$\bar{b}\bar{b}ud$ Tetraquark Resonances in the Born-Oppenheimer Approximation using Lattice QCD Potentials

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Phys. Rev. D **96**, 054510 (2017), [arXiv:1704.02383 [hep-lat]]

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

HIC for FAIR Helmholtz International Center

Motivation

Why investigating heavy tetraquark resonances?

- understand exotic hadrons
- observed tetraquark candidates: resonances
- heavy tetraquarks like Z_c or Z_b have been found in recent years

J. Vijande, F. Fernandez, A. Valcarce, and B. Silvestre-Brac, Eur. Phys. J. A19, 383 (2004), arXiv:hepph/0310007 [hep-ph].
G. Bali and M. Hetzenegger (QCDSF), PoS LATTICE2010, 142 (2010), arXiv:1011.0571 [hep-lat].

Outline

- two static heavy antiquarks: $\bar{b}\bar{b}$
- two light quarks: ud

$\Rightarrow$ $\bar{b}\bar{b}ud$ in Born-Oppenheimer approximation

M. Born and R. Oppenheimer, Annalen der Physik 389, 457 (1927).

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- using effective potential $V(r)$ for $\bar{b}\bar{b}$, computed by lattice QCD
- apply emergent wave method

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- using effective potential $V(r)$ for $\bar{b}\bar{b}$, computed by lattice QCD
- apply emergent wave method
- extract phase shifts, S and T matrix

pole in T matrix $\Leftrightarrow$ Resonance (in second Riemann sheet)

P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M. Wagner: Phys. Rev. D **96**, 054510 (2017)[arXiv:1704.02383 [hep-lat]].

Lattice QCD potentials (1)

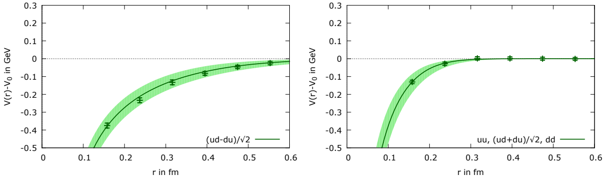
potential has been computed in:

P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M. Wagner: Phys. Rev. D **92**, 014507 (2015)[arXiv:1505.00613 [hep-lat]].
P. Bicudo, K. Cichy, A. Peters, M. Wagner: Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441 [hep-lat]].

two attractive potentials:

- (I = 0, j = 0) $\rightarrow$ scalar Iso-singlet (left)
- (I = 1, j = 1) $\rightarrow$ vector Iso-triplet (right)

(j: total angular momentum of light quarks; I: isospin)



$a = 0.079$ fm and $m_\pi = 340$ MeV

Lattice QCD potentials (2)

screened Coloumb potential

$$V(r) = \underbrace{-\frac{\alpha}{r}}_{\text{Coloumb term}} \times \underbrace{e^{-r^2/d^2}}_{\text{screening term}}$$

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I	j	α	d in fm
0	0	$0.34^{+0.03}_{-0.03}$	$0.45^{+0.12}_{-0.10}$
1	1	$0.29^{+0.05}_{-0.06}$	$0.16^{+0.05}_{-0.02}$

To physical Pion mass extrapolated parameters α and d for the screened Coloumb potential

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solving the Schrödinger equation:

bound state for $I(J^P) = 0(1^+)$ with $E_B = -90^{+43}_{-36}$ MeV

W. Detmold, K. Orginos, and M. J. Savage, Phys. Rev.D **76**, 114503 (2007), [arXiv:hep-lat/0703009 [hep-lat]].
Z. S. Brown and K. Orginos, Phys. Rev.D **86**, 114506 (2012), [arXiv:1210.1953 [hep-lat]].
P. Bicudo and M. Wagner, Phys. Rev.D **87**, 114511 (2013) [arXiv:1209.6274 [hep-ph]].
P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, and M. Wagner: Phys. Rev.D **92** , 014507 (2015) [arXiv:1505.00613 [hep-lat]].
P. Bicudo, J. Scheunert, and M. Wagner: Phys. Rev. D **95**, 034502 (2017), [arXiv:1612.02758 [hep-lat]].
A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman: Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214 [hep-lat]].

