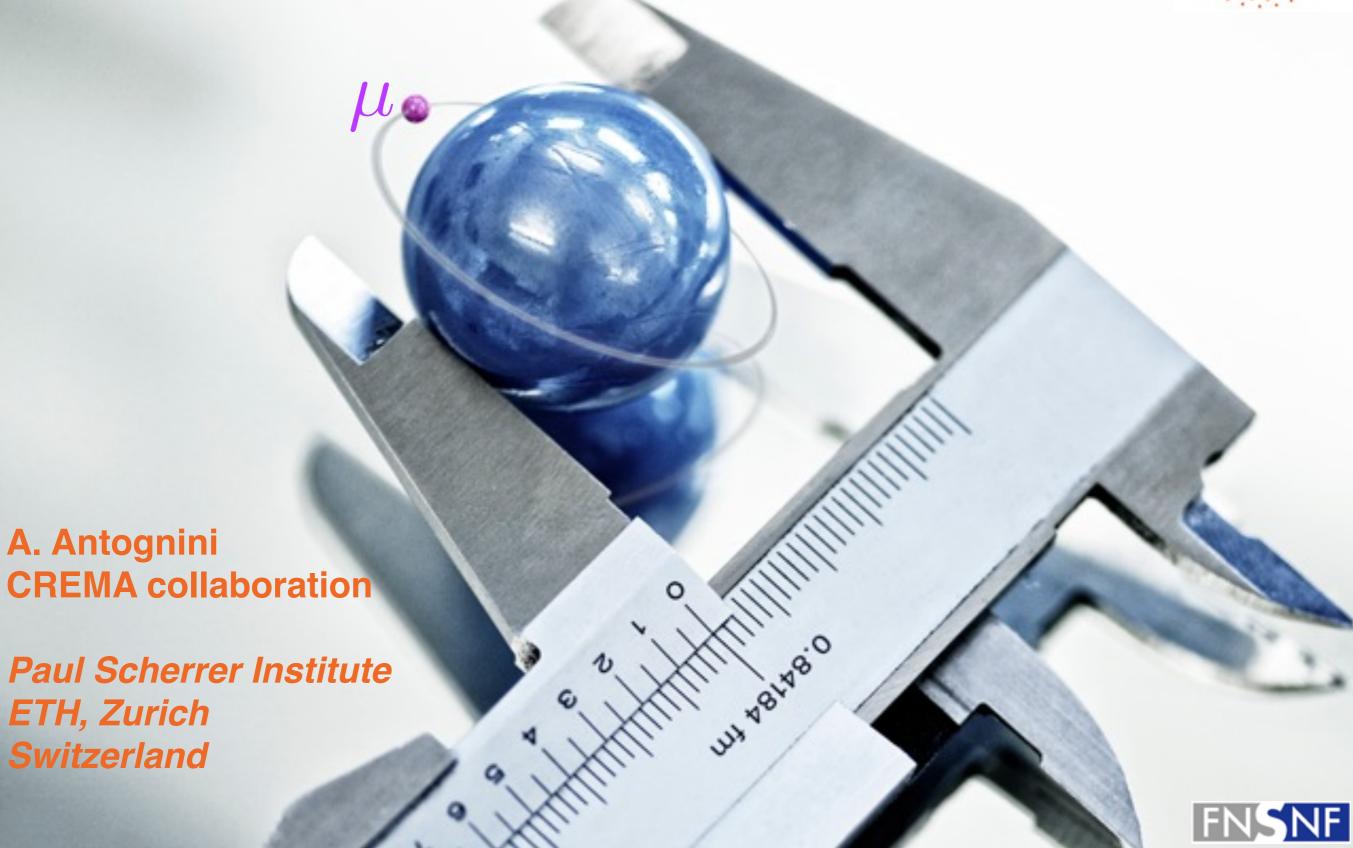
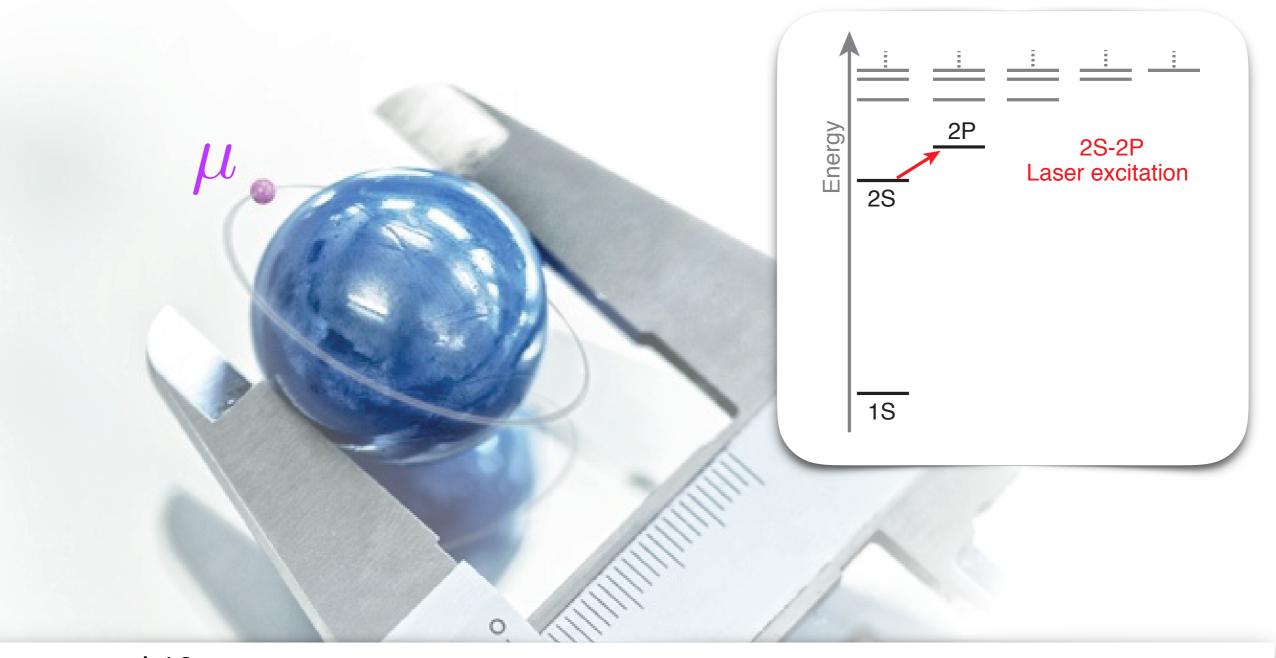
Spectroscopy of muonic atoms

erc

and the proton radius puzzle



Laser spectroscopy of muonic atoms



We measured 10 2S-2P transitions in μp , μd , $\mu^3 He^+$, $\mu^4 He^+$

Theoretical predictions: QED + Nuclear structure

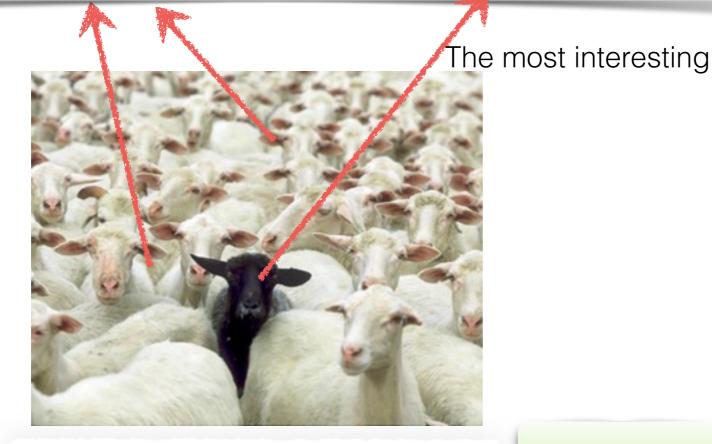


p, d, ³He, ⁴He charge radii

Extracting the proton radius from µp

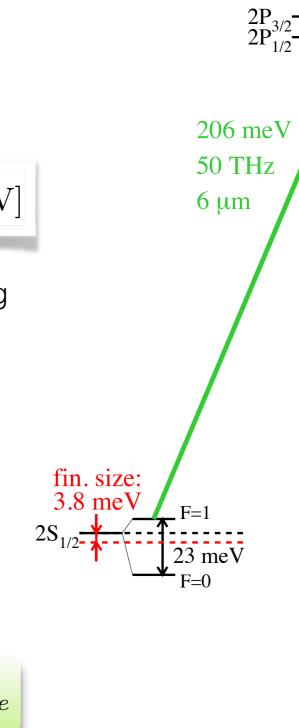
Measure 2S-2P splitting (20 ppm) and compare with theory → proton radius

$$\Delta E_{2P-2S}^{\text{th}} = 206.0336(15) - \frac{5.2275(10)}{p} r_{p}^{2} + 0.0332(20) \text{ [meV]}$$



$$\Delta E_{\text{size}} = \frac{2\pi (Z\alpha)}{3} \, r_{\text{p}}^{2} \, |\Psi_{nl}(0)|^{2}$$
$$= \frac{2(Z\alpha)^{4}}{3n^{3}} m_{r}^{3} \, r_{\text{p}}^{2} \, \delta_{l0}$$

Aldo Antognini



$$m_{\mu} \approx 200 m_e$$

$$r_{\rm p}^2 = -6 \frac{dG_E(Q^2)}{dQ^2} \Big|_{{\rm Q}^2=0}$$

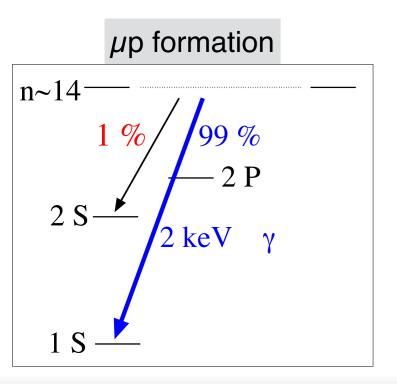
Principle of the μ p 2S-2P experiment

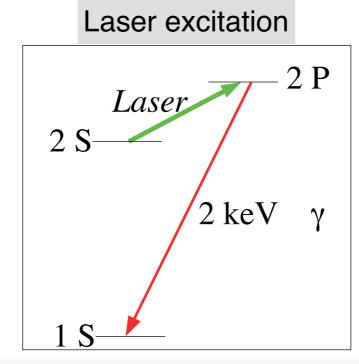
Produce many μ - at keV energy

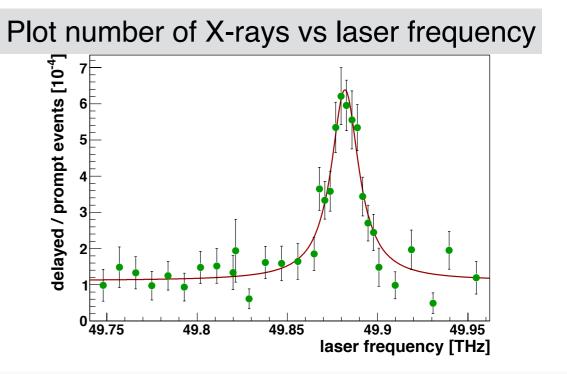
Form μ p by stopping μ - in 1 mbar H₂ gas

Fire laser to induce the 2S-2P transition

Measure the 2 keV X-rays from 2P-1S decay

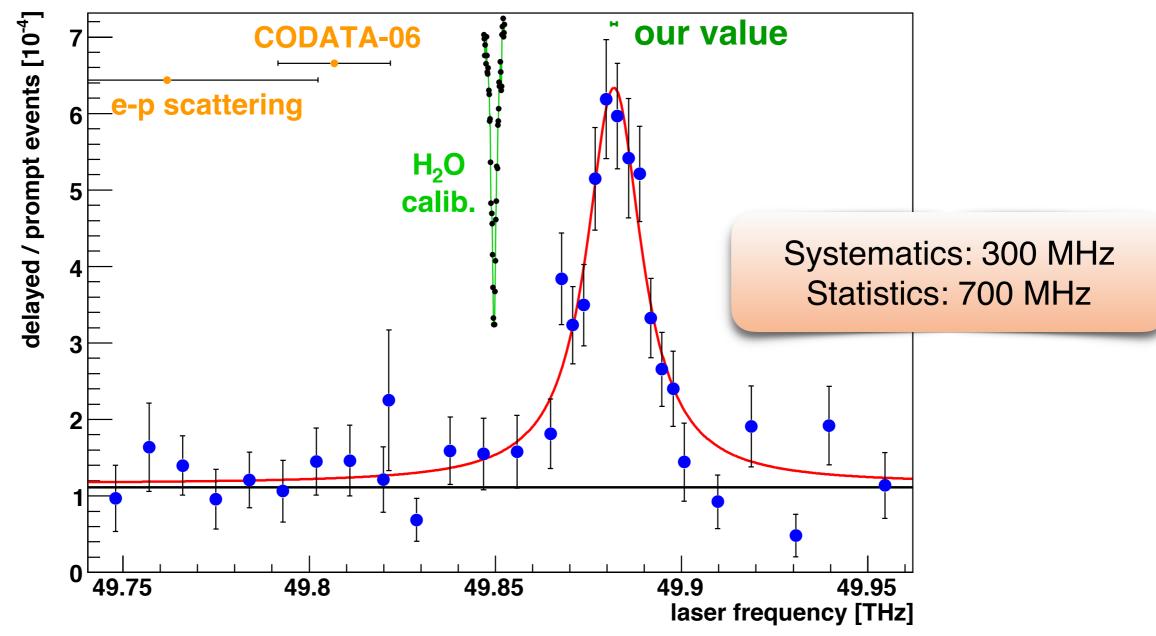






The first µp resonance (2010)

