

*Composite operator and condensate in the
 $SU(N)$ Yang-Mills theory with $U(N - 1)$
stability group*

Matthias Warschinke¹

in collaboration with

R. Matsudo², S. Nishino¹, T. Shinohara¹, K.-I. Kondo^{1,2}

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¹Department of Physics, Graduate School of Science, Chiba University

²Department of Physics, Graduate School of Science and Engineering

Motivation

- dual superconductivity picture is a promising candidate for explaining confinement
- low energy region of QCD as “Abelian” theory + monopoles
- how do the “non-Abelian” d.o.f decouple? acquire a IR mass!
- evidence on the lattice (e.g. exponential fall-off of the corresponding gluon propagator)
- analytical approach: mass term through condensation of dimension-two operator
- mass term would furthermore imply e.g. stabilization of Savvidy vacuum, quark confinement at low temperatures...

1 Reformulated Yang-Mills Theory

- reformulation of $G = SU(N)$ Yang-Mills theory based on Cho-Faddeev-Niemi decomposition with respect to the stability group H

$$\mathcal{A}_\mu = \mathcal{X}_\mu + \mathcal{V}_\mu \in \text{Lie}(G/H) \oplus \text{Lie}(H) \quad (1)$$

- the residual field \mathcal{V}_μ transforms inhomogeneously and is identified as the IR dominant mode
- the remaining or coset field \mathcal{X}_μ transforms homogeneously and decouples in the IR
- decomposition defined in terms of the normalized color field \mathfrak{n}
- additional symmetry under rotations along \mathfrak{n}
- novel viewpoint: non-linear change of variables [*Kondo et al. '08*]

$$\{\mathcal{A}_\mu\} \implies \{\mathcal{X}_\mu; \mathcal{V}_\mu; \mathfrak{n}\} \quad (2)$$

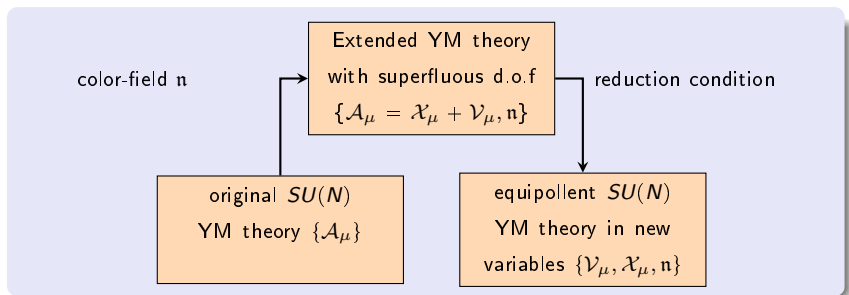
- problem: mismatch between degrees of freedom
- removal of the additional d.o.f through *reduction condition*

$$[\mathfrak{n}, \mathcal{D}_\mu[\mathcal{A}]\mathcal{D}^\mu[\mathcal{A}]\mathfrak{n}] = 0 \iff \mathcal{D}_\mu[\mathcal{V}]\mathcal{X}^\mu = 0 \quad (3)$$

- in addition, this constraint reduces the enlarged gauge symmetry to the original $SU(N)$ symmetry
- incorporate reduction condition in a gauge fixing manner in the path integral

Reformulated $G = SU(N)$ Yang-Mills theory

[Kondo et al. '08]



reduction condition:

$$\mathcal{D}_\mu[\mathcal{V}]\mathcal{X}^\mu = 0 \quad (4)$$

- we adopt the “minimal” choice $H = U(N-1)$, which requires the introduction of only a single color field (other popular choice: $H = U(1)^{N-1}$ related to the maximal Abelian gauge)
- difficult to handle \mathfrak{n} analytically \implies fixed choice: $\mathfrak{n} = T^{N^2-1}$ (last Cartan generator)

