structure of hybrid static potential flux tubes in lattice Yang-Mills-theory

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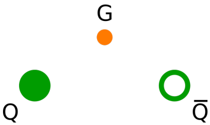
outline

Hybrid static potentials

- Quantum numbers
- Modification of Wilson loop
- Hybrid static potentials



[K. J. Juge, J. Kuti and C. J. Morningstar, "Gluon excitations of the static quark potential and the hybrid quarkonium spectrum," Nucl. Phys. Proc. Suppl. 63, 326 (1998) [hep- lat/9709131]]
[C. Reisinger, S. Capitani, O. Philipsen, M. Wagner : "Computation of hybrid static potentials in SU(3) lattice gauge theory", arXiv:1708.05562 [hep-lat]]

Chromoelectric and Chromomagnetic field

- Computation on the lattice
- Results for ordinary and hybrid static $Q\bar{Q}$

[Lasse Müller, Marc Wagner: "Structure of hybrid static potential flux tubes in SU(2) lattice Yang-Mills theory", (2018) arXiv:1803.11124 [hep-lat]]

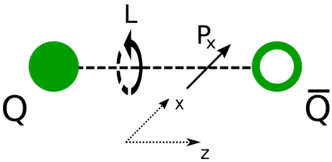
Similar existing work with partially different results:
[M. Cardoso, N. Cardoso, P. Bicudo: "Colour fields of the quark-antiquark excited flux tube", (2018) arXiv:1803.04569 [hep-lat]]

hybrid static potential quantum numbers

Excited gluons contribute to quantum numbers. Hybrid static potentials can be characterized by the following quantum numbers:

- Total angular momentum with respect to the separation axis z: $\Lambda \in \{\Sigma \doteq 0, \Pi \doteq 1, \Delta \doteq 2\}$
- The combination of parity and charge conjugation $(P \circ C)$ $\epsilon \in \{g \doteq +, u \doteq -\}$
- The spatial inversion along an axis perpendicular to z (P_x): $\eta \in \{+, -\}$

E. g.: ordinary $Q\bar{Q}$ corresponds to $\Lambda_\epsilon^\eta = \Sigma_g^+$
$\Rightarrow$ Modification of Wilson loop according to quantum numbers



ordinary Wilson loop

trial state for a ordinary pair of static quarks

$$|\Psi\rangle = \hat{O}\,|\Omega\rangle$$

with

$$\hat{O} = \hat{\bar{Q}}(-R/2)\hat{S}_z\hat{Q}(R/2)$$

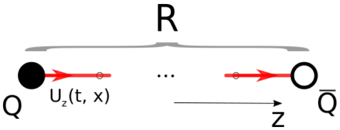
temporal correlator derived via path integral formalism

$$\langle\Psi(t)|\Psi(0)\rangle \propto \langle W\rangle_U \underset{t\to\infty}{\propto} e^{-V_{Q\bar{Q}}t}$$

with the Wilson loop

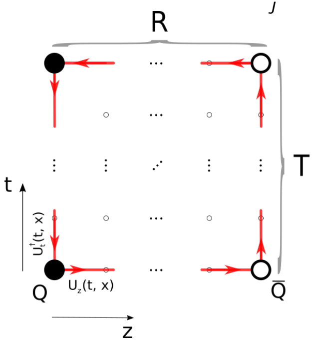
$$W = \mathrm{Tr}\left(S_z \cdot S_t \cdot S_z^\dagger \cdot S_t^\dagger\right)$$

and $\langle\ldots\rangle_U$ an ensemble of gauge link configurations distributed according to e^{-S}



link variable: $U_\mu(t,x) = e^{igaA_\mu(t,x)}$

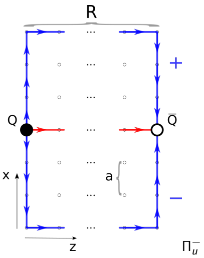
path of links: $S = \prod_j U_j$



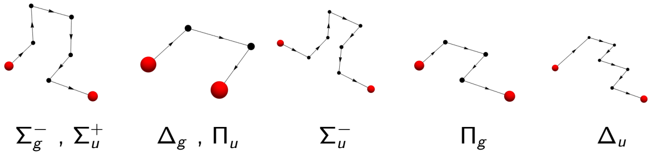
modified Wilson loops to realize hybrid quantum numbers

