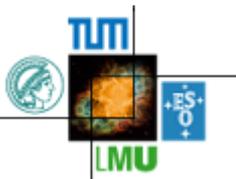


# Dual QCD and Flavour Kaon Physics

*Andrzej J. Buras*  
*(TUM – Institute for Advanced Study)*



**Quark Confinement**  
**Dublin, August 2018**



# Overture

# Present Superstars of K Physics

[NA48  
KTev]

$$\frac{\varepsilon'}{\varepsilon} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$\varepsilon'$ : direct CP violation in  $K_L \rightarrow \pi\pi$   
 $\varepsilon$ : indirect CP violation in  $K_L \rightarrow \pi\pi$

[1964]

$$\varepsilon = e^{i\varphi_\varepsilon} (2.23 \pm 0.01) \cdot 10^{-3}$$

$K^0 - \bar{K}^0$  Mixing

( $\Delta I = 1/2$  Rule)

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 22.35$$

$$A_0 = A(K \rightarrow (\pi\pi)_{I=0})$$

$$A_2 = A(K \rightarrow (\pi\pi)_{I=2})$$

[1955]

[NA62]  
[2019]

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}$$

$\Delta M_K$

[1959]

$K_L - K_S$  Mass difference

# Effective Hamiltonian and OPE

Wilson  
Coefficients

$$H_{\text{eff}} = \sum_i C_i O_i^{\text{SM}} + \sum_j C_j^{\text{NP}} O_j^{\text{NP}}$$

$$C_i = C_i^{\text{SM}} + \Delta_i^{\text{NP}}$$

↑
↑
↑  
C<sub>i</sub><sup>SM</sup>
Absent in SM

$$A(K \rightarrow \pi\pi) = \sum_i \underbrace{C_i(\mu)}_{\text{SD}} \underbrace{\langle \pi\pi | O_i^{\text{SM}}(\mu) | K \rangle}_{\text{LD}} + \sum_j \underbrace{C_j^{\text{NP}}(\mu)}_{\text{SD}} \underbrace{\langle \pi\pi | O_j^{\text{NP}}(\mu) | K \rangle}_{\text{LD}}$$

Example:  $O^{\text{SM}} = (\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{d}\gamma^\mu(1-\gamma_5)d)$

$$O^{\text{NP}} = (\bar{s}(1-\gamma_5)d)(\bar{d}(1-\gamma_5)d)$$

$\mu \approx 1-3 \text{ GeV}$

Renormalization  
scale

# Impact of QCD at SD and LD Scales

(K-physics)

SD

Fully under control: NLO + NNLO

AJB: „Climbing NLO and NNLO Summits of Weak Decays“

(1102.5650; last update 2014)

Including BSM

(Munich, Rome + Gorbahn, Brod,

(early 1990s)

Haisch, Jäger,

Nierste, Cerda-Sevilla)

LD

Lattice QCD

(ETM, SWME, RBC-UKQCD, ...)

(Numerical sophisticated and demanding calculations lasting many years) (from first principles)

BSM operators only  
for  $K^0 - \bar{K}^0$  mixing

Analytic

Dual QCD

(Bardeen, AJB, Gérard, 1985 →)

(Much faster than LQCD, very suitable for non-leptonic transitions)

( $K^0 - \bar{K}^0$  mixing,  $K \rightarrow \pi\pi$ )

SM+

BSM

(2018)

Chiral Perturbation Theory

(Gasser, Leutwyler, 1980 →)

(Much faster than LQCD, very suitable for leptonic, semi-leptonic decays)

( $K_L \rightarrow \pi\ell$ ,  $K \rightarrow \ell\ell$ ,  $K \rightarrow \pi\nu\bar{\nu}$ )

# Plan for next 23 min

**1.**

**Dual QCD**

**2.**

**$\varepsilon'/\varepsilon$  strikes back**

**3.**

**$\varepsilon'/\varepsilon$  Beyond SM**

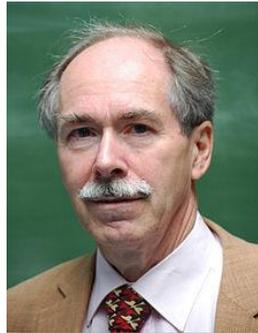
**4.**

**Summary**

# 1.

## Dual QCD

Large N  
QCD



Gerard 't Hooft  
(1974)



Edward Witten  
(1979, 1980)

At Large N QCD becomes a theory of weakly interacting mesons

with coupling  $\frac{1}{f_{\pi}^2} \sim \frac{1}{N}$



In the strict Large N limit QCD becomes a free theory of mesons.



Factorization of hadronic matrix elements



# Dual QCD Approach for Weak Decays

Successful low energy approximation of QCD

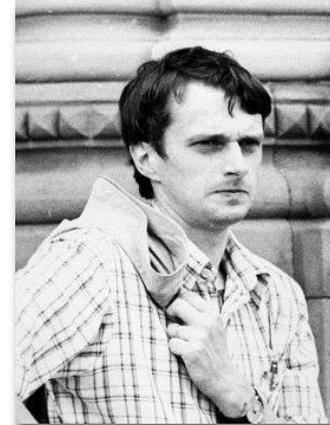
1985



W. Bardeen

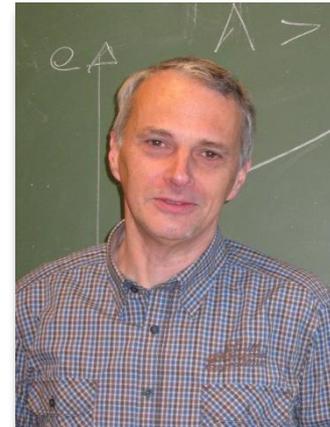
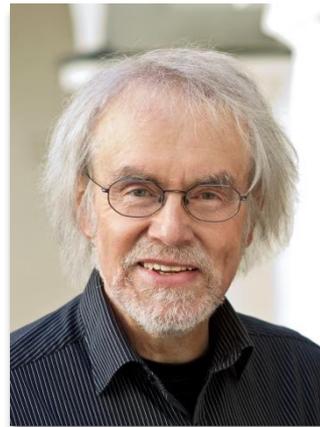


AJB



J.-M. Gérard

2014



# Basics of Dual QCD

Chivukula, Flynn, Georgi  
Bardeen, AJB, Gérard  
(1986)



$$\mathbf{L}_{\text{tr}} = \frac{\mathbf{F}^2}{8} \left[ \text{Tr}(\mathbf{D}_\mu \mathbf{U} \mathbf{D}^\mu \mathbf{U}^+) + r \text{Tr}(m \mathbf{U}^+ + \text{h.c.}) + \mathcal{O}(1/\Lambda_\chi^2) \right]$$

$$\mathbf{U} = \exp \left[ i\sqrt{2} \frac{\boldsymbol{\Pi}}{\mathbf{F}} \right]$$

$$r(\mu) = \frac{2m_K^2}{m_s(\mu) + m_d(\mu)}$$

$\boldsymbol{\Pi}$  = octet of  
pseudoscalar mesons

$$\mathbf{F} \approx \mathbf{F}_\pi \sim \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

$$\bar{\mathbf{q}}_L^b \gamma_\mu \mathbf{q}_L^a = i \frac{\mathbf{F}^2}{8} \left[ (\partial_\mu \mathbf{U}) \mathbf{U}^+ + \mathcal{O}\left(\frac{1}{\Lambda_\chi^2}\right) \right]^{ab}$$

$$\bar{\mathbf{q}}_R^b \mathbf{q}_L^a = -\frac{\mathbf{F}^2}{8} r(\mu) \left[ \mathbf{U} - \frac{1}{\Lambda_\chi^2} \partial^2 \mathbf{U} \right]^{ab}$$

Meson representation  
of

quark-currents

quark-densities

# Hadronic Matrix Elements (Large N Limit)

Simply products of quark currents  
or quark densities  
or tensor structures

$$P_L = (1 - \gamma_5) / 2$$

Example :

$$\langle \pi\pi (I = 0) | (\bar{s}P_L d)(\bar{u}P_L u) | K \rangle = -\frac{F_\pi}{24} \left[ \frac{2m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2$$

SD Scale

This is tree-level calculation in meson theory: factorizable contributions

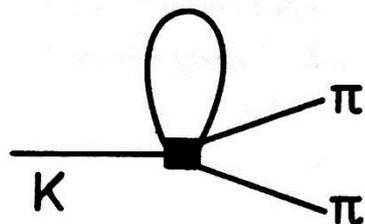
In order to match with short-distance Wilson coefficients  
non-factorizable contributions must be included:  
loops in meson theory

