?
- $\overline{MS}$ is perturbative, cannot answer this

Non-perturbative issues require the lattice

- adjust lattice parameters for hadron spectrum
- read off quark masses and see if $m_u = 0$

Complication: m_d can induce an effective m_u



Mass ratios not renormalization group invariant

$$\frac{m_u}{m_d} \to \frac{m_u + \epsilon m_d}{m_d + \epsilon m_u}$$

“t’Hooft vertex” (1976)

Can we use topology?

- $m_u = 0$ equivalent to vanishing susceptibility

How to define lattice topology?

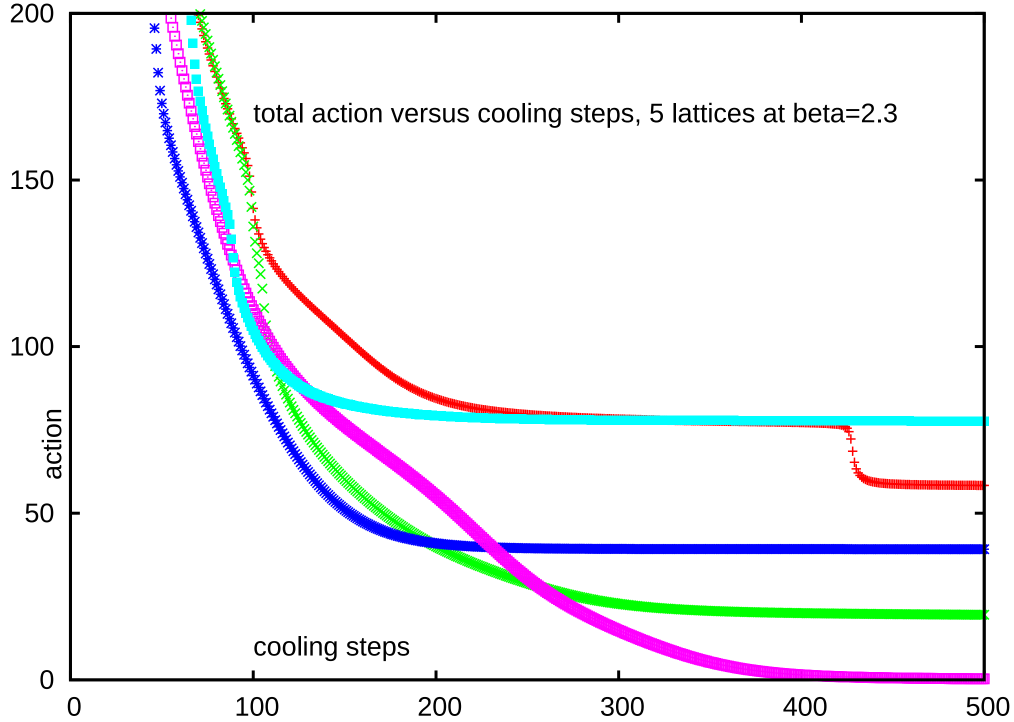
Space of lattice fields simply connected

Topology lost at the outset

- small instantons can fall through the lattice

Cooling (Wilson flow, . . .) to remove UV fluctuations

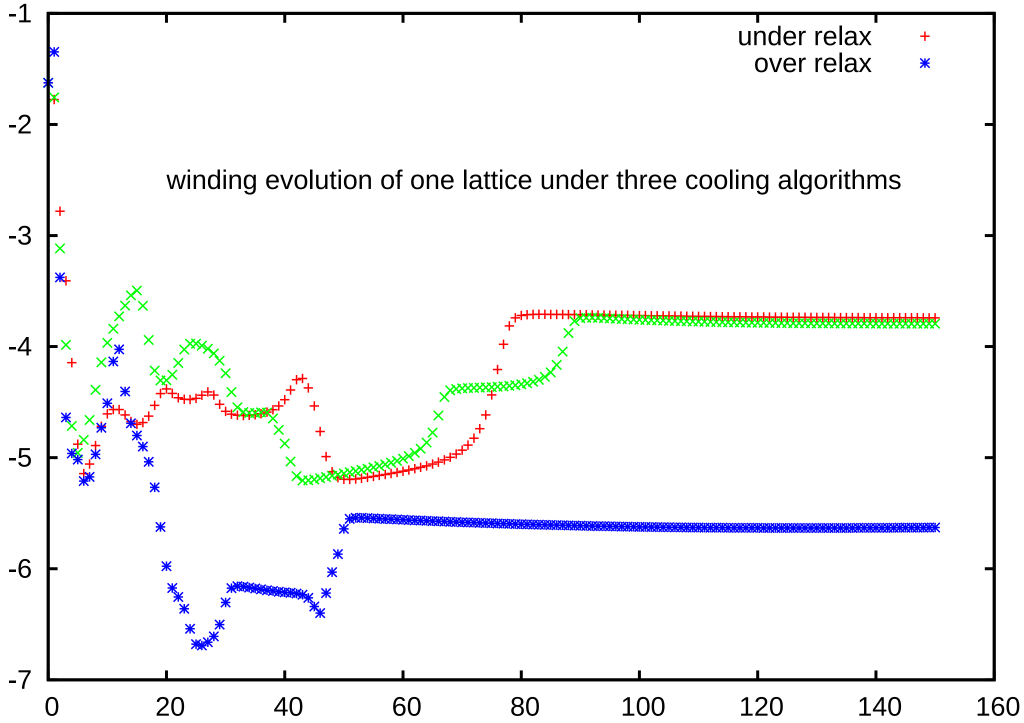
- Action settles to multiple instantons



Many studies over the years: M. Teper (1985); de Forcrand, Garcia-Perez, Stamatescu; Del Debbio, Giusti, Pica; Bruckmann, Gruber, Jansen, Marinkovic, Urbach, Wagner; Ilgenfritz, Martemyanov, Muller-Preussker, Veselov, ...

