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# Variational and Dyson–Schwinger equations of QCD

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# Outline



# QCD Hamiltonian in Coulomb gauge

Schrödinger equation

$$H|\Psi\rangle = E|\Psi\rangle$$

with

$$H = H_{\mathrm{YM}} + \psi^\dagger(\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m - g\boldsymbol{\alpha}\cdot\mathbf{A})\psi$$

$$+ \frac{g^2}{2}\,\psi^\dagger\psi\,\frac{1}{-\partial\cdot D}(-\partial^2)\frac{1}{-\partial\cdot D}\,\psi^\dagger\psi$$

where $D = \partial + gA$.

# Canonical Dyson–Schwinger Equations

## Formal equivalence to Lagrangian approach

Vacuum expectation value

$$\langle f[A, \psi^\dagger, \psi] \rangle = \int \mathcal{D}A \, J_A \, \mathcal{D}\psi^\dagger \, \mathcal{D}\psi \, |\Phi[A, \psi^\dagger, \psi]|^2 \, f[A, \psi^\dagger, \psi]$$

Writing the vacuum wave functional as

$$\left|\Phi[A, \psi^\dagger, \psi]\right|^2 =: \mathrm{e}^{-S[A, \psi^\dagger, \psi]}$$

we have an Euclidean QFT defined by an "action" $S[A, \psi^\dagger, \psi]$.
Kernels of the vacuum wave functional

$$S[A, \psi^\dagger, \psi] = \tfrac{1}{2} \gamma_2 \, A^2 + \tfrac{1}{3!} \gamma_3 \, A^3 + \tfrac{1}{4!} \gamma_4 \, A^4 + \psi^\dagger (\bar{\gamma} + \bar{\Gamma}_0 \, A) \psi + \ldots$$

# Canonical Dyson–Schwinger Equations

## Formal equivalence to usual DSEs

Gluon and quark cDSEs are derived from the identity

$$0 = \int \mathcal{D}A \, \mathcal{D}\psi^\dagger \, \mathcal{D}\psi \, \frac{\delta}{\delta\phi} \Big\{ J_A \; \mathrm{e}^{-S[A,\psi^\dagger,\psi]} \, f[A,\psi^\dagger,\psi] \Big\}$$

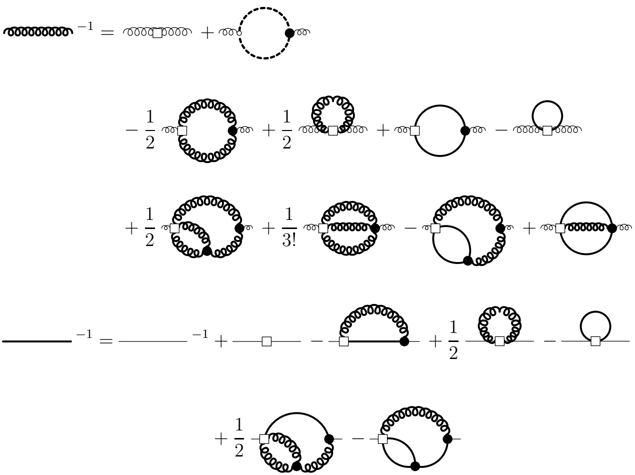
where $\phi \in \{A, \psi^\dagger, \psi\}$.

Not quite equations of motion, rather relations between the Green functions and the — so far undetermined — kernels of the vacuum wave functional.

# Canonical Dyson–Schwinger Equations

## Propagator DSEs



# Canonical Dyson–Schwinger Equations

## Kernels of the vacuum wave functional

$$S[A,\psi^\dagger,\psi] = \tfrac{1}{2}\gamma_2\,A^2 + \tfrac{1}{3!}\gamma_3\,A^3 + \tfrac{1}{4!}\gamma_4\,A^4 + \psi^\dagger(\bar{\gamma} + \bar{\Gamma}_0\,A)\psi + \ldots$$

### "Bare" vertices?

The coefficients in the vacuum wave functional play the role of the bare vertices, but

- have a non-trivial expansion in powers of the coupling
- are (in general) non-local functions
- are not exactly known… $\Rightarrow$ **variational kernels**

# Canonical Dyson–Schwinger Equations

## Variational method

$$\langle H \rangle \to \min$$

- choose an Ansatz for the vacuum wave functional
- evaluate the energy in this state
- use the DSEs to express the energy density as a function of the variational kernels
- minimize the energy by taking functional derivatives w.r.t. the variational kernels

$$\Downarrow$$

This gives a set of gap equations.

Campagnari, Reinhardt, PRD82, 105021 [1009.4599]; PRD92, 065021 [1507.01414]

# Chiral symmetry breaking

## Coulomb gauge pairing model (1980s)

Make BCS-type Ansatz for the wave functional

$$S[A, \psi^\dagger, \psi] = S_{\mathrm{YM}}[A] + \psi^\dagger \, \beta \, s(\mathbf{p}) \, \psi \quad \Leftrightarrow \quad |\mathrm{BCS}\rangle \sim \mathrm{e}^{-s \, b^\dagger d^\dagger} |0\rangle$$

Throw away the gauge field sector and keep only the linearly rising part of the potential

$$H = \psi^\dagger (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m) \psi + \frac{1}{2} \psi^\dagger \psi \, V_{\mathrm{C}} \, \psi^\dagger \psi, \qquad V_{\mathrm{C}} = \sigma_{\mathrm{C}} \, r.$$

Results: $\langle \bar{q} q \rangle = (-185\,\mathrm{MeV})^3$, $M(0) = 135\,\mathrm{MeV}$ (with $\sigma_{\mathrm{C}} = 2.5\,\sigma$).