They vanish at factorization scale:  $0 (m_\pi, m_K) \neq \mu$

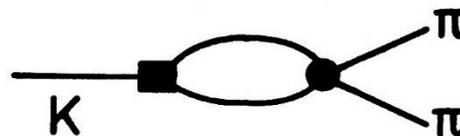


# Meson Evolution

Loops with a physical cutt-off  $\Lambda$  :  $1/N$  non-factorizable contributions

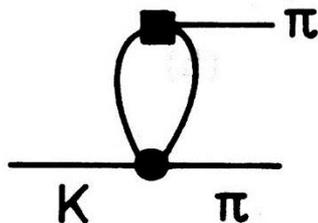


(a)

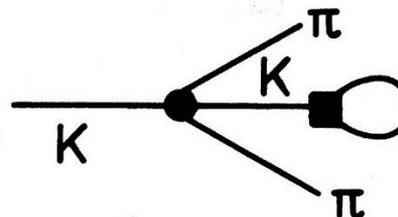


(b)

■ weak  
● strong



(c)



(d)

Very different philosophy from Chiral PTh



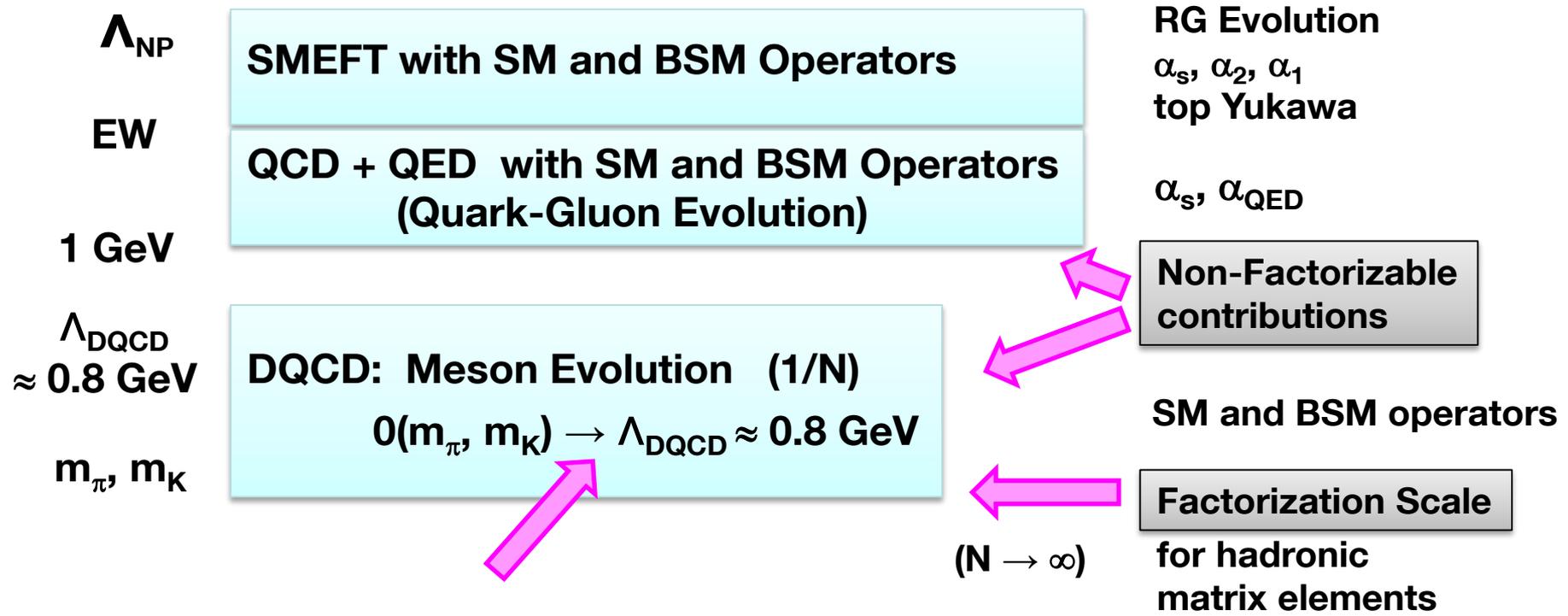
No dimensional regularisation !!!

# Basic Structure of DQCD for $K \rightarrow \pi\pi$ , $K^0 - \bar{K}^0$ mixing

$(\varepsilon'/\varepsilon, \varepsilon, \Delta I = 1/2 \text{ Rule}, \Delta M_K)$

SM and BSM Operators

Reviews: 1401.1385, 1408.4820



**Crucial strong dynamics**  
Responsible for  $\Delta I = 1/2$  Rule,  $\varepsilon'/\varepsilon, \varepsilon, \Delta M_K, K \rightarrow \pi\pi$  in general.

# The Main Role of DQCD

**1.** Efficient approximate method for obtaining results for non-leptonic decays: years, even decades before Lattice QCD.

(1986-2012)

BSM:2018

**2.** Giving insight in numerical results obtained by Lattice QCD at 2-3 GeV.

Progress in  
LQCD  
↓  
(2012 → )

**3.** The only existing QCD method allowing to study analytically the dominant dynamics between  $m_K$  and 1 GeV.

**MESON EVOLUTION** :

The pattern of operator mixing found to agree with SD mixing. both for SM and BSM operators.



# $\Delta I = 1/2$ Rule

$$R = \frac{\text{Re } A_0}{\text{Re } A_2} \approx 22.4$$

Since 1955

Gell-Mann Pais  
 $R = \sqrt{2}$  in Fermi-Theory  
(No QCD)

1974

Octet Enhancement  
(Current-Current Operators)

(Altarelli + Maiani  
Gaillard + Lee)

$$R \approx 3$$

(Asymptotic Freedom)  
SD evolution

1986

Dual QCD (BBG)

(Main Dynamics Behind  $\Delta I = 1/2$  Rule)

Octet Enhancement including LD part (Meson Evolution)  
QCD Penguins at 10%

2014:  $R = 16.0 \pm 1.5$

FSI ? (Pich)  
New Physics ?  
(1404.3824, G')  
AJB, De Fazio, Girrbach

2015

RBC-UKQCD

$$R = 31 \pm 11$$

(2012)

Confirmation of Octet Enhancement

# $\hat{B}_K$ Parameter for $K^0 - \bar{K}^0$ Mixing, $\varepsilon_K$

1986

Donoghue et al  
Pich + Rafael

$$\hat{B}_K \approx 0.33$$

$$\hat{B}_K \approx 0.4$$

Lattice QCD

$$\hat{B}_K \approx 1$$

1987

BBG

$$\hat{B}_K = 0.67 \pm 0.07$$

$$\hat{B}_K = 0.75 \text{ (Large N limit)}$$

2018

BBG

$$\hat{B}_K = 0.73 \pm 0.02$$

Lattice QCD:

$$\hat{B}_K = 0.763 \pm 0.010$$

Gérard:  $\hat{B}_K < 0.75$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{4}{3} F_K^2 m_K B_K(\mu) \quad \hat{B}_K = B_K(\mu) \alpha_s^P(\mu)$$

$$Q = \left[ \bar{s} \gamma_\mu (1 - \gamma_5) d \right]^2$$

# 2018 Results in DQCD

: BSM hadronic  
Matrix elements

★ 1.

Matrix elements of chromomagnetic penguins  
AJB + Gérard 1803.08052 (First on-shell  $K \rightarrow \pi\pi$   
calculation to date)

Confirmation of  $K \rightarrow \pi$  matrix element  
by ETM collaboration 1712.09824

$$B_{\text{CMO}} \approx 1/3$$

Much smaller  
than early  
estimates in  
chiral quark  
model

★ 2.

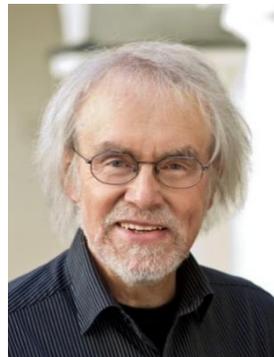
Explanation of BSM  $B_i$  parameters ( $K^0 - \bar{K}^0$  Mixing)  
obtained by Lattice QCD 1804.02401 (AJB + Gérard)

★ 3.