consequences of this symmetry breaking

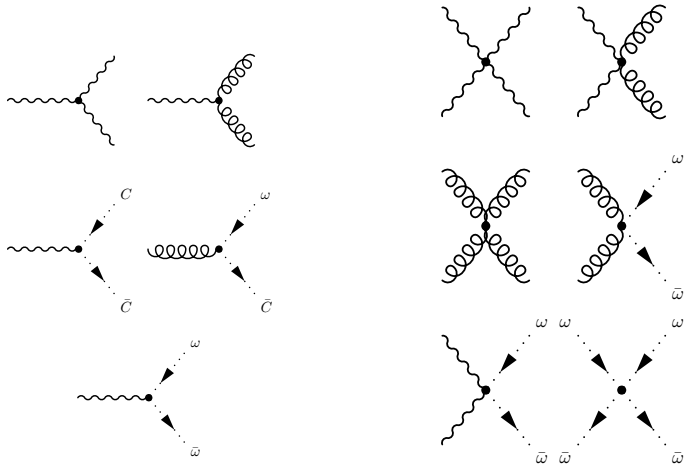
- reduction condition appears as a “gauge fixing” term for the coset gluon $\mathcal{X}_\mu \implies$ breaking $SU(N) \rightarrow U(N-1)$
- despite the symmetry breaking: at least a (on-shell) BRST-invariant dimension-2 operator can be introduced

$$\mathcal{O} = \text{Tr}_{G/H} (\mathcal{X}_\mu \mathcal{X}^\mu - 2i\xi \mathcal{C} \bar{\mathcal{C}}) \quad (5)$$

$$\begin{aligned} \mathcal{L}_{YM} + i\delta_B \bar{\delta}_B \text{Tr}_{G/H} (\mathcal{X}_\mu \mathcal{X}^\mu - i\xi \mathcal{C} \bar{\mathcal{C}}) - i\delta_B \text{Tr}_{U(1)} [\bar{\mathcal{C}} (2\partial_\mu \mathcal{V}^\mu + \alpha \mathcal{N})] \\ - i\delta_B \text{Tr}_{SU(N-1)} [\bar{\mathcal{C}} (2\partial_\mu \mathcal{V}^\mu + \lambda \mathcal{N})] \end{aligned}$$

2 One-loop analysis and multiplicative renormalizability

Based on the set of vertices...



...the one-loop RG functions were obtained ✓

For $N = 2$ (MAG): agreement with [e.g. *Shinohara et al. '03*]

$$\begin{aligned}
 \gamma_X &= \frac{g^2}{(4\pi)^2} \frac{N}{2} \left(\frac{17}{6} - \frac{\xi}{2} - \frac{\alpha + (N-2)\lambda}{N-1} \right) & \beta_g &= -\frac{g^3}{(4\pi)^2} \frac{11}{3} N, \\
 \gamma_V &= \frac{g^2}{(4\pi)^2} \left(\frac{13N+9}{6} - \frac{\lambda}{2} (N-1) \right), & \tilde{\gamma}_V &= \frac{g^2}{(4\pi)^2} \frac{11}{3} N, \\
 \gamma_\omega = \gamma_{\tilde{\omega}} &= \frac{g^2}{(4\pi)^2} \frac{N}{2} \left(3 - \frac{\alpha + (N-2)\lambda}{N-1} \right), & \tilde{\gamma}_C &= -\tilde{\gamma}_{\bar{C}} = \frac{g^2}{(4\pi)^2} \frac{N}{2} (3 + \xi), \\
 \gamma_C &= \frac{g^2}{(4\pi)^2} \left[\frac{N}{2} (3 + \xi) - \lambda(N-1) \right], & \gamma_{\bar{C}} &= -\frac{g^2}{(4\pi)^2} \frac{1}{2} [N\xi + 3 - \lambda(N-1)], \\
 \gamma_\lambda &= \frac{g^2}{(4\pi)^2} \left(\frac{13N+9}{3} - \lambda(N-1) \right), & \gamma_\alpha &= \frac{g^2}{(4\pi)^2} \frac{22}{3} N,
 \end{aligned}$$

$$\mu \partial_\mu \xi = \xi \gamma_\xi = \frac{g^2}{(4\pi)^2} \left(\frac{4}{3} \xi - \xi^2 - 3 \right) N \quad (6)$$