The emergent wave method (1)

Consider Schrödinger equation:

$$(H_0 + V(r))\,\Psi = E\Psi \tag{1}$$

split wave function into two parts:

$$\Psi = \underbrace{\Psi_0}_{\text{incident wave}} + \underbrace{X}_{\text{emergent wave}} \; ; \qquad H_0\Psi_0 = E\Psi_0 \tag{2}$$

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Inserting (2) in (1):

$$(H_0 + V(r) - E)\,X = -V(r)\Psi_0 \tag{3}$$

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Partial wave decomposition

express Ψ_0 as sum of spherical waves:

$$\Psi_0 = e^{ik\cdot r} = \sum_l (2l+1) i^l j_l(kr) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}) \tag{4}$$

j_l: spherical Bessel function; P_l: Legendre polynomials.

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for any arbitrary E, we can compute $\chi_l(r)$:

$$\Rightarrow \left(-\frac{1}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r) - E \right) \chi_l(r) = -V(r)\, kr\, j_l(kr) \quad (6)$$

The emergent wave method (3)

- for large separations $r \geq R$ we assume: $V(r) \approx 0$
  $\rightarrow$ emergent wave as superposition of outgoing spherical waves

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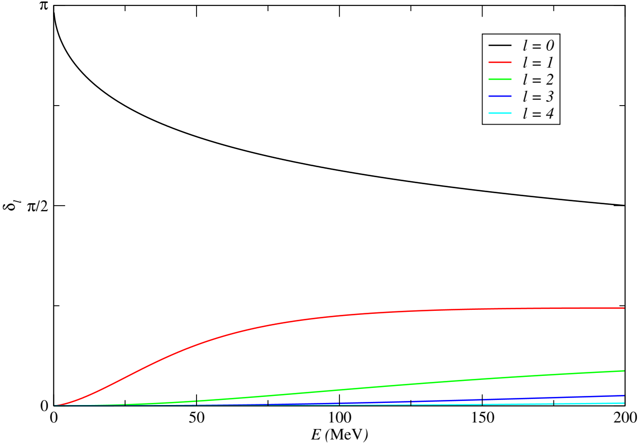
- use complex energies E
- Pole of $t_l(E)$ $\Leftrightarrow$ complex resonances energy E
  Re(E): resonance mass   Im(E): decay width

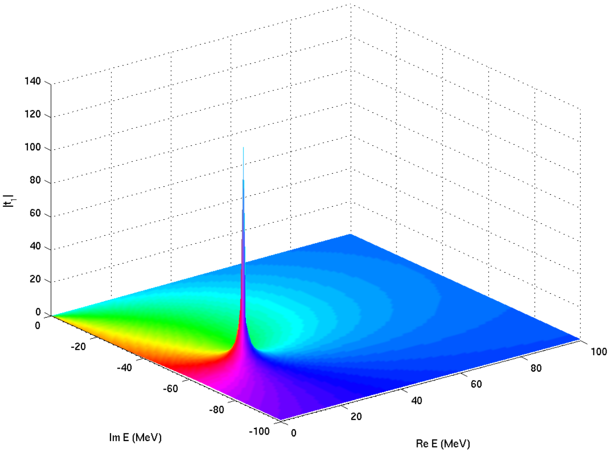
Results for Phase shifts, T Matrix poles and Resonances (1)

clear signal of resonance: fast increasing of δ_l from 0 to $\approx \pi$



Phase shift δ_l for scalar Iso-singlet (I = 0, j = 0) and different orbital angular momenta l.

Results for Phase shifts, T Matrix poles and Resonances (2)



T matrix eigenvalue t_1 as a function of the complex energy E for $(I = 0, j = 0)$, $l = 1$. Vertical axis: norm $|t_1|$. colour: phase $\arg(t_l)$.