$K \rightarrow \pi\pi$  matrix elements of all BSM 4-quark operators



Jason Aebischer



AJB



J.-M. Gérard

(1807.01709)



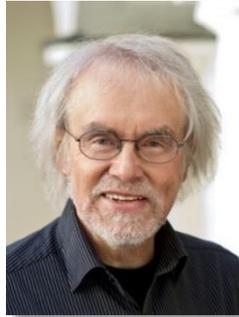
## Master Formula for $\varepsilon'/\varepsilon$ Beyond SM (1807.02520)



Jason Aebischer



Christoph Bobeth



AJB



Jean-Marc Gérard



David Straub



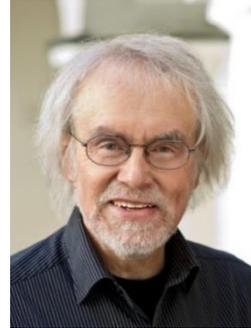
## Anatomy of $\varepsilon'/\varepsilon$ Beyond SM (1808.xxxxx) First SMEFT analysis



Jason Aebischer



Christoph Bobeth



AJB



David Straub

# **Section 2**

## **$\varepsilon'/\varepsilon$ strikes back**

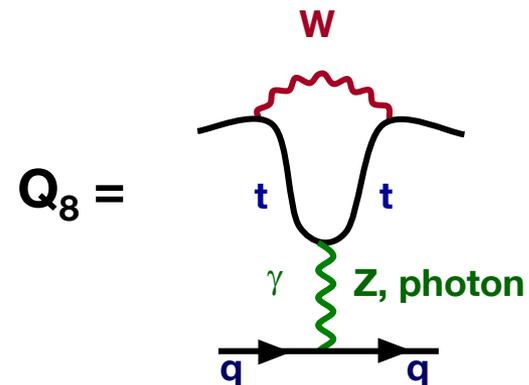
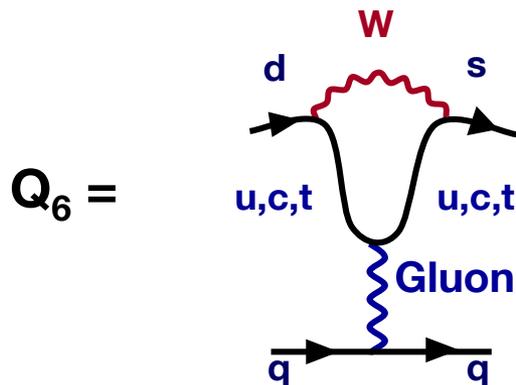
# Main Actors in $\varepsilon'/\varepsilon$ in SM

$Q_6$  – QCD Penguin operator

$Q_8$  – Electroweak Penguin operator

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_8 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A}$$



# QCD and Electroweak Penguin Matrix Elements

1986

BBG strict Large N limit

$$B_6^{1/2} = B_8^{3/2} = 1 \quad (\mu \approx 0(m_\pi))$$

$$\langle Q_6(\mu) \rangle_0 = -4 \left[ \frac{m_K^2}{m_s(\mu)} \right]^2 (F_K - F_\pi) B_6^{(1/2)}(\mu)$$
$$\langle Q_8(\mu) \rangle_2 = \sqrt{2} \left[ \frac{m_K^2}{m_s(\mu)} \right]^2 F_\pi B_8^{(1/2)}(\mu)$$

$B_6(\mu), B_8(\mu)$ : very weak  $\mu$  dependence for  $\mu > 1\text{GeV}$

# Four dominant contributions to $\varepsilon'/\varepsilon$ in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

$$(\varepsilon'/\varepsilon)_{\text{SM}} = \left[ \frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} \left[ -3.6 + 21.4 \cdot B_6^{(1/2)} + 1.2 - 10.4 \cdot B_8^{(3/2)} \right]$$

(NLO)      (Q<sub>4</sub>)

(V-A) ⊗ (V-A) QCD Penguins	(V-A) ⊗ (V+A) QCD Penguins	(V-A) ⊗ (V-A) EW Penguins	(V-A) ⊗ (V+A) EW Penguins
-------------------------------	-------------------------------	------------------------------	------------------------------

From ReA<sub>0</sub> (points to -3.6)  
 From ReA<sub>2</sub> (points to 1.2)

Assumes that ReA<sub>0</sub> and ReA<sub>2</sub> ( $\Delta I=1/2$  Rule) fully described by SM (includes isospin breaking corrections)

Extracted from



**RBC-UKQCD**

$B_6^{(1/2)} = B_8^{(3/2)} = 1$  in the large N limit

$B_6^{(1/2)} = 0.57 \pm 0.19$

$B_8^{(3/2)} = 0.76 \pm 0.05$

**Why  $B_6^{(1/2)} < B_8^{(3/2)} < 1$  ?**

and not  $B_6^{(1/2)} > 1$  ,  $B_8^{(3/2)} < 1$  (Pallante, Pich... FSI  
2000)

**Answer in Large N (Dual QCD) Approach**

AJB + Gérard (1507.06326)

Before 2015 it was wrongly assumed that

$$B_6^{(1/2)} = B_8^{(3/2)} = 1 \text{ at } \mu \approx 0(1 \text{ GeV})$$

But  $B_6^{(1/2)} = B_8^{(3/2)} = 1$  is large N prediction  
for  $\mu = m_\pi$  not  $\mu = 0(1 \text{ GeV})$

Meson evolution  $m_\pi \rightarrow \mu = 0(1 \text{ GeV})$  suppresses  
 $B_6^{(1/2)}$  and  $B_8^{(3/2)}$  below 1 and  $B_6^{(1/2)}$  stronger than  $B_8^{(3/2)}$   
in accordance with quark evolution for  $\mu > 1 \text{ GeV}$

1986

BBG strict Large N limit

$$\mathbf{B}_6^{1/2} = \mathbf{B}_8^{3/2} = 1$$

( $\mu \approx 0(m_\pi)$ )

2015

AJB + Gérard  
1507.06326

Including 1/N  
(meson evolution  
for  $B_6, B_8$ )

$$\mathbf{B}_6^{1/2} < \mathbf{B}_8^{3/2} < 1$$

at  $\mu \geq 1\text{GeV}$

RBC-UKQCD

$$\mathbf{B}_6^{(1/2)} = 1 - 0.66 \ln \left( 1 + \frac{\Lambda^2}{\tilde{m}_6^2} \right) \Rightarrow \mathbf{B}_6^{(1/2)} < 0.7$$

$$\mathbf{B}_6^{(1/2)} = 0.57 \pm 0.19$$

$$\mathbf{B}_8^{(3/2)} = 1 - 0.17 \ln \left( 1 + \frac{\Lambda^2}{\tilde{m}_8^2} \right) \Rightarrow \mathbf{B}_8^{(3/2)} \approx 0.8 \pm 0.1$$

$$\mathbf{B}_8^{(3/2)} = 0.76 \pm 0.05$$

$$\tilde{m}_{6,8} < \Lambda$$

$$(\varepsilon'/\varepsilon)_{\text{SM}} < (6.0 \pm 2.4) \cdot 10^{-4}$$

(2015)



**RBC-UKQCD**  
(1505.07863)

$$\left(\varepsilon'/\varepsilon\right)_{\text{SM}} = (1.4 \pm 6.9) \cdot 10^{-4}$$

No isospin breaking  
correction (IB)

AJB, Gorbahn, Jäger  
Jamin  
(1507.06345)

$$\left(\varepsilon'/\varepsilon\right)_{\text{SM}} = (1.9 \pm 4.5) \cdot 10^{-4}$$

Lattice results + IB

AJB + Gérard  
(1507.06326)

$$\left(\varepsilon'/\varepsilon\right)_{\text{SM}} < (6.0 \pm 2.4) \cdot 10^{-4}$$

Dual QCD bound

Kitahara, Nierste, Tremper  
(1607.06727)

$$\left(\varepsilon'/\varepsilon\right)_{\text{SM}} = (1.1 \pm 5.1) \cdot 10^{-4}$$

Lattice results + IB

Gisbert, Pich  
(1712.06147)

$$\left(\varepsilon'/\varepsilon\right)_{\text{SM}} = (15 \pm 7) \cdot 10^{-4}$$

Chiral Pert. Th.

Experiment  
(NA48, KTeV)

$$\left(\varepsilon'/\varepsilon\right)^{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

**NNLO** :

Maria Cerda-Sevilla, Martin Gorbahn, Sebastian Jäger,  
Ahmet Kokulu

In progress

# FSI in $K \rightarrow \pi\pi$

AJB, Gérard 1603.05686

**Relevant for  $\Delta I=1/2$  Rule**  
**(in agreement with Pallante, Pich,...)**

**Less important for  $\varepsilon'/\varepsilon$**   
**(in variance with Pallante, Pich,...)**

# As the existence of Meson Evolution has been questioned over last 30 years by some Chiral Experts by some Lattice Experts

Let me demonstrate its existence  
by considering BSM operators in  $(K^0 - \bar{K}^0 \text{ Mixing})$

Very  
good  
test !

