Localisation, chiral symmetry and confinement in QCD and related theories

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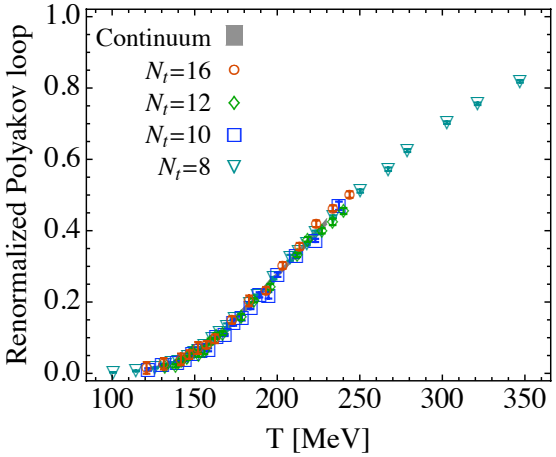
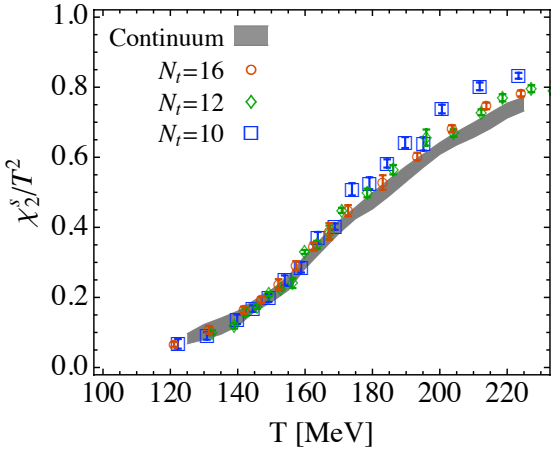
Including work done in collaboration with
S.D. Katz, T.G. Kovács and F. Pittler

QCD at Finite Temperature

QCD displays two dramatic phenomena at around the same pseudo-critical temperature T_c: deconfinement and chiral symmetry restoration





Figures from [Budapest-Wuppertal collaboration (2010)]

Analytic crossover in the range $T \simeq 145 - 165$ MeV, no sharply defined T_c, but peaks of chiral susceptibility and quark entropy ($\sim$ deconfinement) at compatible Ts [Bazavov *et al.* (2016)]

See A. Bazavov's plenary

This happens also in other QCD-like theories

3+1 and 2+1 SU(3) pure gauge, $N_T = 4$ unimproved staggered fermions...

Relation between the two phenomena still not fully clear

Chiral Symmetry and the Dirac Spectrum

χSB $\sim$ accumulation of Dirac modes around the origin [Banks, Casher (1980)]:

$$\langle\bar{\psi}\psi\rangle = \frac{1}{V_4}\int_0^\infty d\lambda\,\rho(\lambda)\frac{2m}{\lambda^2+m^2} \qquad \rho(\lambda) \equiv \langle\sum_i \delta(\lambda-\lambda_i)\rangle$$

Spectral density behaves differently near the origin in the two phases:

- $T < T_c$: $\rho(0) \neq 0$
- $T > T_c$: $\rho(0) = 0$

Is such an accumulation or lack thereof related to the confining properties?

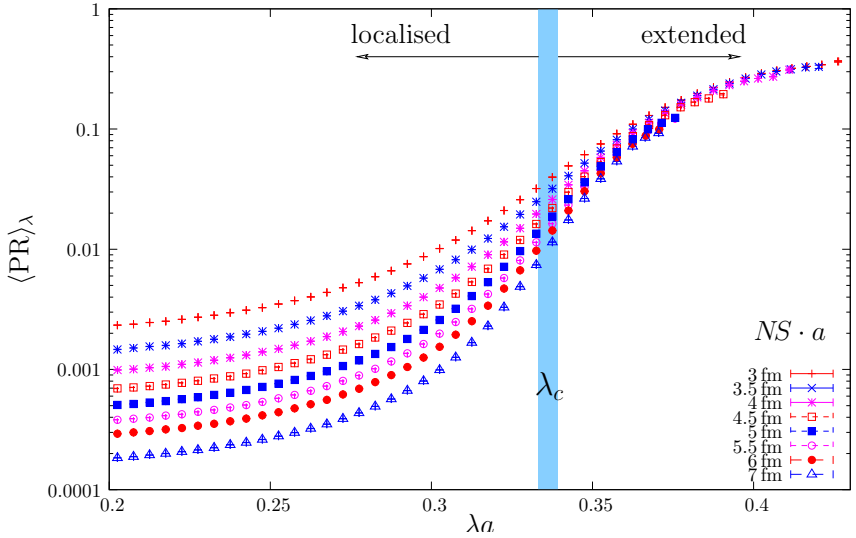
See also E. Shuryak's talk

Possible link: localisation of Dirac modes

- $T < T_c$: all modes extended
- $T > T_c$: low modes localised on the scale T^{-1}

[García-García, Osborn (2007), Kovács (2010), Kovács, Pittler (2010), Kovács, Pittler (2012), Cossu, Hashimoto (2016)]

Localisation in the Dirac Spectrum



Data for $T \simeq 2.6 T_c$ from [MG, Kovács, Pittler (2014)]

PR: fraction of volume occupied by an eigenmode

$$\mathrm{PR} = \left[\sum_x |\psi(x)|^4 \cdot V_4\right]^{-1}$$

- PR $\sim$ constant as $V \to \infty$ for an extended mode
- PR $\to 0$ as $V \to \infty$ for a localised mode

$T < T_c$: no localised modes in the chirally broken phase

$T > T_c$: modes localised for $\lambda < \lambda_c(T)$, phase transition at $\lambda_c(T)$

- Loc. modes seen with staggered [García-García, Osborn (2007), Kovács, Pittler (2012)], overlap [Kovács (2010)], domain-wall fermions [Cossu, Hashimoto (2016)], survive continuum limit [Kovács, Pittler (2012)]
- Second-order phase transition with divergent correlation length, same exponent as 3D unitary Anderson model [MG, Kovács, Pittler (2014)]

Tools: disordered Hamiltonians, lattice gauge theory, random matrix theory

Localisation and Disordered Hamiltonians

Anderson model: tight-binding Hamiltonian for "dirty" conductors

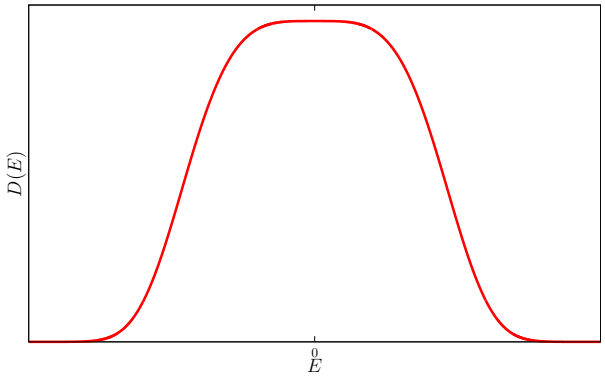
[Anderson (1958)]

$$H_{\vec{x},\vec{y}}^{\mathrm{AM}} = \varepsilon_{\vec{x}}\delta_{\vec{x},\vec{y}} + \sum_{\mu=1}^{3}(\delta_{\vec{x}+\hat{\mu},\vec{y}} + \delta_{\vec{x}-\hat{\mu},\vec{y}})$$