A trial state with defined quantum numbers Λ_ϵ^η is constructed by

$$|\Psi_{hybrid}\rangle_{S,\Lambda_\epsilon^\eta} = \frac{1}{4} \underbrace{\left(1+\eta(\mathcal{P}\circ\mathcal{C})+\epsilon\mathcal{P}_x+\eta\epsilon(\mathcal{P}\circ\mathcal{C})\mathcal{P}_x\right)}_{\text{behavior under } (\mathcal{P}\circ\mathcal{C}) \text{ and } \mathcal{P}_x \text{ transformations}} \underbrace{\sum_{k=0}^{3} \exp\left(i\Lambda k\frac{\pi}{2}\right)\hat{R}\left(k\frac{\pi}{2}\right)}_{\text{rotations weighted according to } \Lambda} \hat{O}\,|\Omega\rangle$$

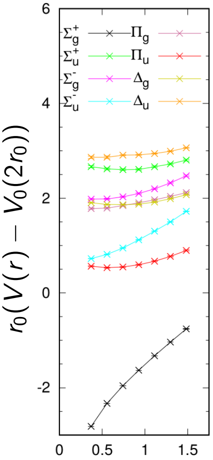


with $\hat{O}$ as before but with a more complex spatial path between quark and antiquark

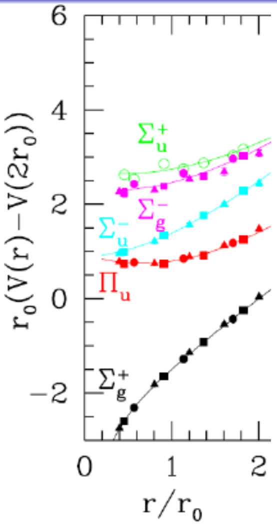


optimized operators to construct hybrid quantum number trial states

hybrid static potentials from optimized operators



Our results



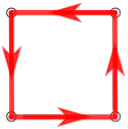
Previous results
[K.J.Juge, J. Kuti, C. Morningstar.
Gluon excitations of the static quark potential and the hybrid quarkonium spectrum.
Nucl.Phys.Proc.Suppl., 63:326-331, 1998 [hep-lat/9709131]]

- optimized operators with large ground state overlaps
- plateaus are reached at small t with favorable signal-to-noise ratio
- we observe lower-lying potentials
- qualitative difference in the Σ_u^+ potential

chromoelectric and chromomagnetic field

On the lattice field strength tensor corresponds to plaquette

$$P_{\mu\nu} = \mathrm{Tr}\left[e^{igaF_{\mu\nu}}\right] \Rightarrow Tr\left(F_{\mu\nu}^2\right) \approx \frac{2}{g^2a^2}\left(2 - P_{\mu\nu}\right)$$



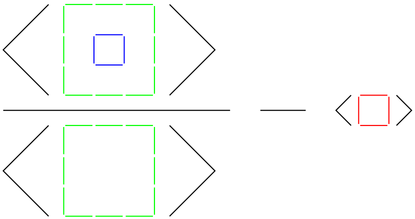
plaquette

$\Rightarrow E^2$ and B^2 are gauge invariant quantities

$$\Delta B_j^2 \equiv \langle B_j(\vec{x})^2\rangle_{Q\bar{Q}} - \langle B_j^2\rangle_{vac} = \frac{2}{g^2a^2}\left[\langle P_{kl}\rangle - \frac{\langle W \cdot P_{kl}(T/2,\vec{x})\rangle}{\langle W\rangle}\right]$$

$$\Delta E_j^2 \equiv \langle E_j(\vec{x})^2\rangle_{Q\bar{Q}} - \langle E_j^2\rangle_{vac} = \frac{2}{g^2a^2}\left[\frac{\langle W \cdot P_{0j}(T/2,\vec{x})\rangle}{\langle W\rangle} - \langle P_{0j}\rangle\right]$$