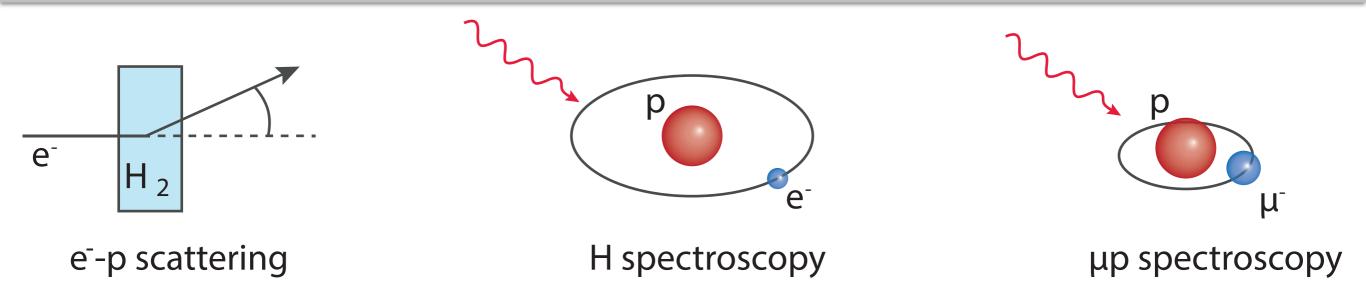
Discrepancy:
$$5.0\,\sigma \leftrightarrow 75~\mathrm{GHz} \leftrightarrow \delta\nu/\nu = 1.5\times10^{-3}$$

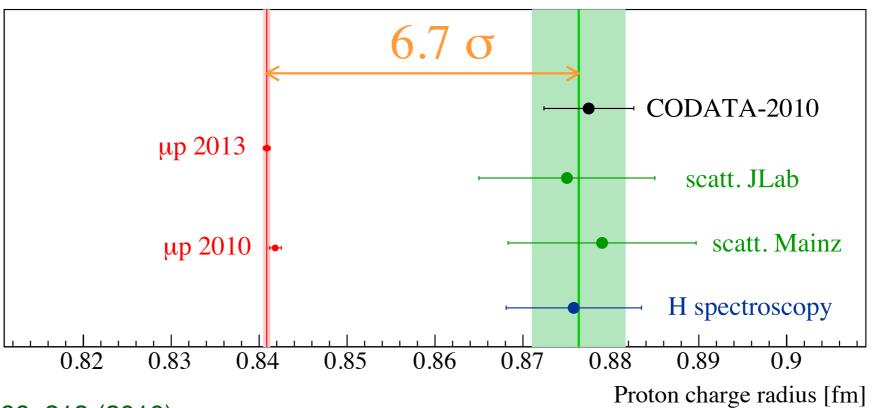


Pohl et al., Nature 466, 213 (2010)



Three ways to the proton radius





Pohl et al., Nature 466, 213 (2010) Antognini et al., Science 339, 417 (2013) Pohl et al., Science 353, 669 (2016)



µp experiment

μp theory

H experiments

BSM physics

e-p scattering

Rarely criticised since:

 $m_{\mu} \approx 200 m_e$

sensitive to the radius

$$\sim m^3 R_p^2$$



insensitive to systematical effects

$$\sim 1/m$$



μp experiment

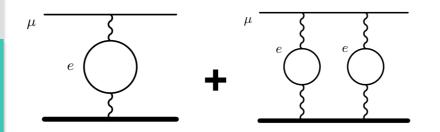
µp theory

H experiments

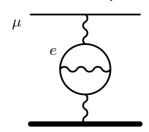
BSM physics

e-p scattering





Pachucki, Borie, Eides, Karschenboim, Jentschura, Martynenko, Indelicato Pineda, Peset...

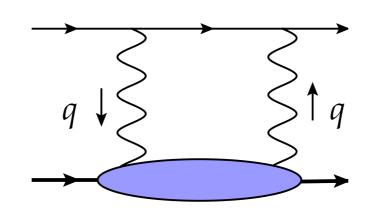


many

Finite-size contributions

$$= -5.2275(10)r_p^2$$

Two-photon exchange





Pachucki, Carlson, Birse, McGovern, Pineda, Peset, Gorchtein, Pascalutsa, Vanderhaeghen, Alarcon, Miller, Paz, Hill, Hagelstein, Tomalak...

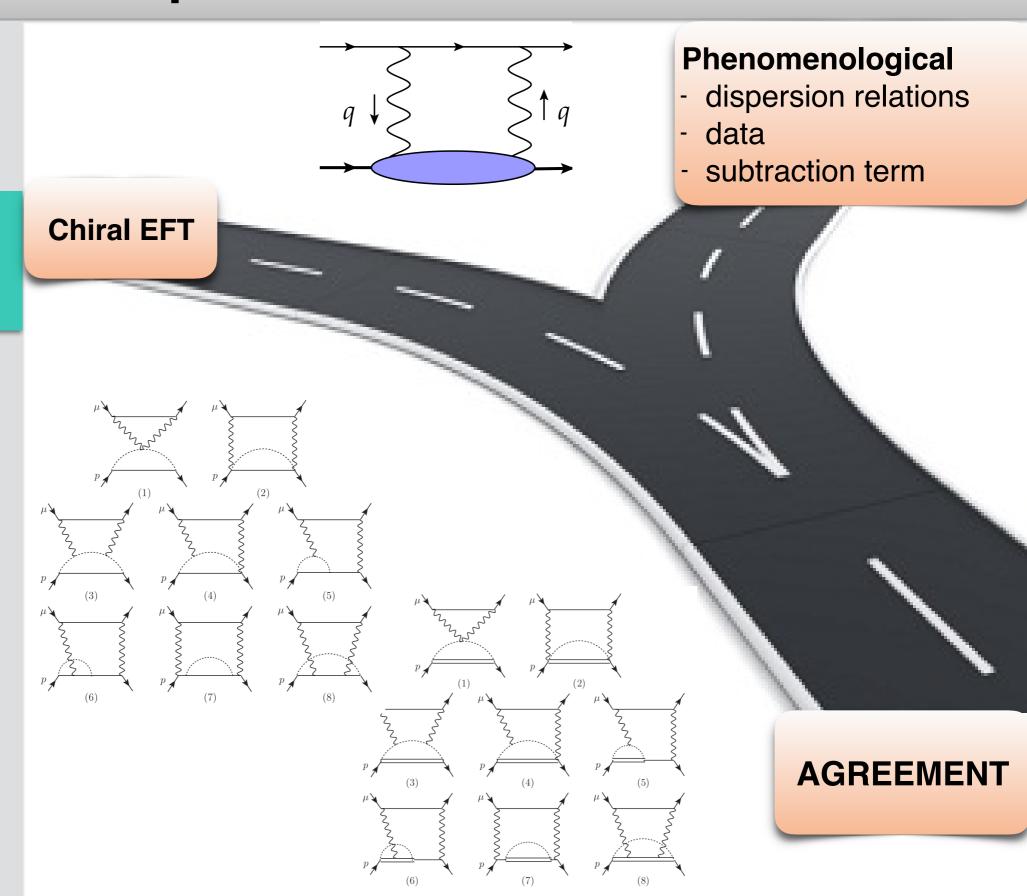
μp experiment

μp theory

H experiments

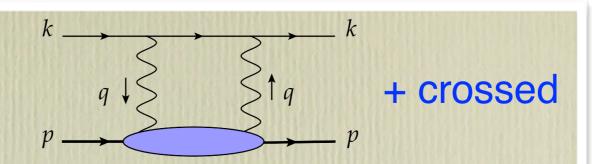
BSM physics

e-p scattering



Technicalities on TPE in µp

Kinematics: 2 loop variables q^2 and v=(pq)/M



$$\mathcal{M} = e^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4} \bar{u}(k) \left[\gamma^{\nu} \frac{1}{\not{k} - \not{q} - m_l + i\epsilon} \gamma^{\mu} + \gamma^{\mu} \frac{1}{\not{k} + \not{q} - m_l + i\epsilon} \gamma^{\nu} \right] u(k) T_{\mu\nu}$$

Forward virtual Compton amplitude

$$T^{\mu\nu} = \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p|T j^{\mu}(x)j^{\nu}(0)|p\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2} (p - \frac{pq}{q^2}q)^{\mu} (p - \frac{pq}{q^2}q)^{\nu} T_2(\nu, Q^2)$$

Lamb shift (nS-nP)

$$\Delta E = -\frac{\alpha^2}{2\pi m_l M_d} \phi_n^2(0) \int d^4q \frac{(q^2 + 2\nu^2) T_1(\nu, q^2) - (q^2 - \nu^2) T_2(\nu, q^2)}{q^4 [(q^2/2m_l)^2 - \nu^2]}$$

Slide stolen from Gorchtein



Technicalities on TPE in µp

T₁, T₂ - the imaginary parts known (Optical theorem)

$$\operatorname{Im} T_1(\nu,Q^2) = \frac{1}{4M} F_1(\nu,Q^2)$$
 Inelastic structure functions = data (real and virtual photoabsorption, FF)

Real parts - from forward dispersion relation

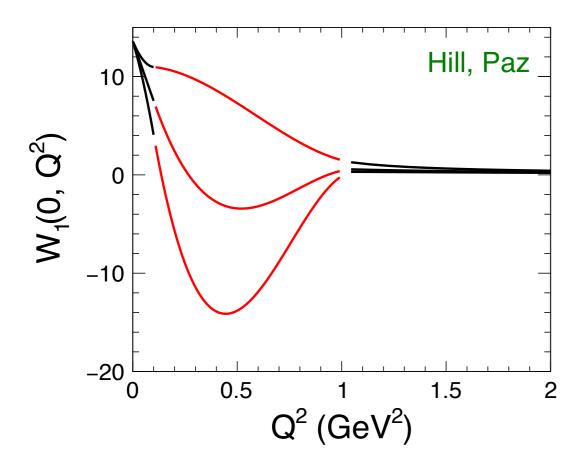
$$F_1(
u o \infty, q^2) \sim
u^{1+\epsilon}$$
 - subtraction needed $F_2(
u o \infty, q^2) \sim
u^{\epsilon}$ - no subtraction

$$\operatorname{Re}T_{1}(\nu, Q^{2}) = \bar{T}_{1}(0, Q^{2}) + T_{1}^{pole}(\nu, Q^{2}) + \frac{\nu^{2}}{2\pi M} \int_{\nu_{0}}^{\infty} \frac{d\nu'}{\nu(\nu'^{2} - \nu^{2})} F_{1}(\nu', Q^{2})$$

$$\operatorname{Re}T_{2}(\nu, Q^{2}) = T_{2}^{pole}(\nu, Q^{2}) + \frac{1}{2\pi} \int_{\nu_{0}}^{\infty} \frac{d\nu'}{\nu'^{2} - \nu^{2}} F_{2}(\nu', Q^{2})$$

Slide stolen from Gorchtein

How reliable is the TPE in μp ?



Pachucki, PRA 60, 3593 (1999)
Nevado, Pineda, PRC 77, 035202 (2008)
Peset, Pineda, EPJA 51, 32 (2015)
Peset, Pineda, NPB 887, 69 (2014)
Carlson, Vanderhaeghen, PRA 84, 020102 (2011)
Hill, Paz, PRL 107, 160402 (2011)
Miller, arXiv:1209.4667 (2012)
Birse, McGovern, EPJA 48, 120 (2012)
Miller, PLB 718, 1078 (2013)
Gorchtein et al., PRA 87, 052501 (2013)]
Alarcon, Lensky, Pascalutsa, EPJC 74, 2852 (2014)
Tomalak, Vanderhaeghen, PRD 90, 013006 (2014)
Tomalak, Vanderhaeghen, EPJC 76, 125 (2016)
Hill, Paz, PRD 95, 094017 (2017)

Subtraction term:

- low Q²: NRQED + LEC

- medium Q²: unknown

- high Q²: OPE expansion

µp experiment

μp theory

H experiments

BSM physics

e-p scattering

Uncertainties and discrepancy

0.3	meV	Discrepancy
0.0015	meV: meV:	TPE uncertainty conservatively (Hill, Pineda) TPE uncertainty (McGovern, Pascalutsa) QED+other uncertainties 3y uncertainty (Pachucki)
0.0023	meV:	Muonic hydrogen measurement uncertainty

Pachucki, Carlson, Birse, McGovern, Pineda, Peset, Gorchtein, Pascalutsa, Hagelstein, Vanderhaeghen, Tomalak, Martynenko, Alarcon, Miller, Paz, Hill, Llanes-Estrada, Szczepaniak...

