A Model for Soft and Hard Interactions based on the CGC/Saturation Approach and the BFKL Pomeron

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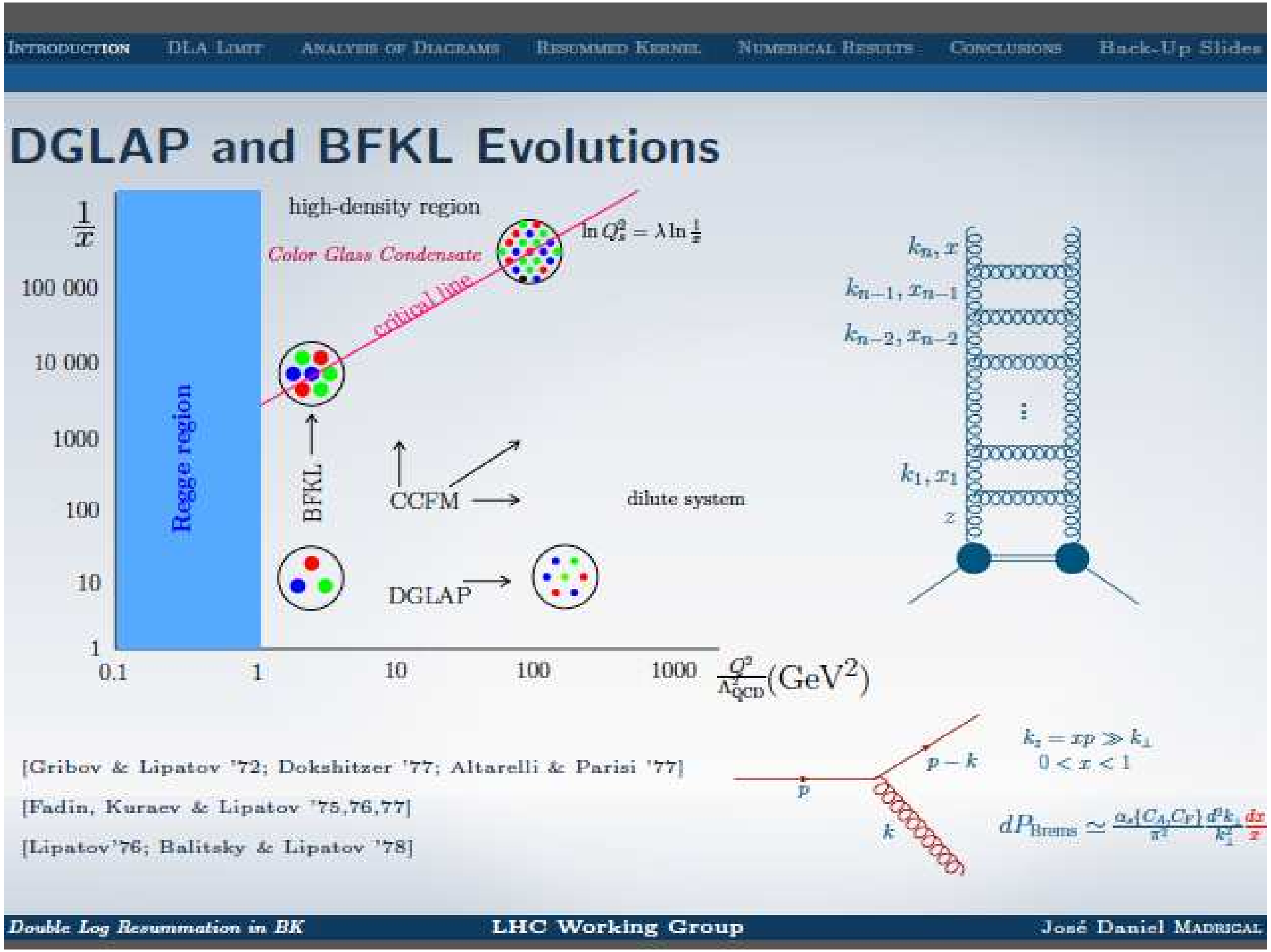
(work done with Genya Levin and Irina Potashnikova)

- Develop a model for SOFT and HARD interactions at high energy based on the BFKL Pomeron and the CGC/saturation approach
- Green function for the Pomeron is calculated in framework of the CGC/saturation approach - replacing Pomeron Calculus
- BFKL Pomeron describes both hard AND soft interactions at high energy

Guide to the Various Regions



Scattering near the Unitarity Limit

1. In the Regge limit of pQCD, when $s \gg \Lambda_{hard}$, as the energy increases the parton density becomes more dense, and the scattering amplitude A(s,t) grows.

2. As long as densities are NOT TOO HIGH, growth is described by BFKL evolution equation.

3. Density becomes higher as A(s,t) $\rightarrow$ 1, and one enters a regime called SATURATION, where the BFKL evolution FAILS.