Random potential $|\varepsilon_{\vec{x}}| \leq \frac{W}{2}$

- No disorder ($W = 0$): delocalised modes
- With disorder ($W \neq 0$): localised modes at the band edge beyond critical energy $E_c(W)$ (mobility edge)
- E_c moves towards the band center as W increases, for $W > W_c$ all the states are localised: metal-insulator transition

  In QCD $\lambda_c(T)$, all modes extended below T_c
- Second-order phase transition at E_c with divergent correlation length $\xi \sim |E - E_c|^{-\nu}$ (Anderson transition)



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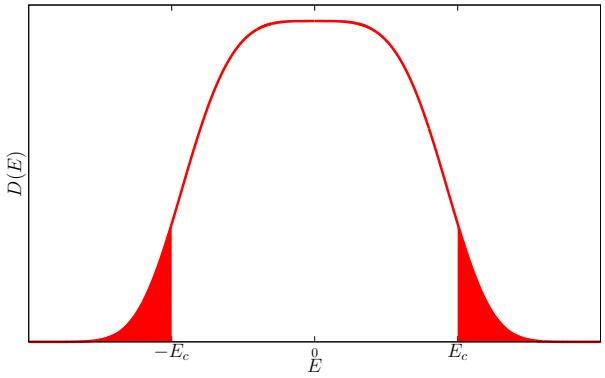

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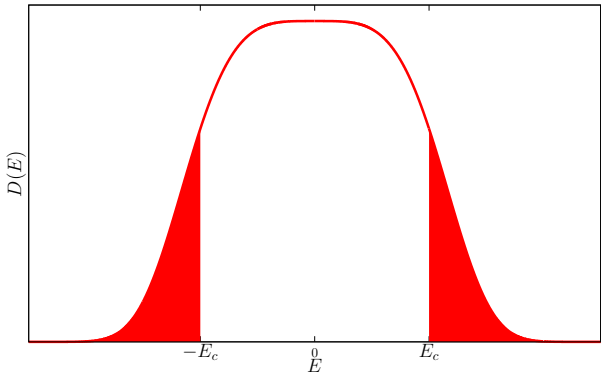

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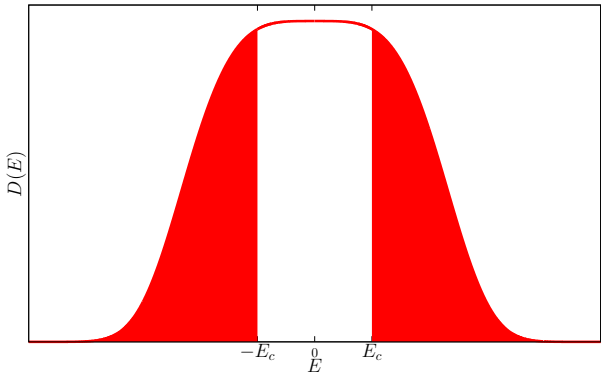

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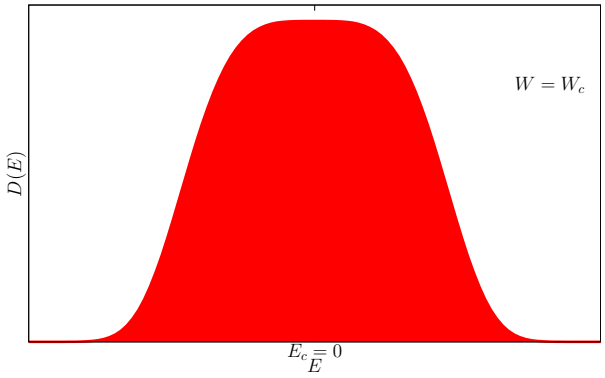

Localisation and Disordered Hamiltonians

Anderson model: tight-binding Hamiltonian for "dirty" conductors

[Anderson (1958)]

$$H^{\mathrm{UAM}}_{\vec{x},\vec{y}} = \varepsilon_{\vec{x}}\delta_{\vec{x},\vec{y}} + \sum_{\mu=1}^{3}(\delta_{\vec{x}+\hat{\mu},\vec{y}} + \delta_{\vec{x}-\hat{\mu},\vec{y}})e^{i\phi_{\vec{x},\vec{y}}}$$

Random potential $|\varepsilon_{\vec{x}}| \leq \frac{W}{2}$, random phases $\phi_{\vec{y},\vec{x}} = -\phi_{\vec{x},\vec{y}}$ (unitary AM)

- No disorder ($W = 0$): delocalised modes
- With disorder ($W \neq 0$): localised modes at the band edge beyond critical energy $E_c(W)$ (mobility edge)
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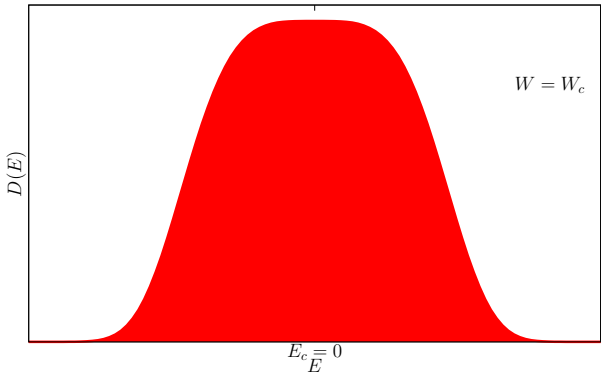


Lattice Gauge Theory

LGT: gauge theory Euclidean functional integral regularised by discretising it on a finite lattice, typically a hypercube of spacing a:

- gauge fields $A_\mu(x)$ replaced by parallel transporters $U_\mu(x)$ on the lattice edges $(x, x + a\hat{\mu})$, integrated over with Haar measure
- fermion fields $\psi(x), \bar{\psi}(x)$ attached to lattice sites
- finite temperature: fixed temporal extent $1/T = N_T a$, pbc/abc for bosons/fermions

From the practical point of view:

- the partition function becomes that of a statistical system with an exact gauge symmetry

$$Z = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-S_{\rm gauge}[U] + \bar{\psi}(D_{\rm lat}[U]+m)\psi}$$

- the staggered Dirac operator is (i times) the Hamiltonian of a quantum mechanical system with off-diagonal disorder (gauge links)

$$[D_{\rm stag}]_{xy} = \frac{1}{2} \sum_\mu \eta_\mu(x) \left[U_\mu(x) \delta_{x+\mu.y} - U_{-\mu}(x) \delta_{x-\mu.y} \right]$$

Random Matrix Theory

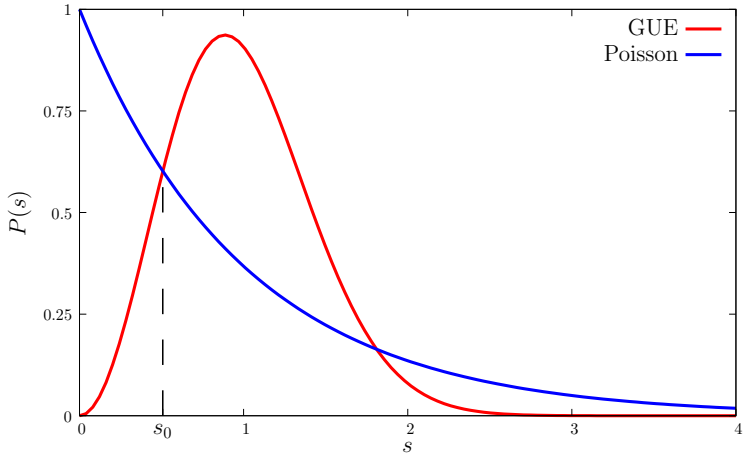
Random matrix: matrix with random entries, distributed according to some probability distribution

Who would have guessed?

