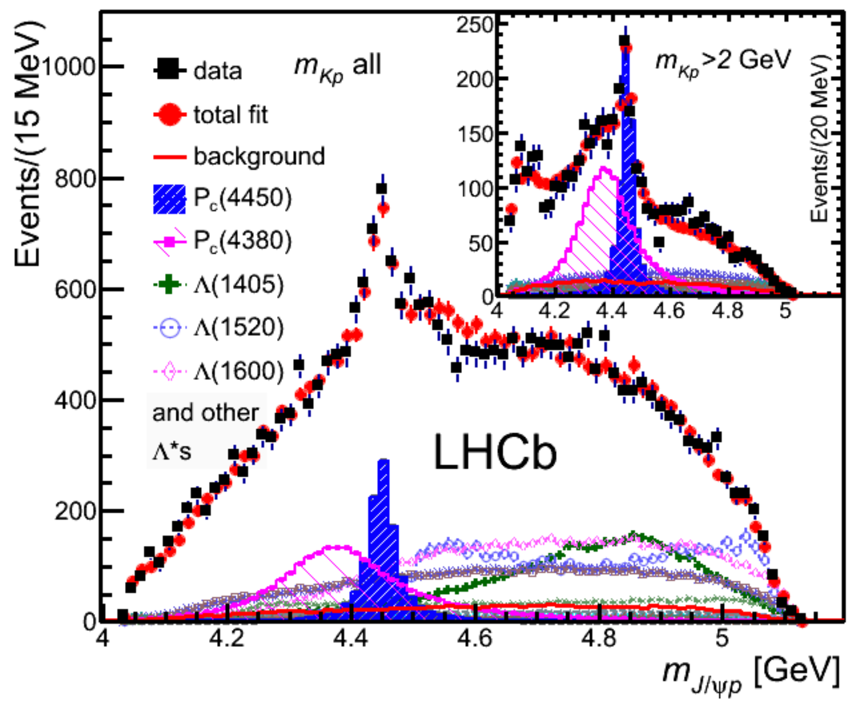
Near Threshold Exotic Hadrons with Two Heavy Quarks*



* Tetraquarks or pentaquarks with Either QQ or $Q\bar{Q}$

TDC
Yiming Cai
(In preparation)

An overview

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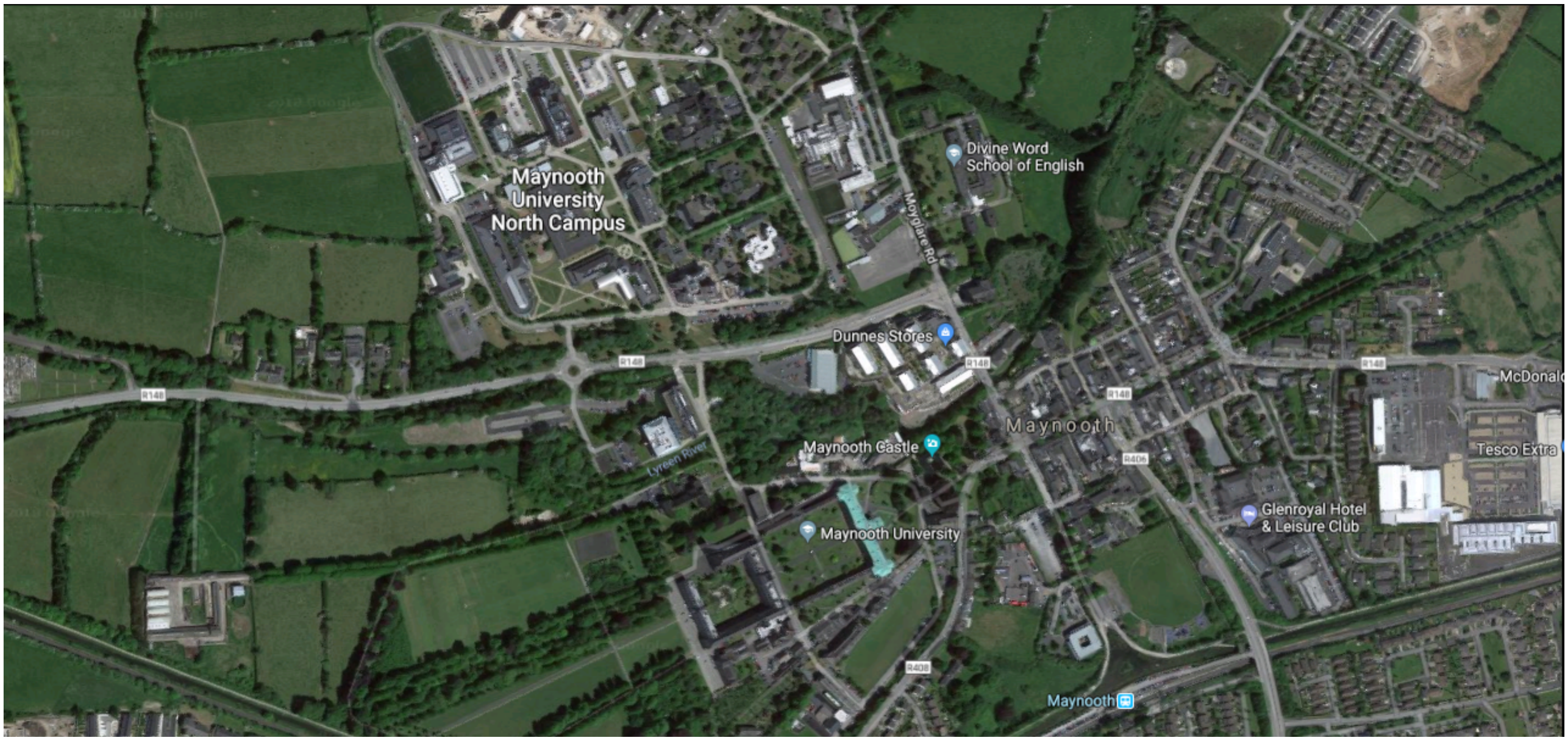