- problematic: “gauge-fixing” parameter corresponding to the reduction condition has no fixed point

Multiplicative renormalizability of the composite operator

- mixing with other operators of same quantum number has to be taken into account

$$\begin{pmatrix} \left[\frac{1}{2} X_\mu^a X_a^\mu \right] \\ \left[\frac{1}{2} V_\mu^j V_j^\mu \right] \\ [i\omega^a \bar{\omega}^a] \\ [iC^j \bar{C}^j] \\ \left[\frac{1}{2} V_\mu^\gamma V_\mu^\gamma \right] \\ [iC^\gamma \bar{C}^\gamma] \end{pmatrix} = \begin{pmatrix} 1 - Z_1^{(1)} & 0 & -Z_3^{(1)} & 0 & 0 & 0 \\ -Z_7^{(1)} & 1 - Z_8^{(1)} & -Z_9^{(1)} & 0 & 0 & 0 \\ -Z_{13}^{(1)} & 0 & 1 - Z_{15}^{(1)} & 0 & 0 & 0 \\ 0 & -Z_{20}^{(1)} & 0 & 1 & 0 & 0 \\ -Z_{25}^{(1)} & 0 & -Z_{27}^{(1)} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \left[\frac{1}{2} X_\mu^a X_a^\mu \right]_R \\ \left[\frac{1}{2} V_\mu^j V_j^\mu \right]_R \\ [i\omega^a \bar{\omega}^a]_R \\ [iC^j \bar{C}^j]_R \\ \left[\frac{1}{2} V_\mu^\gamma V_\mu^\gamma \right]_R \\ [iC^\gamma \bar{C}^\gamma]_R \end{pmatrix}$$

The requirement of multiplicative renormalizability translates into

$$-Z_1^{(1)} + Z_X^{(1)} + \xi Z_{13}^{(1)} \stackrel{!}{=} Z_\xi^{(1)} - Z_{15}^{(1)} + Z_\omega^{(1)} + \frac{1}{\xi} Z_3^{(1)} \quad (7)$$

Indeed, we find

$$-Z_1^{(1)} + Z_X^{(1)} + \xi Z_{13}^{(1)} = \frac{g^2 \mu^{-2\epsilon}}{(4\pi)^2 \epsilon} \left[\frac{N}{6} (13 - 3\xi) \right] \quad (8)$$

$$Z_\xi^{(1)} - Z_{15}^{(1)} + Z_\omega^{(1)} + \frac{1}{\xi} Z_3^{(1)} = \frac{g^2 \mu^{-2\epsilon}}{(4\pi)^2 \epsilon} \left[\frac{N}{6} (13 - 3\xi) \right] \quad (9)$$

The composite operator is multiplicatively renormalizable ✓

$$\gamma_O = \frac{g^2}{(4\pi)^2} \frac{N}{3} (13 - 3\xi) \quad (10)$$

For $N=2$ (MAG): agreement with [\[Dudal et al. '03\]](#)

3 LCO formalism and existence of the condensate

- developed by [Knecht, Verschelde; '95, '01] to tame new divergences coming from composite operator source term $\sim J\mathcal{O}$
- starting point: add additional piece to bare Lagrangian

$$\mathcal{L}_{LCO} = \frac{1}{2}(\kappa + \delta\kappa)J^2 \quad (11)$$

- requiring RG invariance of this additional term and assuming that $\kappa[g^2(\mu), \xi(\mu)]$ yields an ODE for the parameter κ^3 ,

$$\left[2\epsilon + 2\gamma_{\mathcal{O}} - \beta_{g^2} \frac{\partial}{\partial g^2} - \xi\gamma_{\xi} \frac{\partial}{\partial \xi} \right] (\kappa + \delta\kappa) = 0 \quad (12)$$

- this can, in principle, be solved order by order

$$\kappa(g^2, \xi) = \frac{\kappa_0}{g^2} + \hbar\kappa_1(\xi) + \hbar^2 g^2 \kappa_2(\xi) + \dots \quad (13)$$

³From now on, $\alpha = \lambda = 0$ is adopted as they are fixed points.