Dense random matrices: universal spectral properties depend only on symmetry class

Distribution $P(s)$ of unfolded level spacing:

- RMT statistics: *Wigner surmise* $P(s) = as^b e^{-cs^2}$ with class-dependent constants a, b, c
- Independent eigenvalues (Poisson statistics): exponential distribution $P(s) = e^{-s}$



Unfolding: local rescaling of eigenvalues that yields unit spectral density $\lambda_i \to x_i = \int^{\lambda_i} d\lambda\, \rho(\lambda)$

Corresponds to rescaling level spacings by the local average level spacing

The Anderson model Hamiltonian and the lattice Dirac operator are *sparse* random matrices: what are the statistical properties of their spectra?

Localisation and Spectral Statistics in High-T QCD

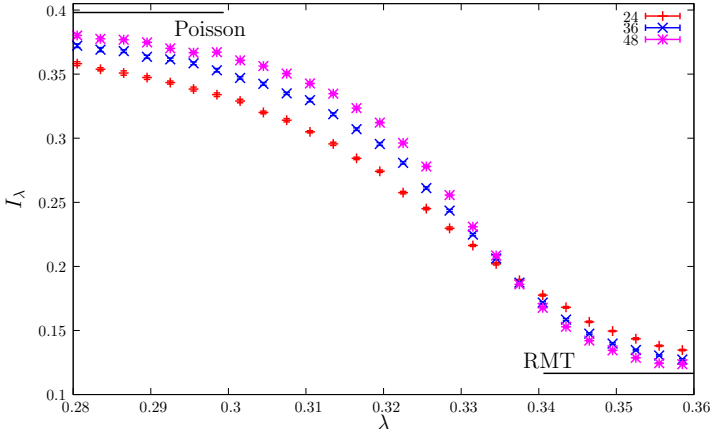
Local spectral statistics depend on localisation properties of modes:

- extended modes $\Rightarrow$ RMT statistics
- localised modes $\Rightarrow$ Poisson statistics

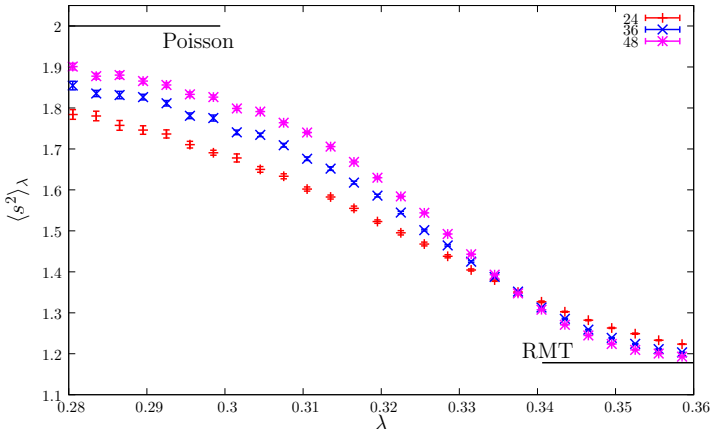
Used to determine λ_c and the correlation-length critical exponent ν of the localisation/delocalisation transition [Shklovskii et al. (1993)]

2+1 rooted 2stout improved staggered fermions [Budapest-Wuppertal collaboration (2010)]

$N_T = 4$, $\beta = 3.75 \to T = 394\ \mathrm{MeV} = 2.6\,T_c$, $a = 0.125\ \mathrm{fm}$; physical quark masses



$$I_\lambda = \int_0^{s_0} ds\, P_\lambda(s)\,, \quad s_0 \simeq 0.5$$



$$\langle s^2 \rangle_\lambda = \int_0^\infty ds\, P_\lambda(s)\, s^2$$

Data and ν from [MG, Kovács, Pittler (2014)]

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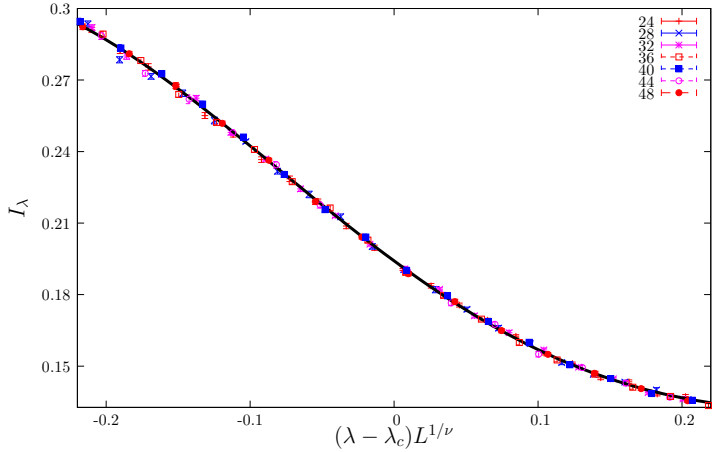


$$I_\lambda(L) = f(\xi(\lambda)/L) = f(|\lambda - \lambda_c|^{-\nu}/L) = F((\lambda - \lambda_c)L^{1/\nu})\,, \qquad \nu = 1.43(6)$$

Data and ν from [MG, Kovács, Pittler (2014)]

Mobility Edge: Temperature Dependence

Mobility edge λ_c characterises the spectrum $\rightarrow$

λ_c renormalises as a mass, λ_c/m_{ud} is RG invariant

[Giusti, Lüscher (2009), Kovács, Pittler (2012)]