**Important**

: The controversial issue of  
Final State interactions is  
absent here !!!

and four parameters to our disposal  $B_2, B_3, B_4, B_5$

# BSM $\Delta S = 2$ Operators $K^0 - \bar{K}^0$ Mixing

BSM

$$O_2 = (\bar{s}^\alpha P_L d^\alpha) (\bar{s}^\beta P_L d^\beta) \rightarrow B_2$$

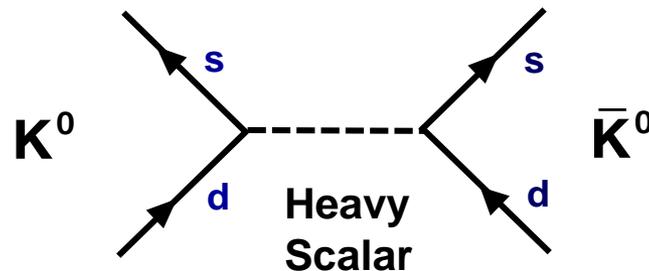
$$O_3 = (\bar{s}^\alpha P_L d^\beta) (\bar{s}^\beta P_L d^\alpha) \rightarrow B_3$$

$$O_4 = (\bar{s}^\alpha P_L d^\alpha) (\bar{s}^\beta P_R d^\beta) \rightarrow B_4$$

$$O_5 = (\bar{s}^\alpha P_L d^\beta) (\bar{s}^\beta P_R d^\alpha) \rightarrow B_5$$

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$$

$$\langle O_i(\mu) \rangle \approx \frac{B_i(\mu)}{m_s^2(\mu)}$$



# DQCD

## Explaining Values for $B_2, B_3, B_4, B_5$ from Lattice QCD

(AJB + Gérard, 1804.02401)

(ETM15, SWME, RBC-QCD)

$\mu$	$B_2$	$B_3$	$B_4$	$B_5$	$K^0 - \bar{K}^0$ mixing
3 GeV	0.49	0.77	0.90	0.65	AJB Lattice Average ( $\pm 5\%$ )
Quark Gluon Evolution ↓ 1 GeV					<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <math>B_2=B_3=B_4=B_5=1</math> </div> <div style="border: 1px solid black; padding: 2px; display: inline-block;">                     Vacuum insertion                 </div>
} gap : Vector mesons					
(0.70) GeV					Meson Evolution in the chiral limit
Meson Evolution ↑					
Factorization Scale $\approx 0$					← $N \rightarrow \infty$

# DQCD

## Explaining Values for $B_2, B_3, B_4, B_5$ from Lattice QCD

(AJB + Gérard, 1804.02401)

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Quark Gluon Evolution $\downarrow$ 1 GeV					$B_2=B_3=B_4=B_5=1$ Vacuum insertion
} gap : Vector mesons					
(0.70) GeV					Meson Evolution in the chiral limit
Meson Evolution $\uparrow$					
Factorization Scale $\approx 0$	1.2	3.0	1.0	0.23	$\leftarrow N \rightarrow \infty$

# DQCD

## Explaining Values for $B_2, B_3, B_4, B_5$ from Lattice QCD

(AJB + Gérard, 1804.02401)

(ETM15, SWME, RBC-QCD)

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3 GeV	0.49	0.77	0.90	0.65	AJB Lattice Average ( $\pm 5\%$ )
Quark Gluon Evolution ↓ 1 GeV	0.62	1.10	0.90	0.45	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <math>B_2=B_3=B_4=B_5=1</math> </div> <div style="border: 1px solid black; padding: 2px; display: inline-block;">                     Vacuum insertion                 </div>
} gap : Vector mesons					
(0.70) GeV Meson Evolution ↑					Meson Evolution in the chiral limit
Factorization Scale $\approx 0$	1.2	3.0	1.0	0.23	← $N \rightarrow \infty$

# DQCD

## Explaining Values for $B_2, B_3, B_4, B_5$ from Lattice QCD

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(ETM15, SWME, RBC-QCD)

$\mu$	$B_2$	$B_3$	$B_4$	$B_5$	$K^0 - \bar{K}^0$ mixing
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} gap : Vector mesons					
(0.70) GeV Meson Evolution ↑	0.79	0.96	0.83	0.30	Meson Evolution in the chiral limit
Factorization Scale $\approx 0$	1.2	3.0	1.0	0.23	← $N \rightarrow \infty$

**Important !**

$$\langle \mathbf{0}_i(\mu) \rangle \approx \frac{B_i(\mu)}{m_s^2(\mu)}$$

Similar to  $B_6$  and  $B_8$

No FSI  
Meson evolution  
exhibited much  
clearer than in  
 $K \rightarrow \pi\pi$

This insight in  $B_i$  values from Lattice QCD has been obtained from DQCD without ANY input beyond  $\Lambda \approx m_\rho$  (Only pseudoscalar masses,  $F_K$  and  $\alpha_{\text{QCD}}$  involved)

No low-energy constants  $L_i$  etc. familiar from Chiral Pert. Th.

**Question** : Can this insight be obtained from Chiral Pert. Th. ?



Support for  $\varepsilon'/\varepsilon$  anomaly from DQCD

## Summary on $\varepsilon'/\varepsilon$ in the SM (DQCD)

**$\varepsilon'/\varepsilon$  anomaly could turn out to be the largest anomaly in flavour physics !**

$$(\varepsilon'/\varepsilon)_{\text{SM}} < (6.0 \pm 2.4) \cdot 10^{-4}$$

Dual  
QCD

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

AJB Expectation  
(May 2018)  
1805.11096

$$(\varepsilon'/\varepsilon)_{\text{SM}} = (5 \pm 2) \cdot 10^{-4}$$

**Will new RBC-UKQCD results support this claim?**

# Section 3

## $\epsilon'/\epsilon$ Beyond the SM

Review: [AJB \(1805.11096\)](#) (update soon)

# Effective Hamiltonian and OPE

$$H_{\text{eff}} = \sum_i C_i O_i^{\text{SM}} + \sum_j C_j^{\text{NP}} O_j^{\text{NP}}$$

$$C_i = C_i^{\text{SM}} + \Delta_i^{\text{NP}}$$

↑
↑  
Absent in SM

$\mu \approx 1 - 3 \text{ GeV}$

Renormalization scale

$$A(K \rightarrow \pi\pi) = \sum_i \underbrace{C_i(\mu)}_{\text{SD}} \underbrace{\langle \pi\pi | O_i^{\text{SM}}(\mu) | K \rangle}_{\text{LD}} + \sum_j \underbrace{C_j^{\text{NP}}(\mu)}_{\text{SD}} \underbrace{\langle \pi\pi | O_j^{\text{NP}}(\mu) | K \rangle}_{\text{LD}}$$

Already calculated by Lattice + DQCD

Calculated in July 2018 in DQCD

(1807.01709)

# All Dimension 6 BSM Four-Quark Operators (linearly independent)

ABG (1807.01709) ABBS (1808.xxxxx)

QCD x QED  
invariant

$$\text{SM} : 7$$

$$\text{BSM} : 33 = 7' + \underbrace{13 + 13}'$$

Obtained from  
SM : L ↔ R  
Matrix Elements  
reverse sign

Operators  
unrelated  
to SM  
operators

Matrix Elements  
only calculated  
in DQCD

SMEFT  
invariant  
under SM  
gauge Group

$$\text{BSM} : 21 = 7' + \underbrace{7 + 7}'$$

# Magnificent Seven

ABG (1807.01709)