Maynooth University North Campus
Divine Word School of English
Moyglare Rd
Dunnes Stores
R148
Maynooth
Maynooth Castle
Lyreen River
R406
McDonald
Tesco Extra
Glenroyal Hotel & Leisure Club
Maynooth University
R408
Maynooth

An overview

- Introduction: QCD vs the Quark Model
  - What exotics teach us
  - The value of generic arguments for the existence of exotics in limits of QCD
    - Existence of narrow hybrids at large Nc, Tetraquarks at large Nc (AS), QQ tetraquarks at large m_Q.

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  - The value of generic arguments for the existence of exotics in limits of QCD
    - Existence of narrow hybrids at large Nc, Tetraquarks at large Nc (AS), QQ tetraquarks at large m_Q.

- It is shown in a model independent way that parametrically narrow doubly heavy (QQ or $Q\overline{Q}$) tetraquarks and pentaquarks exist in the heavy quark limit of QCD; some of these are parametrically close to threshold.

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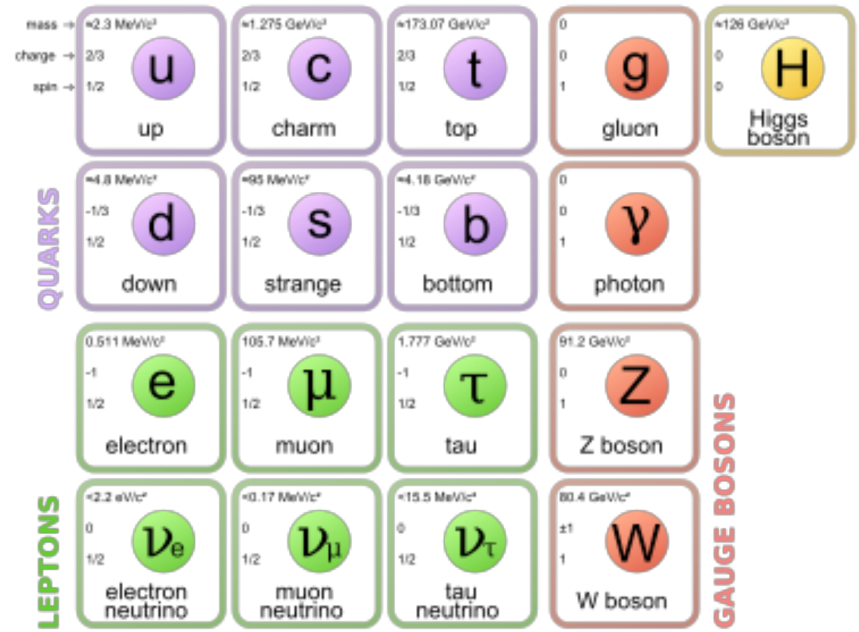
All these meanings are fundamentally different

In what sense is the quark model a “model”?