1**S**

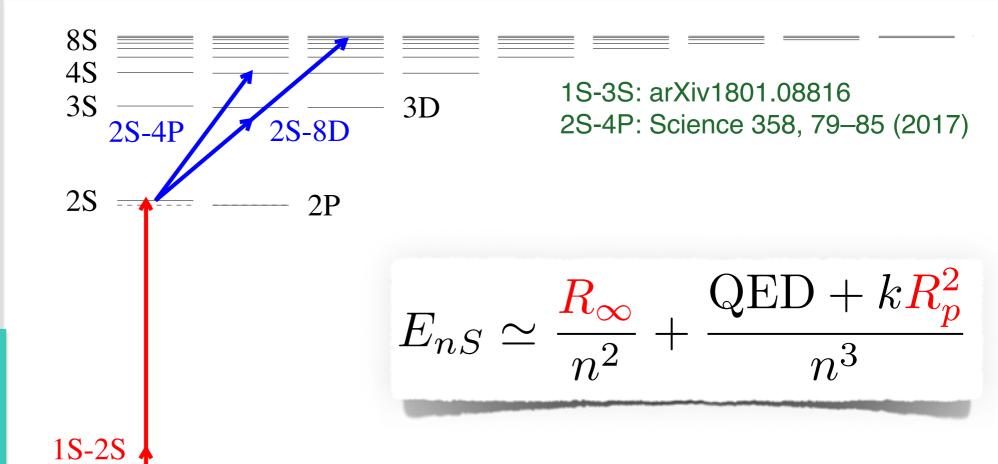
μp experiment

μp theory

H experiments

BSM physics

e-p scattering



· Two unknown: R_{∞} , R_{p}

Two groups of measurements:

- 1S-2S: 10⁻¹⁵ rel. accuracy



- others: <10⁻¹³ rel. accuracy broader lines, more prone to systematics



The proton radius puzzle (2010)

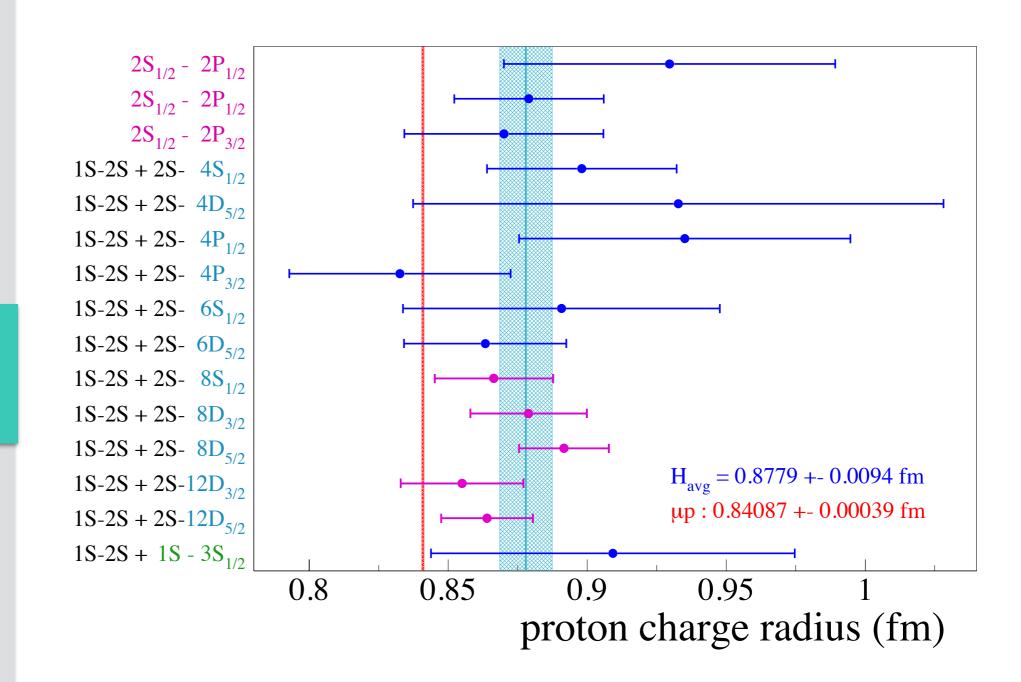
µp experiment

μp theory

H experiments

BSM physics

e-p scattering

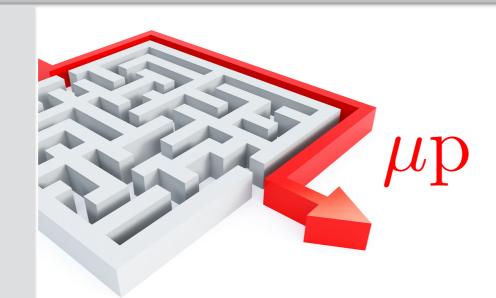


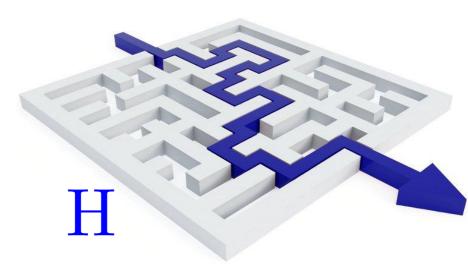
 4σ only when averaging



μp experiment

μp theory





H experiments

Large sensitivity to rp ⇒ requires low-precision meas. Large insensitivity to systematics But difficult to see the signal

Low sensitivity to rp ⇒ requires high-precision But "easy" to see the signal

BSM physics

e-p scattering

Explain the discrepancy by shifting the							
$\mu p (2S-2P)$	100σ	75 GHz	4Γ				
H (1S-2S)	$4'000\sigma$	40 kHz	40Γ				
H (2S-4P)	$< 1.5 \sigma$	9 kHz	$7 \cdot 10^{-4} \Gamma$				
H (2S-2P)	$< 1.5 \sigma$	5 kHz	$7 \cdot 10^{-4} \Gamma$				

exp accuracy

line width



XIIIth Quark Confinement

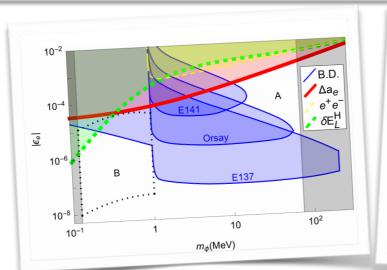
μp experiment

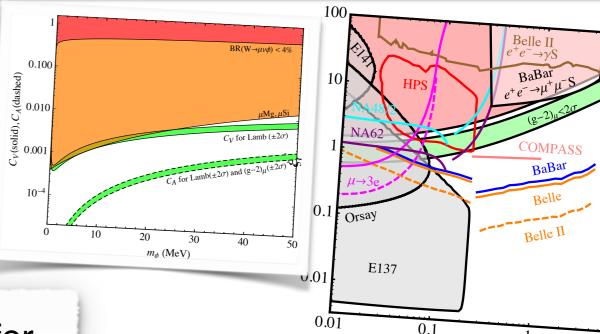
μp theory

H experiments

BSM physics

e-p scattering





Some open regions for MeV force carrier still resist

Martens & Ralston (201%)(GeV) Liu, McKeen & Miller (2016), Batell et. al (2016), Carlson (2014)

- Tuning (e.g. vector vs axial-vector)
- Preferential coupling to μ and p
- No UV completion and no full SM gauge inv.

BSM explanations "unnatural" and small window! BUT more natural extensions for

$$R_H < R_{\mu p} < R_{\rm scatt}$$

Pospelov



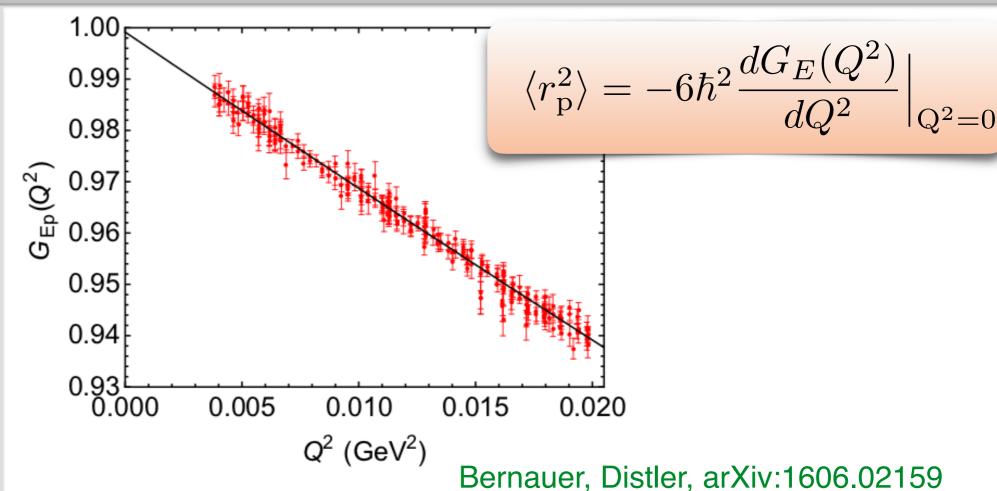
µp experiment

μp theory

H experiments

BSM physics

e-p scattering



Sick, Trautmann, arXiv:1701.01809
Lee, Arrington, Hill, arXiv:1505.01489
Hoferichter et al., EPJA 52, 331 (2016)
Alarcon, Weiss, arXiv:1710.06430
Sick, arXiv:1801.01746

Higinbotham et al.,, arXiv: 1510.01293

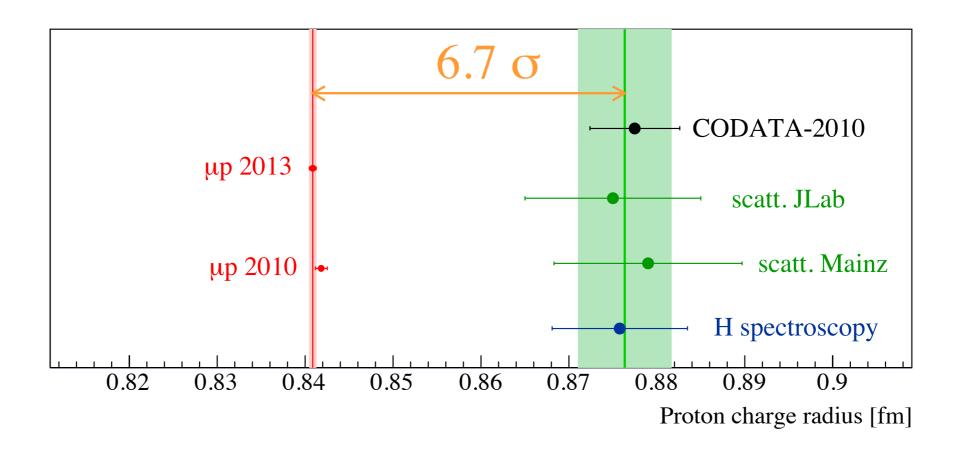
Griffioen et al., arXiv:1509.06676

Lorenz et al., PRD 91, 014023 (2015)

