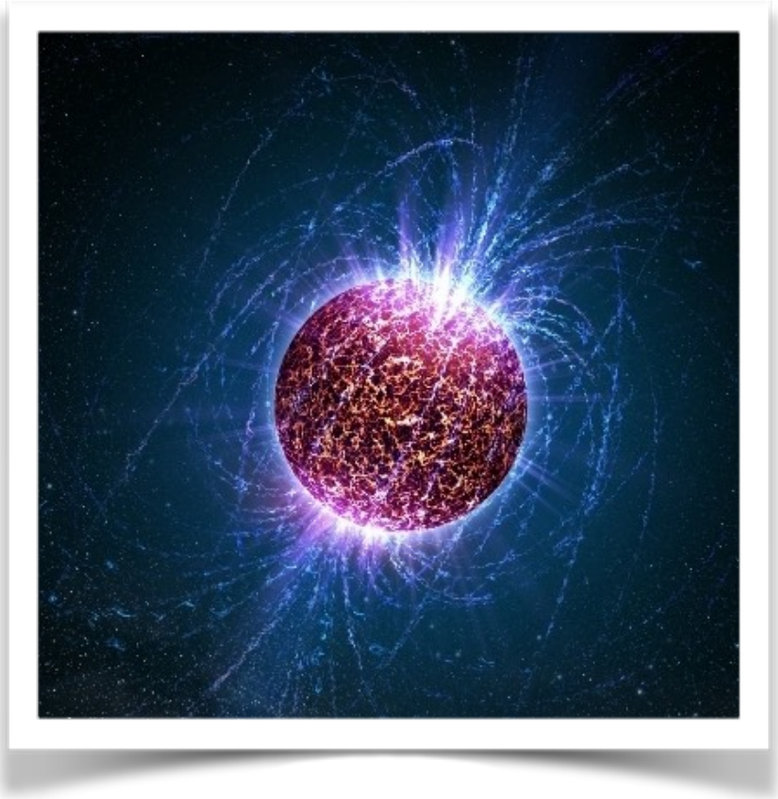
Z3 gauge theory with matter at finite densities

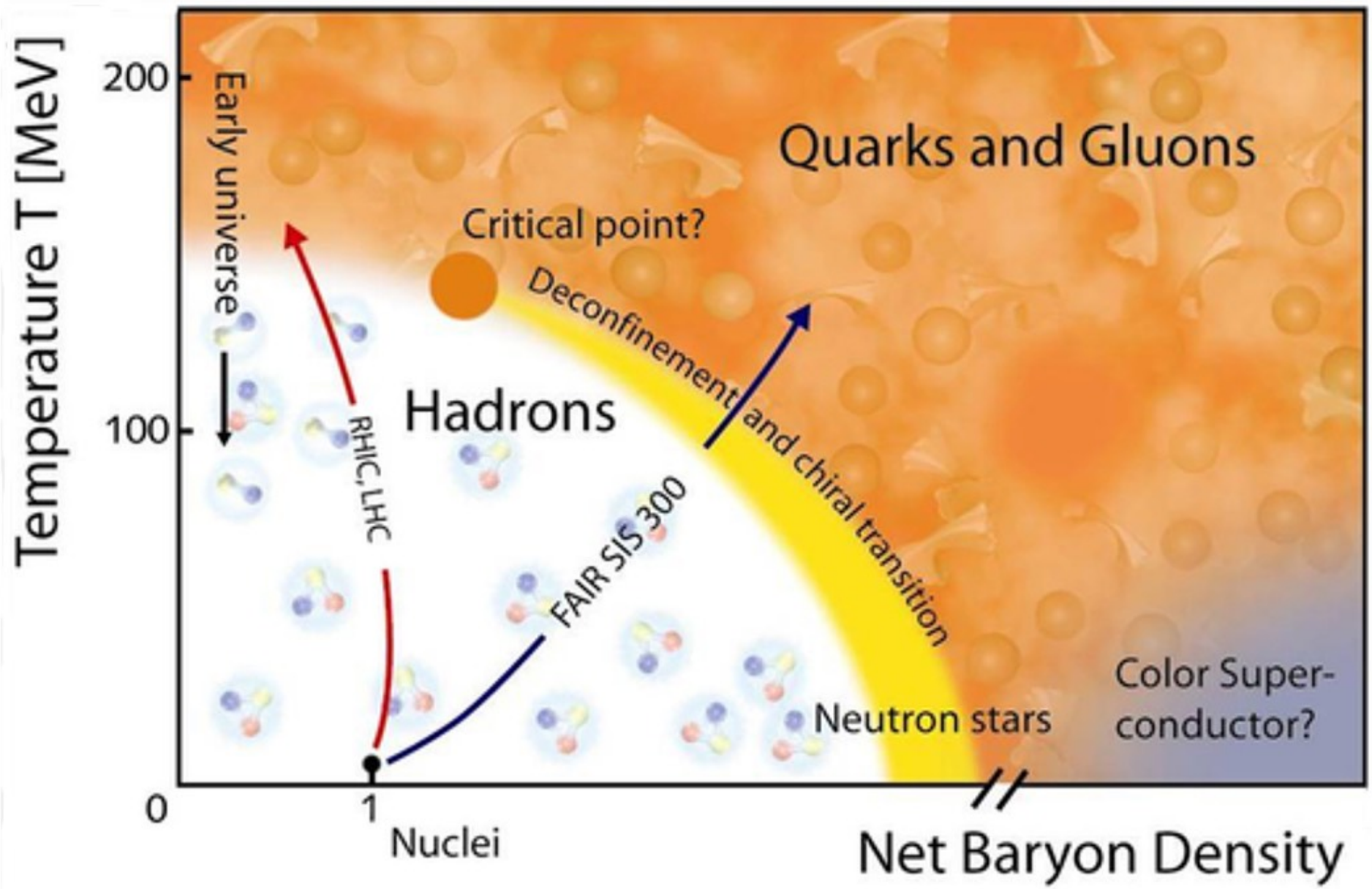


Kurt Langfeld (Liverpool University)