Often stable but ambiguities appear

- winding can depend on cooling algorithm
- with which action should we cool? How long?



Can we use the index theorem?

- count small eigenvalues of the Wilson operator
  - at finite cutoff not exact zeros
  - how to define "small"?
  - depends on eigenvalues in first Wilson "circle"

Count zero modes of the overlap operator

- operator not unique: "domain wall height"
- reverts to Wilson eigenvalue distribution

Should we care if topology is ambiguous?

- not measured in laboratory experiments
- concentrate on $M_{\eta'}$, which is physical
  - Witten-Veneziano formula a large N_c result

Equivalent to the $m_u = 0$ issue

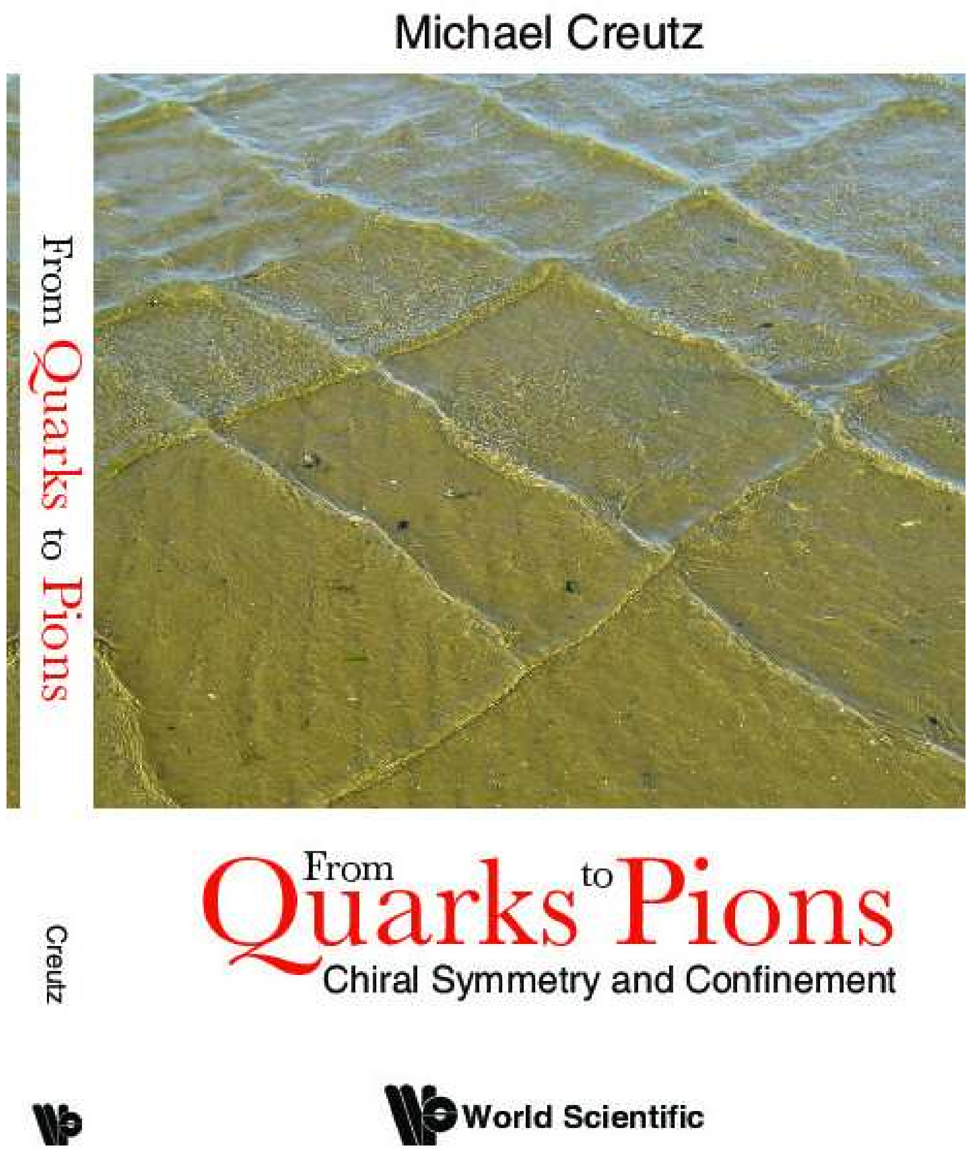
Summary

QCD has $N_f + 1$ mass parameters

- including one CP violating parameter
- Θ not visible in perturbation theory

Experiment shows no evidence of non-zero Θ

- a puzzle for unification
  - fine tuning? $m_u = 0$ not natural
  - axion relaxes Θ to zero?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