Allowed by SMEFT :  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

$$A = (\bar{s}\gamma^\mu P_L d)(\bar{d}\gamma_\mu P_L d) - (\bar{s}\gamma^\mu P_L d)(\bar{s}\gamma_\mu P_L s)$$



$$B_2 = (\bar{s}P_R d)(\bar{d}P_L d) - (\bar{s}P_R d)(\bar{s}P_L d)$$

$$C_2 = (\bar{s}\gamma^\mu P_L d)(\bar{d}\gamma_\mu P_R d) - (\bar{s}\gamma^\mu P_L d)(\bar{s}\gamma_\mu P_R s)$$

$$D_1 = (\bar{s}P_L u)(\bar{u}P_L d)$$



$$D_1^* = -(\bar{s}\sigma^{\mu\nu} P_L u)(\bar{u}\sigma_{\mu\nu} P_L d)$$

$$D_2 = (\bar{s}P_L d)(\bar{u}P_L u)$$



$$D_2^* = -(\bar{s}\sigma^{\mu\nu} P_L d)(\bar{u}\sigma_{\mu\nu} P_L u)$$

All  
colour singlet  
quark bilinears



Largest  
Matrix Elements

# NP Models and $\epsilon'/\epsilon$ Anomaly (Only SM operators)

<b>Littlest Higgs (T parity)</b>	<b>Blanke, AJB, Recksiegel (1507.06316)</b>
<b>Z-FCNC</b>	<b>AJB (1601.00005), Bobeth, AJB, Celis, Jung (1703.04753) Endo, Kitahara, Mishima, Yamamoto (1612.08839)</b>
<b>Z'-Models</b>	<b>AJB (1601.00005), AJB, Buttazzo, Knegjens (1507.08672)</b>
<b>331- Models</b>	<b>AJB, De Fazio (1512.02869, 1604.02344)</b>
<b>Vector-Like Quarks</b>	<b>Bobeth, AJB, Celis, Jung (1609.04783)</b>
<b>SUSY</b>	<b>Tanimoto, Yamamoto (1603.07960) Kitahara, Nierste, Tremper (1604.07400) Endo, Mishima, Ueda, Yamamoto (1608.01444) Crivellin, D'Ambrosio, Kitahara, Nierste (1703.05786) Endo, Goto, Kitahara, Mishima, Ueda, Yamamoto (1712.04959)</b>
<b>Right-handed Currents</b>	<b>Cirigliano, Dekens, De Vries, Meraghetti (1703.04751)</b>
<b><math>SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}</math></b>	<b>Haba, Umeeda, Yamada (1802.09903, 1806.03424)</b>
<b>Leptoquark Models</b>	<b>Bobeth, AJB (1712.01295)</b>

# Master Formula for $\varepsilon'/\varepsilon$ Beyond SM

$$\left(\frac{\varepsilon'}{\varepsilon}\right) = \left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{NP}}$$

Valid in  
ANY extension  
of SM



$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{NP}} = \sum_i P_i(\mu_{\text{ew}}) \text{Im} \left[ C_i(\mu_{\text{ew}}) - C'_i(\mu_{\text{ew}}) \right]$$


$i = 1, \dots, 40$

4-quark operators + 1 dipole operator

$(P_L \leftrightarrow P_R)$

Model

independent:  $P_i(\mu_{\text{ew}})$  - Include hadronic elements + renormalization group effects from  $\mu \approx 0$  (1 GeV) to  $\mu_{\text{ew}} \approx 0$  ( $M_w$ )

All  
listed  
in

J. Aebischer, C. Bobeth, AJB, J.-M. Gérard, D. Straub  
1807.02520

# **Section 4**

# **Summary**

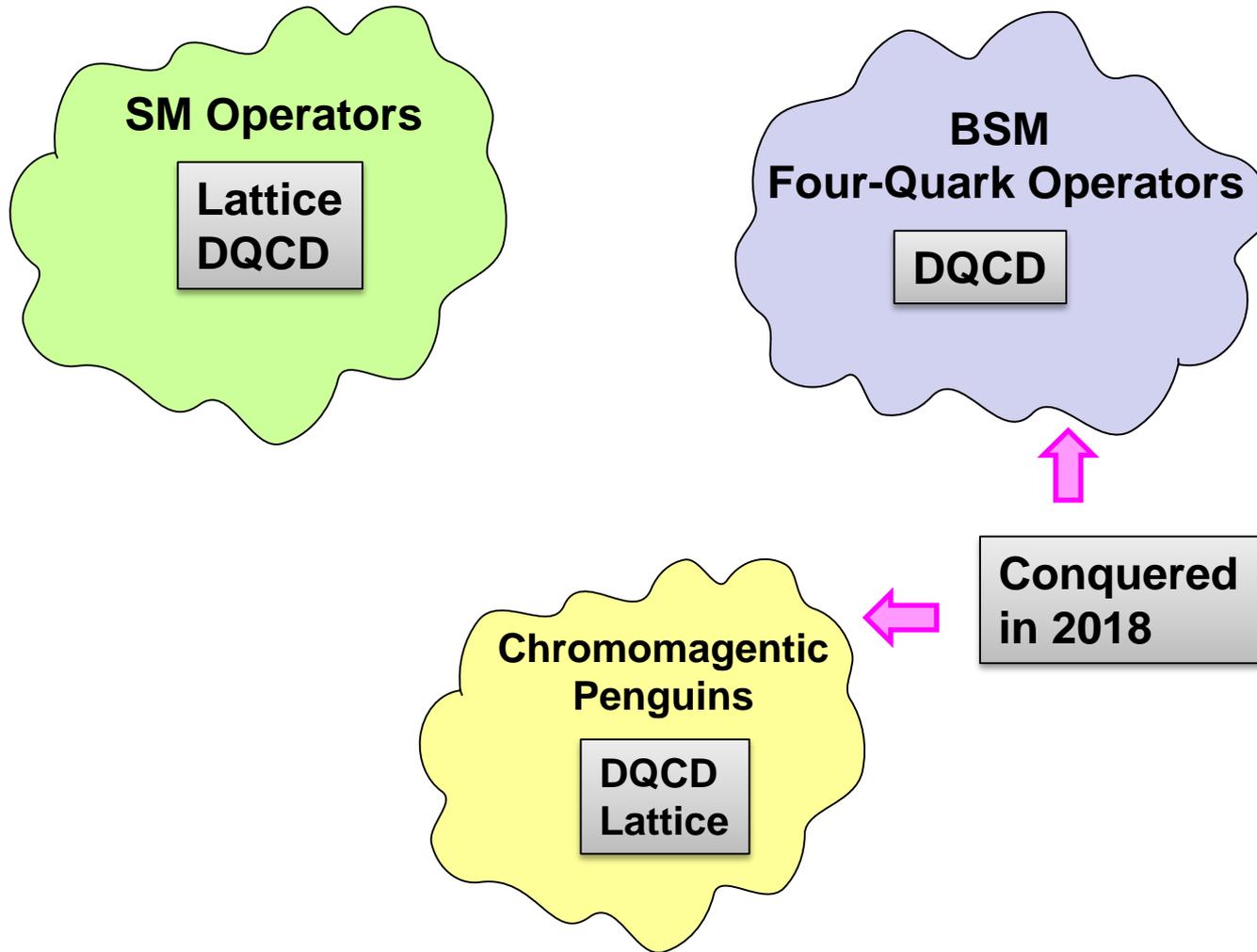
# Main Messages from this Talk



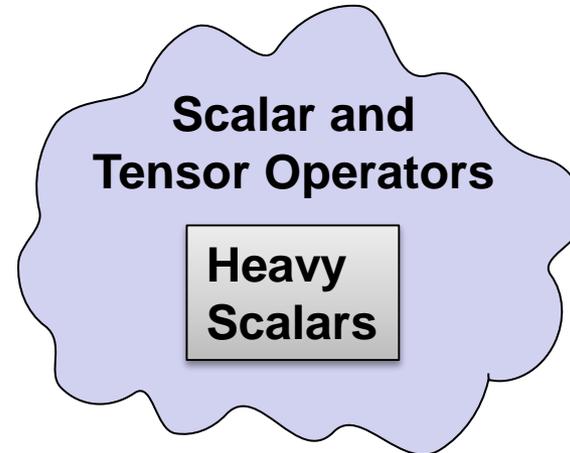
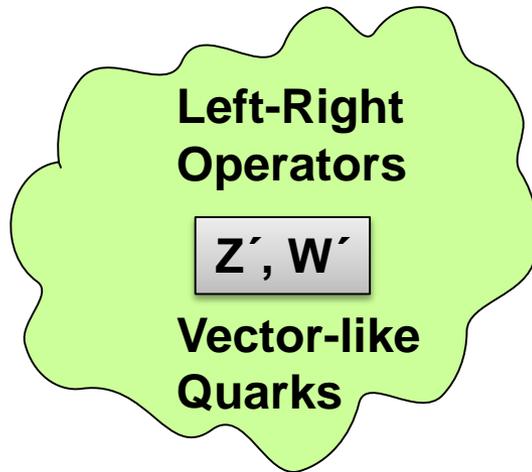
**The inclusion of meson evolution in the phenomenology of any non-leptonic transition like  $K^0 - \bar{K}^0$  mixing,  $K \rightarrow \pi\pi$  decays ( $\Delta I = 1/2$  Rule,  $\varepsilon'/\varepsilon$ ) is mandatory !**

**Meson Evolution is hidden in LQCD results but among analytic approaches only DQCD takes this important QCD dynamics into account.**