- however, this implies that in order to obtain the n - loop part of κ , we need all other quantities to $n + 1$ - loop order, e.g.

$$[\xi\gamma_{\xi,1}\partial_{\xi} - 2\gamma_{\mathcal{O},1} - \beta_{g^2,1}] \kappa_0 = 2\delta\kappa_0 \quad (14)$$

- $\delta\kappa_0$ is the coefficient of the one-loop divergent part quadratic in the sources, coming from

$$-\frac{i}{2}d_{G/H} \text{Tr} \log (-p^2 g_{\mu\nu} + (1 - \xi^{-1})p_{\mu}p_{\nu} + g_{\mu\nu}J) + id_{G/H} \text{Tr} \log(-p^2 + \xi J) \quad (15)$$

$$\implies \frac{\delta\kappa_0}{\epsilon} = \frac{d_{G/H}}{2} \frac{3 - \xi^2}{(4\pi)^2 \epsilon} \quad (16)$$

- the solution to the ODE for the tree-level part of κ is then obtained

$$\kappa_0 = \frac{2(N-1)}{N}\xi + C(4\xi - 3\xi^2 - 9) \quad (17)$$

- a choice for the integration constant C is motivated later
- to discuss the existence of the condensate, the tree-level part will be enough
- one more problem to be tackled: the quadratic source term κJ^2 spoils the usual construction of the generating functional

- removal through Hubbard-Stratonovich transformation

$$1 = \int d\sigma \text{Exp} \left[-i \frac{1}{2g^2\kappa} (\sigma - g\mathcal{O} - g\kappa J)^2 \right] \quad (18)$$

- this yields the transition

$$\begin{aligned} \mathcal{L} + \mathcal{O}J + \frac{1}{2}\kappa J^2 + \mathcal{L}_{counter} \\ \implies \mathcal{L} - \frac{\sigma^2}{2g^2\kappa} + \frac{1}{\kappa} \frac{\sigma}{g} \mathcal{O} - \frac{1}{2\kappa} \mathcal{O}^2 + \frac{\sigma}{g} J + \mathcal{L}_{counter} \end{aligned} \quad (19)$$

- nontrivial vev of σ induces tree-level mass term for the coset gluon

$$m_X^2 = \frac{g\langle\sigma\rangle}{\kappa_0} \quad (20)$$

$$\kappa_0 = \frac{2(N-1)}{N}\xi + C(4\xi - 3\xi^2 - 9)$$

- final step: effective potential for σ

$$V(\sigma) = \frac{\sigma^2}{2\kappa_0} - \frac{\kappa_1}{2\kappa_0^2}g^2\sigma^2 - \frac{3}{64\pi^2}2(N-1)\frac{g^2\sigma^2}{\kappa_0^2}\left(\frac{5}{6} - \log\left[\frac{g\sigma}{\kappa_0\bar{\mu}^2}\right]\right) + \frac{1}{64\pi^2}2(N-1)\frac{\xi^2g^2\sigma^2}{\kappa_0^2}\left(\frac{3}{2} - \log\left[\frac{\xi g\sigma}{\kappa_0\bar{\mu}^2}\right]\right) \quad (21)$$