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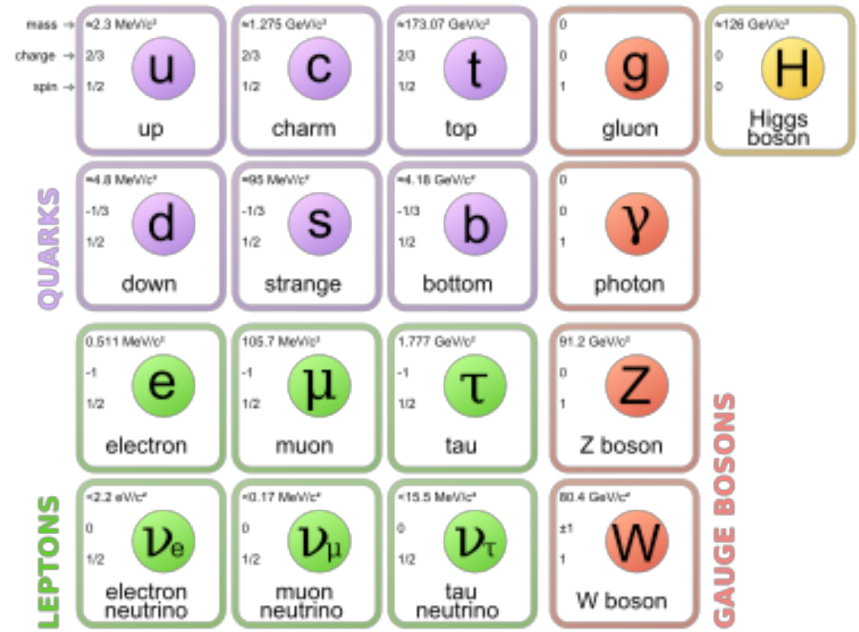

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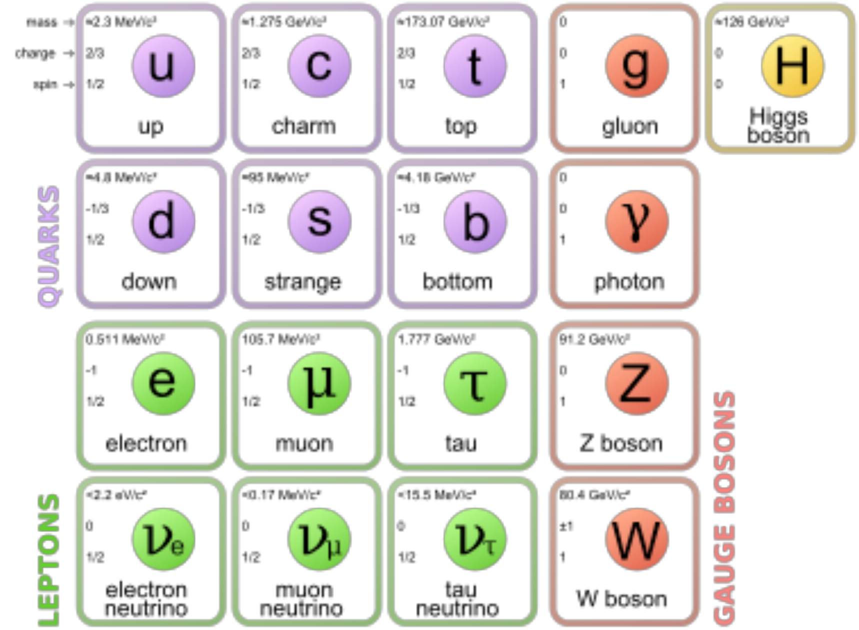

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- Unlike a fashion model, it is not an embodiment of elegance.

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- They are important in that they help clarify what QCD has and the quark model does not.
- There are three types:
    - Cryptoexotics. States which by their quantum numbers can be made in the simple quark model but which dynamically are dominated by components which are not of the quark model type. There are intrinsic ambiguities in identifying these
    - Quantum number exotics. States which by their quantum numbers **cannot** be made in the quark model. (eg. an isospin 2 meson)
    - Heavy quark number exotics. Formally cryptoexotic states that become quantum number exotic if one assumes the number of heavy quarks and antiquarks are separately conserved—which to good approximation they are. (Eg. isospin one states containing an additional charm-anticharm pair.)