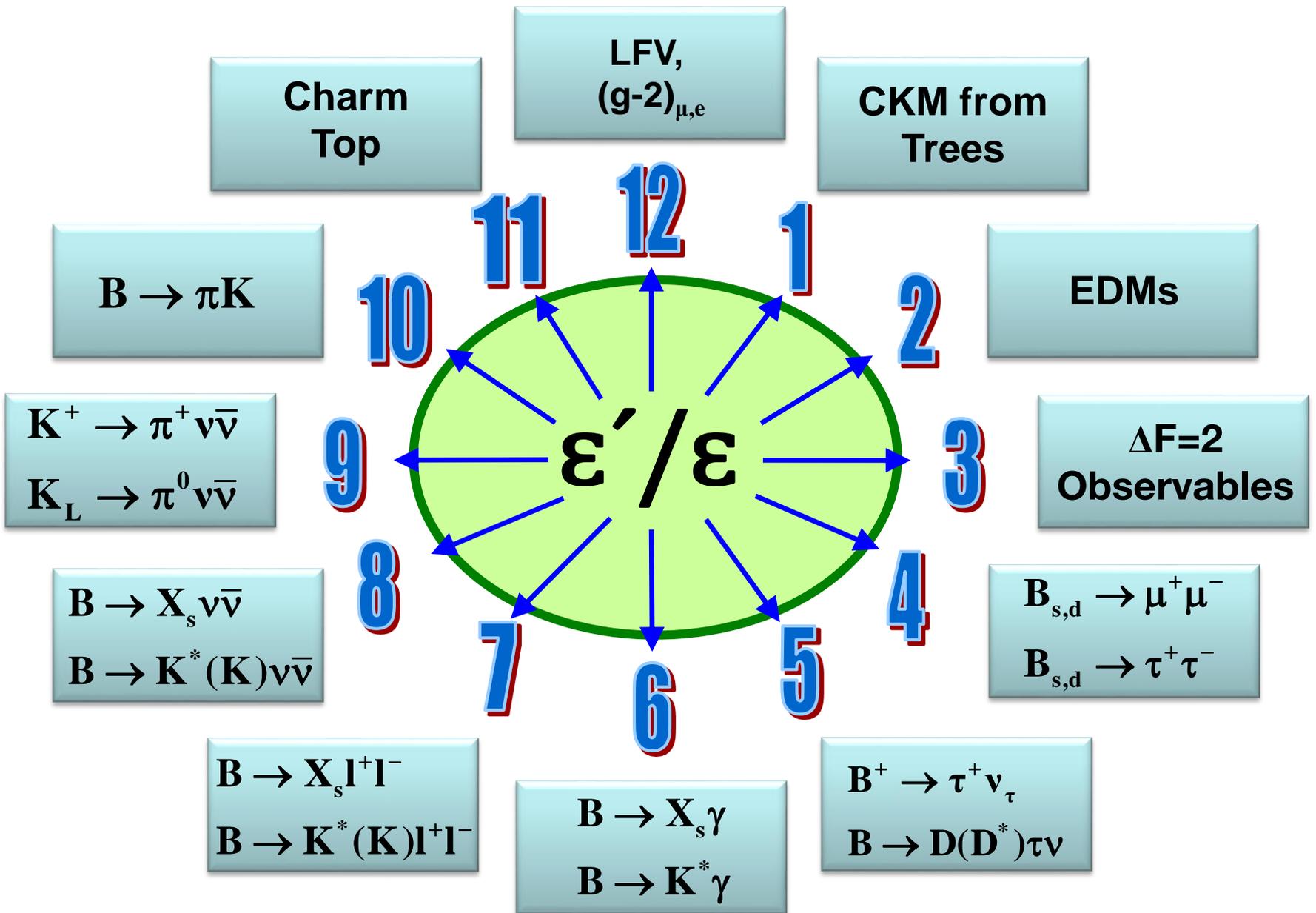
# Islands of $K \rightarrow \pi\pi$ Matrix Elements



# Most important for $\varepsilon'/\varepsilon$ Anomaly



**BSM anatomy of  $\varepsilon'/\varepsilon$ : Aebischer, Bobeth, AJB, Straub  
(1808.xxxxx) SMEFT analysis**



# Open Questions for Coming Years

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})?$$

NA62

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})?$$

KOTO

$$B \rightarrow K(K^*) \nu \bar{\nu}?$$

Belle

$$(\varepsilon'/\varepsilon)_{SM} ?$$

$$(\varepsilon_K)_{SM} ?$$

$$(\Delta M_K)_{SM} ?$$

**New Anomalies in Flavour Physics (B, D, LFV)?**

**New Particles discovered at the LHC?**

**What about  $\Delta I=1/2$  Rule?**

**(New Physics at 10-20% ?)**

**Lattice QCD**

**Can hopefully answer this question.**

**Coming Years**

**: Flavour Precision Era**

**LHC  
Upgrade  
E = 14 TeV  
(CERN)**

**Precision  
B<sub>d,s</sub> – Meson  
Decays  
LHCb, CMS  
KEK (Japan)**

★  
 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  ( $\sim 10^{-10}$ ) (CERN)  
 $K_L \rightarrow \pi^0 \nu \tilde{\nu}$  ( $\sim 3 \cdot 10^{-11}$ ) J-PARC  
(Japan)

**Lepton Flavour  
Violation**

$\mu \rightarrow e \gamma$

$\mu \rightarrow e e e$

$\tau \rightarrow \mu \gamma, \tau \rightarrow 3 \mu$

**Electric  
Dipole  
Moments**

★  
 $(g-2)_\mu$

**Improved  
Lattice  
Gauge Theory  
Calculations**

★  
 $\varepsilon'/\varepsilon$

$\Delta I = 1/2$  Rule,  
 $\Delta M_K$

**Neutrinos**

**Exciting Times are just  
ahead of us !!!**

**Exciting Times are just  
ahead of us !!!**

**Thank You !**

# Backup

# $\varepsilon'/\varepsilon$ strikes back (CP-Violation in $K_L \rightarrow \pi\pi$ )

New results on hadronic matrix elements of QCD penguin ( $B_6$ ) and electroweak penguin ( $B_8$ ) operators

Large N approach to QCD

:  $B_6 < B_8 < 1$



Upper Bound on  $\varepsilon'/\varepsilon$  in the Standard Model

AJB + Gérard (1507.06326)

Supported by Lattice QCD

:  $B_6 = 0.57 \pm 0.19$     $B_8 = 0.76 \pm 0.05$

RBC-UKQCD

Anatomy of  $\varepsilon'/\varepsilon$  in the Standard Model

: NLO

$(\varepsilon'/\varepsilon)_{SM} = (1.9 \pm 4.5) \cdot 10^{-4}$

AJB, Gorbahn, Jäger, Jamin (1507.06345)

$(\varepsilon'/\varepsilon)_{SM} = (6.0 \pm 2.4) \cdot 10^{-4}$  for  $B_6 = B_8 = 0.76$

$(\varepsilon'/\varepsilon)_{exp} = (16.6 \pm 2.3) \cdot 10^{-4}$

Possible New Physics

Implications for  $K \rightarrow \pi\nu\bar{\nu}$

Z' general (AJB, Buttazzo, Knecht, 1507.08672)

Littlest Higgs Model (Blanke, AJB, Recksiegel, 1507.06316)

331 Models (AJB, De Fazio, 1512.02869, 1604.02344)

New Strategy (AJB, 1601.00005)

Vector-like Quarks (Bobeth, AJB, Celis, Jung, 1609.04783)

Leptoquarks (Bobeth, AJB, 1712.01295)

More papers later

(1703.04753)

# General Z' at Work

Can solve anomalies in  $R_K, R_{K^*}, P_5'$   
(many papers)

Here :  $\varepsilon'/\varepsilon, K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}, \Delta M_K$

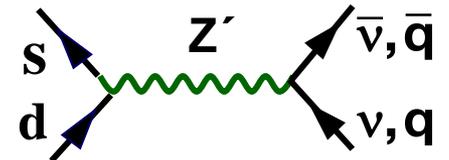


$Q_6, Q_6'$  – QCD Penguin operators

$Q_8, Q_8'$  – Electroweak Penguin operators

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_8 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A}$$



# Strategy (Z')

AJB (1601.00005)

$$\left(\varepsilon'/\varepsilon\right)^{\text{NP}} = \kappa_{\varepsilon'} \cdot 10^{-3}$$

$$0.5 \leq \kappa_{\varepsilon'} \leq 1.5$$

(Im)

$$\varepsilon_{\kappa}^{\text{NP}} = \kappa_{\varepsilon} \cdot 10^{-3}$$

$$0.1 \leq \kappa_{\varepsilon} \leq 0.4$$

(Im, Re)