Results for Phase shifts, T Matrix poles and Resonances (3)

- for orbital angular momentum $l = 1$ and $(I = 0, j = 0)$ we find a T matrix pole
- after detailed statistical and systematic error analysis:

  Re(E)=17^{+4}_{-4} MeV   decay width: Γ= -2Im(E)=112^{+90}_{-103} MeV

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- no clear evidence for resonance pole for $l \neq 1$ or $(I = 1, j = 1)$

Summary and Outlook

- investigated $\bar{b}\bar{b}ud$-tetraquark system in the Born-Oppenheimer approximation

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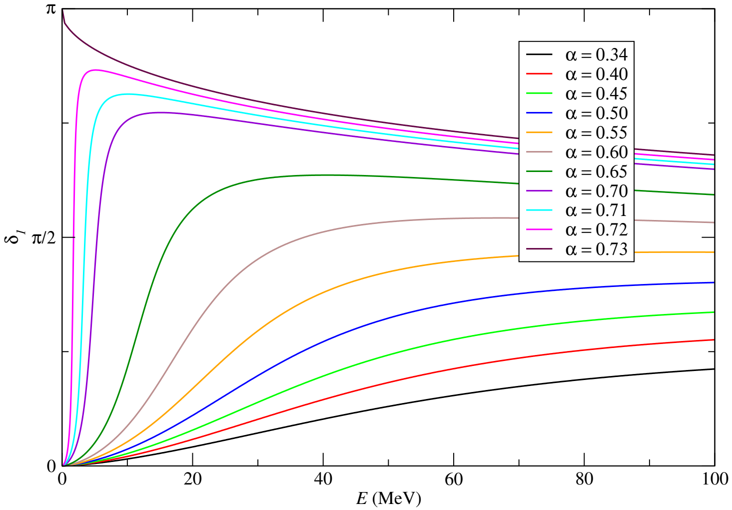
Summary and Outlook

- investigated $\bar{b}\bar{b}ud$-tetraquark system in the Born-Oppenheimer approximation
- clear evidence for a resonance with $I(J^P) = 0(1^-)$ and $m = 2M_B + \mathrm{Re}(E) = 10576^{+4}_{-4}$ MeV
- next step: using dynamic heavy quarks, e.g. in the framework of non-relativistic QCD (NRQCD) to verify the resonance

A. Francis, R. J. Hudspith, R. Lewis, K. Maltman: arXiv:1711.03380v1 [hep-lat].
A. Peters, P. Bicudo, L. Leskovec, S. Meinel, M. Wagner: arXiv:1609.00181v1 [hep-lat].

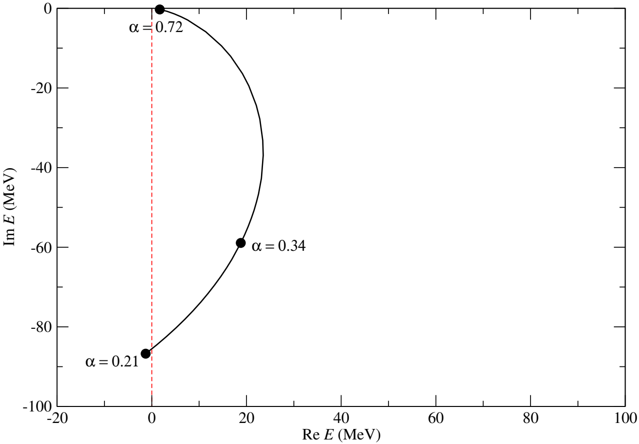
Backup

Results for Phase shifts, T Matrix poles and Resonances (4)



Phase shift δ_1 for different parameters α with fixed $d = 0.45$.

Results for Phase shifts, T Matrix poles and Resonances (5)



Location for the pole of the eigenvalue t_1 of the T matrix in the complex plane for $(I = 0, j = 0)$.