Horbatsch, Hessels, Pineda, arXiv:1610.09760



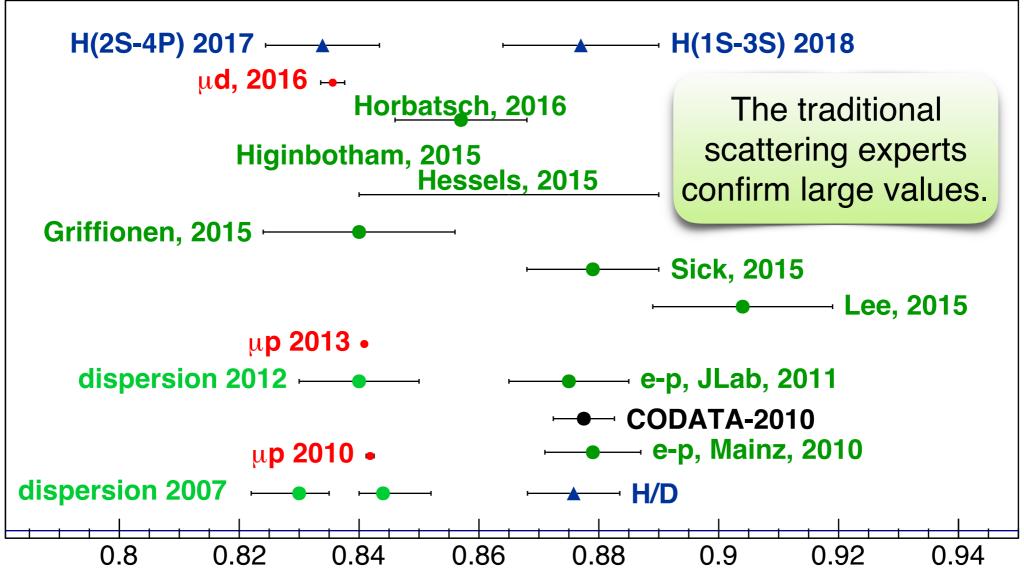
The proton charge radii





The proton charge radii market







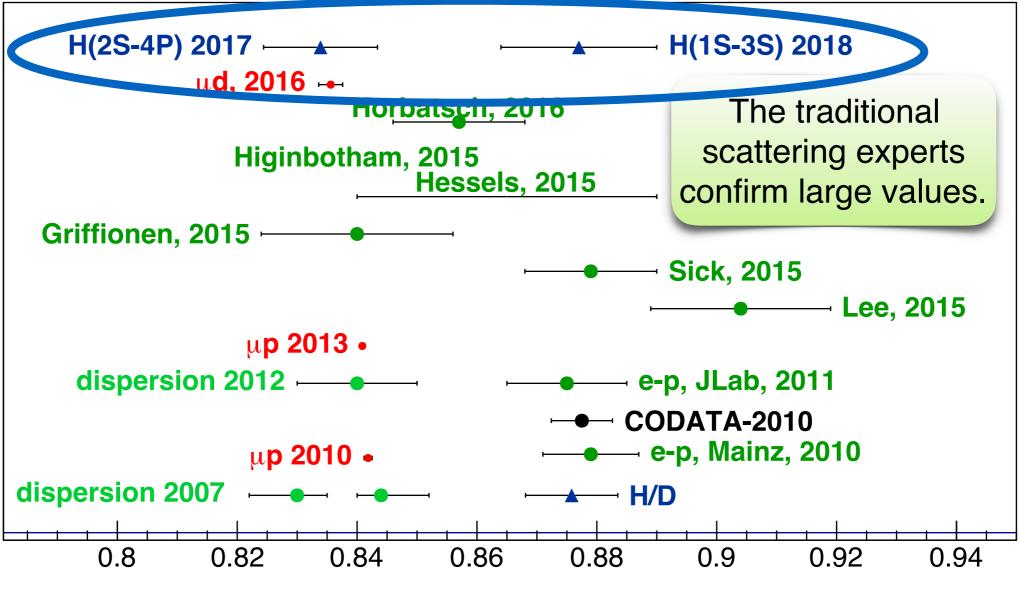
XIIIth Quark Confinement

Proton charge radius [fm]

20

The proton charge radii market







XIIIth Quark Confinement

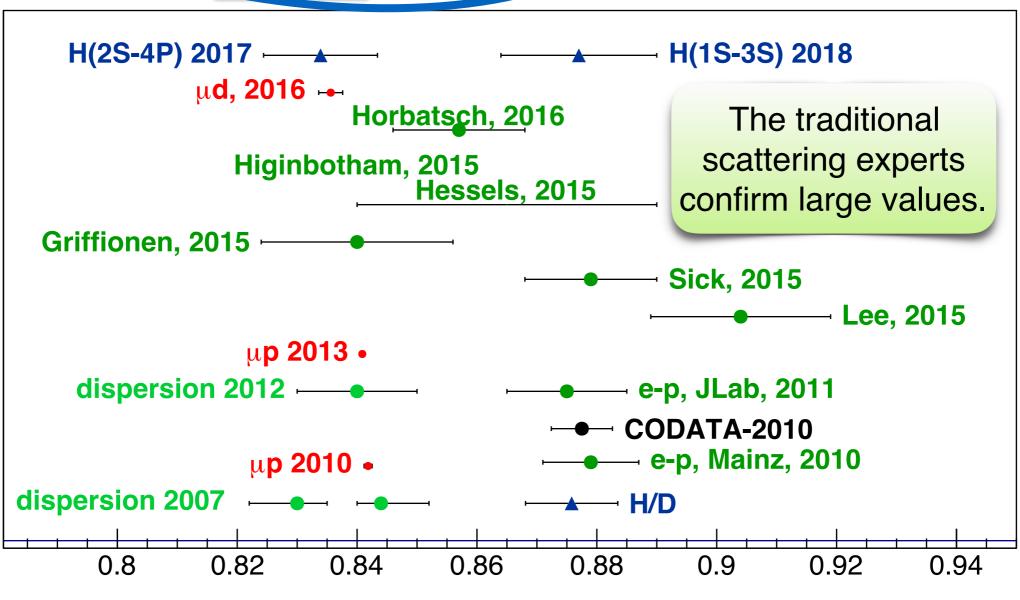
Proton charge radius [fm]

The proton charge radii on the market



New value from e-p scattering will be soon available

Ashot et al., PRad, JLAB



Proton charge radius [fm]



The race to the proton radius solution

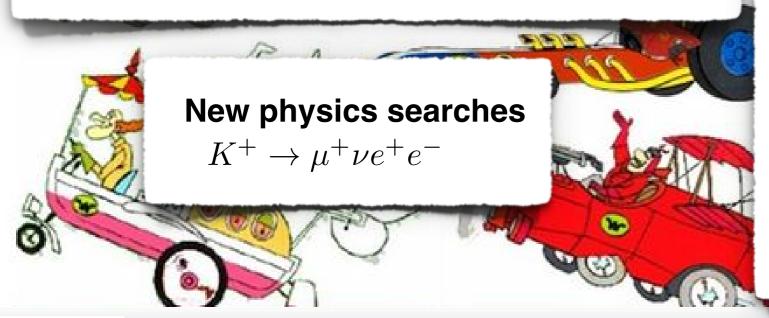




The race to the proton radius solution

Atomic spectroscopy

- · H(2S-2P) (Toronto)
- H(1S-3S) (LKB, MPQ)
- · H(2S-4P) (MPQ)
- H₂, H₂+, HD, HD+,HT (LKB, LaserLaB, ETH)
- He+ (LaserLaB, MPQ)
- He (LaserLab)
- · Li+ (Mainz)
- Muonium (ETH, PSI)
- Positronium (ETH, UC London)
- Rydberg states in H-like ions (NIST)
- Rydberg states in optical lattice (Ann Arbor)





- · µd
- μ^3 He, μ^4 He
- · μp HFS
- μLi ?

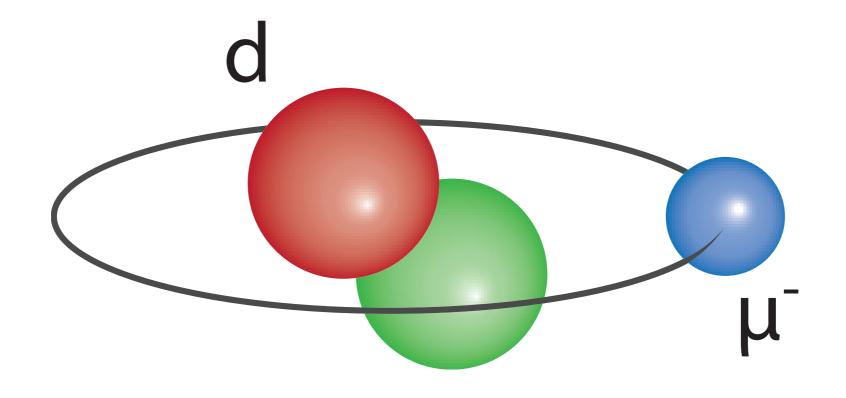


Scattering

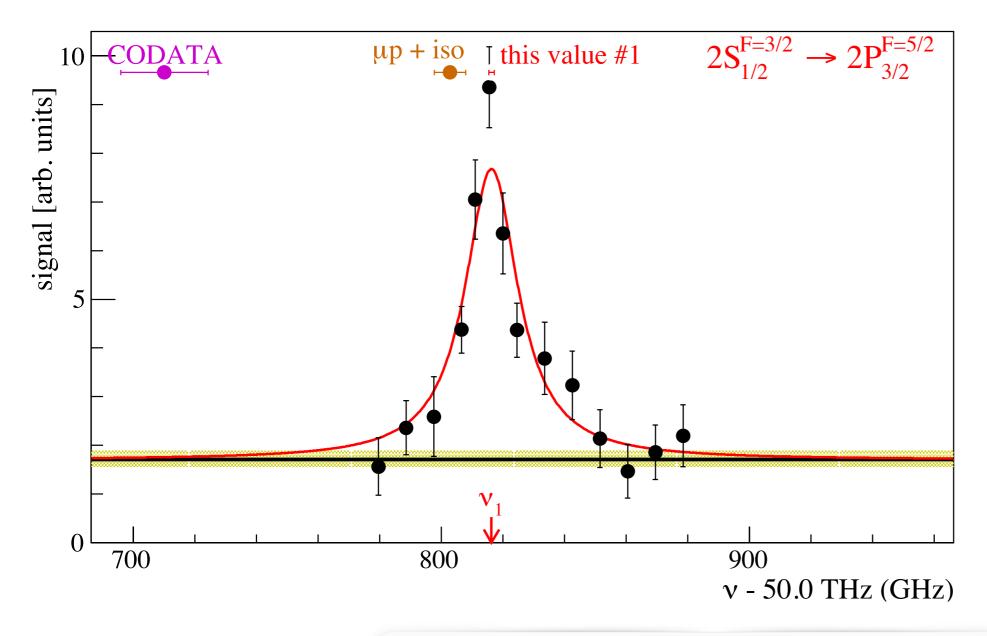
- e-p, PRad (JLAB)
- e-p, ISR & MAGIX (Mainz)
- μ-p, e-p, MUSE (PSI)
- μ-p, COMPASS (CERN)
- e-p, ProRad (Orsay)
- Tohoku, (Sendai)