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- With the advent of “new” heavy flavor spectroscopy of states containing a charm-anticharm pair at Belle, BaBar and LHCb (X,Y,Z mesons and P_c) there is increasingly compelling evidence for exotics. In particular, there is very strong evidence that the Z mesons and P_c which appear to be clear examples of heavy quark number exotics), while others are cryptoexotic.
  - Many of these states appear to be very close to thresholds for break up into two hadrons—one containing a charmed quark and the other and charmed antiquark.

Theory Situation

- It is interesting to find limits of QCD for which one can show in a model independent way exotics of some type will exist as parametically narrow (or strong-interaction-stable states). Such examples illustrate unambiguously the limitation of the naïve quark model to fully capture hadronic physics in QCD.

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  - The large N_c Limit: Glueballs exist as parametrically narrow states ($\Gamma \sim N_c^{-2}$) that decoupled (mixing angle $\sim N_c^{-1/2}$) from ordinary mesons. (G. 't Hooft 1973, E. Witten 1979)

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  - Heavy quark limit: Doubly heavy (QQ) tetraquarks quarks exist as strong-interaction stable states mesons. (A. V. Manohar and M. B. Wise 1992)

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It is instructive to see if one can deduce the existence of such states directly from QCD in a model-independent way—albeit in a limit.

Present analysis

Interesting results have been obtained, but the analysis is not fully complete:

- The formulation as it stands is highly technical and not totally transparent—it should be possible to simplify.
- Estimates for scaling of resonant widths are preliminary. Argument unambiguously valid for tetraquarks with two heavy quarks or pentaquaks with a heavy quark and antiquark but are upper bounds that may be reduced. Some subtleties remain with argument for tetraquarks with a heavy quark and antiquark or a pentaquark with two heavies. Will neglect these in this talk.


PREVIEWS
OF
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  - But does depend on an *extreme* heavy quark limit
- Key result: in the extreme heavy quark limit, *many* parametrically narrow exotics exist in all doubly heavy channels in which heavy hadrons attract at long distances including some that are parametrically close to threshold.

Principal Result: Scaling Behavior

Two scale problem:

m_Q (heavy quark mass)

Λ (typical hadronic scales including m_π)

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For any attractive doubly heavy channel :

The typical binding energy relative to threshold to break into two heavy mesons: $B \sim \Lambda$

Number of exotic resonances in channel: $N \sim (m_Q / \Lambda)^{1/2}$

Typical level spacing: $\Delta E \sim (\Lambda^3 / m_Q)^{1/2}$

There exist states parametrically close to threshold for dissociation into two heavy hadrons with binding energy $B \sim \Lambda^{2-\varepsilon} / m_Q^{1-\varepsilon}$ for $1 \geq \varepsilon > 0.$

Typical level spacing: $\Delta E \sim (\Lambda^{4-\varepsilon} / m_Q^{2-\varepsilon})^{1/2}$

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Typical resonance width: $\Gamma \sim (\Lambda^3 / m_Q)^{1/2} \sim \Delta E\, (\Lambda / m_Q)^{1/2}$

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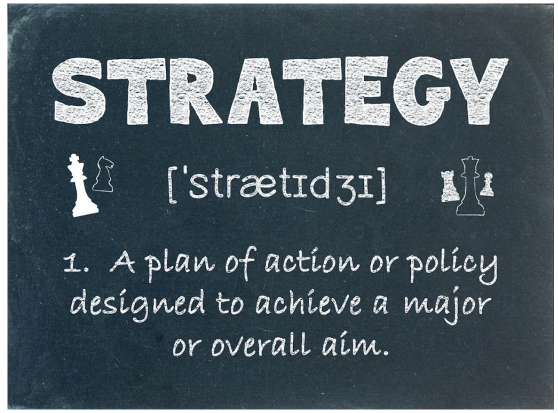
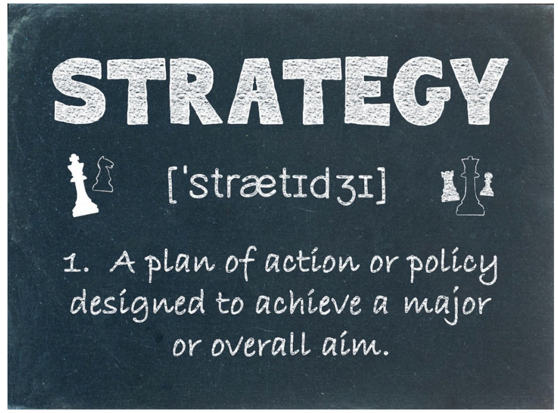
These widths are "parametrically narrow" in the sense that they are parametrically much smaller than both the level spacing and the distance to threshold. They unambiguously correspond to resonances—at least in a world of sufficiently heavy quark masses.