$\varepsilon_{\kappa}$  more important than  $K_L \rightarrow \mu^+\mu^-$  in  $Z'$  models

Re and Im Parts:  $Z'$  Couplings

$\Delta_L^{\text{sd}}(Z')$ ,  $\Delta_R^{\text{sd}}(Z')$

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}, \quad K_L \rightarrow \pi^0 \nu \bar{\nu}, \quad K_L \rightarrow \mu^+ \mu^-, \quad \Delta M_K$$

(Re, Im)      (Im)      (Re)      (Im, Re)

# Basic Structure of NP Contributions

AJB (1601.00005)

$$\begin{array}{ll} (\varepsilon'/\varepsilon)^{\text{NP}} \rightarrow \text{Im} & \varepsilon_{\text{K}}^{\text{NP}} \rightarrow \text{Im} \cdot \text{Re} \\ (\kappa_{\varepsilon'} \geq 0.5) & (\kappa_{\varepsilon} \geq 0.1) \\ \Delta M_{\text{K}}^{\text{NP}} \sim \left[ (\text{Re})^2 - (\text{Im})^2 \right] \end{array}$$

**Dominance of  $Q_6 (Q_6')$   $\Rightarrow$   $\text{Im} \gg \text{Re} \Rightarrow \left\{ \Delta M_{\text{K}}^{\text{NP}} < 0 \right\}$**   
(large)

**Dominance of  $Q_8 (Q_8')$   $\Rightarrow$   $\text{Re} \gg \text{Im} \Rightarrow \left\{ \Delta M_{\text{K}}^{\text{NP}} > 0 \right\}$**   
(small)



**Distinction between  
these scenarios**

# Main Message



**Correlation between  $\varepsilon'/\varepsilon$  and  $K \rightarrow \pi\nu\bar{\nu}$  in  $Z'$  scenarios depends on whether QCP Penguin ( $Q_6$ ) or EWP ( $Q_8$ ) dominates NP in  $\varepsilon'/\varepsilon$**

**Effects in  $K \rightarrow \pi\nu\bar{\nu}$  much larger if  $Q_6$  dominates NP**

## ε'/ε within SM

$$\varepsilon'/\varepsilon \sim \left[ \frac{\text{Re } A_2}{\text{Re } A_0} \text{Im } C_6 \langle Q_6 \rangle_0 - \text{Im } C_8 \langle Q_8 \rangle_2 + \text{smaller contributions} \right]$$

$$\left\{ \frac{\text{Re } A_2}{\text{Re } A_0} \approx \frac{1}{22} \quad \frac{\text{Im } C_6}{\text{Im } C_8} \approx 90 \quad \frac{\langle Q_8 \rangle_2}{\langle Q_6 \rangle_0} \approx 2 \right\} \Rightarrow \text{strong cancellations}$$

## ε'/ε beyond SM

( $Q_6, Q_8, Q'_6, Q'_8$ )

**1.** Generally  $Q_8$  wins over  $Q_6$  because  $\left( \frac{\text{Im } C_6}{\text{Im } C_8} \right)^{\text{NP}} \approx 0(1)$  but can provide  $\Delta(\varepsilon'/\varepsilon) > 0$

**2.**  $Q_6$  wins over  $Q_8$  in the presence of a flavour symmetry forbidding  $Q_8$

**3.** Chromomagnetic operators

# Stars of KAON Flavour Physics

$$\varepsilon_K, \Delta M_K$$

$$\varepsilon'/\varepsilon$$

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

$$K_{L,S} \rightarrow \mu^+ \mu^-$$

$$K_L \rightarrow \pi^0 l^+ l^-$$

$\Delta I = 1/2$  Rule

They all can give some information about very short distance scales but to identify new physics, correlations with  $B_{s,d}$  and D observables, EDMs, Lepton physics crucial

In particular if we want to reach Zeptouniverse without any direct hints from the LHC

**AJB:**

# **Strong Belief in Great Future of Kaon Flavour Physics**

**The Renaissance  
of Kaon Flavour Physics**

**1606.06735**

**The Revival  
of Kaon Flavour Physics**

**1609.05711**

**Kaon Flavour Physics  
strikes back**

**1611.06206**

**The Return  
of Kaon Flavour Physics**

**1805.11096**

# Meson Evolution $B_i(\Lambda)$ in the Chiral Limit

$$\Lambda \approx 0.65 \pm 0.05 \text{ GeV}$$

(Pattern consistent with quark-gluon evolution)

$$B_2(\Lambda) = 1.2 \left[ 1 - \frac{8}{3} \frac{\Lambda^2}{(4\pi F_K)^2} \right] \quad \text{(strong suppression)}$$

$$B_3(\Lambda) = 3.0 \left[ 1 - \frac{16}{3} \frac{\Lambda^2}{(4\pi F_K)^2} \right] \quad \text{(even stronger suppression)}$$

$$B_4(\Lambda) = 1.0 \left[ 1 - \frac{4}{3} \frac{\Lambda^2}{(4\pi F_K)^2} \right] \quad \text{(moderate suppression)}$$

$$B_5(\Lambda) = 0.23 \left[ 1 + 4 \frac{\Lambda^2}{(4\pi F_K)^2} \right] \quad \text{(strong enhancement)}$$

$\Lambda =$  Cut-off, physical separates the non-factorizable meson evolution from the quark-gluon evolution.

# Main Messages on LQs in $\varepsilon'/\varepsilon$ and rare K Decays

If improved lattice calculations will confirm the  $\varepsilon'/\varepsilon$  anomaly at the level  $(\varepsilon'/\varepsilon)_{\text{NP}} \geq 5 \cdot 10^{-4}$  LQs are likely not responsible for it.

But if  $\varepsilon'/\varepsilon$  anomaly disappears large NP effects from LQs in rare K decays still possible.

(Need non-zero couplings to first generation!!)

(Need imaginary couplings!)

(Need both left-handed and right-handed couplings!)

In contrast to most explanations of B-anomalies

# 2016 Standard Model Results

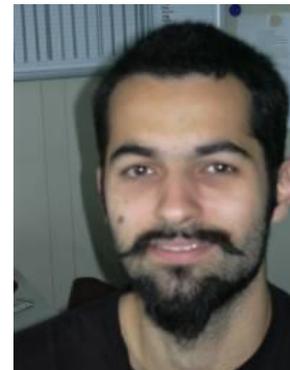
Teppei Kitahara



Ulrich Nierste



Paul Tremper



NLO

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\text{SM}} = (1 \pm 5) \cdot 10^{-4}$$

1607.06727

2018

First NNLO Result for  $(\epsilon'/\epsilon)_{\text{SM}}$

Maria Cerda-Sevilla



Martin Gorbahn



Sebastian Jäger



Ahmet Kokulu



New  
IAS  
Postdoc

New  
TUM  
Postdoc

# All Dimension 6 Operators beyond SM

(lineary independent)

SM : 7

BSM : 7' obtained from SM through  $L \leftrightarrow R$  (primed operators)

QCD \* QED  
invariant

{ 13 operators unrelated to SM operators  
13' primed operators  $L \leftrightarrow R$  }

33

new operators with 33 new Wilson coefficients  
calculable in a given NP model

SMEFT

$(13 + 13') \rightarrow (7 + 7')$  operators unrelated to SM

Invariance  
under SM  
gauge group

→

21

new Wilson coefficients

# How many new hadronic matrix elements have to be calculated (contributing to $A_0$ and $A_2$ )?

QCD \* QED  
invariant

In principle :  $2 \times 33 = 66$  matrix elements

But in reality :  $2 \times 13 = 26$  (40 matrix elements obtained from  $L \leftrightarrow R$  by just reversing the sign)

SMEFT

:  $2 \times 7 = 14$

Message

: More work for Lattice QCD (26 or 14)