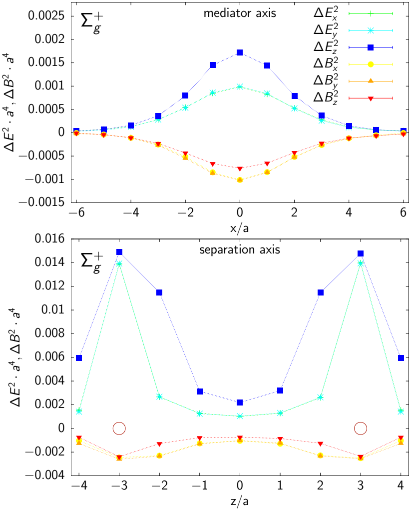
$\langle E^2\rangle$ in presence of $Q\bar{Q}$

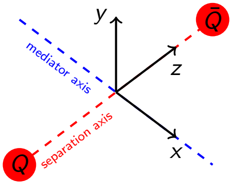


$\langle E^2\rangle$ in vacuum

ordinary static potential flux tubes

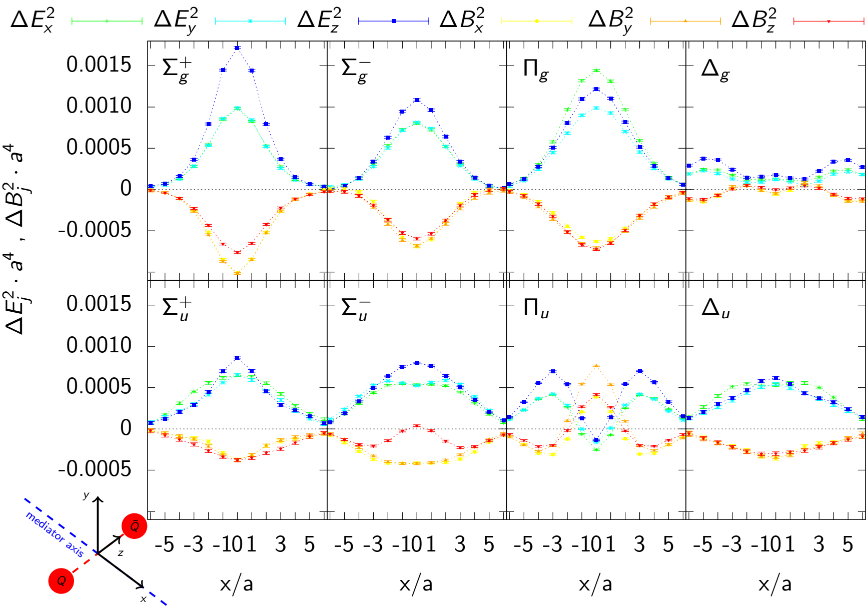
Computed in SU(2) lattice gauge theory with lattice spacing $a \approx 0.073 fm$