4. NON LINEARITIES lead to SATURATION + UNITARIZATION of A(s,t).

5. Balitsky-Kovchegov equation is the simplest and most accurate way to describe the saturation regime of QCD. It is non-linear and resums QCD fan diagrams in the LLA.

Phenomenological Input

1. A deficiency that has to be overcome, is the fact that the BFKL Pomeron does NOT lead to shrinkage of the diffractive peak, and has no slope for the Pomeron trajectory.

2. This can be cured by introducing a non-perturbative correction at large impact parameter, which also assures satisfying the Froissart-Martin bound for σ_{tot}.

3. In our model we fix the large b behaviour by assuming that the SATURATION MOMENTUM has the following form:

   $Q^2_s(b,Y) = Q^2_{0s}(b,Y_0)e^{\lambda(Y-Y_0)}$ and $Q^2_{0s}(b,Y_0) = (m^2)^{(1-\frac{1}{\bar\gamma})}[S(b,m)]^{\frac{1}{\bar\gamma}}$
   $S(b,m) = \frac{m^2}{2\pi}e^{-mb}$ and $\bar\gamma = 0.63 = 1 - \gamma_{cr}$

   The parameter $\lambda = \bar\alpha_S\chi\left(\gamma_{cr}\right)/(1-\gamma_{cr})$,
   in leading order of perturbative QCD ($\lambda = 0.2$ to 0.3)

   The parameter m is introduced to describe the large b behaviour, it determines the typical sizes of dipoles inside the hadron.

Phenomenological Input continued

4. Our model includes two additional scales m_1 and m_2, which describe two typical sizes in the proton wave function.

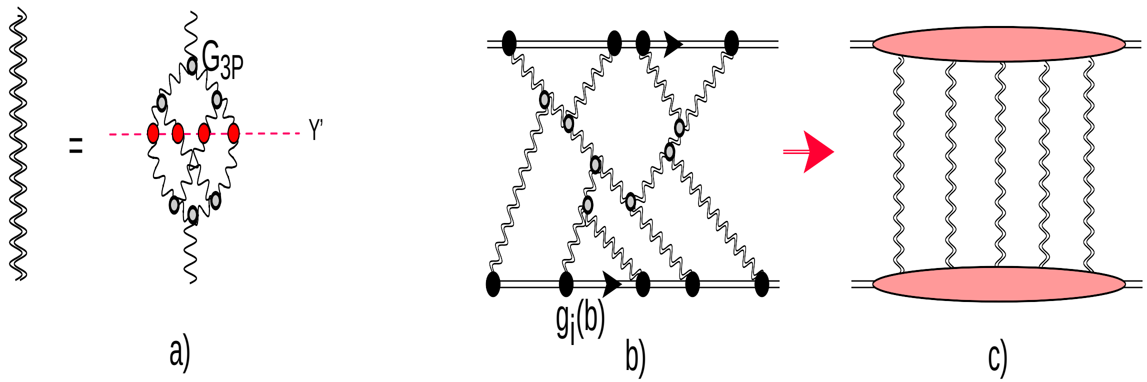
   Can associated these with:

   (i) the distance between the constituent quarks; size of the proton $R_p \approx \frac{1}{m_1}$.

   (ii) m_2 can be associated with the size of the constituent quark; $R_q \approx \frac{1}{m_2}$.

5. Altinoluk et al JHEP 1404, 075 (2014) have proved the equivalence of the CGC/saturation approach and the BFKL Pomeron calculus for a wide range of rapidities
   $Y \leq \frac{2}{\Delta_{\text{BFKL}}} \ln\left(\frac{1}{\Delta^2_{\text{BFKL}}}\right)$.

Dressed Pomeron in MPSI approximation



a) Dressed Pomeron in MPSI approximation and b) Sum of net diagrams
b) reduces to c) after integration over positions of G_{3P} in rapidity.

Wavy lines describe BFKL Pomerons, double wavy show dressed Pomerons
Red blobs denote the amplitude for dipole-dipole interactions.
The grey blobs stand for triple Pomeron vertices,
while black blobs show the hadron-Pomeron vertex $g_i(b)$.
Since the typical rapidity is $O(Y - Y_i) \approx \frac{1}{\Delta_{BFKL}}$, only large Pomeron loops with rapidity $O(Y)$
contribute at high energies $\rightarrow$ can sum such loops using MPSI approximation.