13th Confinement conference,
Maynooth, 2 August 2018

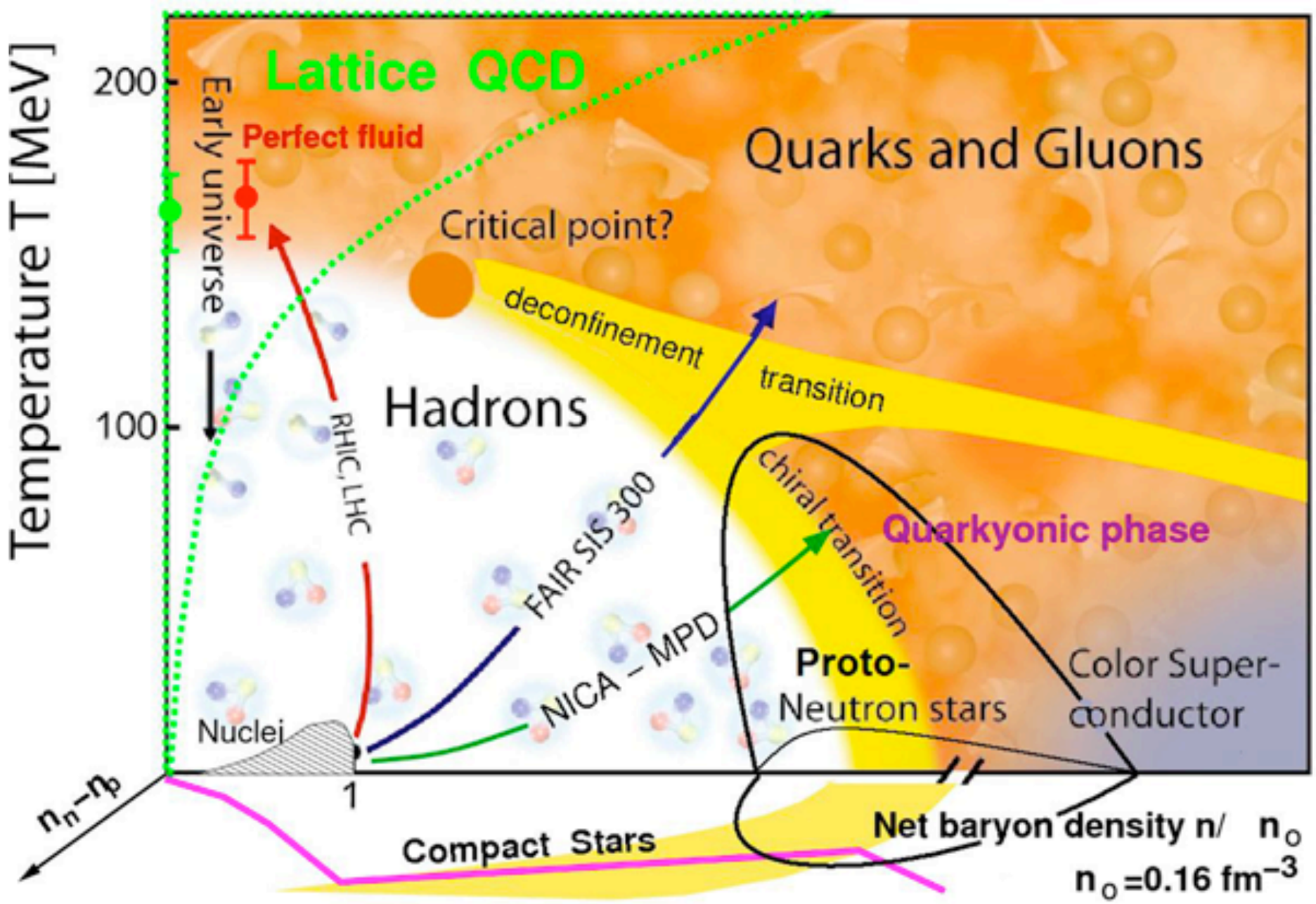
QCD phase diagram:

…10 years ago



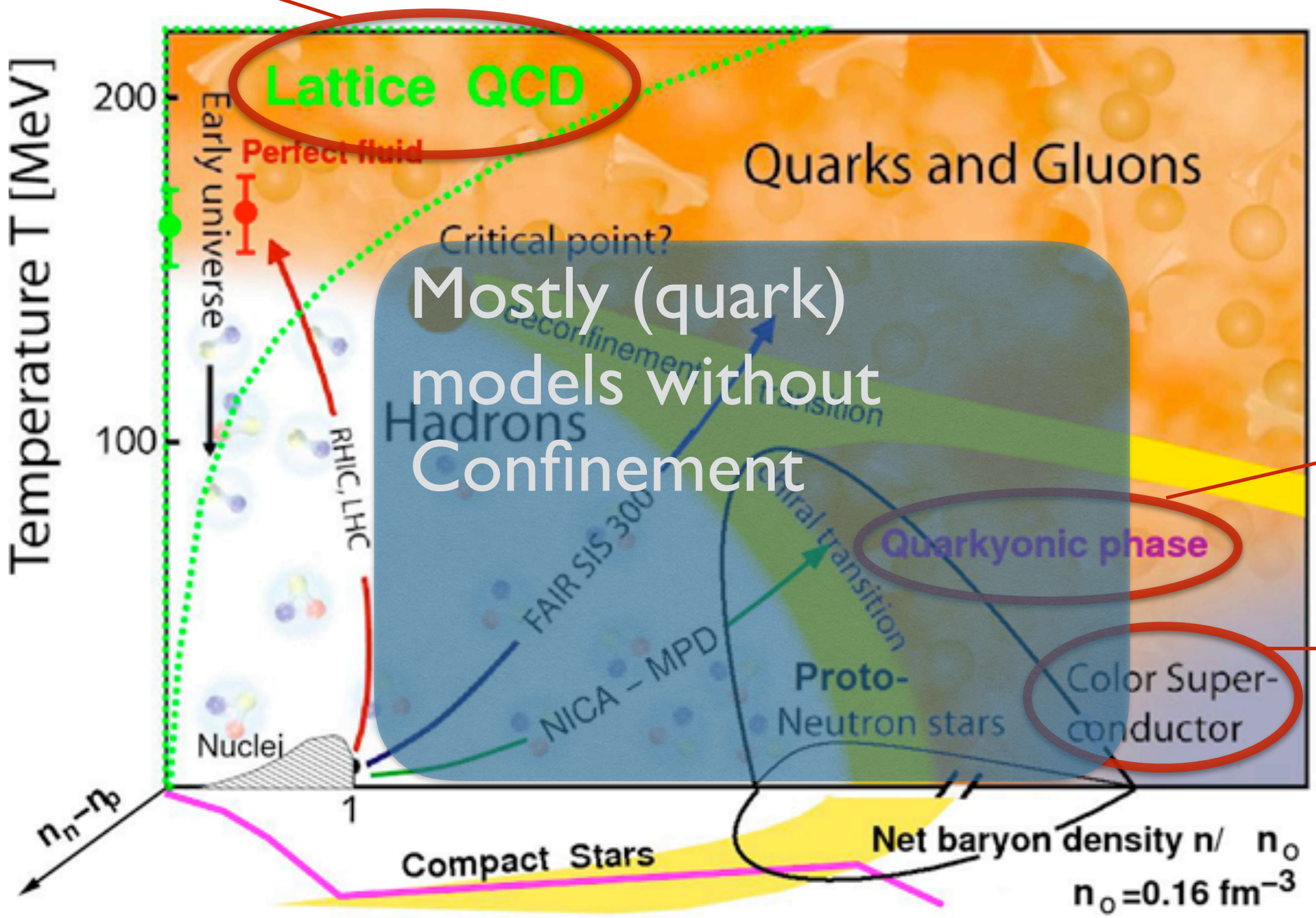
QCD phase diagram:

The Phases of Dense Matter, INT, July 11 - August 12, 2016



What are the theoretical foundations?

