Application of new anomaly to QCD vacua

Yuya Tanizaki

RIKEN BNL Research Center, Brookhaven National Laboratory

August 3, 2018 @ Maynooth

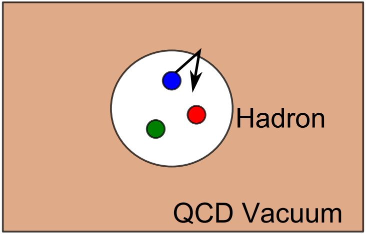
References: 1807.07666[hep-th]

Contents

Chiral symmetry breaking in QCD

Heuristic argument (like the bag model) indicates that

$$\text{Confinement} \Rightarrow \text{Chiral symmetry breaking}$$



How much can we say about chiral symmetry breaking based on QCD?

Anomaly matching

Definition

't Hooft anomaly $\equiv$ Global symmetry that cannot be gauged.

Theorem

't Hooft anomaly is renormalization-group invariant. ('t Hooft, '80)

Classic Example of 't Hooft anomaly

Massless QCD has chiral symmetry $SU(N_f)_L \times SU(N_f)_R \times U(1)_V$ with 't Hooft anomaly ($\neq$ quantum anomaly):

$$D^\mu J^a_\mu = 0, \quad D^\mu J^{5a}_\mu = \frac{N_\mathrm{c}}{4\pi^2} \mathrm{tr}\left(T^a \left\{F_V^2 + \frac{1}{3}F_A^2 + \cdots\right\}\right)$$

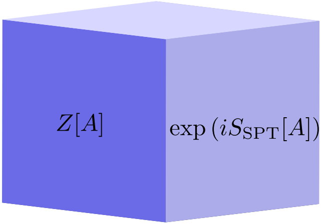
Conservation of J^{5a} is violated by background flavor gauge fields.

"Modern proof" of anomaly matching

A: Background gage field for global symmetry G
$A \mapsto A + \mathrm{d}_A\theta$: G-gauge transformation, but this has an anomaly:

$$Z[A + \mathrm{d}_A\theta] = Z[A]\exp(\mathrm{i}\mathcal{A}[A, \theta]).$$

Combined system with (d+1)D symmetry-protected topological (SPT) phase is G-gauge invariant.



$\Rightarrow$ Our system can be regarded as the surface state of SPT phase, and must be nontrivial.

Perturbative anomaly matching for massless QCD

QCD with massless quarks has the following 't Hooft anomaly:

$$D^\mu J^{5a}_\mu = \frac{N_\mathrm{c}}{4\pi^2}\mathrm{tr}\left(T^a\left\{F_V^2 + \frac{1}{3}F_A^2 + \cdots\right\}\right).$$

Assuming SSB $SU(N_\mathrm{f})_L \times SU(N_\mathrm{f})_R \to SU(N_\mathrm{f})_V$, the pion EFT can reproduce this anomaly by adding the WZ term:

$$\frac{N_\mathrm{c}}{240\pi^2}\int \mathrm{tr}[(U^\dagger \mathrm{d}U)^5].$$

Chiral symmetry breaking is a natural candidate to match the 't Hooft anomaly.

Ordinary and exotic chiral symmetry breaking

Ordinary scenario:

- Order parameter is quark bilinear: $U = \overline{\psi}_L \psi_R$.
- $SU(N_\mathrm{f})_L \times SU(N_\mathrm{f})_R \to SU(N_\mathrm{f})_V$.
- This is chosen in our universe.

Exotic scenario (Stern phase): (Stern, 97, 98)

- Order parameter is quark quartic: $O = \sum_a \mathrm{tr}(T^a U)\mathrm{tr}(T^a U^\dagger)$.
- $SU(N_\mathrm{f})_L \times SU(N_\mathrm{f})_R \to SU(N_\mathrm{f})_V \times (\mathbb{Z}_{N_\mathrm{f}})_L$.
- This is ruled out at the zero-density QCD by QCD inequality. (Kogan, Kovner, Shifman, 98)
- At finite densities, it was unknown if it can exist. Sign problem invalidates both numerical works and QCD inequalities.

Purpose of this talk

We rule out the exotic scenario based only on symmetry and anomaly.