For the BFKL Pomeron $\lambda = 4.88\bar{\alpha}_s$ while $\Delta_{BFKL} = 4ln2\bar{\alpha}_s \approx 0.2$

Dressed Pomeron in MPSI approximation (continued)

The resulting Green function of the Dressed Pomeron is given by:

$$G_{I\!P}^{\text{dressed}}(Y - Y_0, r, R, b) \;=$$

$$a^2 \left\{ 1 \;-\; \exp\left(-T\left(Y - Y_0, r, R, b\right)\right) \right\} + 2a(1-a)\frac{T\left(Y - Y_0, r, R, b\right)}{1 \;+\; T\left(Y - Y_0, r, R, b\right)}$$

$$+\;(1-a)^2 \left\{ 1 - \exp\left(\frac{1}{T\left(Y - Y_0, r, R, b\right)}\right) \frac{1}{T\left(Y - Y_0, r, R, b\right)} \Gamma\left(0, \frac{1}{T\left(Y - Y_0, r, R, b\right)}\right) \right\}$$

$$with \;\; T\left(Y - Y_0, r, R, b\right) \;=\; \frac{\bar{\alpha}_S^2}{4\pi} G_{I\!P}\left(z \to 0\right) \;=\; \phi_0 \left(r^2 Q_s^2\left(R, Y, b\right)\right)^{1-\gamma_{cr}}$$
$$=\; \phi_0 S\left(b\right) e^{\lambda(1-\gamma_{cr})Y}$$

z $= ln(r^2 Q_s^2(b, Y))$, a $=$ 0.65, $\gamma_{cr} \;\approx\;$ 0.37

Parameters of the Model

We need to introduce four constants: g_i and m_i $(i = 1, 2)$, to describe the vertices of the hadron-Pomeron interaction:

$$g_i\,(b) \;=\; g_i\,S_{I\!P}\,(m_i, b) \text{ with } S_{I\!P}\,(m_i, b) \;=\; \frac{m_i^3\,b}{4\pi}\,K_1\,(m_i b)$$

$$S_{I\!P}\,(m_i, b) \xrightarrow{\text{Fourier image}} (\frac{m_i^2}{q^2 + m_i^2})^2$$

$$\Omega_{i,k}\,(Y; b) \;=\; \int d^2 b' \; \frac{g_i\left(\vec{b}'\right)\, g_k\left(\vec{b} - \vec{b}'\right)\, \bar{G}^{\text{dressed}}_{I\!P}\,(Y)}{1 \,+\, 1.29\,\bar{G}^{\text{dressed}}_{I\!P}\,(Y)\left[g_i\left(\vec{b}'\right) + g_k\left(\vec{b} - \vec{b}'\right)\right]},$$

where $\bar{G}^{\text{dressed}}_{I\!P}\,(Y) \;=\; \int d^2\, b''\, G^{\text{dressed}}_{I\!P}\,(Y; b'')\,.$

Basic formalism for Two Channel Model

Following Good-Walker the observed physical hadronic and diffractive states are written

$$\psi_h \;=\; \alpha\,\Psi_1 + \beta\,\Psi_2\,; \qquad \psi_D \;=\; -\beta\,\Psi_1 + \alpha\,\Psi_2; \qquad \text{where} \qquad \alpha^2 + \beta^2 \;=\; 1.$$

Functions ψ_1 and ψ_2 form a complete set of orthogonal functions $\{\psi_i\}$ which diagonalize the interaction matrix $\mathbf{T}$

$$A^{i'k'}_{i,k} =<\psi_i\,\psi_k|\mathbf{T}|\psi_{i'}\,\psi_{k'}>= A_{i,k}\,\delta_{i,i'}\,\delta_{k,k'}.$$

The unitarity constraints can be written as

$$2\,\mathrm{Im}\,A_{i,k}\,(s,b) \;=\; |A_{i,k}\,(s,b)\,|^2 + G^{in}_{i,k}(s,b)$$

At high energies a simple solution to this equation is

$$A_{i,k}(s,b) = i\left(1 - \exp\left(-\frac{\Omega_{i,k}(s,b)}{2}\right)\right)$$

$$G^{in}_{i,k}(s,b) = 1 - \exp\left(-\Omega_{i,k}(s,b)\right).$$

$G^{in}_{i,k}(s,b)$ denotes the contribution of all non-diffractive inelastic processes

Physical Observables for Elastic, and Low Mass Diffraction

elastic amplitude : $a_{el}(s,) = i\left(\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\right);$

elastic observables : $\sigma_{tot} = 2\int d^2b\, a_{el}(s,b); \quad \sigma_{el} = \int d^2b\, |a_{el}(s,b)|^2;$

optical theorem : $2\,\mathrm{Im}A_{i,k}(s,t=0) = 2\int d^2b\,\mathrm{Im}A_{i,k}(s,b) = \sigma_{el} + \sigma_{in} = \sigma_{tot}$

single diffraction : $\sigma_{sd}^{GW} = \int d^2b\left(\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2-\beta^2)A_{1,2} + \beta^2 A_{2,2}\}\right)^2;$

double diffraction : $\sigma_{dd}^{GW} = \int d^2b\,\alpha^4\beta^4\{A_{1,1} - 2A_{1,2} + A_{2,2}\}^2.$