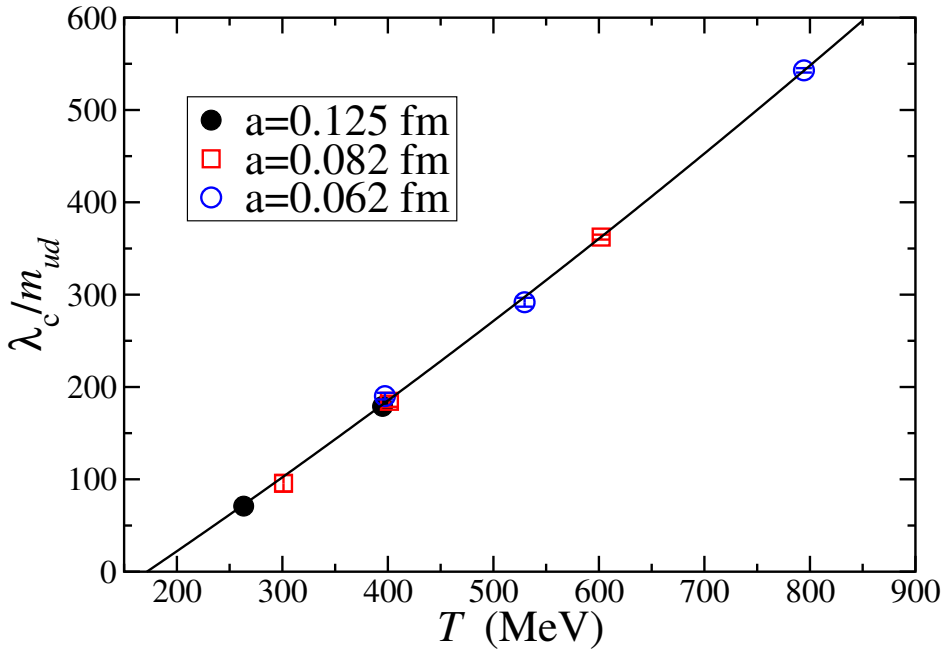


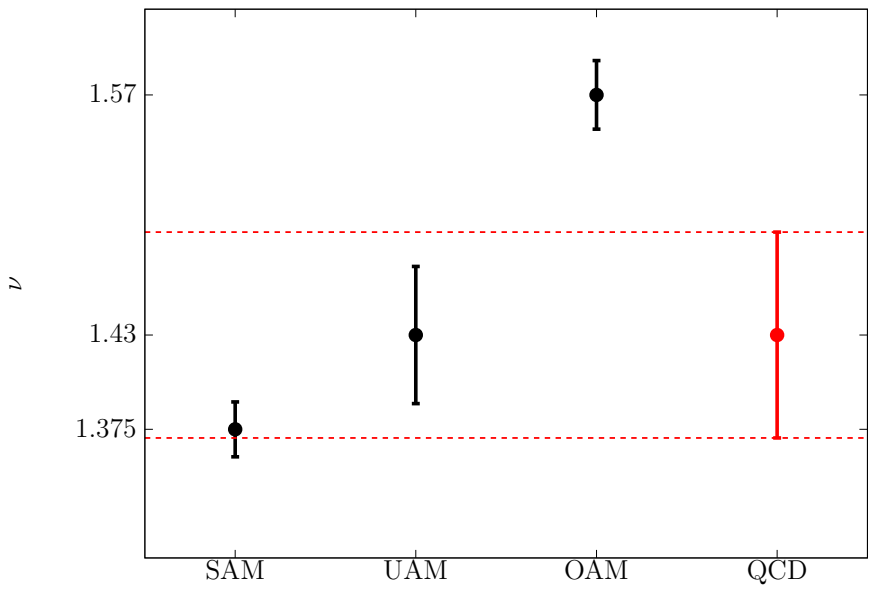
Figure from [Kovács, Pittler (2012)]

Temperature dependence compatible with vanishing $\lambda_c(T)$ at the chiral crossover [Kovács, Pittler (2012)]

Critical Behaviour

Localisation/delocalisation transition in the 3D UAM class:

- same ν [MG, Kovács, Pittler (2014)] (QCD) [Slevin, Ohtsuki (1999)] (UAM)
- same critical multifractality [Ujfalusi, MG, Pittler, Kovács, Varga (2015)]



Data from [MG, Kovács, Pittler (2014), Slevin, Ohtsuki (1997), (1999), Asada *et al.*, (2005)]

How come?

- Unitary class appropriate for SU(3) staggered fermions
- 3D from dimensional reduction, but where is the diagonal disorder?

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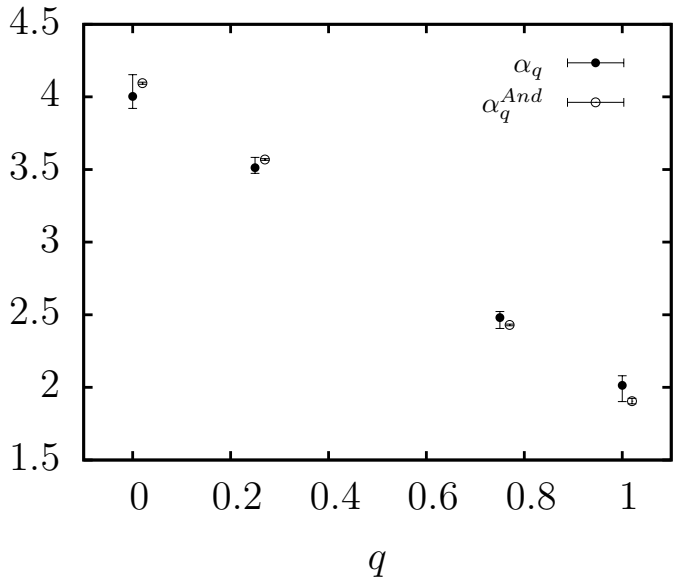

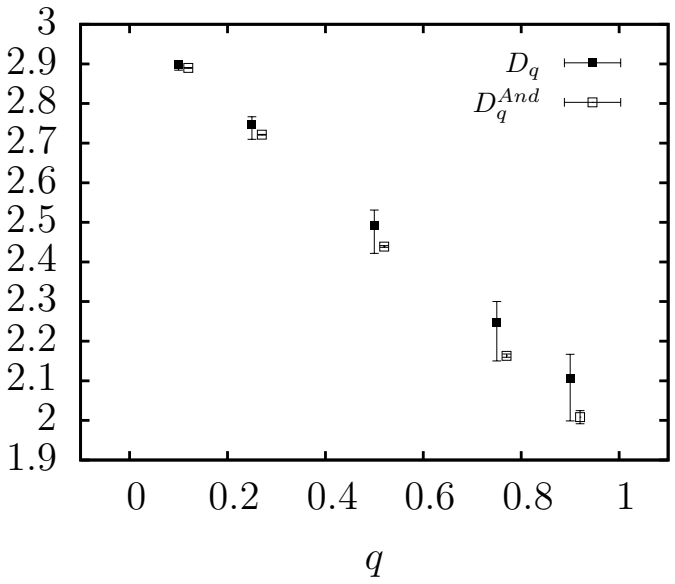

Figures from [Ujfalusi, MG, Pittler, Kovács, Varga (2015)]

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Polyakov Line Fluctuations

Qualitatively: spatial fluctuations of Polyakov lines provide the diagonal disorder [Bruckmann, Kovács, Schierenberg (2011), MG, Kovács, Pittler (2015)]

Above T_c, $P(\vec{x})$ gets ordered along $\mathbf{1}$ with "islands" of $P(\vec{x}) \neq \mathbf{1}$

- Ordering produces an effective gap in the spectrum
- "Wrong" $P(\vec{x})$ allows for smaller $\lambda \Rightarrow$ localising "trap" for eigenmodes

Seen to work in SU(2) [Bruckmann, Kovács, Schierenberg (2011)], toy model [MG, Kovács, Pittler (2015)], QCD-DW fermions [Cossu, Hashimoto (2016)]

Why does it not work below T_c?

- No "traps" there – but none in the AM either
- Below T_c we have effectively a 4D AM with off-diagonal disorder

Formally: staggered Dirac operator equivalent to N_T coupled 3D Anderson models [MG, Kovács, Pittler (2016), MG, Kovács, Pittler (2017)]

Dirac-Anderson Hamiltonian

Set $H = -iD_{\mathrm{stag}} = H_0 + H_I$ and diagonalise H_0 (temporal hoppings)

- diagonal disorder H_0 = fluctuations of the Polyakov line phases $\phi_a(\vec{x})$

$$H_{0\,ak\vec{x},bl\vec{y}} = \eta_4(\vec{x}) \sin \omega_{ak}(\vec{x}) \delta_{ab} \delta_{kl} \delta_{\vec{x}\vec{y}}$$

  Effective Matsubara frequencies: $\omega_{ak}(\vec{x}) = \frac{1}{N_T}(\pi + \phi_a(\vec{x}) + 2\pi k)$

  Indices: spatial site $\vec{x}$, colour $a = 1, \ldots, N_c$, temporal momentum $k = 0, \ldots, N_T - 1$

- coupling related to the correlation between time-slices, AMs more or less mixed by H_I (containing off-diagonal disorder from spatial links)

details

Unlike AM, disorder strength bounded, but

- disordered phase: weak correlations, no structure in the diagonal noise
- ordered phase: strong correlations, "islands" of Polyakov lines not aligned to the ordered value provide convenient "localising centres"

Correlations and localising centers both needed

Toy model study supports this picture

[MG, Kovács, Pittler (2016), MG, Kovács, Pittler (2017)]

χSR and Localisation from Deconfinement?