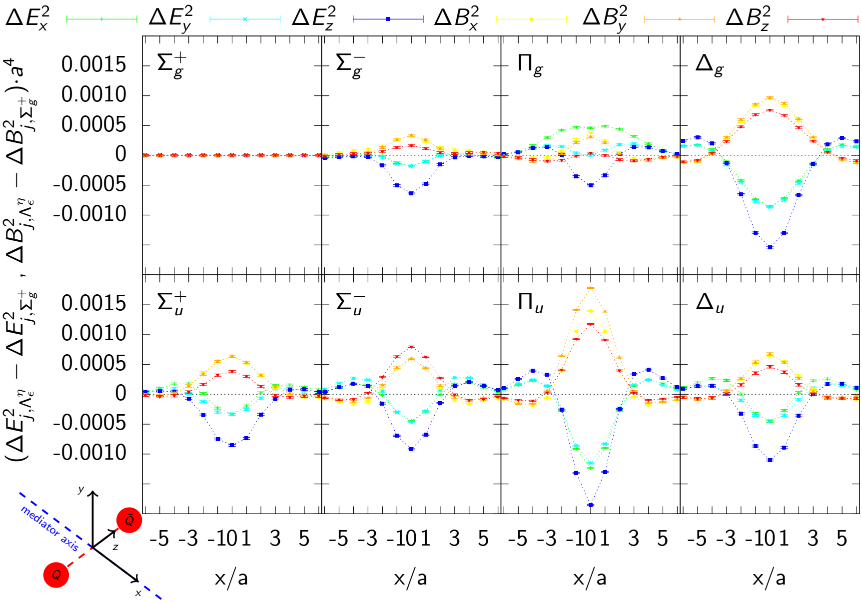


strong discretization errors near charge positions (energy density $\rightarrow \infty$ there) most interesting results on mediator axis.

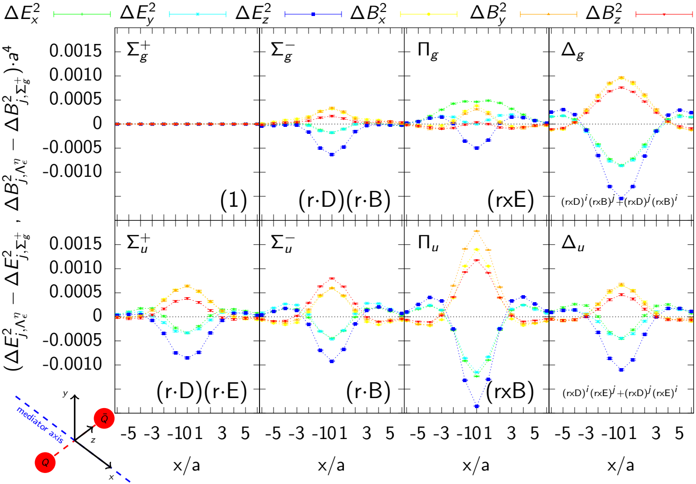
hybrid static potential flux tubes on mediator axis



difference to the ordinary static potential on mediator axis

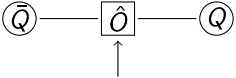


operators for gluonic excitations between static quarks in pNRQCD



Comparison to paper from pNRQCD:

[N. Brambilla, A. Pineda, J. Soto and A. Vairo: "Effective field theories for heavy quarkonium" (2005) [hep-ph/0410047]]

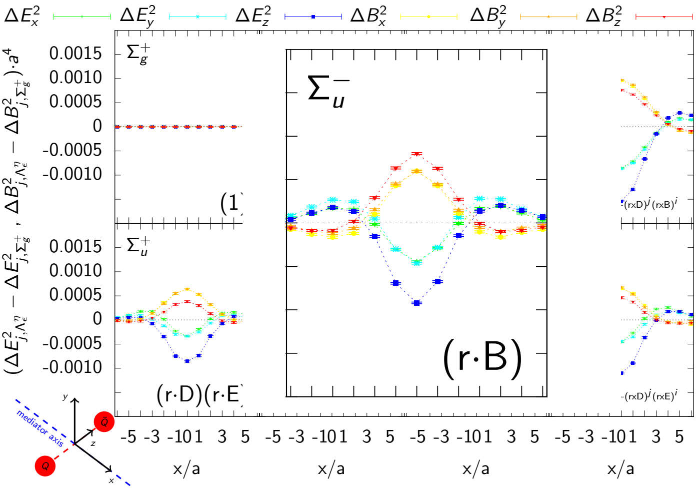


local excitation operator

- · : scalar product
- x : cross product
- D : covariant derivative

$r = (0, 0, R)^T$

operators for gluonic excitations between static quarks in pNRQCD



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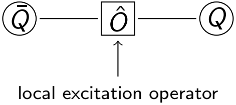