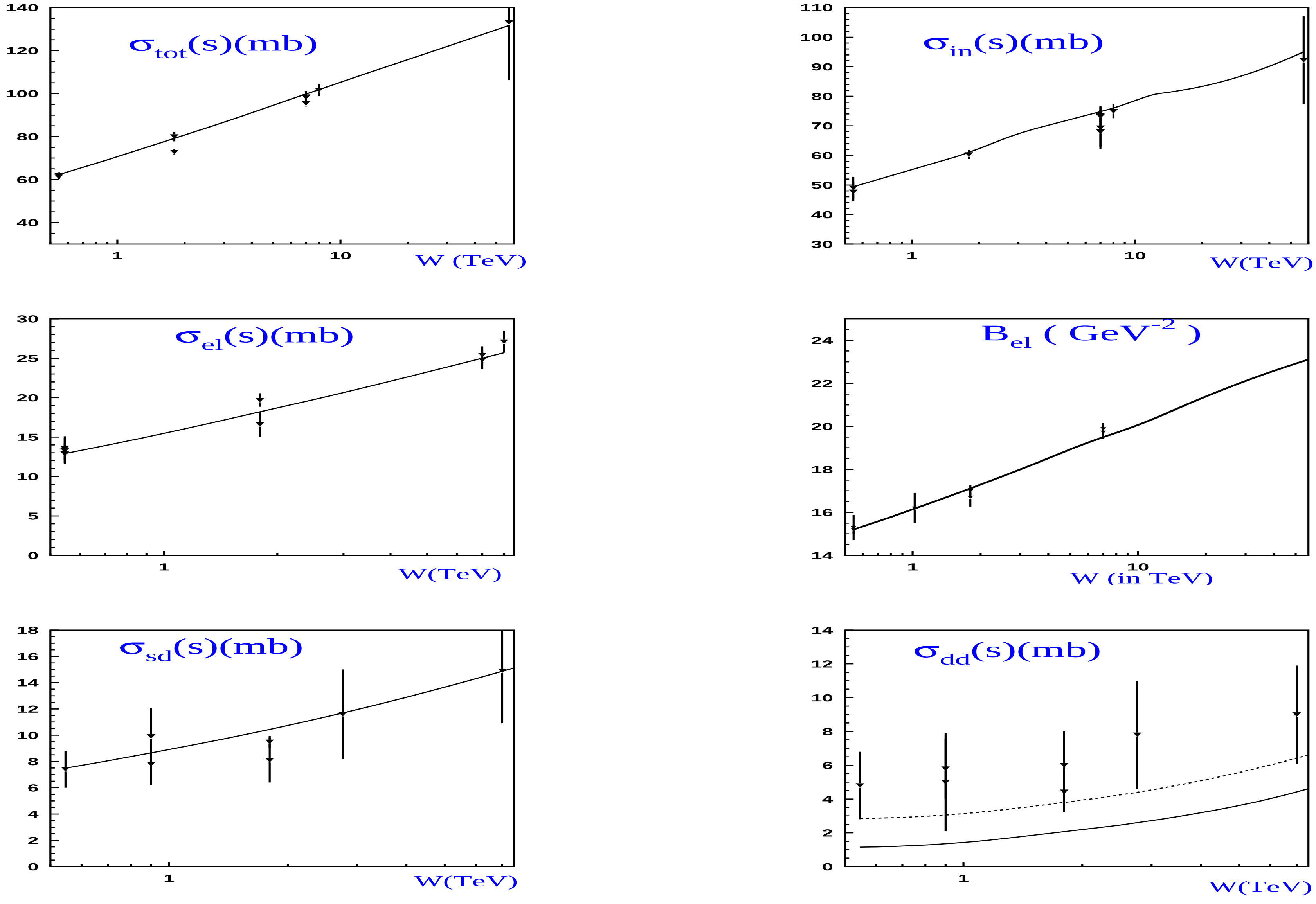
'GW' denotes the Good -Walker component, that is responsible for diffraction in the small mass region.