How are deconfinement, χS restoration and localisation related?

- Deconfinement changes the effective dimensionality of the Dirac-Anderson system and creates an effective gap in the spectrum
  - low-T: Polyakov lines are disordered $\rightarrow$ strongly coupled of AMs, single effectively 4D system with uncorrelated off-diagonal noise
  - high-T: Polyakov lines are ordered $\rightarrow$ weakly coupled AMs, N_T effectively 3D systems with (correlated) diagonal noise
- Localisation requires effective gap and decoupling of AMs
  - "islands" should be "energetically" favourable to localise
  - coupled AMs prevent localisation by mixing modes
- χSB $\approx$ nonzero $\rho(0)$ requires mixing of AMs and finite density of low "unperturbed" modes
  - without mixing, modes are repelled from the origin
  - without low "unperturbed" modes, mixing is not sufficient

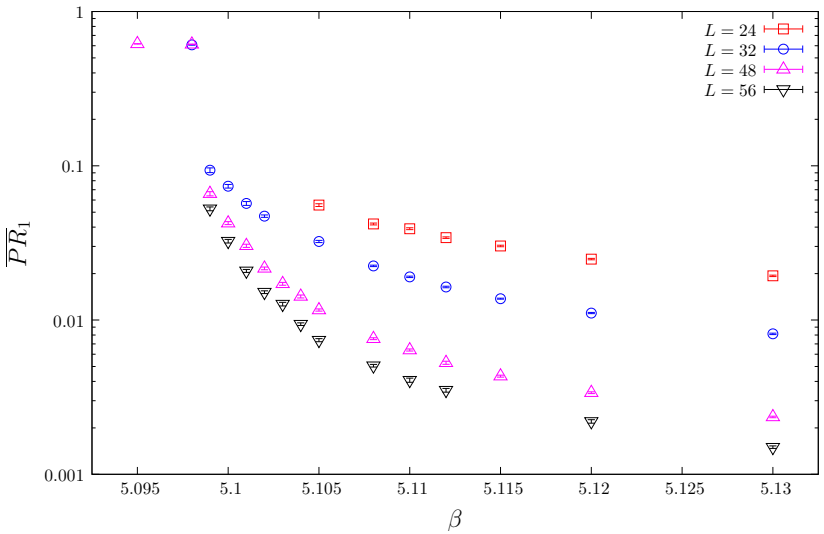
Deconfinement seems to provide the conditions for localisation, and to remove the conditions for χSB: does it happen in general?

QCD-like theories: $N_T = 4$ Unimproved Staggered Fermions

In QCD the transition is a crossover, but what happens to localisation when there is a genuine phase transition?

$N_T = 4$ unimproved staggered fermions: first order phase transition

[De Forcrand, Philipsen (2003)]



Figures from [MG, Katz, Kovács, Pittler (2017)]

- $N_F = 3$ rooted staggered fermions, $m = 0.01 < m_c$
- Symmetries:
  - center $\mathbb{Z}_3$ (broken by fermions)
  - chiral SU(3) (broken by quark masses)
- Transition is a lattice artefact (but OK for our purposes)

Deconfinement, chiral transition and localisation of low modes at the same critical coupling (within errors) [MG, Katz, Kovács, Pittler (2017)]

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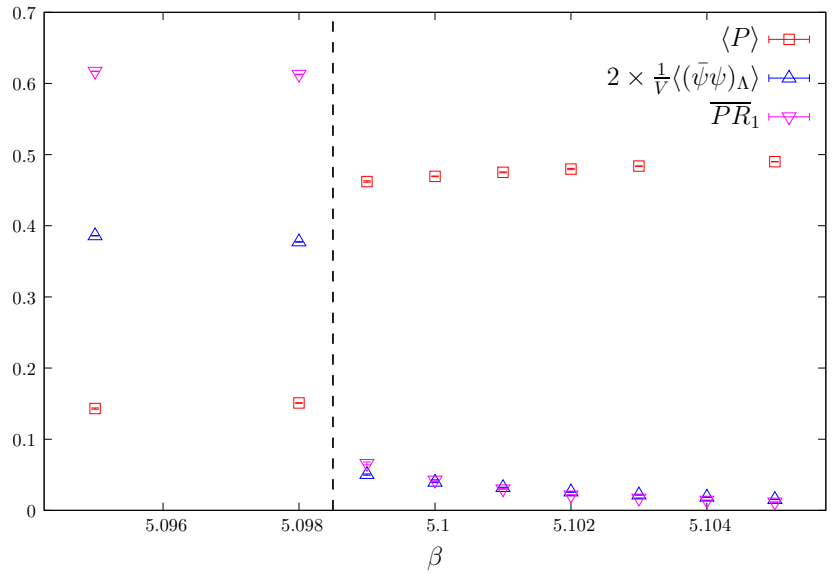

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QCD-like theories: Pure Gauge SU(3)

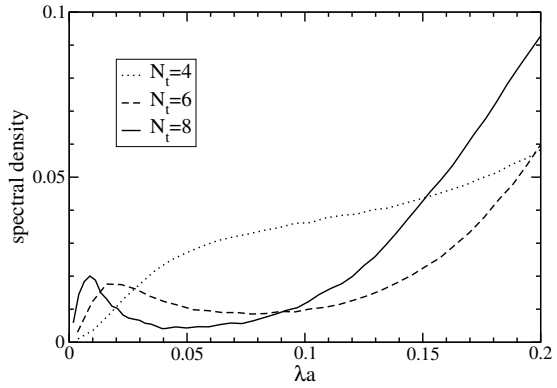
SU(3) pure gauge theory has a first-order deconfining phase transition

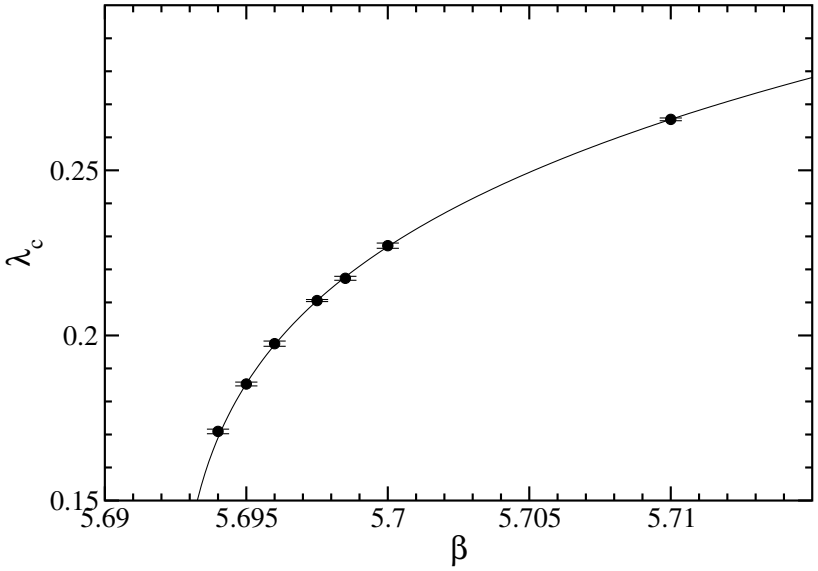
[G. Boyd *et. al.* (1996)]