- the **tree part** should remain positive, at least in the vicinity of $\xi = 0 \implies$ restricts the choice of the integration constant in κ_0
- for later convenience:

$$C = -\frac{1}{11}\frac{N-1}{N} \quad (22)$$

- indeed, from the one-loop effective potential one finds a nontrivial extremum defined by

$$m_X^2 = \frac{g\sigma_*}{\kappa_0} = \bar{\mu}^2 \text{Exp} \left[\frac{H_1}{g^2} + H_2 \right] \quad (23)$$

$$V(\sigma_*) = -(3 - \xi^2) \frac{2(N-1)}{128\pi^2} m_X^4 \quad (24)$$

with

$$H_1(\xi, \kappa_0) = -\frac{1}{(3 - \xi^2)} \frac{32\pi^2}{2(N-1)} \kappa_0$$

$$H_2(\xi, \kappa_1) = \frac{1}{(3 - \xi^2)} \left(\frac{32\pi^2}{2(N-1)} \kappa_1 + 1 + \frac{1}{2} \xi^2 \log \xi^2 - \xi^2 \right)$$

For $N=2$ (MAG): agreement with [\[Dudal et al. '03\]](#)

$$m_X^2 = \bar{\mu}^2 \text{Exp} \left[\frac{H_1}{g^2} + H_2 \right]$$

$$H_1(\xi, \kappa_0) = -\frac{1}{(3-\xi^2)} \frac{32\pi^2}{2(N-1)} \kappa_0$$

- to obtain the correct UV limit $m_X^2 \xrightarrow{g^2 \rightarrow 0} 0$, H_1 must be negative
- since $\kappa_0 > 0$ this restricts $\xi^2 < 3$, which is consistent with our interpretation of the reduction condition, namely setting $\xi = 0$ in the end
- then, in particular $V(\sigma_*) = -(3 - \xi^2) \frac{2(N-1)}{128\pi^2} m_X^4 < 0$
- moreover, for $\xi = 0$ the function H_2 becomes an irrelevant constant and the gluon mass can be written as

$$m_X^2 = \text{const} \times \Lambda_{QCD}^2; \quad \Lambda_{QCD} = \bar{\mu} \text{Exp} \left[- \int^g \frac{dg'}{\beta_g(g')} \right]$$

The condensate shows up at the confinement scale ✓

4 Conclusion and Outlook

- one-loop analysis of $SU(N)$ Yang-Mills theory with $U(N-1)$ stability group has been performed
- in particular, the multiplicative renormalizability of a certain BRST invariant dimension-2 composite operator has been shown
- based on these results, the existence of the corresponding condensate was discussed within the LCO formalism
- for the “physical” choice $\xi = 0$, the condensate shows up at the QCD scale, generating a tree level mass for the coset gluon \mathcal{X}_μ
- this can be seen as evidence for the “Abelian dominance” as one of the key properties of the dual superconductivity picture

However,

- the treatment of the “gauge parameter” ξ is still unsatisfying, as we set it to zero despite having no fixed point
- the question whether the coset gluon can obtain a IR mass or not is generically nonperturbative
- indeed, the problem is currently reinvestigated within the framework of the functional renormalization group

- calculating the flow according to the Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_t R_k \right], \quad t = \log(k/\Lambda). \quad (25)$$

- naive input for the flowing effective average action

$$\begin{aligned} \Gamma_k = & \Gamma_{YM} + \Gamma_{GF+FP}^{res} + \Gamma_{GF+FP}^{red} + U_k[\sigma] + \frac{Z_{\sigma,k}}{2} \partial_\mu \sigma \partial_\mu \sigma \\ & + \frac{h_{X,k}}{2} \sigma X_\mu^a X_\mu^a - i h_{\omega,k} \xi \sigma \omega^a \bar{\omega}^a + \frac{\ell_k}{8} X_\mu^a X_\mu^a X_\nu^b X_\nu^b - i \xi \frac{n_k}{2} X_\mu^a X_\mu^a \omega^b \bar{\omega}^b - \frac{y_k}{2} \xi^2 \bar{\omega}^a \omega^a \bar{\omega}^b \omega^b \end{aligned} \quad (26)$$

- does $U_k[\sigma]$ develop a non-trivial minimum as well?

Thank you for your attention.