Finger & Mandula; Adler & Davis; Alkofer & Amundsen.

# Chiral symmetry breaking

## Introduce coupling to transverse gluons

In the exponent of the wave functional

$$S[A,\psi^\dagger,\psi] = S_{\mathrm{YM}}[A] + \psi^\dagger(\bar{\gamma} + \bar{\Gamma}_0\, A)\psi$$

take the most simple Dirac and colour structure

$$\bar{\gamma} \sim \beta\, s(\mathbf{p}), \qquad \bar{\Gamma}_0 \sim \alpha_i\, V(\mathbf{p},\mathbf{q})$$

with $s(\mathbf{p})$ and $V(\mathbf{p},\mathbf{q})$ being scalar variational kernels.

Bare Vertex Approximation

# Chiral symmetry breaking

## Bare vertex approximation

No meaningful $\chi$SB in Landau gauge with bare vertex!

Landau gauge: gauge invariance
$\Rightarrow$ Slavnov–Taylor identity
$\Rightarrow$ fixes longitudinal part of vertex

Coulomb gauge: gauge invariance
$\Rightarrow$ Gauß law
$\Rightarrow$ Coulomb term

The Coulomb term describes the interaction of the temporal/longitudinal gluons!

Motivation, no proof! Quark-gluon vertex under investigation.

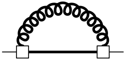
# Chiral symmetry breaking

## The quark gap equation

Replace Coulomb kernel by its Yang–Mills expectation value

$$V_{\mathrm{C}}(r) = -\frac{g^2}{4\pi r} + \sigma_{\mathrm{C}}\, r$$

Variation of the energy density gives the quark gap equation

$s(\mathbf{p}) \sim$  $+$ 

Looks meaningful, but

- bad, bad linear divergences
- quark propagator not multiplicatively renormalizable

**Are we missing something?**

# Chiral symmetry breaking

## Vector kernel with non-trivial Dirac component

More general ansatz for the quark-gluon coupling

$$\bar{\Gamma}_0 = \alpha_i \, V(\mathbf{p}, \mathbf{q}) + \beta \, \alpha_i \, W(\mathbf{p}, \mathbf{q})$$

Turns out this is quite relevant!

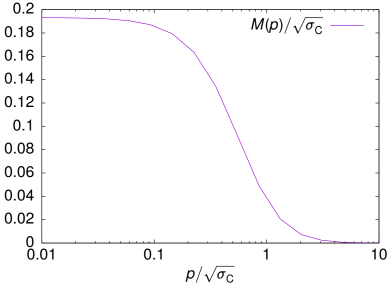
- exists only in chirally broken phase
- linearly divergent terms cancel
- quark propagator multiplicatively renormalizable

Campagnari, Ebadati, Reinhardt, Vastag, PRD94, 074027 [1608.06820]
Campagnari, Reinhardt, PRD97, 054027 [1801.02045]

# Chiral symmetry breaking

## Mass function and chiral condensate




$\overline{\text{MS}}$ scheme, $\mu = 2\,\text{GeV}$:

$$\langle \bar{q}q \rangle = \left(-0.31\,\sqrt{\sigma_C}\right)^3$$

$$M(0) = 0.19\,\sqrt{\sigma_C}$$

Exact value of $\sqrt{\sigma_C}$ uncertain:

$\langle \bar{q}q \rangle = (-216\,\text{MeV})^3$ to $(-270\,\text{MeV})^3$ looks OK

$M(0) = 135\,\text{MeV}$ to $170\,\text{MeV}$ seems somewhat small

# Chiral symmetry breaking

## Static vs. time-dependent propagator

Quark propagator in Landau gauge

$$S(p) = \frac{1}{-i\not{p}A(p^2) + B(p^2)} = Z(p^2)\,\frac{i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

Equal-time propagator

$$S_3(\mathbf{p}) = \int \frac{dp_4}{2\pi}\, S(p)$$

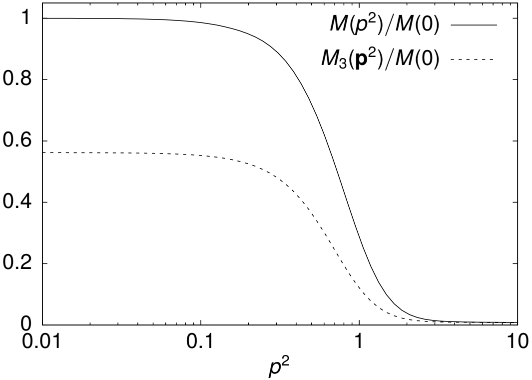
The "mass function" of the static equal-time propagator is

$$M_3(\mathbf{p}) = \frac{\displaystyle\int_0^\infty dp_4\, \frac{Z(p^2)\,M(p^2)}{p^2 + M^2(p^2)}}{\displaystyle\int_0^\infty dp_4\, \frac{Z(p^2)}{p^2 + M^2(p^2)}}$$

# Chiral symmetry breaking

## Static vs. time-dependent propagator




Landau gauge data courtesy of Markus Q. Huber

$$\frac{M_3(0)}{M(0)} = 0.5 \ldots 0.6$$

# Conclusions

## Done

- ✓ standard DSE techniques can be used to treat coupled quark-gluon system in Hamiltonian approach
- ✓ spurious divergences are now under control
- ✓ small IR value of mass function plausible
- ✓ $\sigma_C$ is the only free parameter

## To do

- quark-gluon vertex
- finite temperature & chemical potential