Z3 gauge theory with matter at finite densities

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QCD phase diagram:

...10 years ago
QCD phase diagram:

The Phases of Dense Matter, INT, July 11 - August 12, 2016
What are the theoretical foundations?

“sign problem”

Lattice QCD

Mostly (quark) models without Confinement

Large number of colours (???)

Perturbation theory (???)

Net baryon density \( n_b/n_o = 0.16 \text{ fm}^{-3} \)
QCD phase diagram:

Need to understand better Confinement! [don’t we?]
What is quark confinement?

- Use (heavy) quarks as probe
- It is a feature of pure gauge theory!

Linearly rising Potential

[SU(3) improved action, KL, PRD 76 (2007) 094502]
How can quark confinement understood in gauge theories?

Hierarchy of groups

- Breakdown
- OPE & condensates
- Perturbation theory

Hierarchy of scales

- Low energies
- High energies

$\mathbb{Z}_3$ subgroup

Full $SU(3)$

Gauge fixing + projection
Isolate dofs responsible for confinement

- no confinement without them
- weakly interacting

Use them to understand non-perturb. physics

- deconfinement at finite temperatures
- spontaneous chiral symmetry breaking
- origin of the OPE condensates

Vortex picture of MCG

[Greensite, Faber, Olejnik, Del Debbio…. 1998]

[Engelhardt, Langfeld, Reinhardt, …. 1999]
What do we know about the phase diagram?

Propose a model theory study

- With confinement of charges (with non-trivial triality)
- With dynamical matter (characterised by triality, will be bosonic)
- That can be studied at finite densities of matter

Z3 gauge theory with Z3 matter

(centre) vortex confinement mechanism

Sign-problem solved by real dual simulation: open 2-branes
Details of the model:

Group $Z_3$:

$$Z_3 = \{1, z, z^\dagger\}$$

$$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$U \in Z_3$:

$$UU^\dagger = 1 \quad U^3 = 1 \quad \sum_{U \in Z_3} U = 0$$

Algebraic properties:

$$U^4 = U^3 U = U \quad U^{11} = [U^3]^3 U^2 = U^2 = U^3 U^\dagger = U^\dagger$$

Real function, admits Taylor expansion

$$f(U, U^\dagger) = a + bU + cU^\dagger$$

$a, b, c, \in \mathbb{R}$
Partition function:
\[ Z = \sum_{\sigma, U_\mu \in Z_3} \exp\left\{ \beta S_g[U] + \kappa S_f[\sigma, U] \right\} \]

Plaquette action:
\[ S_g = \sum_{x, \mu > \nu} \text{Re} \left[ U_\mu(x) U_\nu(x + \mu) U_\mu^\dagger(x + \nu) U_\nu^\dagger(x) \right] \]

“glue” - matter interaction:
\[ S_f = 2 \sum_{x, \mu = 1 \ldots 3} \text{Re} \sigma^\dagger(x) U_\mu(x) \sigma(x + \mu) + \sum_{x, \mu = 4} \left[ e^\mu \sigma^\dagger(x) U_\mu(x) \sigma(x + \mu) + e^{-\mu} \sigma^\dagger(x) U_\mu^\dagger(x - \mu) \sigma(x - \mu) \right] \]

Chemical potential

Sign problem
Solving the sign-problem: 2-brane theory rising

Re-writing the pure gauge theory:

\[ S_g = \sum_{x, \mu > \nu} \text{Re} \left[ U_\mu(x) U_\nu(x + \mu) U_\mu^\dagger(x + \nu) U_\nu^\dagger(x) \right] \]

\[ \exp\{\beta S_g\} = \prod_p c(\beta) \left[ 1 + t(\beta) (P_p + P_p^\dagger) \right] \]

**crucial:** \( t(\beta) \geq 0 \)

- Expand brackets
- Sum over \( U_\mu(x) \in \mathbb{Z}_3 \)
Introduce “brane” variables:

\[
\sum_{n_p = -1, 0, 1} \left[ \delta_{n_p, 0} + \delta_{n_p, 1} tP_p + \delta_{n_p, -1} tP_p^\dagger \right]
\]

“Integrate link variables”

remember:

\[
\sum_{U \in \mathbb{Z}_3} U = 0
\]

Non-trivial plaquettes form closed surfaces!
Result for the pure gauge theory:

\[ Z(\beta) = 3^{N_e} c(\beta)^{N_p} \sum_{\{n_p\}} t(\beta)^{S[\{n\}]} \]

\[ S[\{n\}] \] Total area of closed surfaces

Application: plaquette expectation value of pure Z3 gauge theory

\[ \langle P \rangle = \frac{d}{d\beta} Z(\beta) \]
Pure Z3 gauge theory in 2-brane formulation:

pure Z3, flux simulation, $20^4$
Adding dynamical $\mathbb{Z}_3$ matter & a finite chemical potential

$$K_\ell = \sigma^\dagger(x) U_\mu(x) \sigma(x)$$

$$\exp\{\kappa S_f\} = \prod_\ell c_f \left\{ 1 + t_f \left[ \Omega K_\ell + \frac{1}{\Omega} K_\ell^\dagger \right] \right\}$$

\(\Omega = 1\) for \(\mu = 0\)

“Integrate matter fields”

remember:

$$\sum_{\sigma \in \mathbb{Z}_3} \sigma = 0$$
Two proto-type possibilities

“mesonic” matter worldline

“Integrate gluon fields”

\[ \sigma^\dagger U_\mu \sigma \]

Each matter segment comes with a gluon link!

