Constraints on the QCD EOS from neutron-star mergers

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Plan of the talk

✴ The richness of merging binary NSs
✴ Anatomy of GW signal: frequencies and EOS
✴ Constraints from GW170817
✴ Quark-matter before/after merger
✴ Viscous effects at nuclear densities
✴ r-process nucleosynthesis and kilonovae
The two-body problem in GR

• For BHs we know what to expect:
  \[ \text{BH} + \text{BH} \rightarrow \text{BH} + \text{GWs} \]

• For NSs the question is more subtle: the merger leads to a hyper-massive neutron star (HMNS), i.e., a metastable equilibrium:
  \[ \text{NS} + \text{NS} \rightarrow \text{HMNS}+\ldots?\rightarrow \text{BH+torus+}\ldots?\rightarrow \text{BH + GWs} \]

• HMNS phase can provide clear information on EOS
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The two-body problem in GR

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  BH + BH $\rightarrow$ BH + GWs

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  NS + NS $\rightarrow$ HMNS+... $?\rightarrow$ BH+torus+... $?\rightarrow$ BH + GWs

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The equations of numerical relativity

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}, \quad \text{(field equations)} \]

\[ \nabla_\mu T^{\mu\nu} = 0, \quad \text{(cons. energy/momentum)} \]

\[ \nabla_\mu (\rho u^\mu) = 0, \quad \text{(cons. rest mass)} \]

\[ p = p(\rho, \epsilon, Y_e, \ldots), \quad \text{(equation of state)} \]

\[ \nabla_\nu F^{\mu\nu} = I^\mu, \quad \nabla^*_\nu F^{\mu\nu} = 0, \quad \text{(Maxwell equations)} \]

\[ T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{\text{EM}}_{\mu\nu} + \ldots \quad \text{(energy – momentum tensor)} \]

In GR these equations do not possess an analytic solution in the regimes we are interested in.
merger $\rightarrow$ HMNS $\rightarrow$ BH + torus
How to constrain the EOS from the GWs
Anatomy of the GW signal
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binary black holes (2006)

Chirp signal
Anatomy of the GW signal

Chirp signal

black-hole ringdown

binary black holes (2006)
Anatomy of the GW signal

$\text{GNH3, } \bar{M} = 1.350M_\odot$
Anatomy of the GW signal

**Inspiral**: well approximated by PN/EOB; tidal effects important
Anatomy of the GW signal

Merger: highly nonlinear but analytic description possible
Anatomy of the GW signal

*post-merger* (HMNS)

**post-merger**: quasi-periodic emission of bar-deformed HMNS

GNH3, $\tilde{M} = 1.350 M_\odot$
Anatomy of the GW signal

Collapse-ringdown: signal essentially shuts off.
What we can do nowadays

Extracting information from the EOS

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There are lines! Logically not different from emission lines from stellar atmospheres.
This is GW spectroscopy!
A new approach to constrain the EOS

A spectroscopic approach to the EOS

Semi-analytical modelling and MC analysis

Modelling the post-merger signal analytically, it is possible to run Monte Carlo simulations and estimate error in radius

- stiff EOSs: $|\Delta R/\langle R \rangle| < 10\%$ for $N \sim 20$ detections
- soft EOSs: $|\Delta R/\langle R \rangle| \sim 10\%$ for $N \sim 50$ detections
- soft EOSs will inevitably have larger uncertainties
- golden binary: SNR $\sim 6$ at 30 Mpc
  \[ |\Delta R/\langle R \rangle| \lesssim 2\% \text{ at 90\% confidence} \]
GW170817, maximum mass, radii and tidal deformabilities

LR, Most, Weih (2018)
Most, Weih, LR, Schaffner-Bielich (2018)
The outcome of GW170817

- The product of GW170817 was likely a hypermassive star, i.e. a differentially rotating object with initial gravitational mass \( M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot \)

- Sequences of equilibrium models of nonrotating stars will have a maximum mass: \( M_{\text{TOV}} \)

- This is true also for uniformly rotating stars at mass shedding limit: \( M_{\text{max}} \)

- \( M_{\text{max}} \) simple and quasi-universal function of \( M_{\text{TOV}} \) (Breu & LR 2016)

\[
M_{\text{max}} = \left(1.20^{+0.02}_{-0.05}\right) M_{\text{TOV}}
\]
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• **Salmon** region is for differentially rotating equilibrium models.
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The outcome of GW170817

- GW170817 produced object as "x"; GRB implies a BH has been formed: "x" followed two possible tracks: fast (2) and slow (1).