Parameters and Predictions for the Model

model	λ	ϕ_0	g_1 (GeV^{-1})	g_2 (GeV^{-1})	$m(GeV)$	$m_1(GeV)$	$m_2(GeV)$	β
2 channel	0.38	0.0019	110.2	11.2	5.25	0.92	1.9	0.[illegible]8

W (TeV)	σ_{tot} (mb)	σ_{el}(mb) (mb)	B_{el} (GeV^{-2})	single σ_{sd}^{LM} (mb)	diffraction σ_{sd}^{HM} (mb)	double σ_{dd}^{LM} (mb)	diffraction σ_{dd}^{HM} (mb)
0.546	62.3	12.9	15.2	5.64	1.85	0.7	0.46
0.9	69.2	15	16	6.25	2.39	0.77	0.67
1.8	79.2	18.2	17.1	7.1	3.35	0.89	1.17
2.74	85.5	20.2	17.8	7.6	4.07	0.97	1.62
7	99.8	25	19.5	8.7	6.2	1.15	3.27
8	101.8	25.7	19.7	8.82	6.55	1.17	3.63
13	109.3	28.3	20.6	9.36	8.08	1.27	5.11
14	110.5	28.7	20.7	9.44	8.34	1.27	5.4
57	131.7	36.2	23.1	10.85	15.02	1.56	13.7

Results for Two channel Model



Deep Inelastic Scattering [Hard Scattering]

To introduce "Hard Interactions" we consider DIS: where the physical observables are:

$$\sigma_{T,L}\left(Q,Y\right) \;=\; 2\int d^2b\;N_{T,L}\left(Q,Y;b\right)$$

$$F_2\left(Q,Y\right) \;=\; \frac{Q^2}{4\pi^2\alpha_{\text{e.m.}}}\Big\{\sigma_T\,+\,\sigma_L\Big\}$$

Q is the photon virtuality.
and $Y\;=\;\ln\left(1/x_{Bj}\right)$, where x_{Bj} is the Bjorken x.

Deep Inelastic Scattering [Hard Scattering] 2

The observables of DIS can be re-written using

$$N_{T,L}\left(Q,Y;b\right) \;=\; \int \frac{d^2 r}{4\,\pi} \int_0^1 dz\, |\Psi^{\gamma^*}_{T,L}\left(Q,r,z\right)|^2\, N\left(r,Y;b\right)$$

$N\left(r,Y;b\right)$ is the scattering amplitude of the dipole.
z is the fraction of energy carried by quark.

b is the impact parameter for the scattering of the colorless dipole of size r with the proton.

$|\Psi^{\gamma^*}_{T,L}\left(Q,r,z\right)|^2$ is the probability to find a dipole of size r in a photon with the virtuality Q, and with transverse or longitudinal polarization.
The wave functions are:

$$(\Psi^*\Psi)^{\gamma^*}_T = \frac{2N_c}{\pi}\alpha_{\rm em}\sum_f e_f^2\left\{\left[z^2+(1-z)^2\right]\epsilon^2 K_1^2(\epsilon r)+m_f^2 K_0^2(\epsilon r)\right\},$$

$$(\Psi^*\Psi)^{\gamma^*}_L = \frac{8N_c}{\pi}\alpha_{\rm em}\sum_f e_f^2 Q^2 z^2(1-z)^2 K_0^2(\epsilon r),$$

and $\epsilon^2 \;=\; m_f^2 + z(1-z)Q^2$.

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Since we take into account the contribution of the heavy c-quark we introduce a correction due to large mass of this quark:

$$x_{Bj} \rightarrow x_{Bj}\left(\frac{1}{1+\frac{4m_c^2}{Q^2}}\right) \qquad \text{or} \qquad Y_c = Y - \ln\left(1 + 4\,m_c^2/Q^2\right)$$

For DIS we take into account the running QCD coupling:

$$F_2\left(Q,Y\right) \;=\; \frac{Q^2}{4\pi^2\alpha_{\text{e.m.}}}\left\{\frac{\bar{\alpha}_S\left(Q^2\right)}{\bar{\alpha}_S\left(\mu^2\right)}\,\sigma^{\text{light q}}\left(Q,Y\right) + \frac{\bar{\alpha}_S\left(Q^2+4m_c^2\right)}{\bar{\alpha}_S\left(\mu^2\right)}\sigma^{\text{charm q}}\left(Q,Y_c\right)\right\}$$

where μ denotes the typical mass of the soft strong interaction $\mu \sim 1\,GeV$ and

$$\frac{\bar{\alpha}_S\left(Q^2\right)}{\bar{\alpha}_S\left(\mu^2\right)} \;=\; \frac{1}{1 \,+\, \beta\bar{\alpha}_S\left(\mu^2\right)\,\ln\left(Q^2/\mu^2\right)}$$

with $\beta = 3/4$.

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We consider the strong interaction data for energies $W \geq 0.546\, TeV$,
while the experimental data from HERA were measured for lower energies
$10.7 \leq W \leq 301$ GeV.
Therefore, it is necessary to include SECONDARY REGGEONS which give a substantial contribution.

$$\sigma_{I\!R}\,(Q,Y) \;=\; \int \frac{d^2 r}{4\,\pi}\left\{(\Psi^*\Psi)^{\gamma^*}_T \;+\; (\Psi^*\Psi)^{\gamma^*}_L\right\} A_{I\!R}\, r^2\, \left(\frac{Q^2}{x_{Bj}\,Q_0^2}\right)^{\alpha_{I\!R}(0)-1}$$

with $Q_0 = 1\, GeV$.