"sign problem"

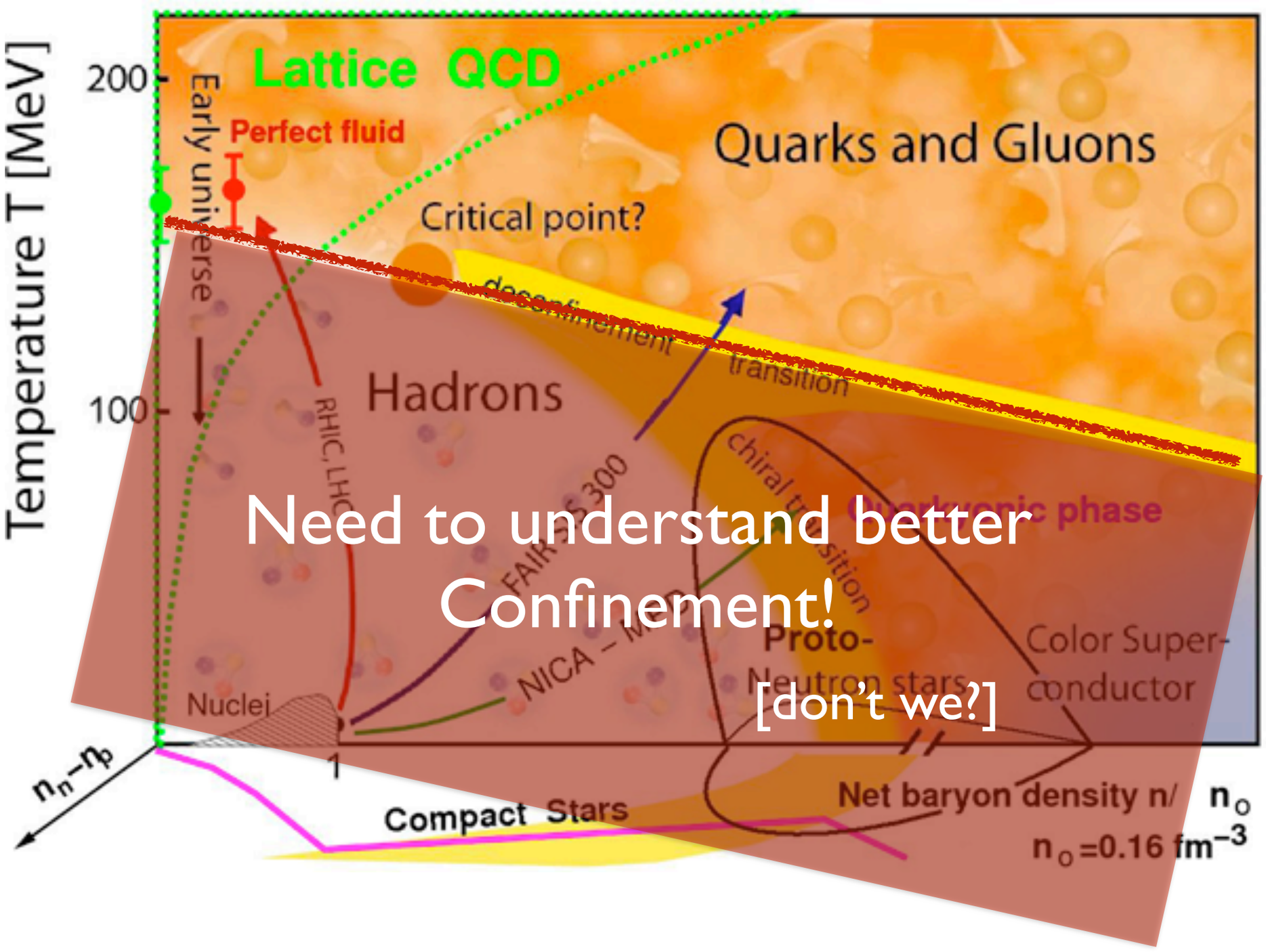


Large number of colours (???)

Perturbation theory (???)

QCD phase diagram:

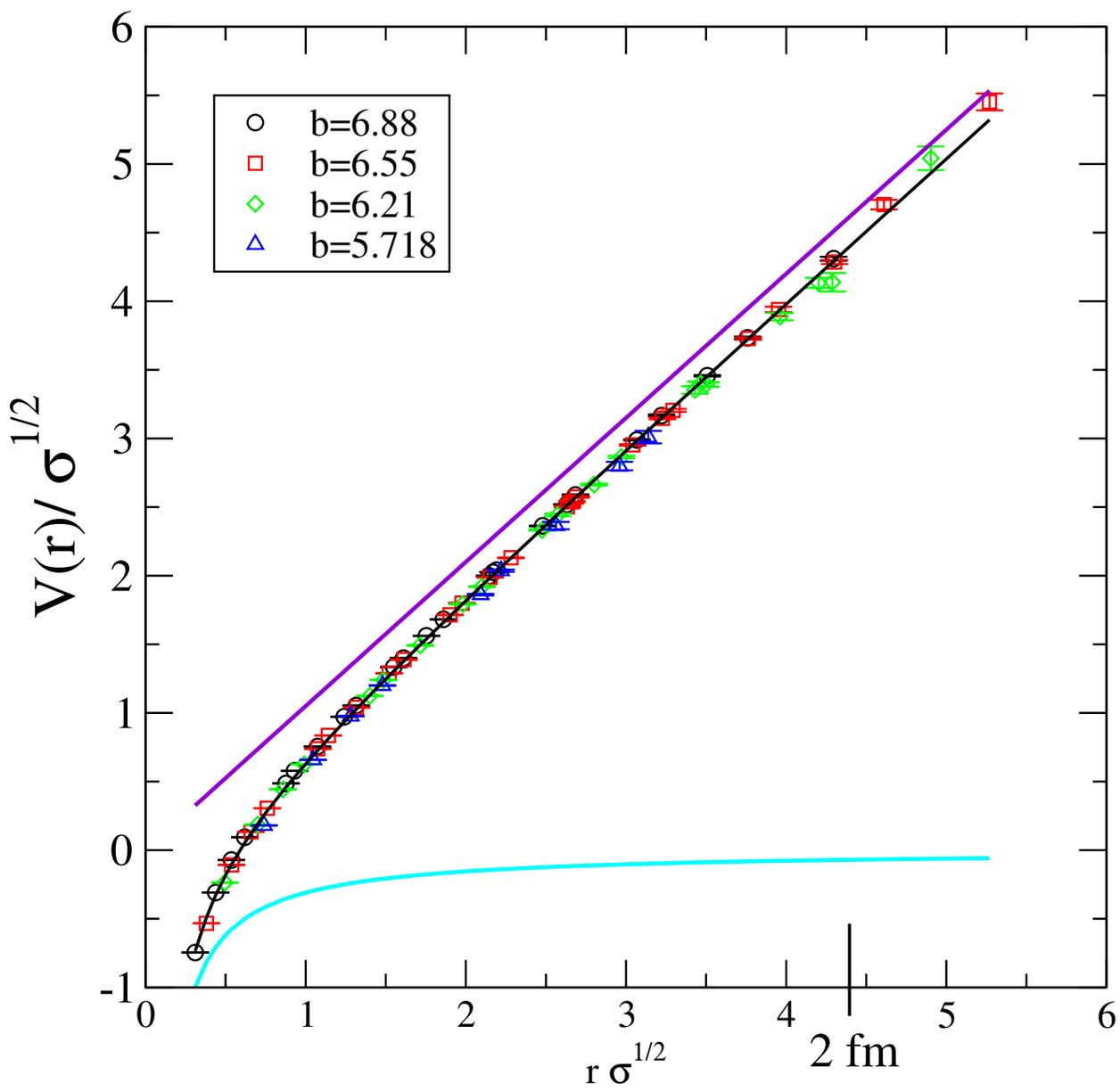
The Phases of Dense Matter, INT, July 11 - August 12, 2016



Need to understand better Confinement!

[don't we?]

What is quark confinement?