- It rapidly produced a BH when still differentially rotating (2).

- It lost differential rotation leading to a uniformly rotating core (1).

- (1) is more likely because of large ejected mass (long lived).

- Final mass is near $M_{\text{max}}$ and we know this is universal!
let’s recap…

• The merger product of GW170817 was initially **differentially** rotating but collapsed as **uniformly** rotating object.

• Measured **gravitational mass** of GW170817
  
  \[ M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot \]

• Ejected **rest mass** deduced from kilonova emission
  
  \[ M_{\text{ej}}^{\text{blue}} = 0.014^{+0.010}_{-0.010} M_\odot \]

• Use **universal relations** and account errors to obtain

\[ 2.01^{+0.04}_{-0.04} \leq M_{\text{TOV}}/M_\odot \lesssim 2.16^{+0.17}_{-0.15} \]
Limits on radii and deformabilities

• Constraining NS radii of neutron stars is an effort with thousands of papers published over the last 40 years.
• Question is deeply related with EOS of nuclear matter.
• Can new constraints be set by GW170817?

• Ignorance can be parameterised and EOSs can be built arbitrarily as long as they satisfy specific constraints on low and high densities.
Mass-radius relations

- We have produced $10^6$ EOSs with about $10^9$ stellar models.

- Can impose differential constraints from the maximum mass and from the tidal deformability from GW170817.
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one-dimensional cuts

- Closer look at a mass of $M = 1.40 \, M_\odot$
- Can play with different constraints on maximum mass and tidal deformability.
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• Overall distribution is very robust

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$R_{1.4} = 12.45 \, \text{km}$

- When considering strong phase transitions, small stars can be found

$8.53 < R_{1.4}/\text{km} < 13.74$

$\bar{R}_{1.4} = 13.06 \, \text{km}$
Constraining tidal deformability

- Can explore statistics of all properties of our $10^9$ models.
- In particular can study PDF of tidal deformability: $\tilde{\Lambda}$

- LIGO has already set upper limit:
  $$\tilde{\Lambda}_{1.4} \lesssim 800$$
- Our sample naturally sets a lower limit:
  $$\tilde{\Lambda}_{1.4} > 375$$
Constraining tidal deformability: PTs

- Can repeat considerations with EOSs having PTs
- Lower limit much weaker: $\tilde{\Lambda}_{1.4} \gtrsim 35$
- Large masses have sharp cut-off on upper limit:
  $\tilde{\Lambda}_{1.7} \lesssim 460$

Hence, detection with $\tilde{\Lambda}_{1.7} \sim 600$ would rule out twin stars!
Phase transitions and their signatures

Most, Papenfort, Dexheimer, Hanauske, Schramm, Stoecker, LR (2018)
$t - t_{mgr} = -2.5 \text{ ms}$
• EOS based on Chiral Mean Field (CMF) model, based on a nonlinear SU(3) sigma model.
• Quarks appear at sufficiently large temperatures and densities.
• For EOS without quarks, the dynamics is very similar, but no PT.
Comparing with the phase diagram

- Reported are the evolution of the max. temperature and density.
- Quarks appear already early on, but only in small fractions.
- Once sufficient density is reached, a full phase transition takes place.
Gravitational-wave emission

“low-mass” binary

“high-mass” binary

- **In low-mass binary**, after \( \sim 5 \) ms, quark fraction is large enough to change quadrupole moment and yield differences in the waveforms.
- **In high-mass binary**, phase transition takes place rapidly after \( \sim 5 \) ms. Waveforms are similar but ringdown is different (free fall for PT).
- Mismatch between inspiral and postmerger: clear **signature** of a PT.
Conclusions

✴ Spectra of post-merger shows peaks, some "quasi-universal".
✴ When used together with tens of observations, they will set tight constraints on EOS: radius known with $\sim 1$ km precision.
✴ GW170817 provided new limits on maximum mass and radii:

$$2.01^{+0.04}_{-0.04} \leq \frac{M_{\text{TOV}}}{M_{\odot}} \lesssim 2.16^{+0.17}_{-0.15}$$

$$12.00 < \bar{R}_{1.4}/\text{km} < 13.45 \quad \bar{R}_{1.4} = 12.45 \text{ km}$$

$$8.53 < \bar{R}_{1.4}/\text{km} < 13.74 \quad \bar{R}_{1.4} = 13.06 \text{ km}$$

✴ Phase transition can take place after merger leading to clear signatures: mismatch between inspiral and postmerger.

Neutron stars and their mergers have great potential to provide complementary constraints on QCD. More to come…