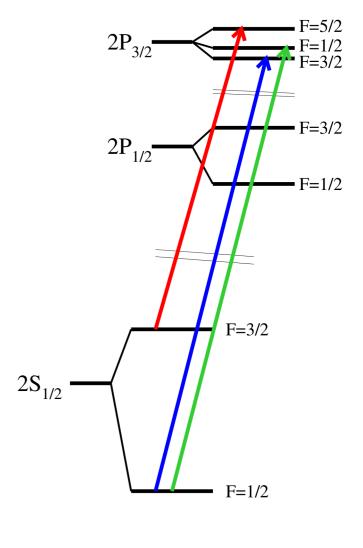






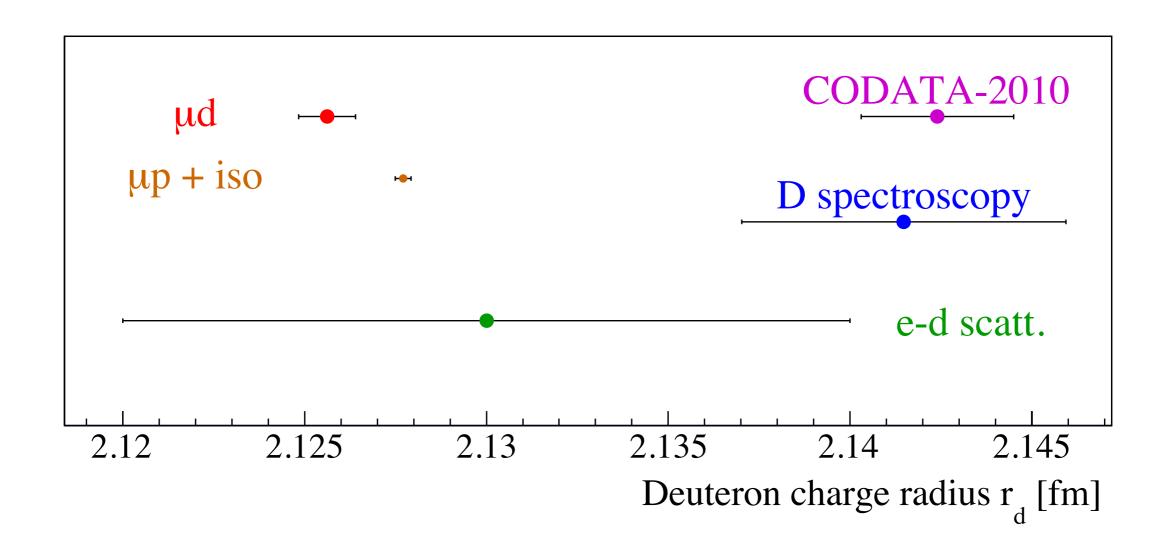






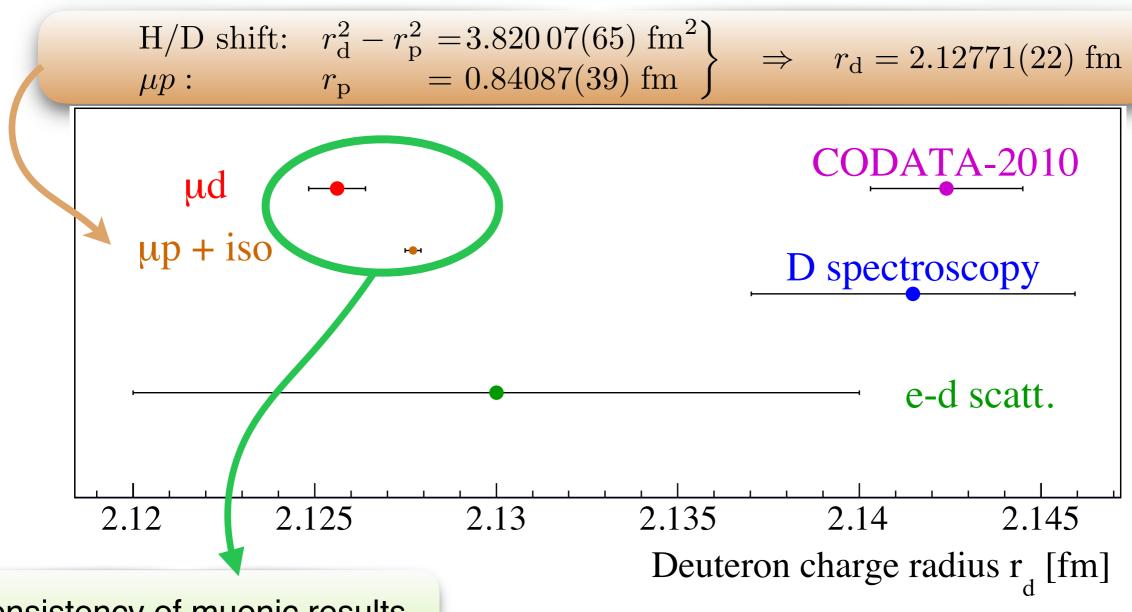
	$\mu p [meV]$	$\mu d [meV]$	
QED	206	229	×1.1
$k\langle r^2 \rangle$	4	28	$\times 7$
TPE	0.03	1.7	$\times 56$





Pohl et al., Science 353, 669 (2016) Krauth et al., Ann. Phys. 336 168 (2016) Hernandez et. al., PLB 736, 344 (2014) Pachucki et al., PRA 91, 040503(R) (2015)



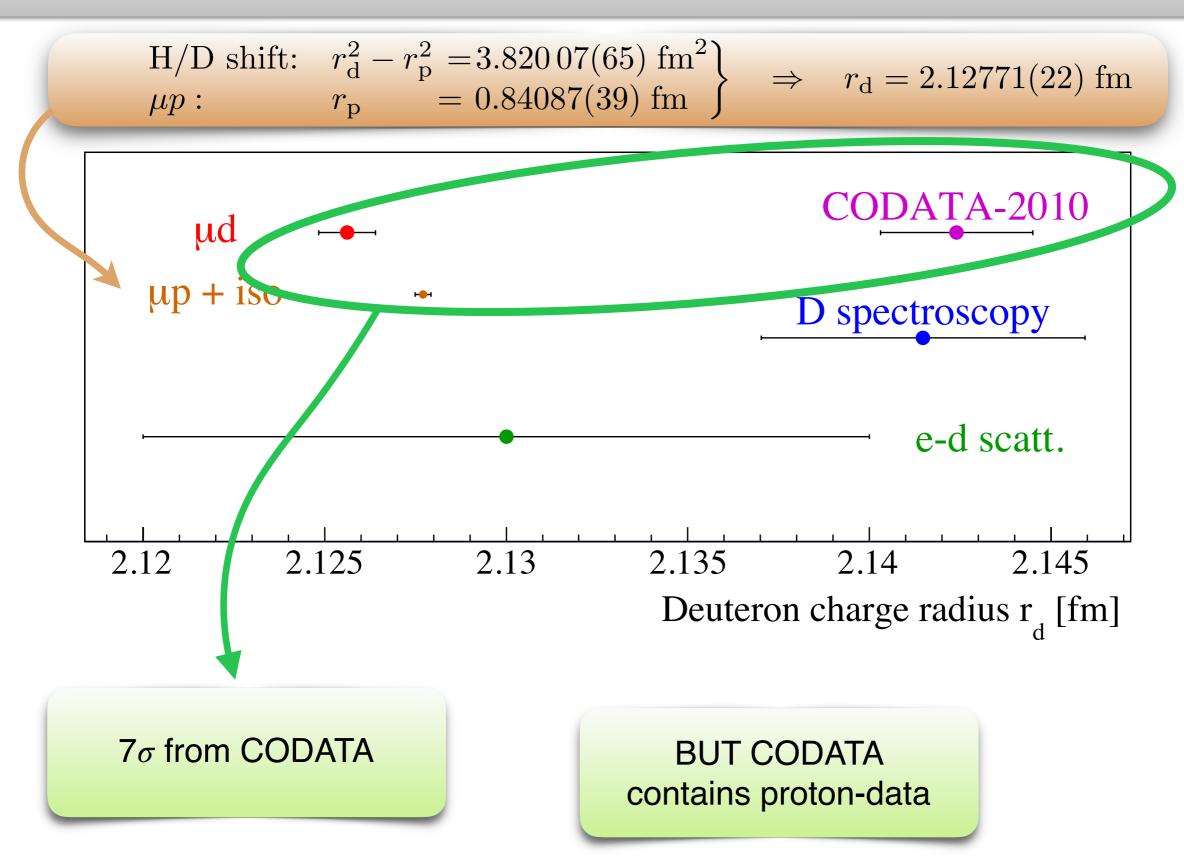


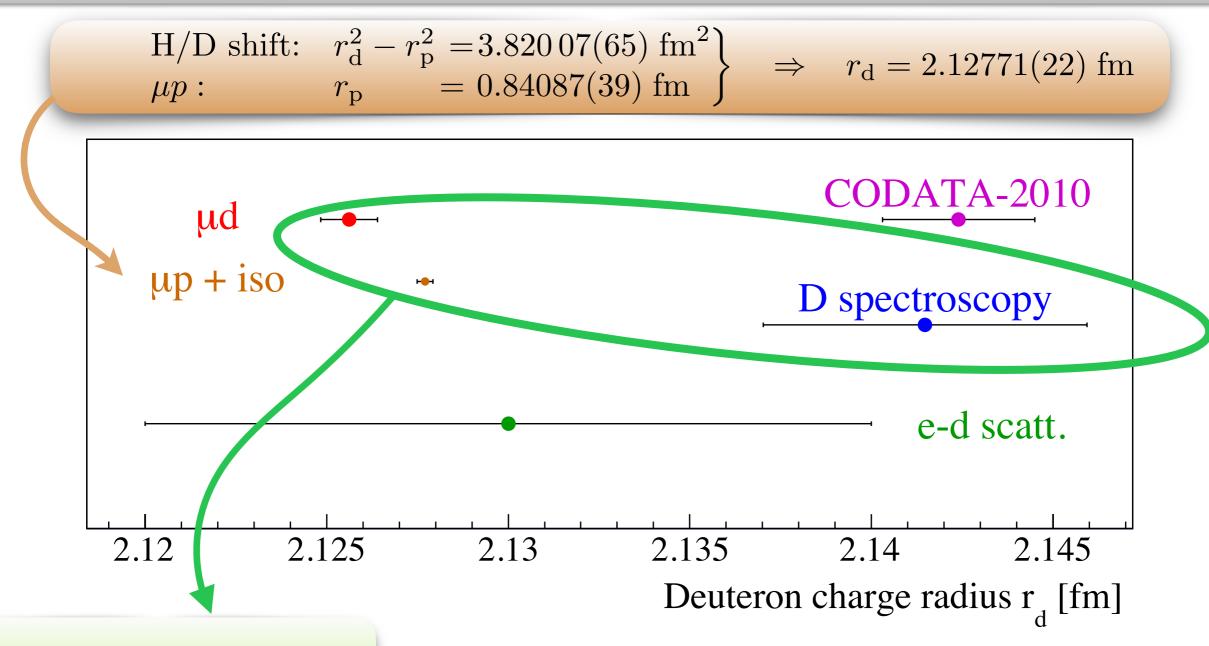
Consistency of muonic results with 1S-2S H/D isotopic-shift

Pachucki, Bacca, Barnea, Gorchtein, Carlson....

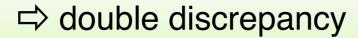
The 2.5σ difference:

- incomplete nuclear polarizabilty?
- BSM physics NOT coupling to n (reduced mass effect)?





 3.5σ from ONLY D-data

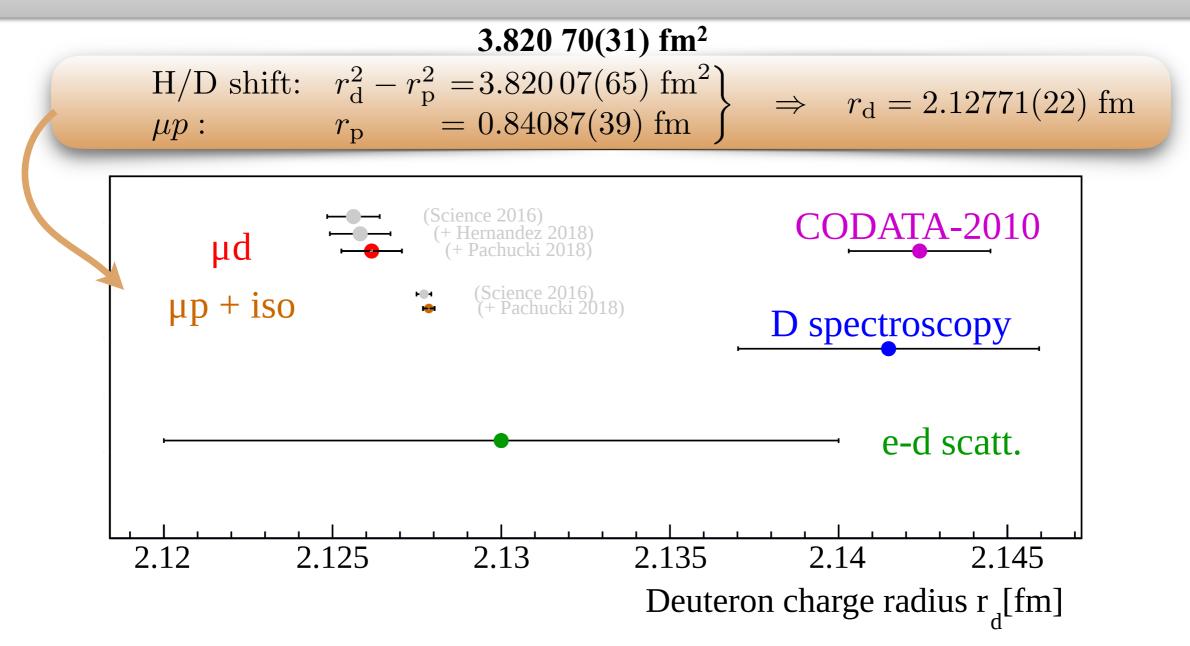


- proton sector
- deuteron sector

- ⇒ Problem with H/D exp (R_∞)?
- ⇒ Problem with H/D th.?
- ⇒ BSM with no coupling to n?