Symmetry of massless QCD

QCD Lagrangian:

$$S = \frac{1}{2g^2}\int \mathrm{tr}(G \wedge * G) + \int \sum_{f=1}^{N_F} \overline{q}_f \gamma_\mu D_\mu q.$$

Symmetry of massless QCD (i.e. Lagrangian + $\mathcal{D}\overline{q}\mathcal{D}q$):

$$G = \frac{SU(N_F)_L \times SU(N_F)_R \times U(1)_V}{\mathbb{Z}_{N_C} \times (\mathbb{Z}_{N_F})_V}$$

- For later purpose, I evade the double counting correctly
- Due to the gauge invariance, symmetry must be divided by $\mathbb{Z}_{N_C} \subset SU(N_C) \times U(1)_V$: Correct $U(1)$ symmetry is $U(1)_B = U(1)_V / \mathbb{Z}_{N_c}$ but not $U(1)_V$.

Further comments on chiral symmetry

$U(1)_A$ is explicitly broken to $(\mathbb{Z}_{2N_F})_A$ by instantons (or quantum anomaly).

This is a subgroup of the continuous flavor symmetry:

$$(\mathbb{Z}_{2N_F})_A \subset SU(N_F)_L \times SU(N_F)_R \times U(1)_V.$$

Indeed, the generator of $(\mathbb{Z}_{2N_F})_A$ can be written as

$$\mathrm{e}^{\frac{2\pi}{2N_F}\mathrm{i}\gamma_5}\mathbf{1}_{N_F} = \exp\left(\frac{2\pi}{N_F}\mathrm{i}\frac{1+\gamma_5}{2}\mathrm{diag}[1,\dots,1,1-N_F]\right)\mathrm{e}^{-2\pi\mathrm{i}/(2N)}$$

Quick message: SSB of $(\mathbb{Z}_{2N_F})_A$ implies continuous ChSB, but the converse is not true.

Contents of background gauge fields

To emphasize the role of discrete axial symmetry, we look at

$$G^{\rm sub} = \frac{SU(N_F)_V \times U(1)_V}{(\mathbb{Z}_{N_{\rm c}}) \times (\mathbb{Z}_{N_{\rm f}})} \times (\mathbb{Z}_{N_{\rm f}})_L.$$

To detect the 't Hooft anomaly of $G^{\rm sub}$, we introduce the background gauge fields (1710.08923, 1711.10487) (ref. Kapustin, Seiberg (2014));

- $SU(N_F)_V$ one-form gauge field: A_f
- $U(1)_V$ one-form gauge field: A_V
- $(\mathbb{Z}_{N_{\rm f}})_L$ one-form symmetry: A_χ
- $(\mathbb{Z}_{N_C})$ two-form gauge field: B_c
- $(\mathbb{Z}_{N_F})$ two-form gauge field: B_f

Role of two-form gauge fields

Why do we need two-form gauge fields?

Gauging $U(1)_V/\mathbb{Z}_{N_c}$
Gauge group $SU(N_c)$ and the $U(1)_V$ phase rotation of quark have the common elements $\mathbb{Z}_{N_c}$ since q has the charge $(\Box, 1)$.

$$\begin{aligned} & \text{Gauging } U(1)_B = U(1)_V/\mathbb{Z}_{N_c} \\ \Leftrightarrow \quad & U(1)_V \text{ gauge field} + \text{constraint: possible charges are } (\Box^k, k). \end{aligned}$$

B_c gives the constraint on the possible gauge charges correctly!
The $U(1)_B$ gauge field A_B is given by

$$\mathrm{d}A_B = N_c(\mathrm{d}A_V + B_c).$$

Discrete 't Hooft anomaly for massless QCD

After introducing the background gauge fields, the partition function $\mathcal{Z}_{\rm QCD}$ is no longer gauge invariant.