- Use (heavy) quarks as probe
- It is a feature of pure gauge theory!

Linearly rising Potential

[SU(3) improved action, KL, PRD 76 (2007) 094502]

How can quark confinement understood in gauge theories?

Hierarchy of groups → Hierarchy of scales

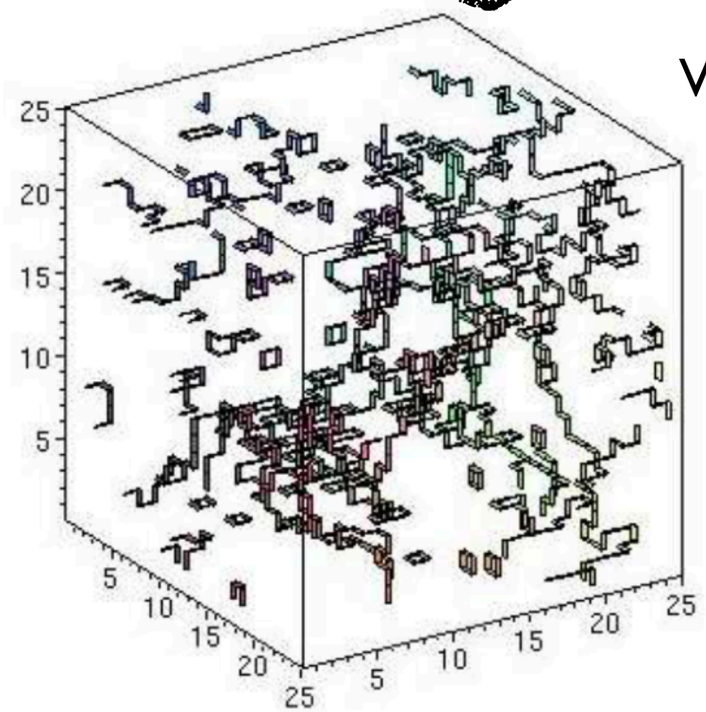
Breakdown — *OPE & condensates* — *Perturbation theory*

Low energies → High energies

Z3 subgroup ← Full SU(3)

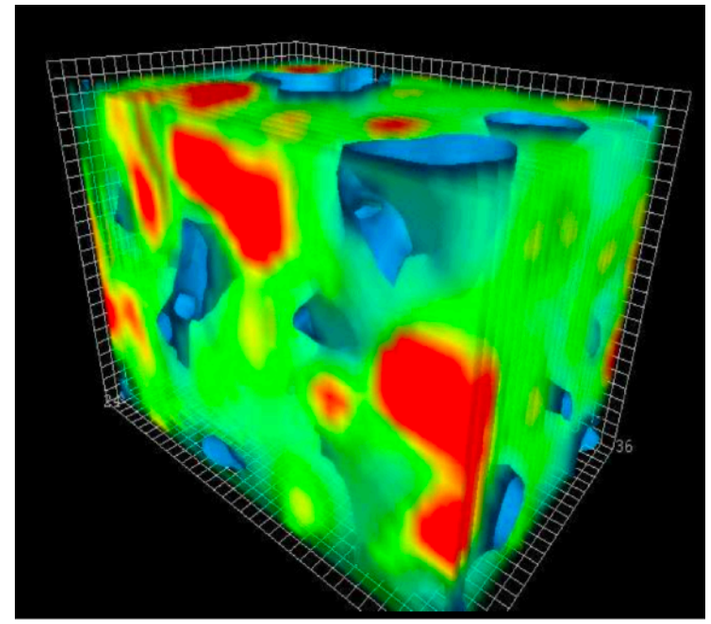
Gauge fixing + projection

Z3 subgroup



Vortex theory

Full SU(3)



CSSM Adelaide

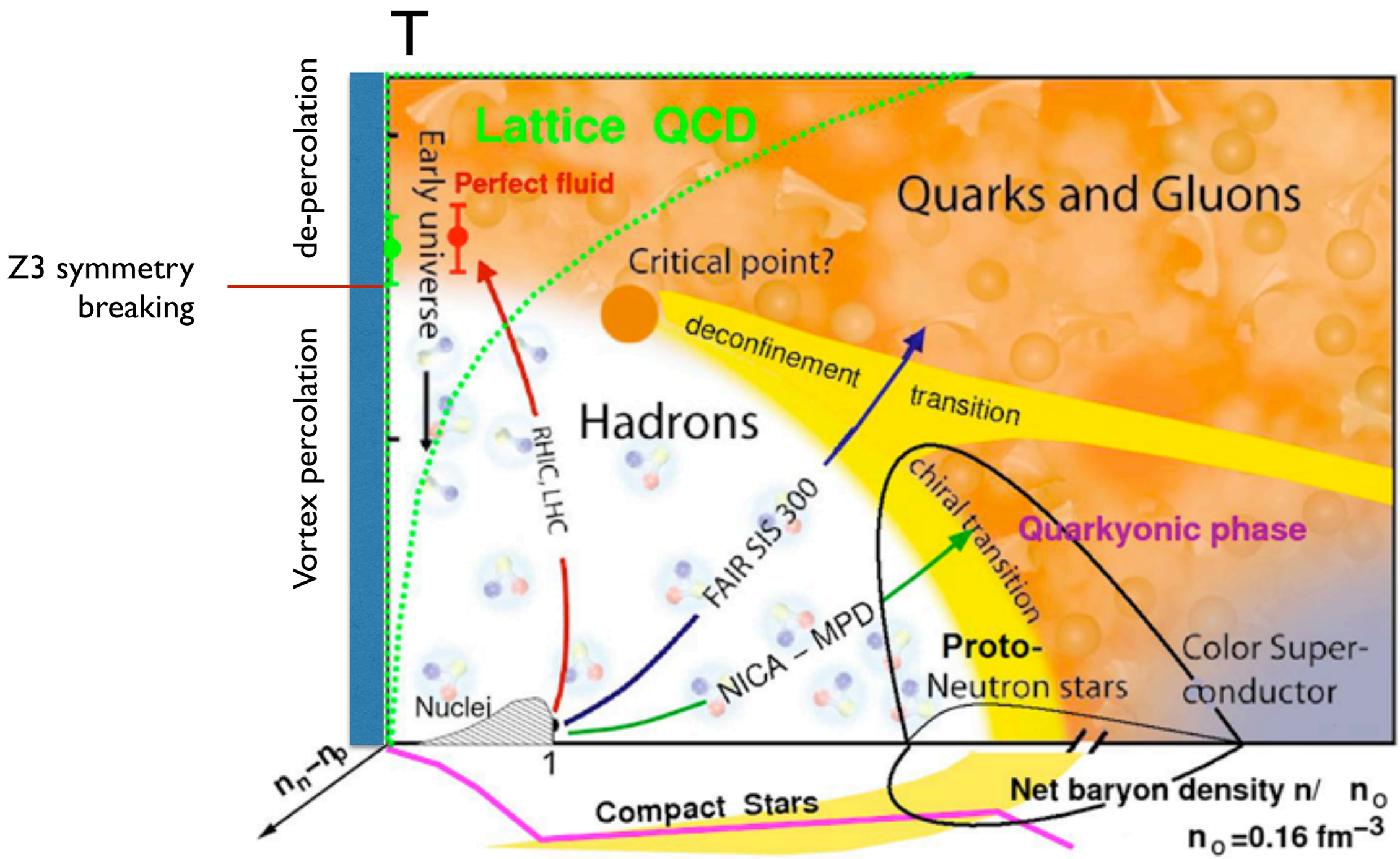
- Isolate dofs responsible for confinement
  - → *no confinement without them* ✓
  - → *weakly interacting* ✓
- Use them to understand non–perturb. physics
  - → *deconfinement at finite temperatures* ✓
  - → *spontaneous chiral symmetry breaking* ✓
  - → *origin of the OPE condensates* ?