Valence quark condensate shows a first-order transition as well

Spectral density right above the transition

Near-zero peak: topology-related near-zero modes





Localised modes appear near the origin at the critical β

[Kovács, Vig (2018)]

2stout improved staggered operator, $N_T = 4$ lattice

Figures from [Kovács, Vig (2018)]

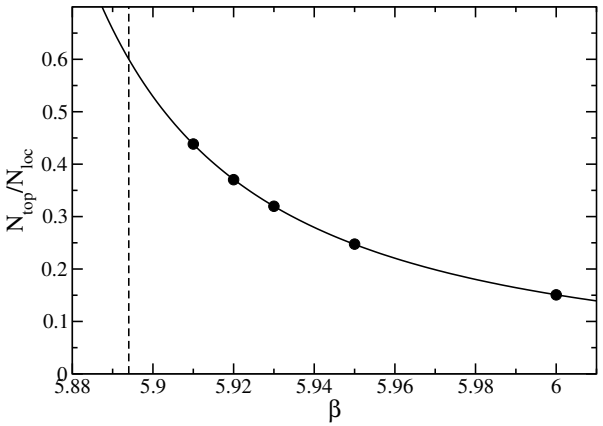
Topology

Localised modes related to topological objects?

- Instantons?

  [García-García, Osborn (2007)]
- SU(3): instantons are not enough (only 60% at T_c)

  Instanton modes counted from near-zero peak [Kovács, Vig (2018)]



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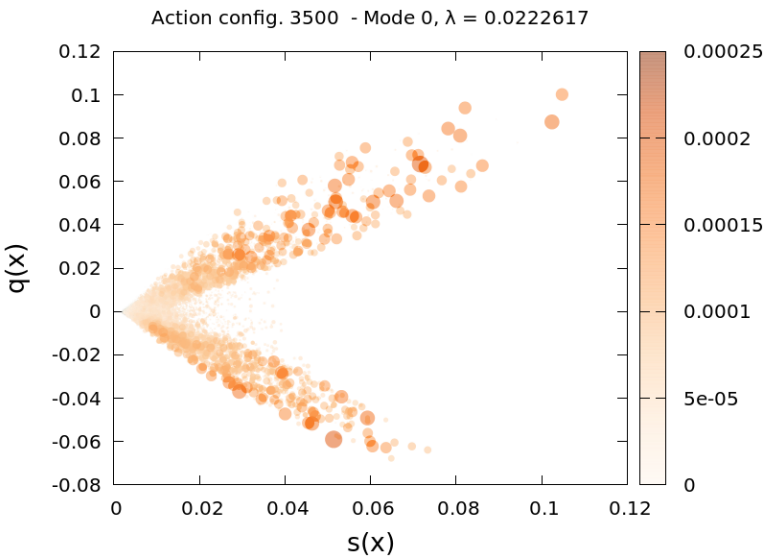

Figures from [Cossu, Hashimoto (2016)]

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  - high modes are delocalised
- Localise on $L\bar{L}$ monopole pairs?
  - match PL of best "islands"
  - related to deconfinement?

[Cossu, Hashimoto (2016)]

Topology

Localised modes related to topological objects?

- Instantons?

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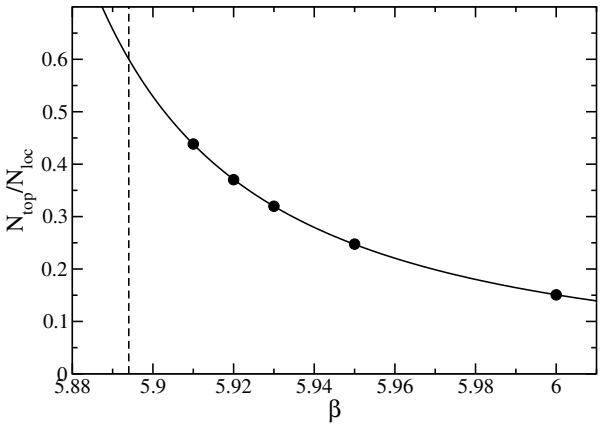

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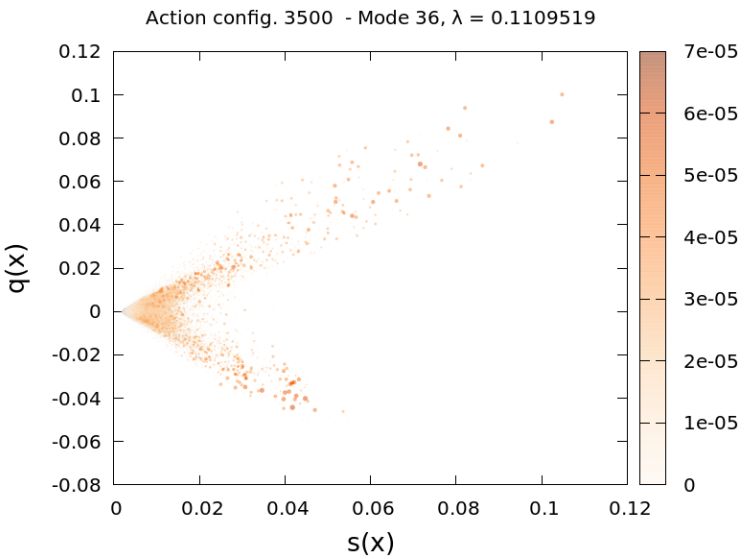

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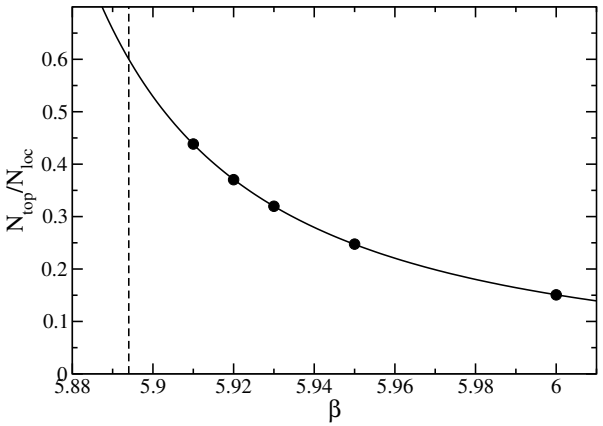

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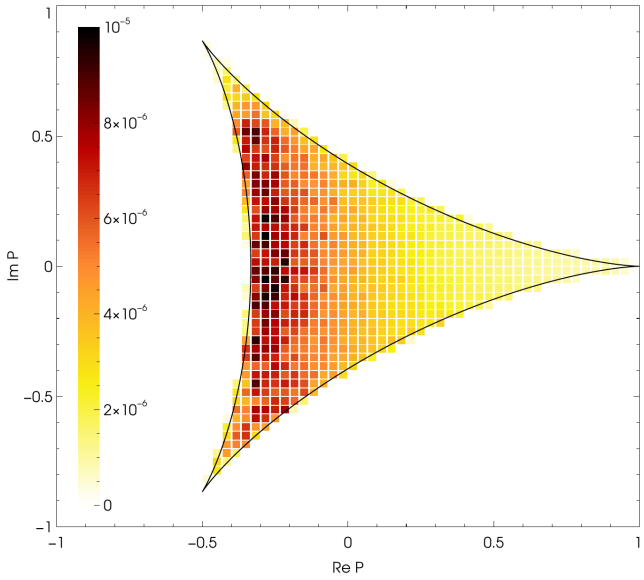