STRATEGY
[ˈstrætɪdʒɪ]
1. A plan of action or policy designed to achieve a major or overall aim.

Strategy to derive scaling rules:

- Work in a regime where heavy quarks are sufficiently heavy number of heavy quarks & antiquarks are separately conserved.



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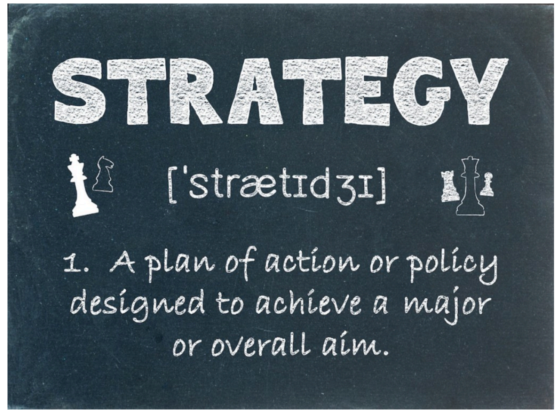
- Start from a physical picture in which a system with fixed tetraquark quantum numbers is described in a Schrödinger equation as a bound state of two heavy mesons.
  - Find the properties of the optimal states in QCD corresponding to such a description.
  - Note that long distance interaction is one-pion-exchange
  - Analyze the Schrödinger equation semi-classically to find properties of bound states.



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  - Analyze the Schrödinger Equation semi-classically to find properties of bound states.
- Use a Hamiltonian approach in Coloumb gauge.
- Estimate widths that are due to exclude parts of QCD.

Some ingredients in the analysis

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Large heavy quark mass: $M_Q >> \Lambda$

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Scale Separation: $p_Q \sim (\Lambda M_Q)^{1/2}$, $v_Q \sim (\Lambda/M_Q)^{1/2}$

- *Justifies semi-classical (WKB)approximation.*
  Eg. number of bound states from E to E+ΔE in Schrödinger description

$$N(E_0, \Delta E) \approx \frac{\int_{E_0}^{E_0+\Delta E} \mathrm{d}E \int \mathrm{d}p\, \mathrm{d}q\, \delta(E - H(p,q))}{2\pi}$$
$$= \frac{\int \mathrm{d}p\, \mathrm{d}q\, \Theta(E_0 + \Delta E - H(p,q))\Theta(H(p,q) - E_0)}{2\pi}$$

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Longest range interaction: *One-pion-exchange*

Isospin of the 2heavy mesons

Spin of light quarks in the 2 heavy mesons

$$\tilde{V}_{\rm long}(q^2) = -\vec{I}_1 \cdot \vec{I}_2 \frac{4g^2}{f_\pi^2} \frac{\vec{s}_{l1} \cdot \vec{q}\; \vec{s}_{l2} \cdot \vec{q}}{q^2 - m_\pi^2}$$

Coupling constant

- *Valid since pion is lightest. Correct even away from regime where χPT is valid.*
- *Combined with semi-classical analysis this implies that the number of bound states with binding energy less than B is* $n_B > \frac{\sqrt{m_Q B}}{\sqrt{2}\pi\, m_\pi} \sim \frac{\sqrt{m_Q B}}{\Lambda}$ *which implies the existence of states with* $B \sim \Lambda^{2-\varepsilon} / m_Q^{1-\varepsilon}$

Some ingredients in the analysis

Effective Schrödinger equation derived from Hamiltonian treatment of QCD restricted to a class of states with fixed quantum numbers.