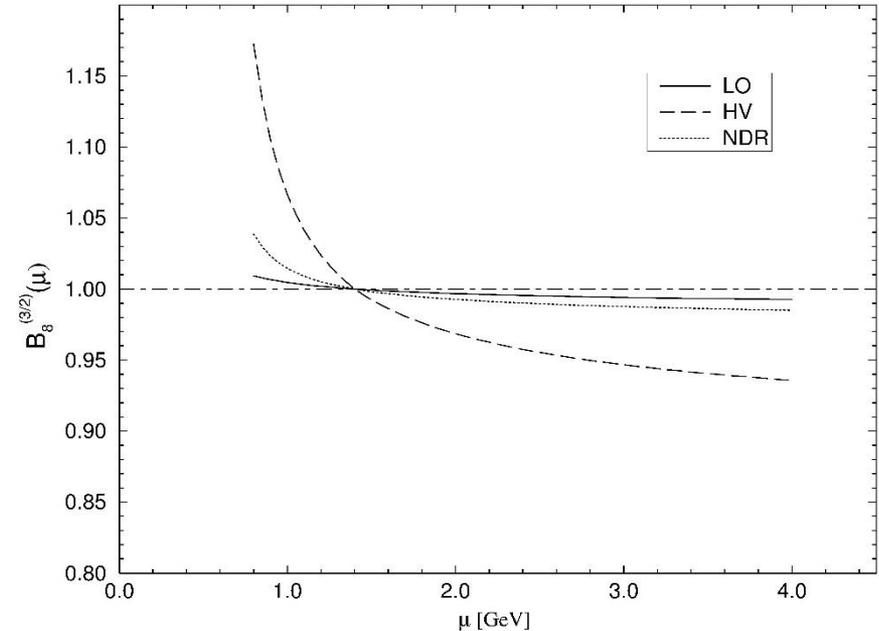
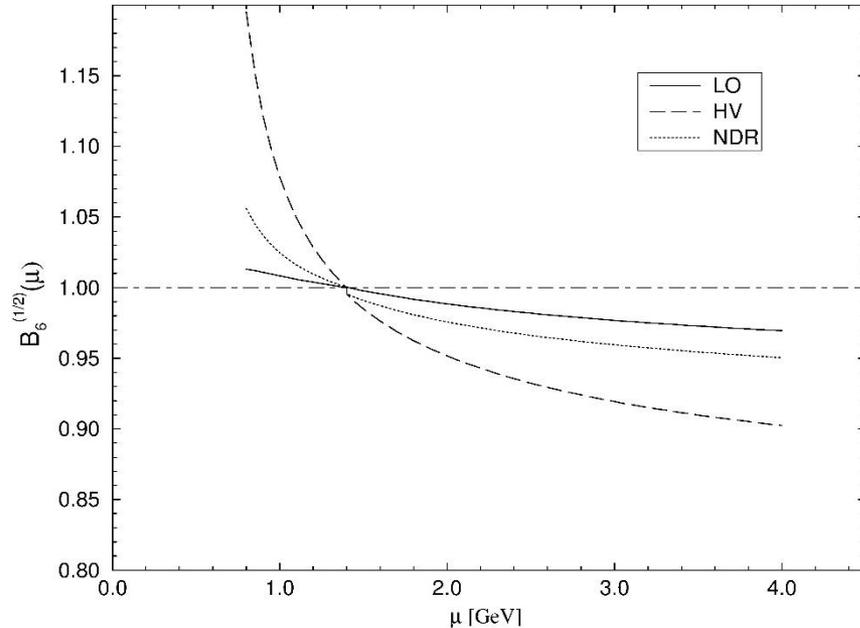
No more work for DQCD :

Aebischer, AJB, Gérard  
(1807.xxxxx)

all 26 matrix elements recently  
calculated including meson evolution  
in the chiral limit

# $B_6$ and $B_8$ in the Perturbative Regime (1993!)

AJB, Jamin, Lautenbacher, (9303284)

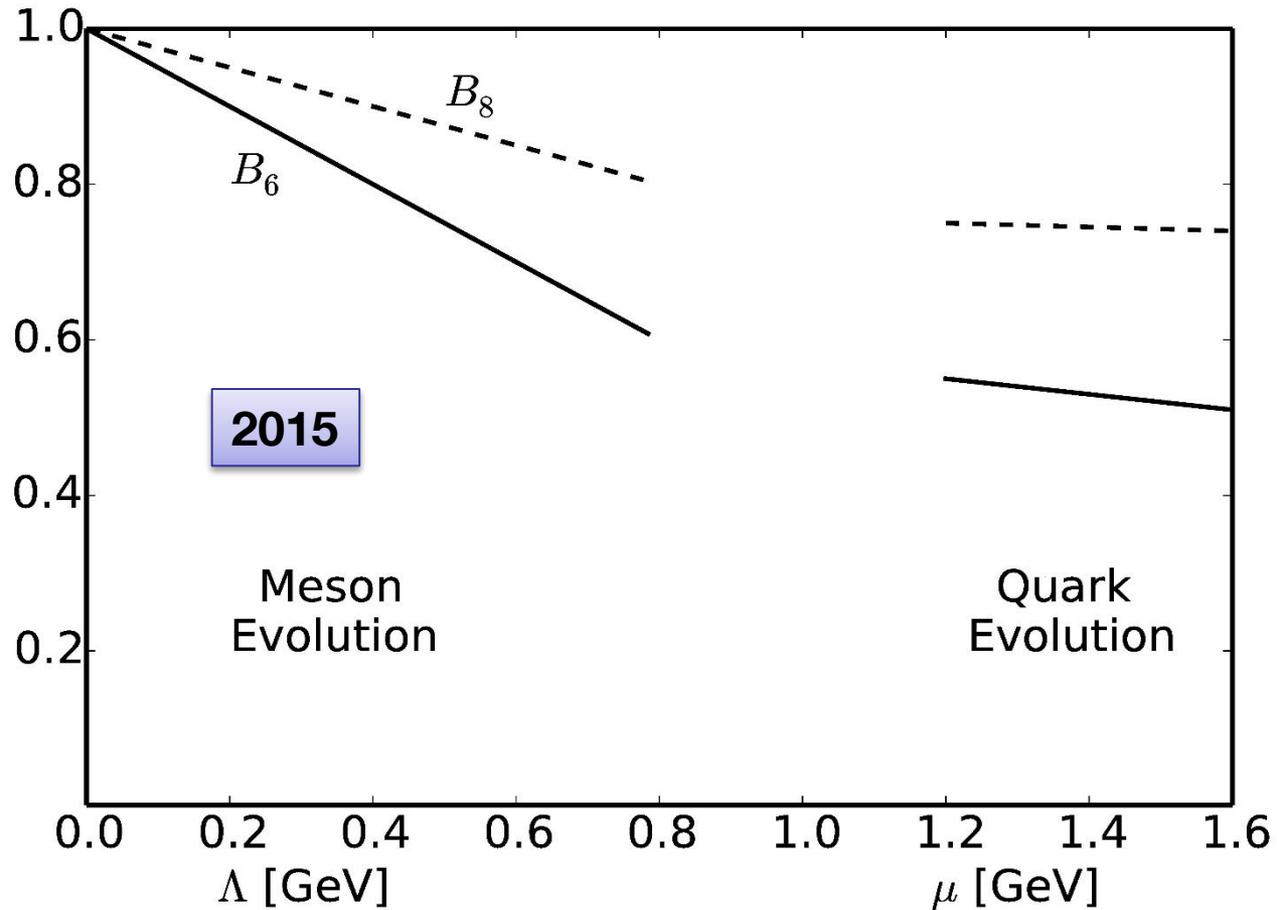


**$B_6$  and  $B_8$  decrease with increasing  $\mu$ !**

**Note  $B_6 = B_8 = 1$  at  $\mu = m_c$  wrong!!**

# Scale Dependence of $B_6$ and $B_8$

AjB+ Gerard (1507.06326)





**Tensor-Tensor and scalar-scalar operators could be behind the  $\varepsilon'/\varepsilon$  anomaly**



**The most likely new particles behind These operators are heavy scalars (colourless + coloured)**

**But new heavy gauge bosons,  $Z'$ ,  $W'$  Could also be important**

**BSM Anatomy : Aebischer, Bobeth, AJB, Straub  
(1808.xxxxx)**

# Forbidden by SMEFT but allowed by QCD\*QED

ABG (1807.01709)



$$\mathbf{B}_1 = (\bar{s}\mathbf{P}_R d)(\bar{u}\mathbf{P}_L u)$$

$$\mathbf{C}_1 = (\bar{s}\gamma^\mu \mathbf{P}_L u)(\bar{u}\gamma_\mu \mathbf{P}_R d)$$

$$\mathbf{D}_3 = (\bar{s}\mathbf{P}_L d)(\bar{d}\mathbf{P}_L d)$$



$$\mathbf{D}_3^* = -(\bar{s}\sigma^{\mu\nu} \mathbf{P}_L d)(\bar{d}\sigma_{\mu\nu} \mathbf{P}_L d)$$

$$\mathbf{D}_4 = (\bar{s}\mathbf{P}_L d)(\bar{s}\mathbf{P}_L s)$$

$$\mathbf{D}_4^* = -(\bar{s}\sigma^{\mu\nu} \mathbf{P}_L d)(\bar{s}\sigma_{\mu\nu} \mathbf{P}_L s)$$



**Largest  
Matrix Elements**