The final equation for F_2 takes a form:

$$F_2\,(Q,Y) \;=\; \frac{Q^2}{4\pi^2\alpha_{\text{e.m.}}}\left\{\frac{\bar{\alpha}_S\left(Q^2\right)}{\bar{\alpha}_S\left(\mu^2\right)}\,\sigma^{\text{light q}}\,(Q,Y)\right.$$

$$\left. +\; \frac{\bar{\alpha}_S\left(Q^2+4m_c^2\right)}{\bar{\alpha}_S\left(\mu^2\right)}\sigma^{\text{charm q}}\,(Q,Y_c) \;+\; \sigma_{I\!R}\,(Q,Y)\right\}$$

DIS 5: Parameters of the fit to DIS data

We introduce a set of new parameters for DIS: m_q-mass of the light quark,
which we assume to be of the order of the constituent quark mass ($\sim 300\,MeV$),
and the mass of charm quark ($m_c = 1.2 \div 1.5\,GeV$),
μ which we believe will be of the order of 1 GeV,
and we introduce two new parameters $A_{I\!R}$ and $\alpha_{I\!R}(0)$ for the secondary Reggeon contribution.

Fit	$m_q(GeV$	$m_c(GeV)$	$\alpha_s(\mu)$	$\mu\ (GeV)$	$A_{I\!R}(GeV^2$	$\alpha_{I\!R}(0))$
I	0.3	1.25	0.263	1.2	2.34	0.55
II	0.2	1.2	0.34	1.25	5.44	0.56

Fitted parameters for DIS

Fit I fix parmeter values from PREVIOUS Soft Fit,
and only determine values of NEW parameters associated with F_2.

Fit II is a JOINT fit to Soft and DIS data.

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Fit	λ	ϕ_0 (GeV^{-2})	g_1 (GeV^{-1})	g_2 (GeV^{-1})	$m(GeV)$	$m_1(GeV)$	$m_2(GeV)$
I(soft int.)	0.38	0.0019	110.2	11.2	5.25	0.92	1.9
II:(soft + DIS)	0.38	0.0022	96.9	20.96	5.25	0.86	1.76

where for Fit I $\beta = 0.58$ and for Fit II $\beta = 0.66$

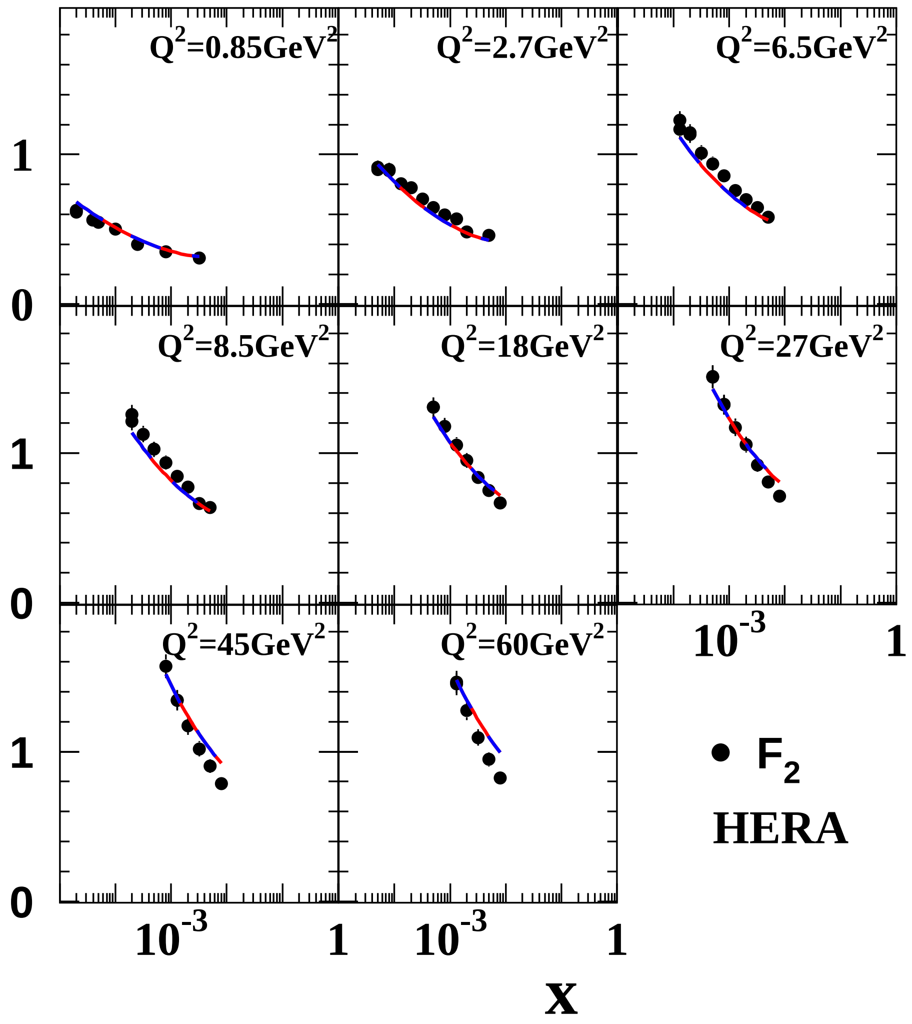
Fit I: parameters for the soft interaction at high energy were FIXED from previous fit to soft data ONLY.
The additional parameters for DIS were found by fitting to the F_2 structure function.