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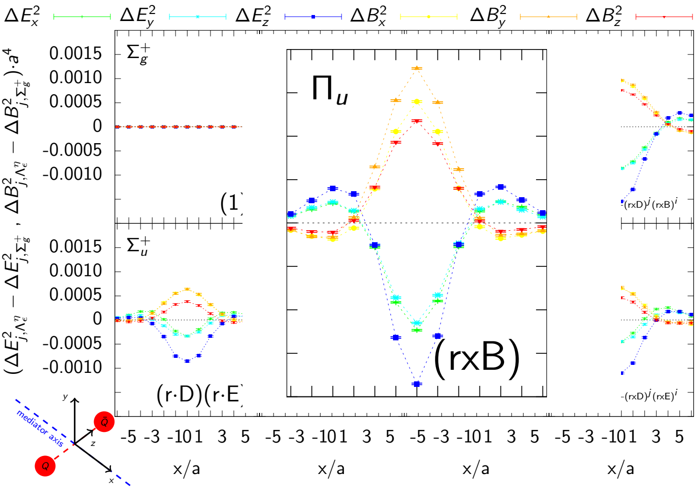

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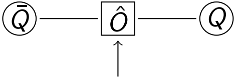

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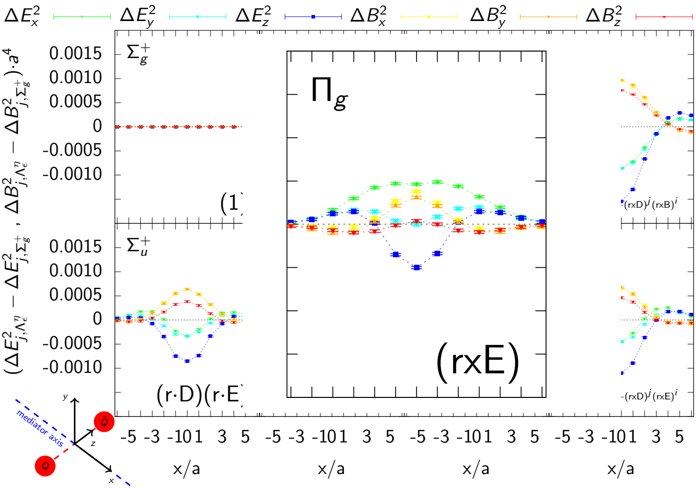

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$\bar{Q}$ — $\hat{O}$ — Q

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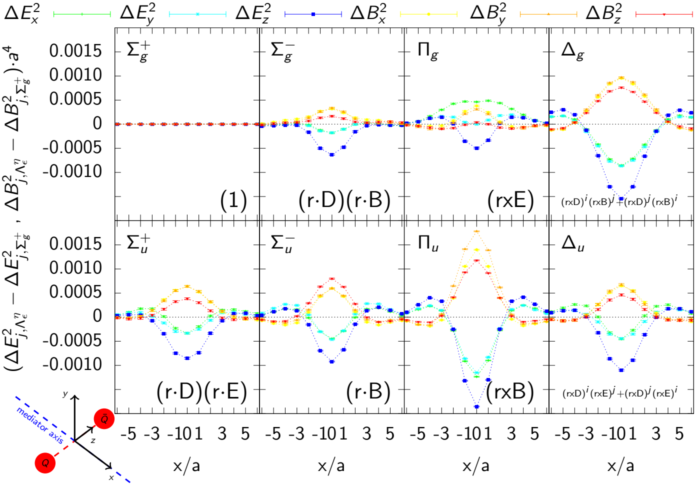

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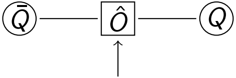

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conclusion and outlook

Very recently flux tubes for hybrid static potentials have been investigated for the first time in
[M. Cardoso, N. Cardoso, P. Bicudo: "Colour fields of the quark-antiquark excited flux tube", (2018) arXiv:1803.04569 [hep-lat]]
and
[Lasse Müller, Marc Wagner: "Structure of hybrid static potential flux tubes in SU(2) lattice Yang-Mills theory", (2018) arXiv:1803.11124 [hep-lat]]
The only major difference in the computations were SU(3) rather than SU(2) lattice gauge theory. Still there are some qualitative discrepancies in the results.

Our studies involved treating the E- and B-field components separately which led to results consistent with analytic approaches in
[N. Brambilla, A. Pineda, J. Soto and A. Vairo: "Effective field theories for heavy quarkonium" (2005) [hep-ph/0410047]].

The next step will be to compute chromoelectric and chromomagnetic field strength components in SU(3) Lattice gauge theory