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QCD-like Theories in 2+1 D

In 2+1 D gauge theories display deconfinement transitions, but there is no chiral symmetry

2d spinors, no "γ_5"

With an even number of massless flavours N_f there is a sort of $\mathrm{U}(N_f)$ "chiral" flavour symmetry, broken by a mass term [Jackiw, Templeton (1981)]

$N_f/2$ pairs, each represented by a 4d spinor $\rightarrow$ usual γ^5

1 staggered fermion $\Rightarrow N_f = 2$

Plausible symmetry breaking $\mathrm{U}(N_f) \rightarrow \mathrm{U}(N_f/2) \times \mathrm{U}(N_f/2)$, signalled by the formation of a condensate [Pisarski (1984)]

Is there still a relation between deconfinement, "χ"SR and localisation?

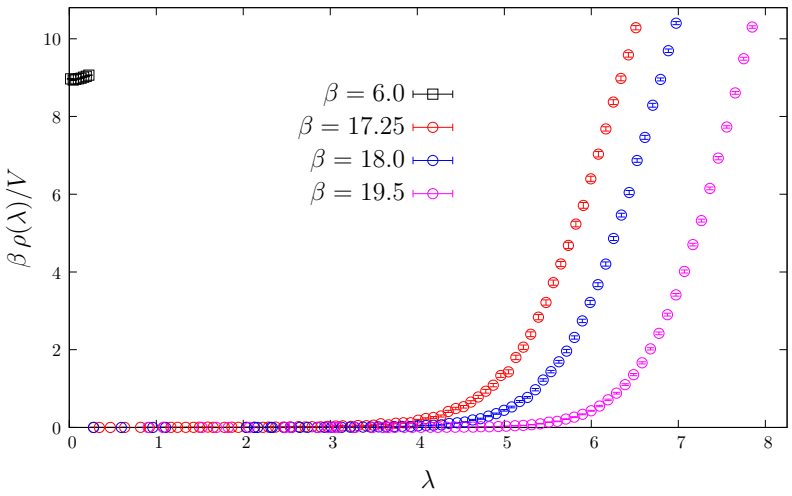
$D = 2$ is special for AMs:

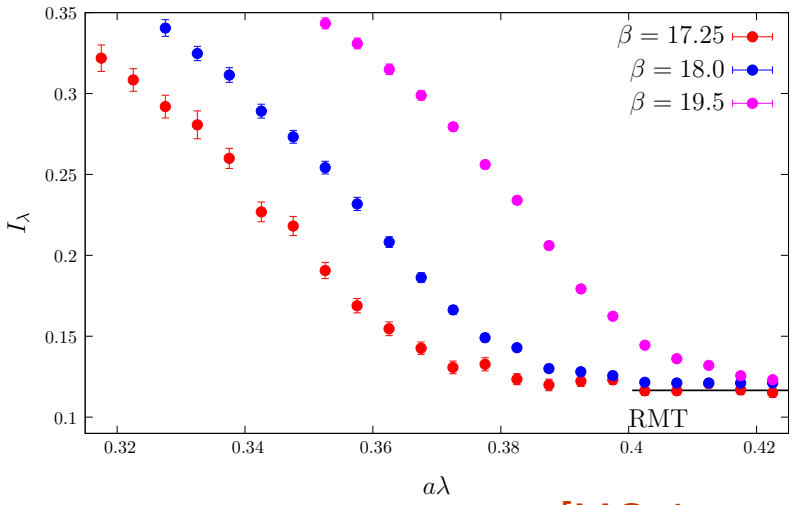
- $D > 2$: Anderson transition always exist
- $D = 1$: all modes are always localised
- $D = 2$: details of the model matter

2+1 D SU(3) Pure Gauge at Finite Temperature

Deconfinement (2nd order) and "χ"SR happen together at $\beta_c \simeq 15$

[Damgaard *et al.* (1998)]





[MG, in progress]

No localised modes in the confined phase, transition from localised to delocalised modes in the deconfined phase

Trivial vacuum picked by hand, mimicking the presence of fermions

AT is a BKT transition in the analogous 2D AM [Xie, Wang, Liu (1997)]

At λ_c the I_λ curves for different volumes merge together $\Rightarrow$ supports BKT

λ_c decreases going down towards β_c, does it vanish at β_c?

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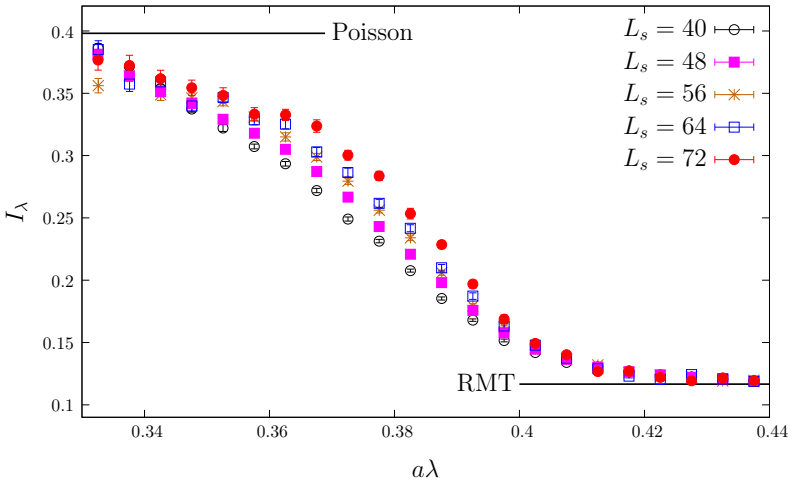