Key operator:

$$\hat{R} \equiv \frac{\int \mathrm{d}^3x\, \mathrm{d}^3y\, |y|\, \hat{Q}^\dagger(\vec{x})\hat{Q}^\dagger(\vec{x}+\vec{y})\hat{Q}(\vec{x}+\vec{y})\hat{Q}(\vec{x})}{2}$$ For QQ

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Gauge invariant operator giving distance between heavy quarks

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Gauge invariant operator giving distance between heavy quarks

Key states:

$|R\rangle$ optimal states with $\hat{R}|R\rangle = |R\rangle$ so that the potential in the effective Schrödinger equation is $V(R) = \langle R|\hat{H}_{\mathrm{QCD}} - \hat{T}_Q|R\rangle + \mathcal{O}\left(\frac{\Lambda^2}{m_Q}\right)$

Heavy quark kinetic energy

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Projection operators:

$\hat{\mathcal{P}}^r = \int dR\, |R\rangle\langle R|$ $\quad\quad \hat{\mathcal{Q}}^r = \hat{1} - \hat{\mathcal{P}}^r$ $\quad\quad$ Feshbach formalism

Some ingredients in the analysis

Divide Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{H}_I$ with

$$\hat{H}_0 \equiv \hat{\mathcal{P}}^{\mathrm{r}} \hat{H} \hat{\mathcal{P}}^{\mathrm{r}} + \hat{\mathcal{Q}}^{\mathrm{r}} \hat{H} \hat{\mathcal{Q}}^{\mathrm{r}}$$

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Find eigenstates of: $\hat{H}_0$

$$\hat{H}_0 |\phi_j\rangle_{\mathrm{r}} = E_j |\phi_j\rangle_{\mathrm{r}}$$ with $$|\phi_j\rangle_{\mathrm{r}} = \int_0^\infty dR \, \phi_j(R) |R\rangle$$ where

$$\left(\frac{\partial_r^2}{m_H} + V(r) + \mathcal{O}\left(\frac{\Lambda^2}{m_Q} \right) \right) \phi_j(r) = E_j \phi_j(r)$$

Some ingredients in the analysis

Obtain width via Imaginary part of Optical potential/Fermi's Golden rule:

$$\Gamma_j = \lim_{\epsilon \to 0+} \mathrm{Im} \left({}_r\langle \phi_j | \hat{H}_I \frac{1}{E_j - \hat{H}_0 + i\epsilon} \hat{H}_I | \phi_j \rangle_r \right)$$

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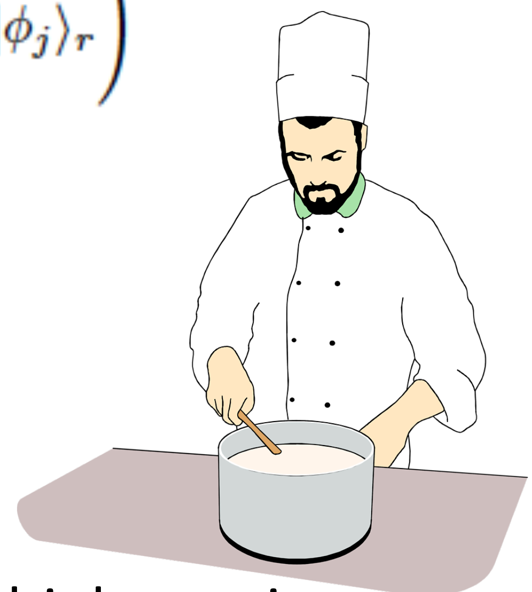

Add ingredients to pot and stir while making conservative assumptions; obtain scaling behavior

In the extreme heavy quark limit, there exist multiple exotic states parametrically close to threshold for dissociation into two heavy hadrons with binding energy $B \sim \Lambda^{2-\varepsilon} / m_Q^{1-\varepsilon}$ for any ε with $1 \geq \varepsilon > 0$.

Typical level spacing: $\Delta E \sim (\Lambda^{4-\varepsilon} / m_Q^{2-\varepsilon})^{1/2}$

Typical resonance width: $\Gamma \sim (\Lambda^{4-\varepsilon} / m_Q^{2-\varepsilon})^{1/2} \sim \Delta E \, (\Lambda / m_Q)^{1/2}$

Caveats

- The argument unambiguously holds for case of tetraquarks with two heavy quarks or pentaquaks with a heavy quark and antiquark
  - May be upper bounds in these case and may be smaller.
- Subtleties remain for the cases tetraquarks with a heavy quark and antiquark or pentaquaks with two heavy quarks.

Open Questions

- Can one use this insight on the scaling as the basis of a tractable EFT to calculate properties of given resonant states?
- Can the analysis be modified to apply away from the extreme heavy quark limit to a regime where the quark mass is heavy on the QCD scale but not heavy enough to induce multiple resonances with the same quantum numbers?
- Can a simple and transparent version of the analysis be formulated?
- Can more stringent scaling be deduced for the widths?