Gauge invariance requires to add 5d SPT phase:

$$\mathcal{Z}_{\rm QCD}\exp\left({N_{\rm f}\over (2\pi)^2}\int A_\chi\wedge \mathrm{d}A_B\wedge B_{\rm f}\right).$$

This says that the baryon number conservation is anomalously violated under $(\mathbb{Z}_{N_{\rm f}})_L$ and $SU(N_{\rm f})/\mathbb{Z}_{N_{\rm f}}$ gauge field:

$$\partial_\mu J^\mu_B={N_{\rm f}\over (2\pi)^2}\mathrm{d}A_\chi\wedge B_{\rm f}.$$

Skyrmions in ordinary chiral SSB

Let us see how the discrete anomaly is matched for ordinary case:

$$G=\frac{SU(N_\mathrm{f})_L\times SU(N_\mathrm{f})_R\times U(1)_V}{\mathbb{Z}_{N_\mathrm{c}}\times\mathbb{Z}_{N_\mathrm{f}}}\to H=\frac{SU(N_\mathrm{f})_V\times U(1)_V}{\mathbb{Z}_{N_\mathrm{c}}\times\mathbb{Z}_{N_\mathrm{f}}}$$

The nonlinear Langrangian has the target space $G/H=SU(N_\mathrm{f})$.

Since $\pi_3(G/H)=\mathbb{Z}$, there are Skyrmions with the skyrmion current

$$J_{\mathrm{skyrmion}}=\frac{1}{24\pi^2}\mathrm{tr}[(U^\dagger \mathrm{d}U)^3].$$

$\Rightarrow \mathrm{d}J_{\mathrm{skyrmion}}[A_\chi,B_\mathrm{f}]=\mathrm{d}J_B[A_\chi,B_\mathrm{f}]$, and the anomaly matching is satisfied. (1807.07666[hep-th])

This is an extension of the $U(1)_V$-$SU(N_\mathrm{f})_L$-$SU(N_\mathrm{f})_L$ anomaly matching in the chiral Lagrangian. (ref. Witten, 1983)

Exotic chiral symmetry breaking

We now consider the Stern phase, where

$$G \to G^{\mathrm{sub}} = \frac{SU(N_F)_V \times U(1)_V}{(\mathbb{Z}_{N_\mathrm{c}}) \times (\mathbb{Z}_{N_\mathrm{f}})} \times (\mathbb{Z}_{N_\mathrm{f}})_L.$$

The target space of the nonlinear Lagrangian is

$$G/G^{\mathrm{sub}} = SU(N_\mathrm{f})/\mathbb{Z}_{N_\mathrm{f}}.$$

$\Rightarrow$ We can obtain the effective theory by gauging $\mathbb{Z}_{N_\mathrm{f}}$ in the ordinary chiral Lagrangian.

U : Nonlinear sigma field $\in SU(N_\mathrm{f}), a_\chi$: $\mathbb{Z}_{N_\mathrm{f}}$ dynamical gauge field

Mismatch of anomaly in Stern phase

Nontrivial homotopy: $\pi_3(G/G^{\mathrm{sub}})=\mathbb{Z}$, $\pi_1(G/G^{\mathrm{sub}})=\mathbb{Z}_{N_\mathrm{f}}$.
There are skyrmions with the current J_{skyrmion}. There are also $\mathbb{Z}_{N_\mathrm{f}}$ vortices.

Under the background gauge fields, the anomalous violation of J_{skyrmion} is

$$\mathrm{d}J_{\mathrm{skyrmion}}=\frac{N_\mathrm{f}}{(2\pi)^2}\mathrm{d}a_\chi\wedge B_\mathrm{f}\neq \mathrm{d}J_B.$$

Therefore, the anomaly matching is not satisfied.

$\Rightarrow$ We rule out the Stern phase from the possible QCD vacua even at finite densities. (1807.07666[hep-th])

Summary

- We find a new 't Hooft anomaly of massless QCD.
  Anomaly matching gives nonperturbative constraints on low-energy physics.
- In the ordinary chiral broken phase, the new discrete anomaly is correctly reproduced by

$$\mathrm{d}J_{\mathrm{skyrmion}} = \mathrm{d}J_B = \frac{N_{\mathrm{f}}}{(2\pi)^2}\mathrm{d}A_\chi \wedge B_{\mathrm{f}}.$$

- In the Stern phase, we show that

$$\mathrm{d}J_{\mathrm{skyrmion}} = \frac{N_{\mathrm{f}}}{(2\pi)^2}\mathrm{d}a_\chi \wedge B_{\mathrm{f}} \neq \mathrm{d}J_B.$$

  Anomaly matching thus rules out the Stern phase even at finite densities.