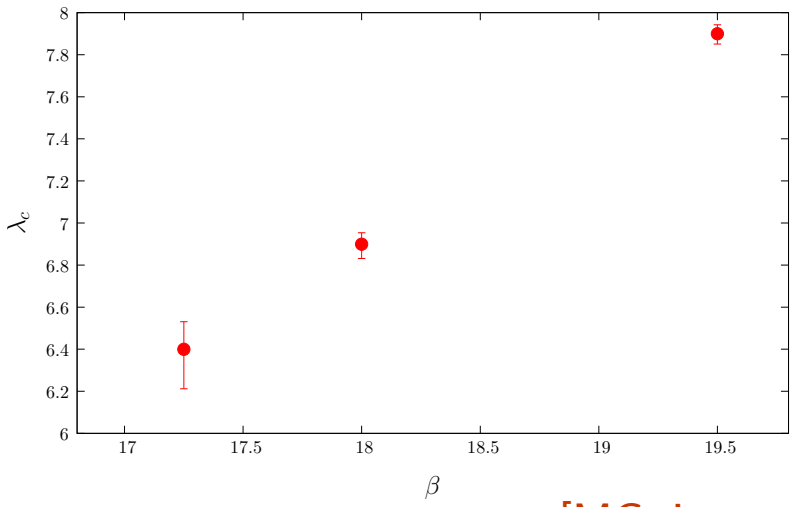

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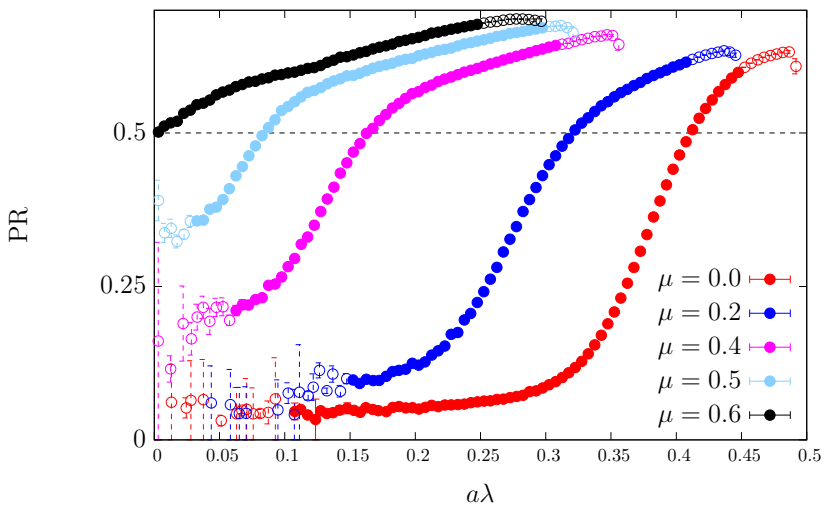
Testing the Sea/Island Picture

Changing the typical Polyakov line phases or the temporal boundary conditions changes the typical effective Matsubara frequencies

Effective gap changes, how does this affect the spectrum and the localisation properties of the eigenmodes?

- 2+1D SU(3) with imaginary chemical potential
  - changes the temporal bc
  - decreases the effective gap
  - λ_c decreases with μ
- 2+1 D U(1) pure gauge theory in the deconfined phase
  - $\langle |\frac{1}{V}\sum_x P(x)| \rangle \neq 0$, phase fluctuates (so does the gap)
  - lowest mode localised for $\phi \approx 0$, delocalised for $|\phi| \approx \pi$



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Results support the "sea" / "island" picture

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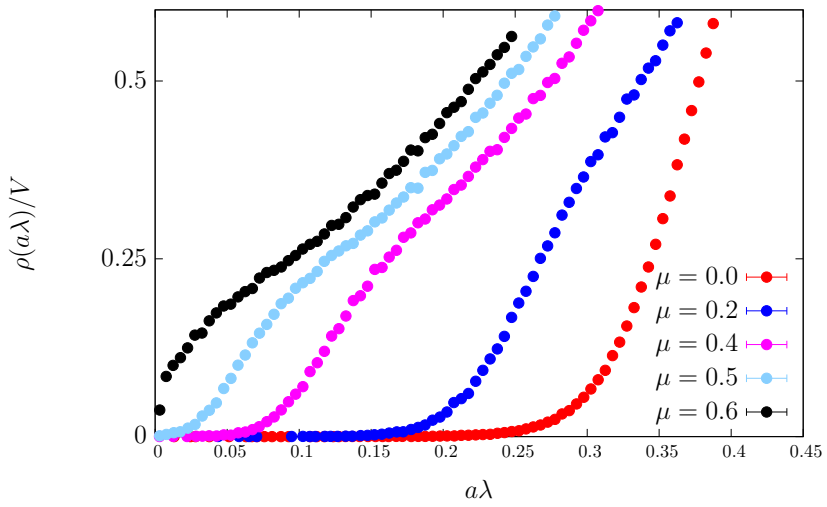

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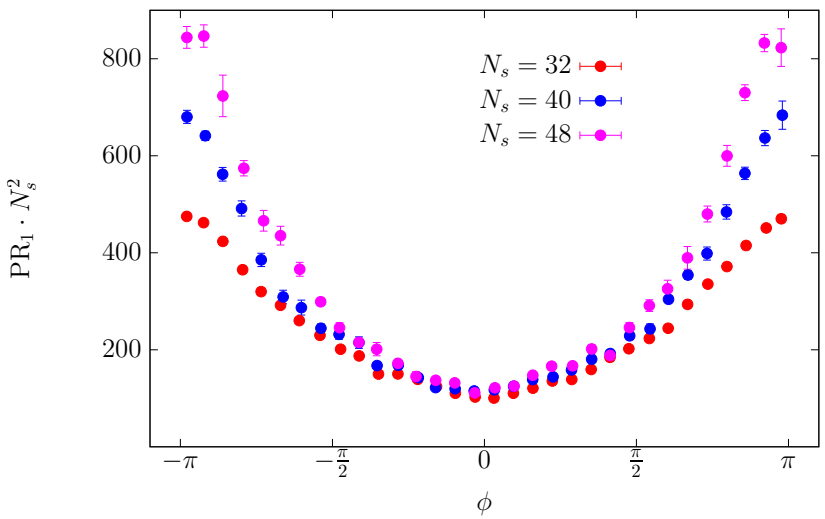

[MG, in progress]

Results support the "sea" / "island" picture

Summary and Outlook

- Deconfinement, chiral symmetry restoration and localisation of low Dirac modes closely connected
- Is deconfinement the driving force behind chiral restoration and localisation? Supported by
  - Dirac-Anderson and sea/island picture
  - numerical results in a quite diverse variety of models

Open issues:

- 2+1 D study in progress
- Imaginary μ in 3+1 D
- Adjoint fermions ($T_{\rm dec} \neq T_{\chi\rm SR}$)
- Topology?

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Dirac-Anderson Hamiltonian

"Hamiltonian" $H = -iD_{\mathrm{stag}}$ split into "free" + "interaction", $H = H_0 + H_I$
Work in the basis of the eigenvectors of H_0 (temporal hoppings)

Uniform temporal diagonal gauge: $U_4(t,\vec{x}) = [P(\vec{x})]^{\frac{1}{N_T}} = \mathrm{diag}(e^{i\frac{\phi_a(\vec{x})}{N_T}})$

$$H_{\vec{x},\vec{y}} = \delta_{\vec{x},\vec{y}} D(\vec{x}) + \sum_{j=1}^{3} \frac{\eta_j(\vec{x})}{2i} \left[\delta_{\vec{x}+\hat{\jmath},\vec{y}} V_{+j}(\vec{x}) - \delta_{\vec{x}-\hat{\jmath},\vec{y}} V_{-j}(\vec{x})\right]$$

Diagonal noise

(random on-site potential)

$$[D(\vec{x})]_{ak,bl} = \eta_4(\vec{x}) \sin \omega_{ak}(\vec{x}) \delta_{ab} \delta_{kl}$$

Off-diagonal noise

(random hoppings)

$$[V_{\pm j}(\vec{x})]_{ak,bl} = \frac{1}{N_T} \sum_{t=0}^{N_T-1} e^{i\frac{2\pi t}{N_T}(l-k)} \times \left[U_{\pm j}^{(\mathrm{utd})}(t,\vec{x})\right]_{ab}$$

Effective Matsubara frequencies $\omega_{ak}(\vec{x}) = \frac{1}{N_T}(\pi + \phi_a(\vec{x}) + 2\pi k)$

Indices: spatial site $\vec{x}$, colour $a = 1, \ldots, N_c$, temporal momentum $k = 0, \ldots, N_T - 1$

back