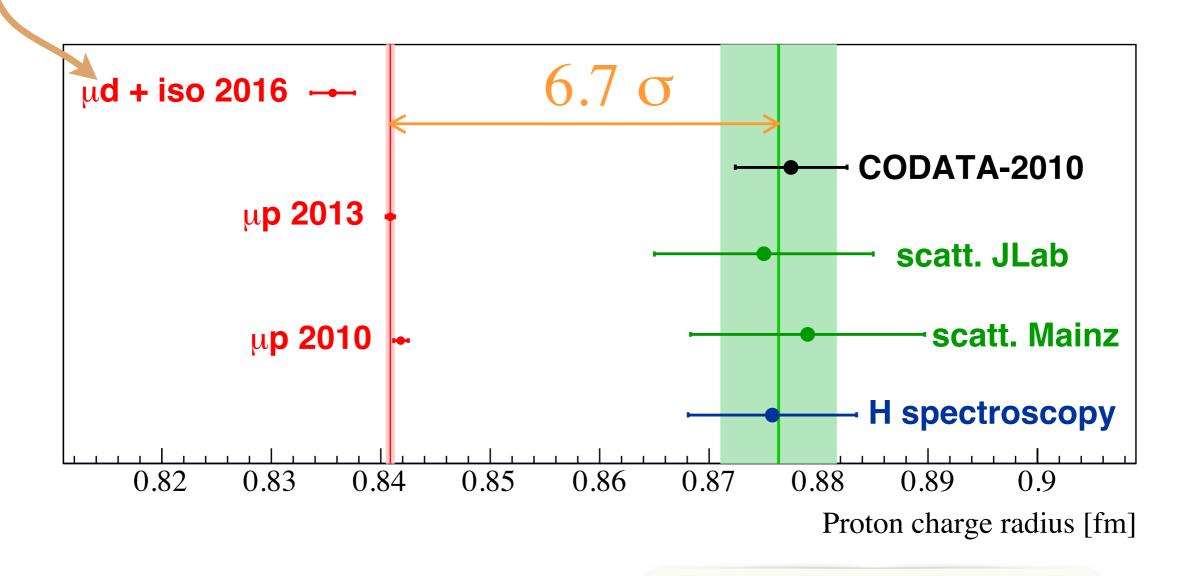
Theory update

Slightly improve the consistency of the muonic results

Hernandez et al, PLB (2018) Pachucki et al., arXiv:1803.10313

The proton charge radius from muonic deuterium

$$H/D \text{ shift:} \quad r_{\rm d}^2 - r_{\rm p}^2 = 3.820\,07(65) \text{ fm}^2$$
 $\mu d: \quad r_{\rm d} = 2.1256(8) \text{ fm}$
 $\Rightarrow \quad r_{\rm p} = 0.8356(20) \text{ fm}$



Pohl et al., Nature 466, 213 (2010) Antognini et al., Science 339, 417 (2013) Pohl et al., Science 353, 669 (2016) Small value of the proton radius is confirmed from μ d

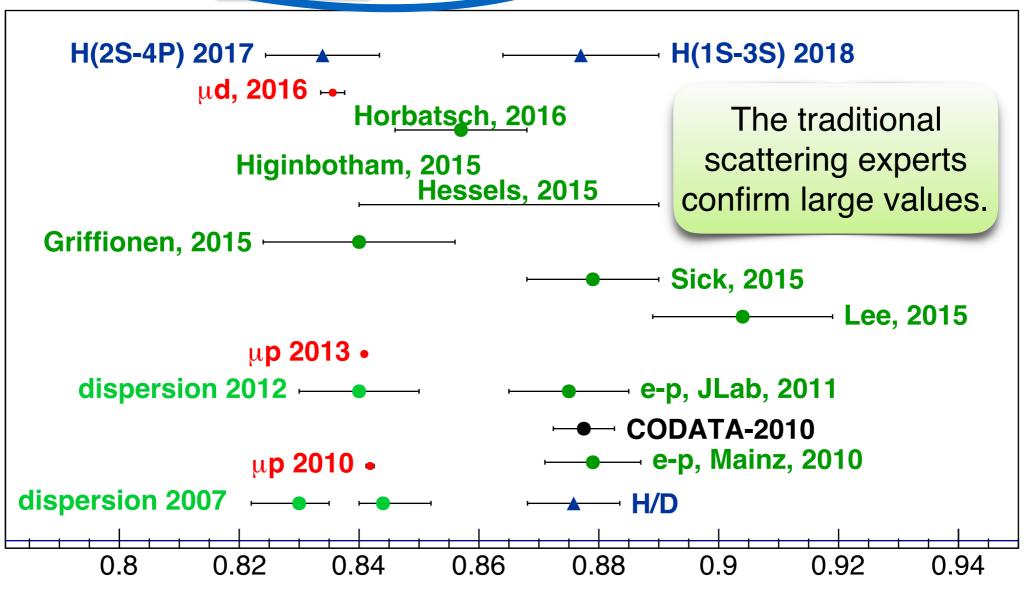


The proton charge radii on the market



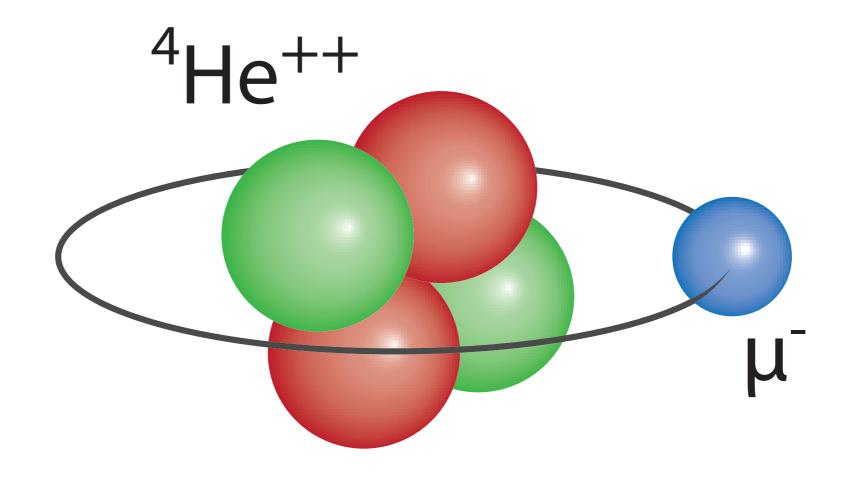
New value from e-p scattering will be soon available

Ashot et al., PRad, JLAB



Proton charge radius [fm]

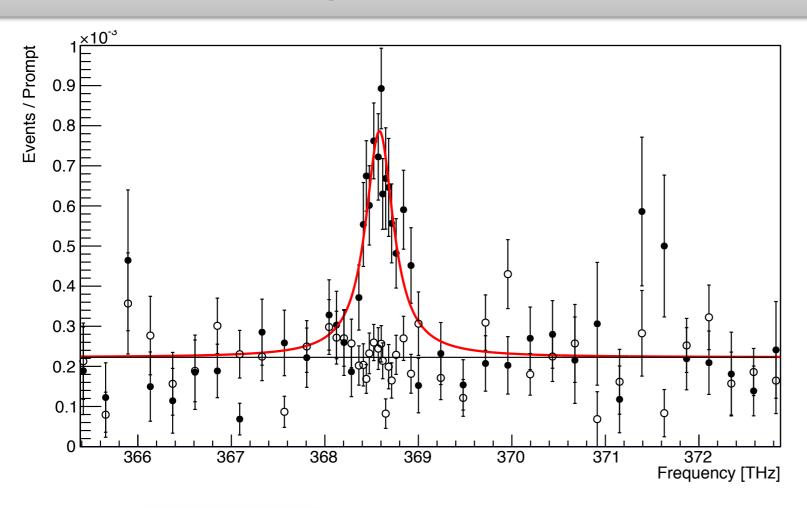


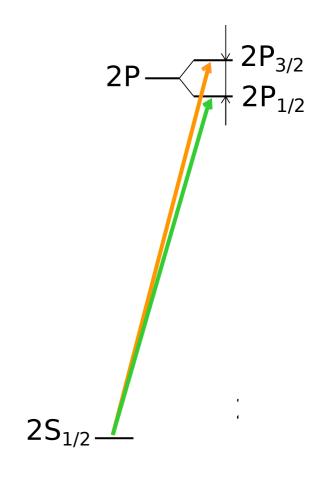






Spectroscopy of muonic Helium (µ4He+)





Experimental accuracy: 17 GHz (0.066 meV)

Statistics / Laser freq. / systematics unc.: 17 GHz / 100 MHz / 10 MHz

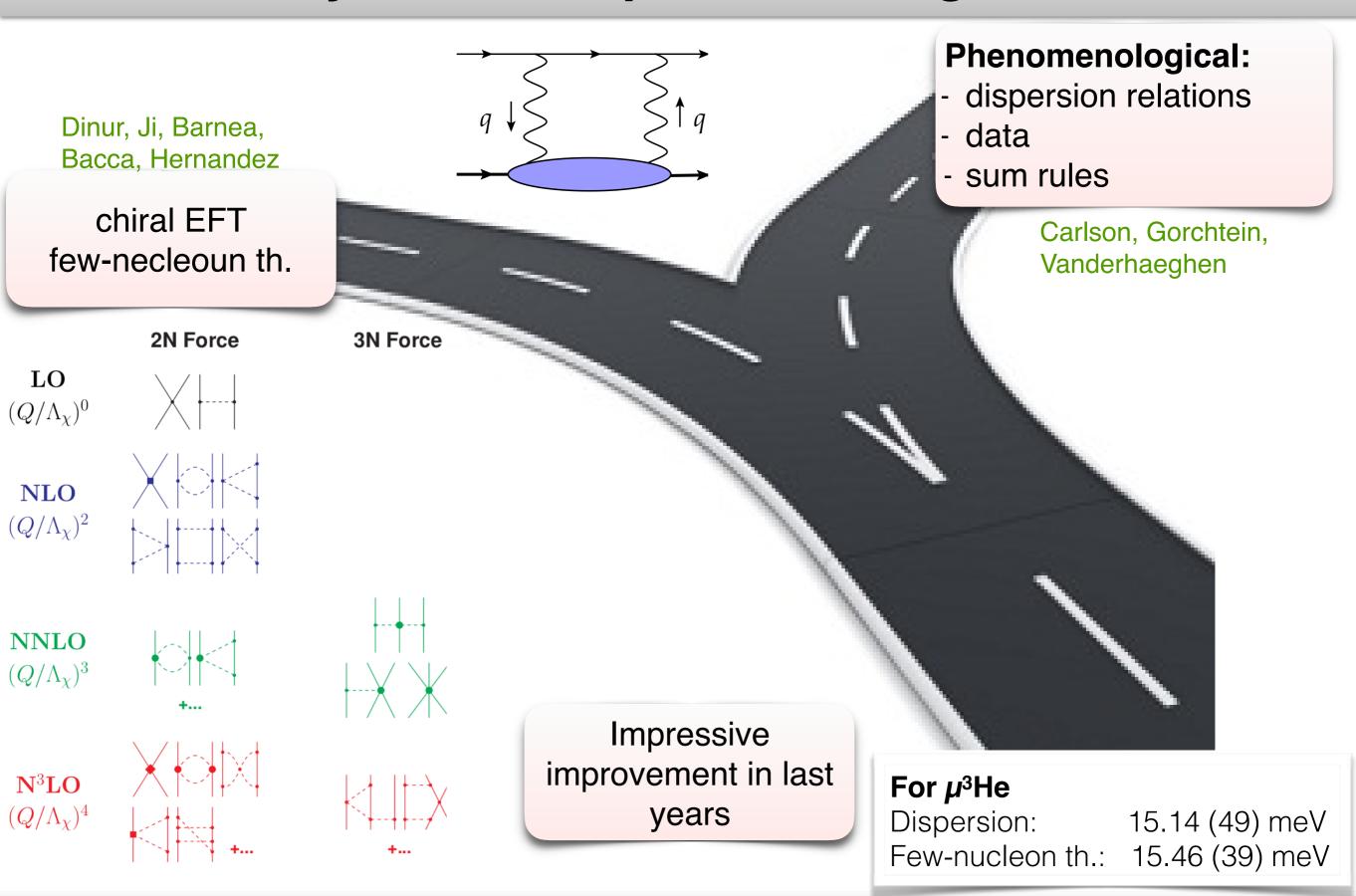
Theory uncertainty: 0.205 meV

Aldo Antognini

$$\Delta E(2S - 2P_{3/2}) = \underbrace{1668.487(14)}_{\text{QED}} - \underbrace{106.358(7)R_E^2}_{\text{finite size}} + \underbrace{6.761(77) + 3.296(189)}_{\text{TPE}} + \underbrace{146.197(12)}_{\text{fine splitting}} \text{ [meV]}$$



TPE: the key to extract precise charge radii



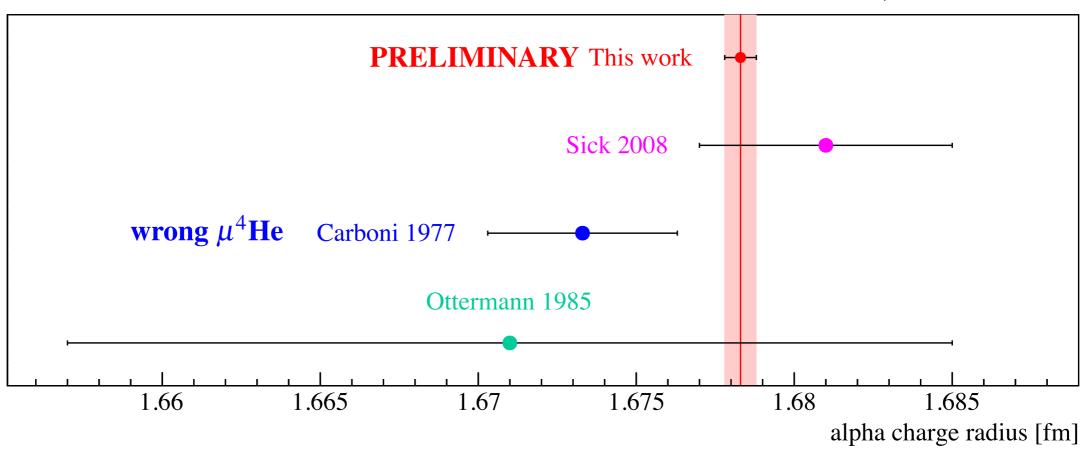




Alpha-particle radius from μ^4 He+ spectroscopy

$$R_E(^4\text{He}) = 1.67xxx(19)_{\text{exp}}(58)_{\text{theo}} \text{ fm}$$
 (muonic helium)
 $R_E(^4\text{He}) = 1.68100(400)$ fm (scattering)