Fit II: joint fit to the soft interaction data at high energy and the DIS data.

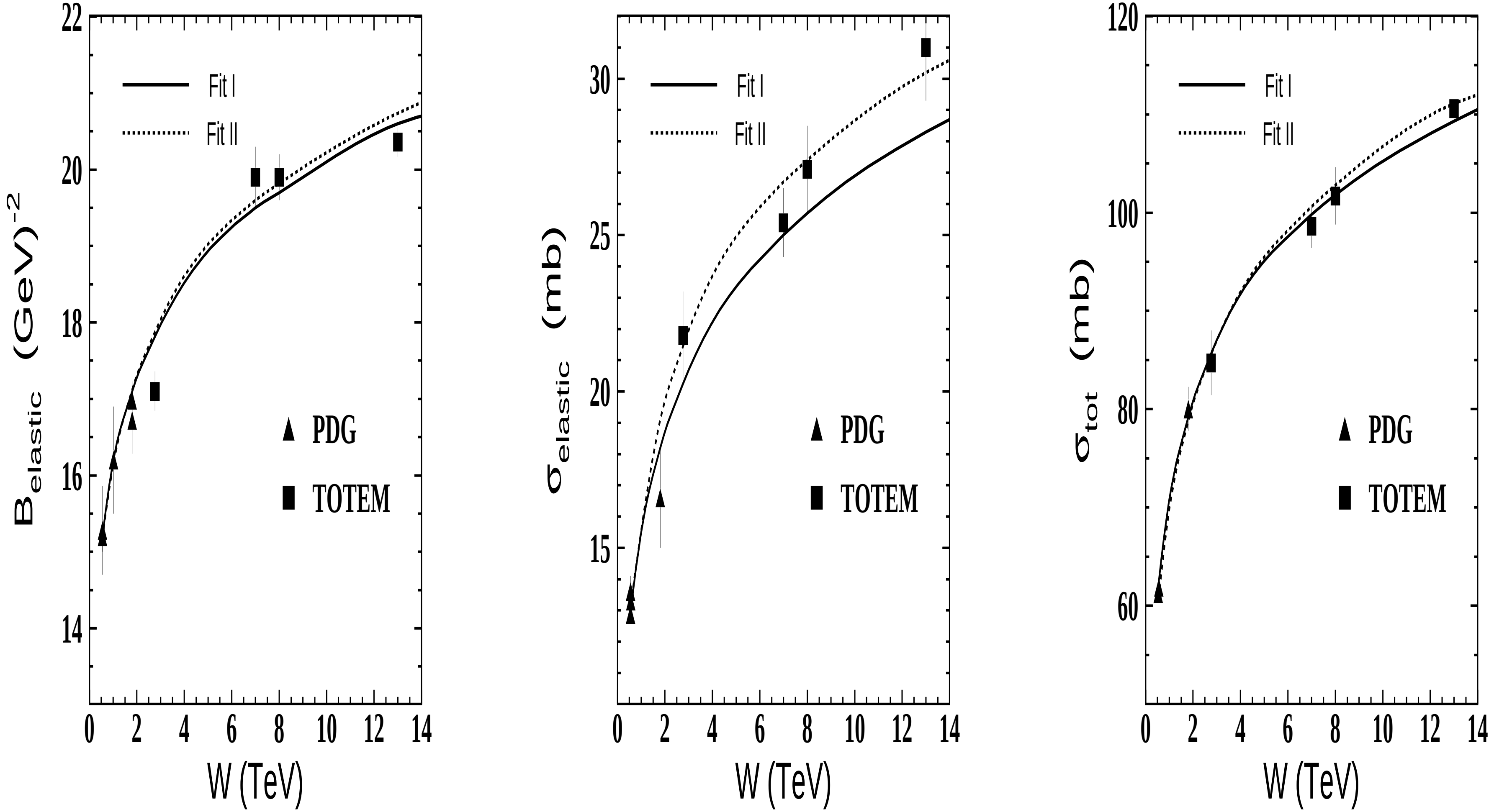
Model Predictions

W	σ_{tot}	σ_{el}(mb)	B_{el}	single	diffraction	double	diffraction
(TeV)	(mb)	(mb)	(GeV^{-2})	$\sigma_{\rm sd}^{\rm smd}$ (mb)	$\sigma_{\rm sd}^{\rm lmd}$ (mb)	$\sigma_{\rm dd}^{\rm smd}$ (mb)	$\sigma_{\rm dd}^{\rm lmd}$ (mb)
0.576	62.3(60.7)	12.9(13.1)	15.2(15.17)	5.64(4.12)	1.85(1.79)	0.7(0.39)	0.46 (0.50)
0.9	69.2(68.07)	15(15.05)	16(15.95)	6.254.67)	2.39(2.35)	0.77(0.46)	0.67 (0.745)
1.8	79.2(78.76)	18.2(19.1)	17.1(17.12)	7.1(5.44)	3.35(3.28)	0.89(0.56)	1.17 (1.30)
2.74	85.5(85.44)	20.2(21.4)	17.8(17.86)	7.6(5.91)	4.07(4.02)	0.97(0.63)	1.62 (1.79)
7	99.8(100.64)	25(26.7)	19.5(19.6)	8.7(6.96)	6.2(6.17)	1.15(0.814)	3.27(3.67[illegible])
8	101.8(102.8)	25.7(27.4)	19.7(19.82)	8.82(7.1)	6.55(6.56)	1.17(0.841)	3.63 (4.05)
13	109.3(111.07)	28.3(30.2)	20.6(20.74)	9.36(7.64)	8.08(8.11)	1.27(0.942)	5.11(5.74)
14	110.5(111.97)	28.7(30.6)	20.7(20.88)	9.44(7.71)	8.34(8.42)	1.27(0.96)	5.4(6.06)
57	131.7(134.0)	36.2(38.5)	23.1(23.0)	10.85(9.15)	15.02(15.01)	1.56(1.26)	13.7(15.6)

The values of cross sections versus energy.
The predictions of fit II, are shown in brackets.



F_2 versus x at fixed Q. The red curve corresponds to Fit I (S.I. only) while the blue one describes Fit II (S.I. + DIS). Data is taken from HERA.

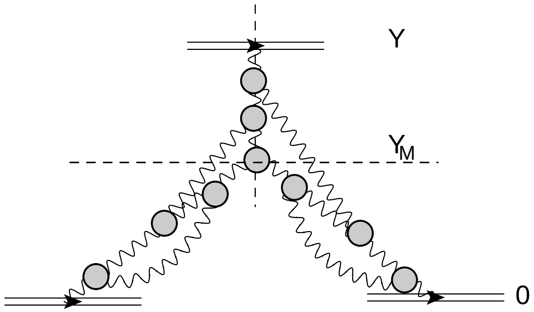


The energy behaviour of σ_{tot}, σ_{el} and the slope B_{el} in our model.
Fit I (soft data only) and Fit II (soft + hard data).

Conclusions