Remember: \[ \sigma^3 = 1 \]

“baryonic” loop
\[ Z(\beta, \kappa, \mu) = 3^{N_\ell + V} c(\beta)^{N_p} c_f(\kappa, \mu)^{N_\ell} \]

\[ \sum_{n_p, k_\ell} t^S(n_p, k_\ell) t^L(k_\ell) \Omega^{t_+(k_\ell) - t_-(k_\ell)} \]

- \( L(k_\ell) \) Total length of matter worldlines
- \( t_{\pm}(k_\ell) \) Total number of matter links in positive (negative) Time direction

Theory of (gluon) surfaces that are either closed or bounded by matter worldlines
Some comments:

Linear confinement (Wilson Loop area law at strong coupling)

Ω(κ, μ)
Independent

Ω(κ, μ)³

Baryonic loop: Important excitation at finite densities
Is the sign problem solved?

\[ t_f(\beta, \kappa, \mu) > 0 \]

\[ \Omega(\beta, \kappa, \mu) > 0 \]
RESULTS

Finite Temperatures

Study \( \langle P_t - P_s \rangle \) \sim \text{entropy density}

\[ \frac{1}{T} = N_t \alpha \]

Lattice spacing

\( 20^3 \times N_t \) lattice
RESULTS

Weak Renormalisability

Study $\langle P_t - P_s \rangle \sim$ entropy density

pure Z3, $\beta=0.427$

$A \exp\{-m/T\}$

Mass Gap!
RESULTS

Scaling analysis  \( m \alpha(\beta) \)

Weak Renormalisability

Strong coupling fixed point

Effective theory ("a" cannot shrink to zero)
Weak Renormalisability

- Effective theory is (largely) cutoff independent (if physical scales are used)
- …as long as the cut-off is smaller than a critical value
  [be aware of rotational symmetry breaking]

Examples:

- QED (merges into GUT)
- Strong coupling lattice QCD
  e.g. [Philipsen, Towards a theoretical description of dense QCD, EPJ Web Conf 137 (2017) 03016]
Let us consider dynamical matter (at finite densities)

- Hopping parameter regulates (matter) mass:
  \[ \kappa \rightarrow \kappa_C \quad m \rightarrow 0 \]

- Access to matter density via:
  \[ \rho(\mu) = \frac{d}{d\mu} \ln Z(\mu) \]

\[ Z(\beta, \kappa, \mu) = 3^{N_\ell+V} \mathrm{c}(\beta)^{N_p} \mathrm{c}_f(\kappa, \mu)^{N_\ell} \sum_{n_p, k_\ell} t_{S(n_p, k_\ell)} t_{f}^{L(k_\ell)} \Omega_{t^+}(k_\ell) - t_{-}(k_\ell) \]

“Silver-Blaze” feature would result from cancellations!
Dense & cold matter: $\beta = 0.43$  $20^4$ lattice  $\kappa = 0.130$

Silver-Blaze feature!

mass gap

Bose-Einstein condensation?
Average cluster size to which a matter link belongs

Massless?

Percolation transition!

Satz type deconfinement

Number of matter clusters

\[ \beta = 0.43 \]

20^4 lattice

massless

Subcritical \( \mu \) effect!
(de spite of SB feature)
**Conclusions:**

- **Studied Z3 gauge theory with dynamical Z3 matter at finite chemical potential**
  - Shares vortex confinement mechanism with QCD (including linear confinement of static charges)
  - Aim: (weakly renormalisable) theory to study the (de-)confinement transition at finite densities exactly

**Results:**

- Mass gap in pure gauge theory
- Matter mass tuneable
- Sign-problem solvable by dualisation (2-brane formulation)
- Silver-Blaze feature
- Satz mechanism for deconfinement at finite densities

*Excellent theory for informing ideas re the QCD phase diagram!*
Outlook:

- Quantify the weakly renormalisability
- Derive the thermodynamics of the (pure) gauge theory
- Study the mu-T phase diagram
- Is there an endpoint?
- Exploring the Satz mechanism: study size of cluster container
- Investigate the “Sc confinement criterion” [Jeff Greensite, Wednesday talk]

Could also fix the gauge and study:

- Fix the gauge and study the mu-dependence of propagators (e.g. in the confinement phase)
- How does the Silver Blaze feature manifest itself in propagators?

Thank you!
\[ V(r)/\sigma^{1/2} \]

- **full SU(2)**
- **vortices only**
- **vortices removed**