Sick, arXiv1505.06924



Excellent agreement between scattering and muonic results

BSM contribution does not have to exceed 3 meV (1σ R_E (Sick, 2015) ⇒ 1.4 meV shift in muonic helium)



Impact of muonic helium (µHe) measurements

Constraints proton radius puzzle

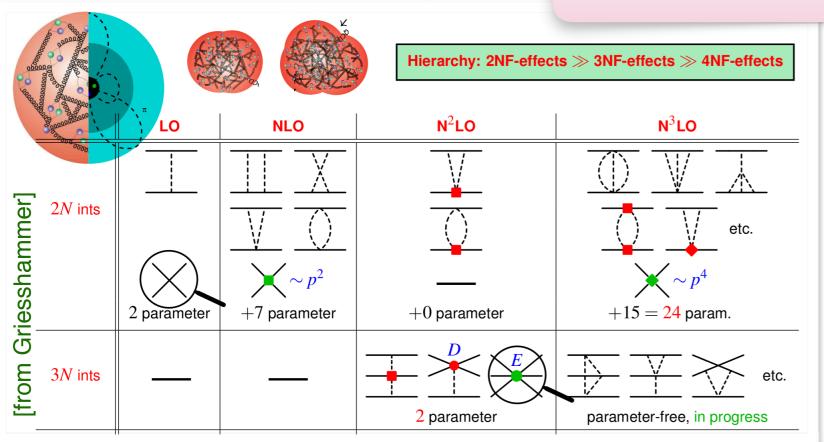
Antognini et al., Can. J. Phys. 89, 47 (2011)

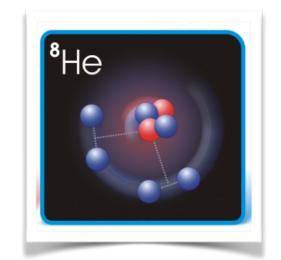
Benchmark for few-nucleon theories

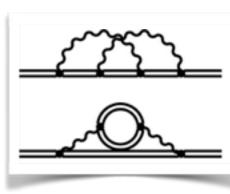
Improve absolute radii of ⁶He and ⁸He

Help understanding the ³He-⁴He charge radii difference

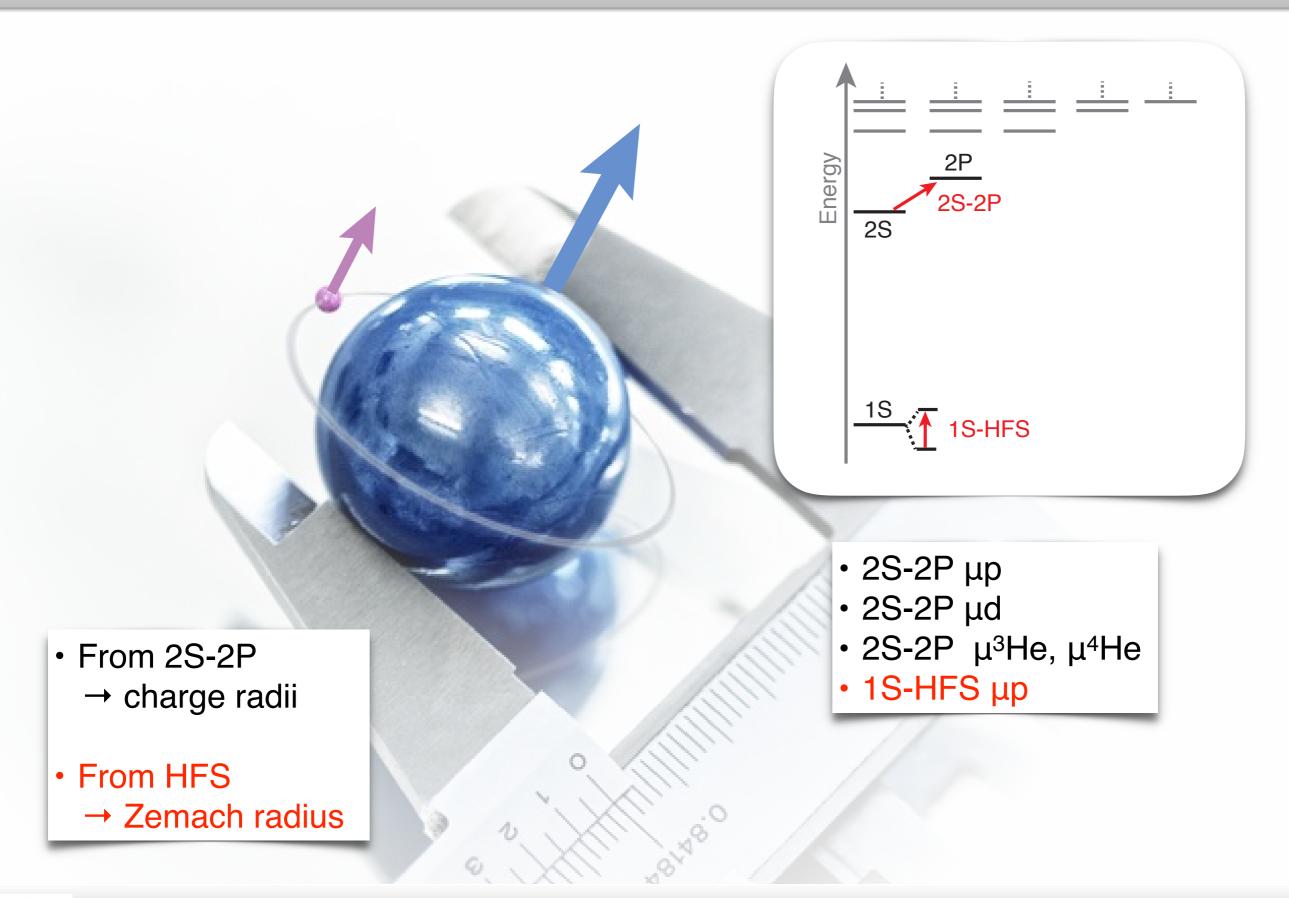
When combined with He and He+ spectroscopy: ⇒Enhanced bound-state QED test, extract R∞







From the 2S-2P to HFS measurements



Hyperfine splitting theory and goals

Measure

the 1S-HFS in μ p with 1-2 ppm accuracy

Goals

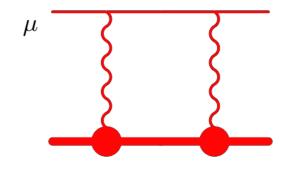
- TPE contribution with 3x10⁻⁴ rel. accuracy
- Zemach radius and polarisability contributions

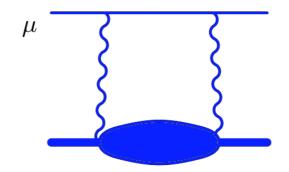
$$\Delta E_{\rm HFS}^{\rm th} = 183.788(7) + 1.0040 \Delta E_{\rm TPE} \,[{\rm meV}]$$

Pineda & Peset (2017)

$$\Delta E_{\text{TPE}} = \Delta E_{\text{Z}} + \Delta E_{\text{Recoil}} + \Delta E_{\text{pol}}$$

$$R_Z = \frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1 + \kappa} - 1 \right]$$







TPE: dispersion based approach

Elastic part (Zemach)

$$\Delta_{\rm Z} = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1 \right] \equiv -2Z\alpha m_r R_{\rm Z},$$

Distler, Bernauer, Sick

Recoil finite-size

$$\Delta_{\text{recoil}} = \frac{Z\alpha}{\pi(1+\kappa)} \int_0^\infty \frac{dQ}{Q} \left\{ \frac{8mM}{v_l + v} \frac{G_M(Q^2)}{Q^2} \left(2F_1(Q^2) + \frac{F_1(Q^2) + 3F_2(Q^2)}{(v_l + 1)(v + 1)} \right) - \frac{8m_r G_M(Q^2) G_E(Q^2)}{Q} - \frac{m}{M} \frac{5 + 4v_l}{(1 + v_l)^2} F_2^2(Q^2) \right\}.$$

Polarisability

$$\Delta_{
m pol.} = rac{Z lpha m}{2\pi (1+\kappa) M} \left[\delta_1 + \delta_2
ight]$$
 =

with:

$$\begin{split} \delta_1 &= 2 \int_0^\infty \frac{\mathrm{d}Q}{Q} \left(\frac{5 + 4v_l}{(v_l + 1)^2} \left[4I_1(Q^2)/Z^2 + F_2^2(Q^2) \right] + \frac{8M^2}{Q^2} \int_0^{x_0} \mathrm{d}x \, g_1(x, Q^2) \right. \\ &\left. \left\{ \frac{4}{v_l + \sqrt{1 + x^2 \tau^{-1}}} \left[1 + \frac{1}{2(v_l + 1)(1 + \sqrt{1 + x^2 \tau^{-1}})} \right] - \frac{5 + 4v_l}{(v_l + 1)^2} \right\} \right), \\ &= 2 \int_0^\infty \frac{\mathrm{d}Q}{Q} \left(\frac{5 + 4v_l}{(v_l + 1)^2} \left[4I_1(Q^2)/Z^2 + F_2^2(Q^2) \right] - \frac{32M^4}{Q^4} \int_0^{x_0} \mathrm{d}x \, x^2 g_1(x, Q^2) \right. \\ &\left. \left\{ \frac{1}{(v_l + \sqrt{1 + x^2 \tau^{-1}})(1 + \sqrt{1 + x^2 \tau^{-1}})(1 + v_l)} \left[4 + \frac{1}{1 + \sqrt{1 + x^2 \tau^{-1}}} + \frac{1}{v_l + 1} \right] \right\} \right) \end{split}$$

$$\delta_2 = 96M^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3} \int_0^{x_0} \mathrm{d}x \, g_2(x,Q^2) \left\{ \frac{1}{v_l + \sqrt{1 + x^2\tau^{-1}}} - \frac{1}{v_l + 1} \right\}. \quad \text{Hagelstein, Pascalutsa, Carlson, Martynenko, Tomalak}$$

Faustov, Vanderhaegen....





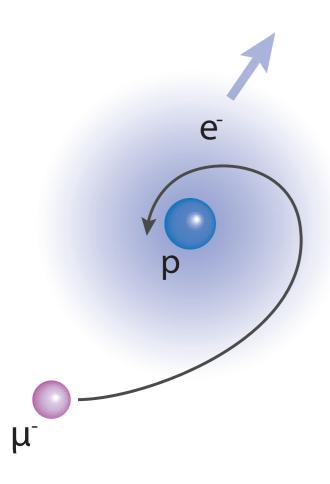
New g₁, g₂ data

almost available

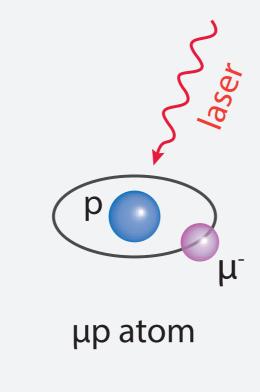
from JLAB

The principle of the µp HFS experiment

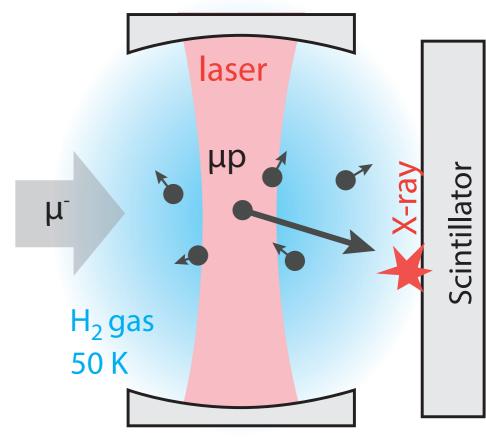
(1) Formation



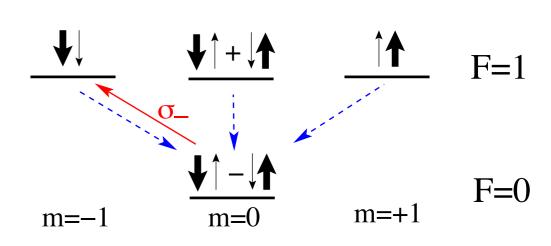
(2) Laser excitation





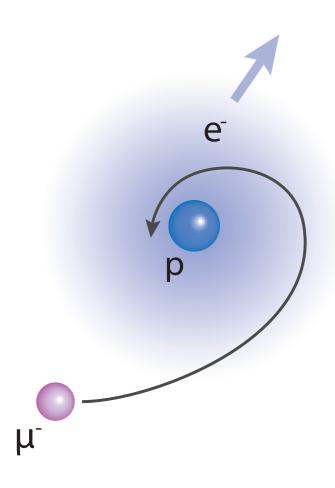


- Laser pulse: $\mu p(F=0) \longrightarrow \mu p(F=1)$
- Collision: $\mu p(F=1) + H_2 \longrightarrow H_2 + \mu p(F=0) + E_{kin}$
- Diffusion: the faster μp reach the target walls
- Resonance: plot number of X-rays vs. frequency