- Constructed a model based on the BFKL Pomeron and the CGC/saturation approach, which successfully describes data in the Regge region, for high energy hadron scattering.
- Do not require that the soft Pomeron to appear as a Regge pole.
- Suggest a procedure where the matching with long distance physics (where confinement of quarks and gluons is essential) can be reached within the CGC/saturation approach.
- Model for soft (long distance) interactions, is able to describe inclusive production.
- Model also successfully describes:
  Long range rapidity correlations [EPJ C75,(2015) 518]
  Survival Probability of central exclusive production [EPJ C76 (2016) 177]
  Long-range elliptic anistropies (ridge structure) in proton -proton collisions [Phys. Rev. D93 (2016) 074029]
  Bose-Einstein correlations in hadron and nucleus collisions [Phys. Rev. D95 (2017) 034005]

Diffractive Scattering based on the Levin-Kovchegov Equation



MPSI approximation: the simplest diagram for single diffraction production. The wavy lines describe BFKL Pomerons. The blobs stand for triple Pomeron vertices. The dashed line denotes the cut Pomeron. $Y_M = \ln\left(M^2/s_0\right)$, where M is the mass of produced particles and s_0 is the scale taken to be of the order of $1\,GeV^2$.

L-K equation has same form as the B-K equation (Nucl.Phys. B577 (2000) 221) for the function

$$G\left(Y, Y_0, r, b\right) \;=\; 2\,N\left(Y, Y_0, r, b\right) \;-\; N_{SD}\left(Y, Y_0, r, b\right),$$

$N\left(Y, Y_0, r, b\right)$ is the imaginary part of the elastic amplitude

The cross section for diffraction production is:

$$\sigma_{diff}\left(Y, Y_0, r\right) \;=\; \int d^2b\, N_{SD}\left(Y, Y_0, r, b\right)$$