Vortex picture of MCG

[Greensite, Faber, Olejnik, Del Debbio…. 1998]

[Engelhardt, Langfeld, Reinhardt, …. 1999]

What do we know about the phase diagram?



[Engelhardt, Langfeld, Reinhardt, Tennert, PRD 61 (2000) 054504]

Propose a ~~model~~ theory study

Z3 gauge theory with Z3 matter

- With confinement of charges
  (with non-trivial triality)

  ← (centre) vortex confinement mechanism

- With dynamical matter
  (characterised by triality, will be bosonic)

- That can be studied at finite densities of matter

  ← Sign-problem solved by real dual simulation: open 2-branes

Details of the model:

Group Z3: $Z_3 = \{1, z, z^\dagger\}$ $\qquad z = -\frac{1}{2} + \frac{\sqrt{3}}{2}\, i$

$U \in Z_3 :$ $\qquad UU^\dagger = 1 \qquad U^3 = 1 \qquad \sum_{U \in Z_3} U = 0$

Algebraic properties: $U^4 = U^3 U = U$

$$U^{11} = [U^3]^3 U^2 = U^2 = U^3 U^\dagger = U^\dagger$$

$$f(U, U^\dagger) = a + bU + cU^\dagger$$

$a, b, c, \in \mathbb{R}$

Real function, admits Taylor expansion

Partition function:

$$Z = \sum_{\sigma, U_\mu \in Z_3} \exp\Big\{ \beta S_g[U] + \kappa\, S_f[\sigma, U] \Big\}$$

$$S_g = \sum_{x,\mu>\nu} \mathrm{Re}\Big[U_\mu(x)\, U_\nu(x+\mu)\, U_\mu^\dagger(x+\nu)\, U_\nu^\dagger(x) \Big]$$

Plaquette action

"glue" - matter interaction

$$S_f = 2 \sum_{x,\mu=1\ldots3} \mathrm{Re}\, \sigma^\dagger(x)\, U_\mu(x)\, \sigma(x+\mu) +$$

h.c.

$$\sum_{x,\mu=4} \Big[\mathrm{e}^{\mu}\, \sigma^\dagger(x)\, U_\mu(x)\, \sigma(x+\mu) + \mathrm{e}^{-\mu}\, \sigma^\dagger(x)\, U_\mu^\dagger(x-\mu)\, \sigma(x-\mu) \Big]$$

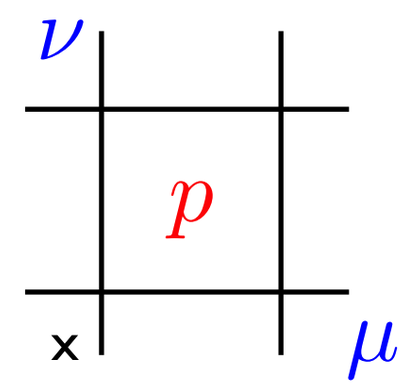
Chemical potential

Solving the sign-problem: 2-brane theory rising

Re-writing the pure gauge theory:

$$S_g = \sum_{x,\mu>\nu} \mathrm{Re}\left[\underbrace{U_\mu(x)\,U_\nu(x+\mu)\,U_\mu^\dagger(x+\nu)\,U_\nu^\dagger(x)}_{P_p \in Z_3}\right]$$

$$\exp\{\beta\, S_g\} = \prod_p c(\beta)\left[1 + t(\beta)\,(P_p + P_p^\dagger)\right]$$



crucial: $t(\beta) \geq 0$

- Expand brackets
- Sum over $U_\mu(x) \in Z_3$

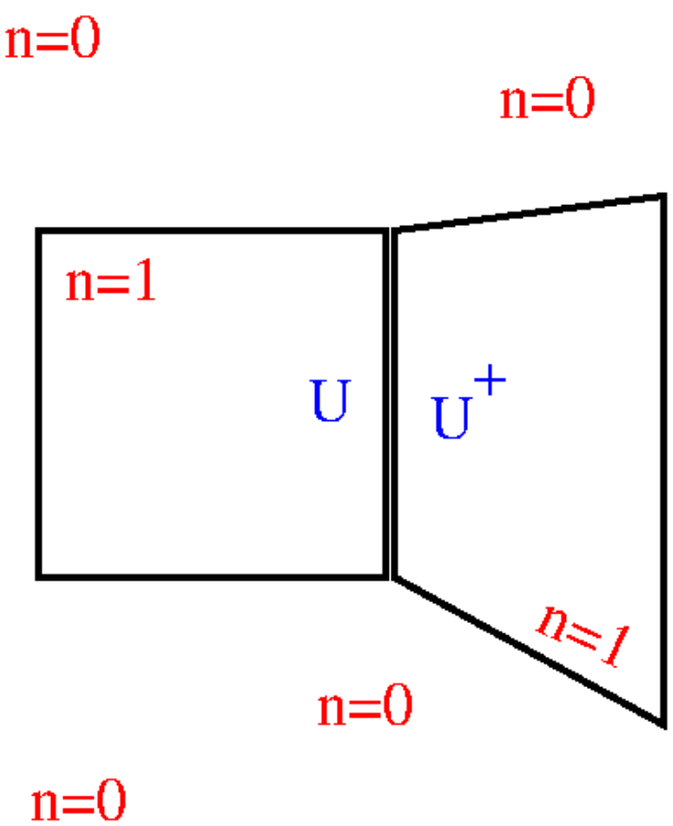
- Introduce "brane" variables:

$$\sum_{n_p=-1,0,1} \left[\delta_{n_p,0} \; + \; \delta_{n_p,1}\, t\, P_p \; + \; \delta_{n_p,-1}\, t P_p^\dagger \right]$$

- "Integrate link variables"

remember:

$$\sum_{U \in Z_3} U = 0$$



Non-trivial plaquettes form closed surfaces!