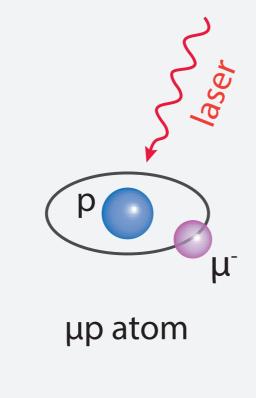


The principle of the µp HFS experiment

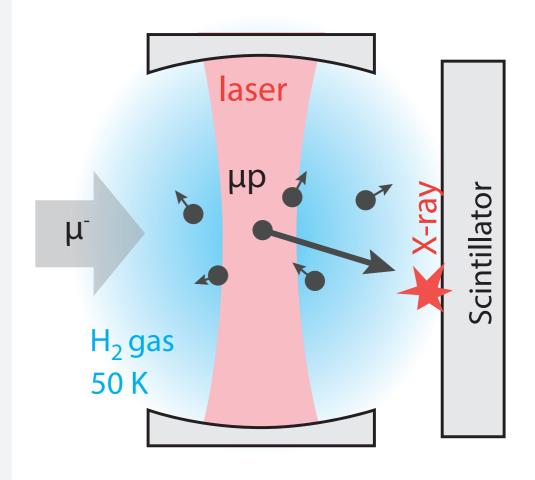
(1) Formation



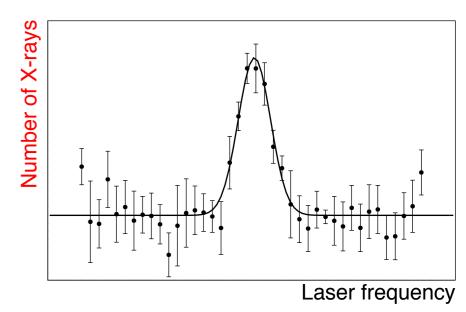
(2) Laser excitation



(3) Detection



- Laser pulse: $\mu p(F=0) \longrightarrow \mu p(F=1)$
- Collision: $\mu p(F=1) + H_2 \longrightarrow H_2 + \mu p(F=0) + E_{kin}$
- Diffusion: the faster μp reach the target walls
- Resonance: plot number of X-rays vs. frequency

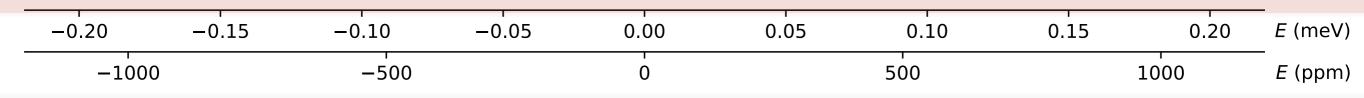


Uncertainties and scanning range

Large BG/Signal ratio

Narrow transition

Large scan range



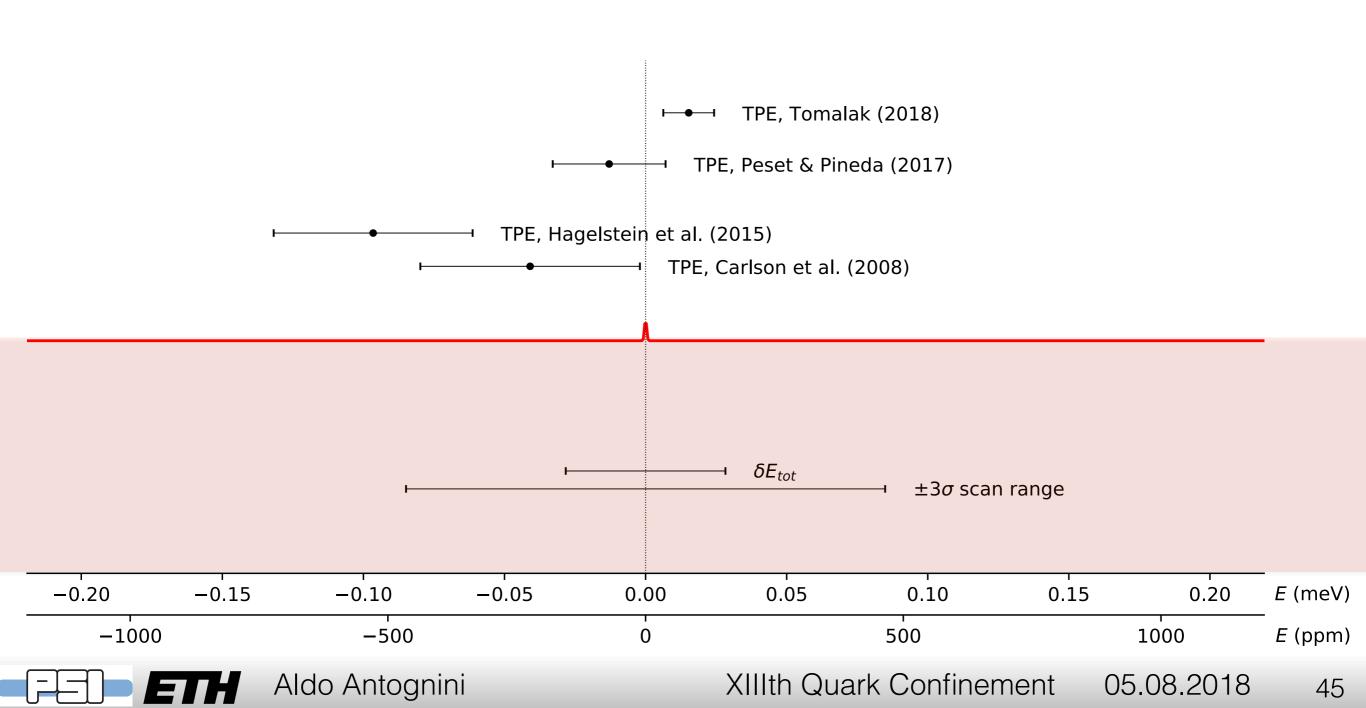




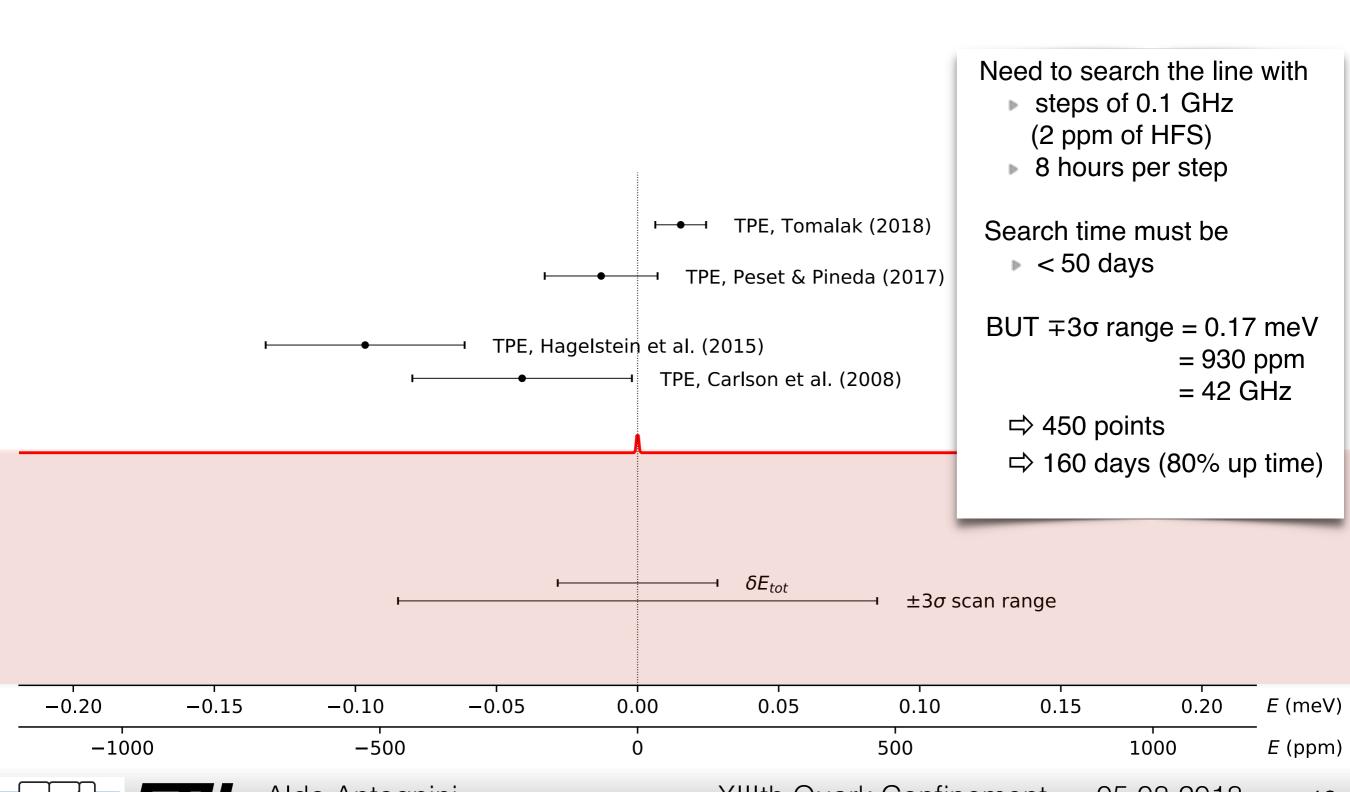
Uncertainties and scanning range

3γ exchange≈50 ppm

Kalinowski, Pachucki



Uncertainties and scanning range



Nucleon Spin Structure at low-Q: A hyperfine view

QED

- check QED contributions in H to improve the TPE(H)
- higher-order QED corrections in μp
- Summary of all contributions would be very helpful (at 1 ppm level).

Zemach radius

- improve determination of Zemach radius, mainly through magnetic FF
- Study correlations R_z vs R_p

Polarisability contribution

- re-evaluate the pol contribution given the new g₁ and g₂ data
- improve chPT prediction also in view of interpretation of HFS measurement
- subtraction term really absent?

A TPE contribution with an accuracy of 25 ppm of HFS is needed to find the line

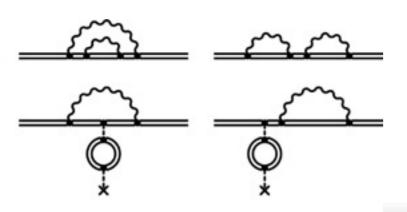


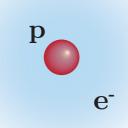


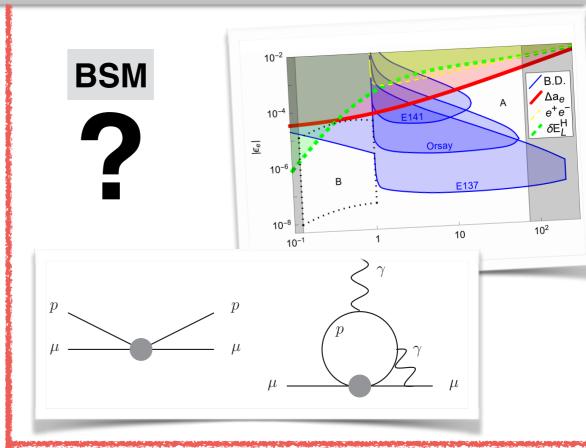
related to...

Chiral PT P 2N Force 3N Force $(Q/\Lambda_{\chi})^0$ Lattice NLO $(Q/\Lambda_{\chi})^2$ NNLO $(Q/\Lambda_{\chi})^3$

Bound-state QED, ions, molecules, Rydberg atoms and R_{∞}







New hadronic effects?

Dispersion-based approaches.

Polarizabilities, form factors and structure functions program

Analysis of e-p scattering

The same issues are critical for the HEP accelerator neutrino program.