- Result for the pure gauge theory:

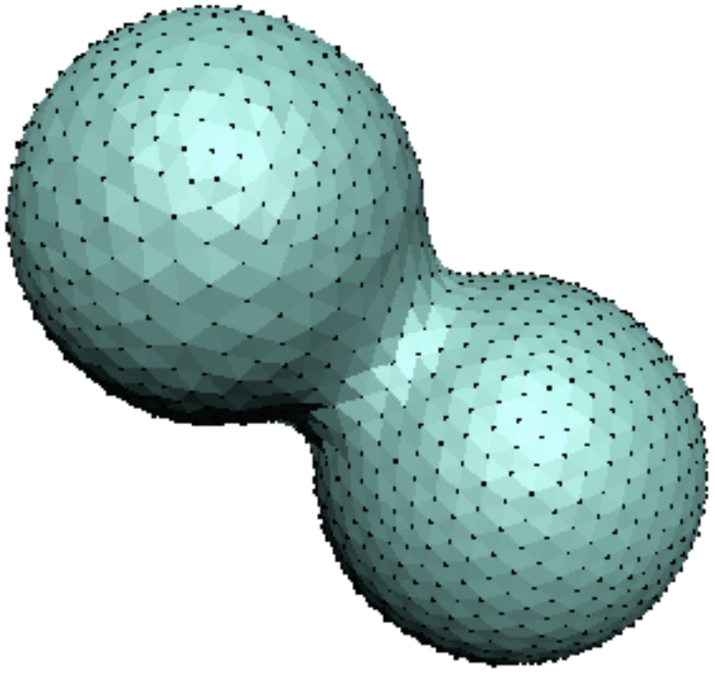
$$Z(\beta) \;=\; 3^{N_\ell}\, c(\beta)^{N_p} \sum_{\{n_p\}} t(\beta)^{S[\{n\}]}$$

Theory of 2D random surfaces in 4D with action proportional to the surface area

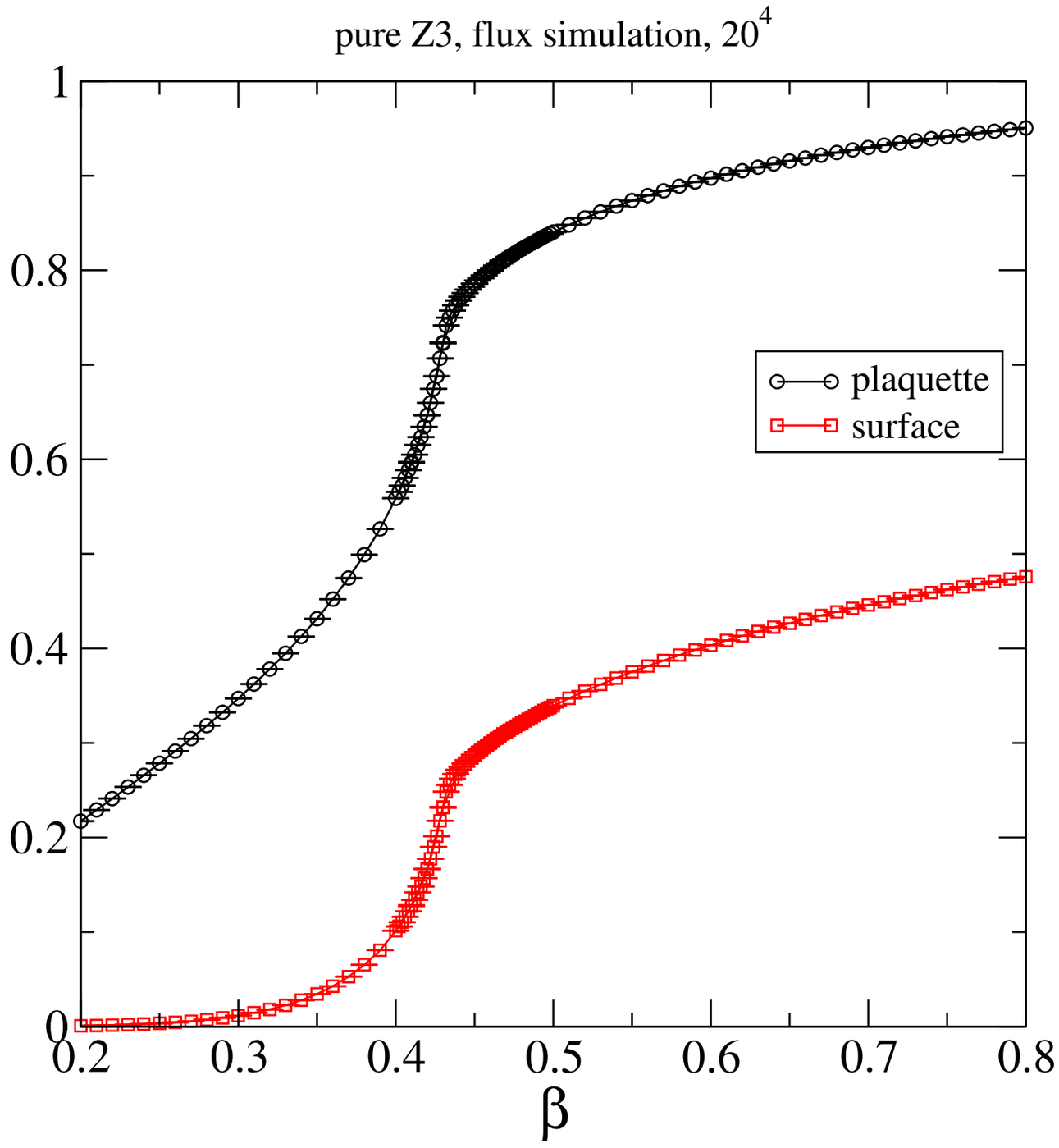
$S[\{n\}]$ Total area of closed surfaces

- Application: plaquette expectation value of pure Z3 gauge theory

$$\langle P \rangle \;=\; \frac{d}{d\beta} Z(\beta)$$



Pure Z3 gauge theory in 2-brane formulation:



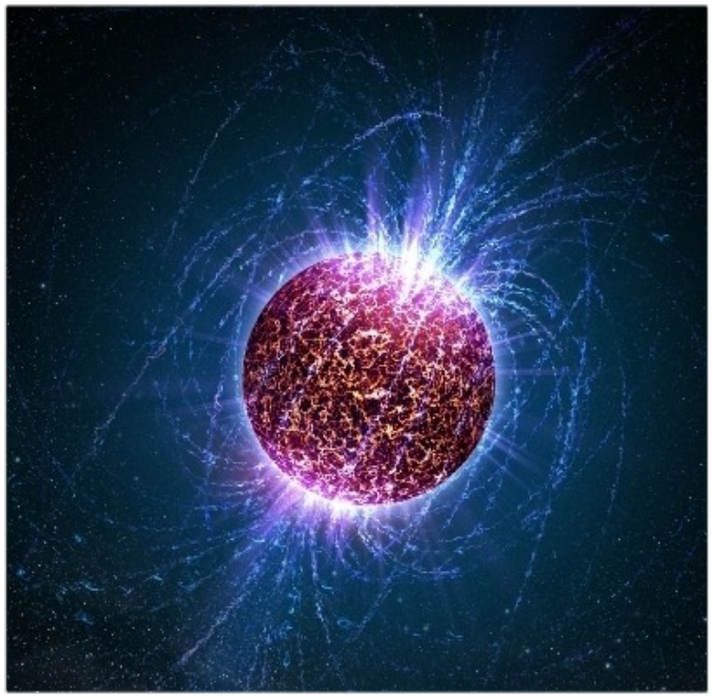
Adding dynamical Z3 matter & a finite chemical potential

"gluon" field

Matter field

$$K_\ell = \underbrace{\sigma^\dagger(x)\, U_\mu(x)\, \sigma(x)}_{\text{Z3}}$$

link: $(x, \mu) \to \ell$



$$\exp\{\kappa S_f\} = \prod_\ell c_f \left\{ 1 + t_f \left[\Omega\, K_\ell + \frac{1}{\Omega} K_\ell^\dagger \right] \right\}$$

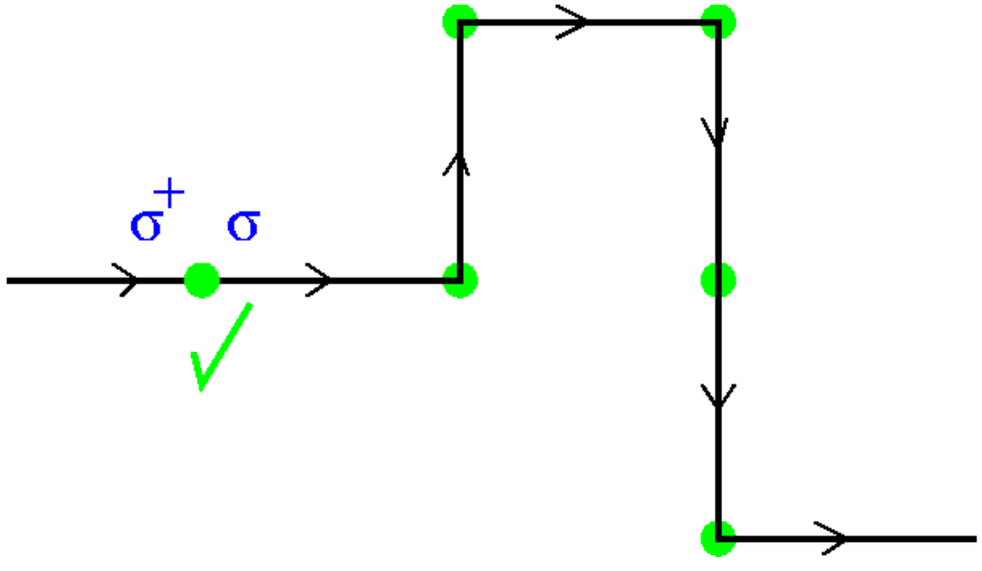
(κ, μ)

$\Omega = 1 \quad \text{for} \quad \mu = 0$

"Integrate matter fields"

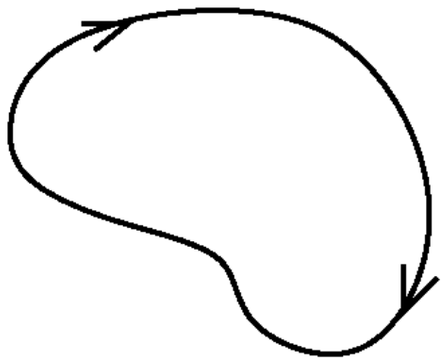
remember:

$$\sum_{\sigma \in Z_3} \sigma = 0$$



Two proto-type possibilities

remember: $\sigma^3 = 1$



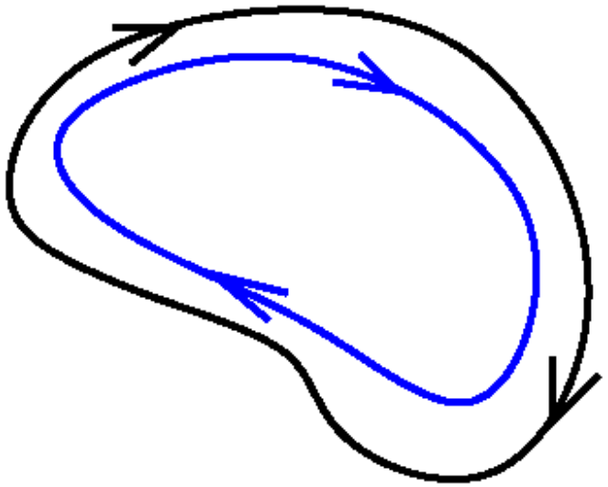
"mesonic" matter worldline

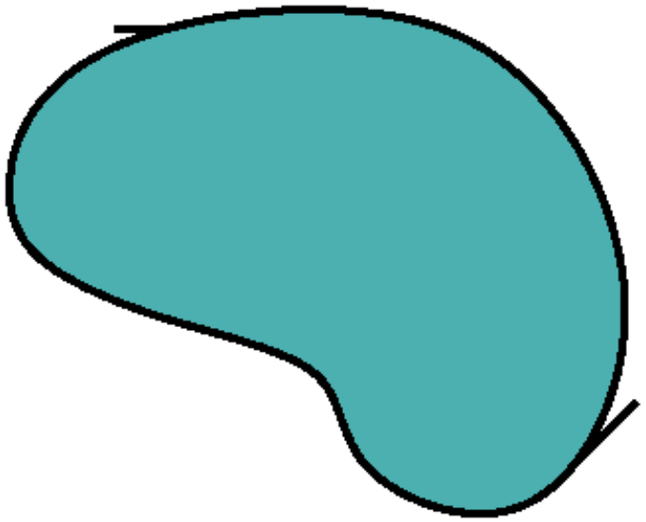
"baryonic" loop

"Integrate gluon fields"

$\sigma^\dagger U_\mu \sigma$

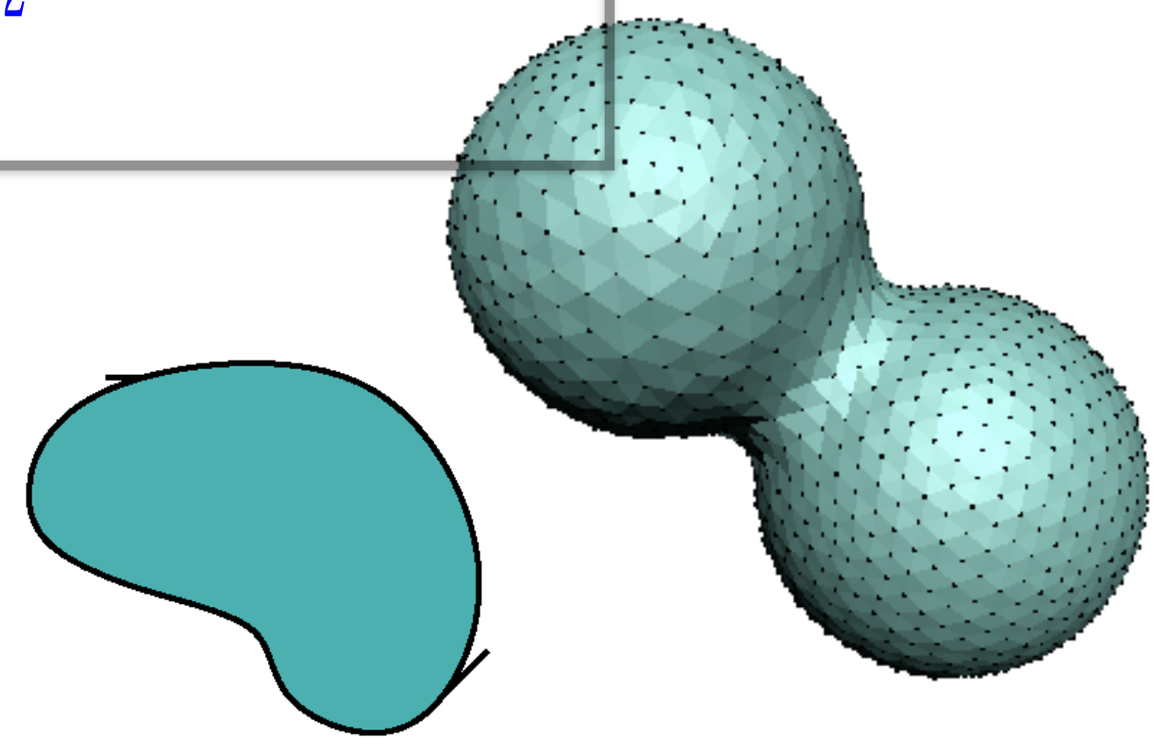
Each matter segment comes with a gluon link!





$$Z(\beta,\kappa,\mu) = 3^{N_\ell+V}\, c(\beta)^{N_p}\, c_f(\kappa,\mu)^{N_\ell}$$

$$\sum_{n_p,k_\ell} t^{S(n_p,k_\ell)}\, t_f^{L(k_\ell)}\, \Omega^{t_+(k_\ell)-t_-(k_\ell)}$$

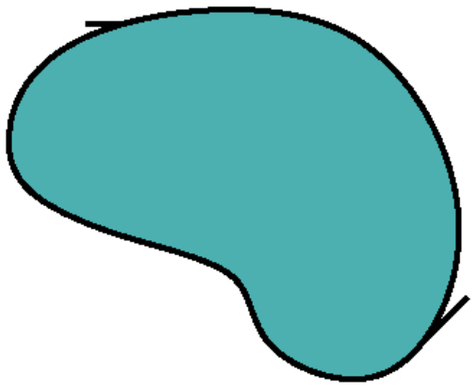


$L(k_\ell)$ Total length of matter worldlines

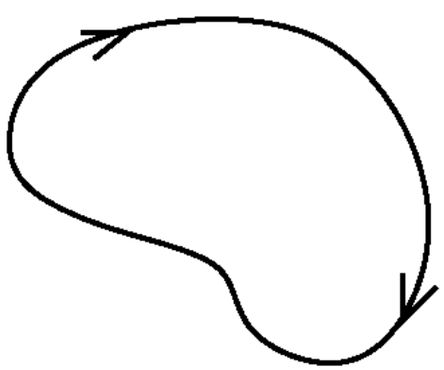
$t_\pm(k_\ell)$ Total number of matter links in positive (negative) Time direction

Theory of (gluon) surfaces that are either closed or bounded by matter worldlines

Some comments:



Linear confinement (Wilson Loop area law at strong coupling)



$\Omega(\kappa, \mu)$

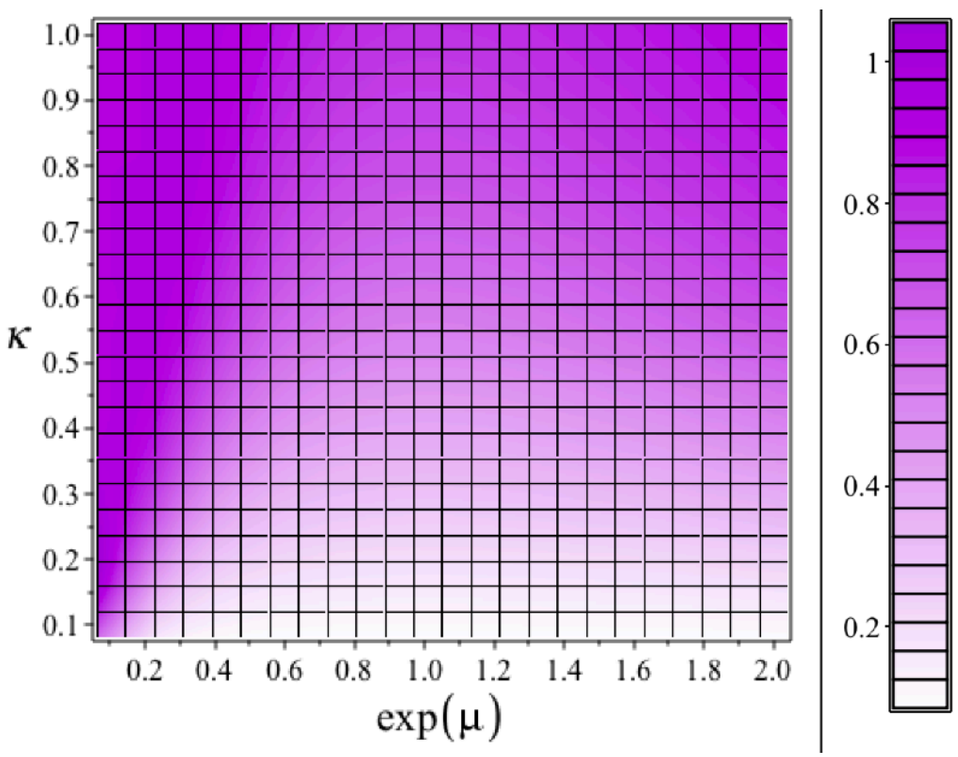
Independent

$\Omega(\kappa, \mu)^3$

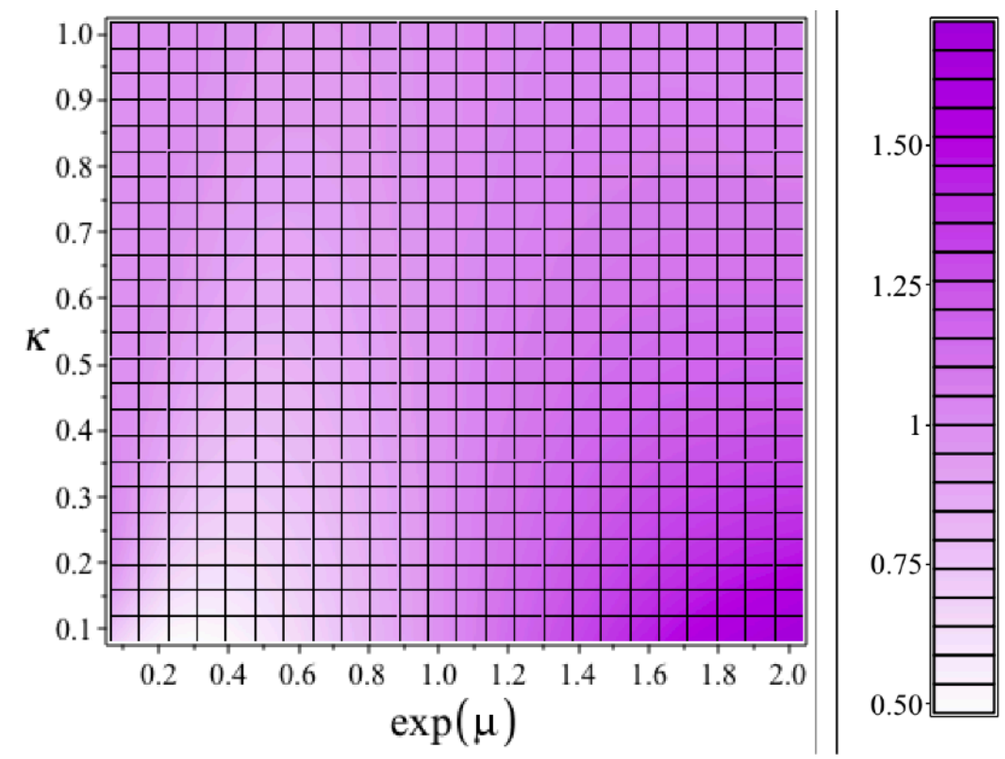
Baryonic loop:
Important excitation at finite densities

Is the sign problem solved?

$t_f(\beta, \kappa, \mu) > 0$

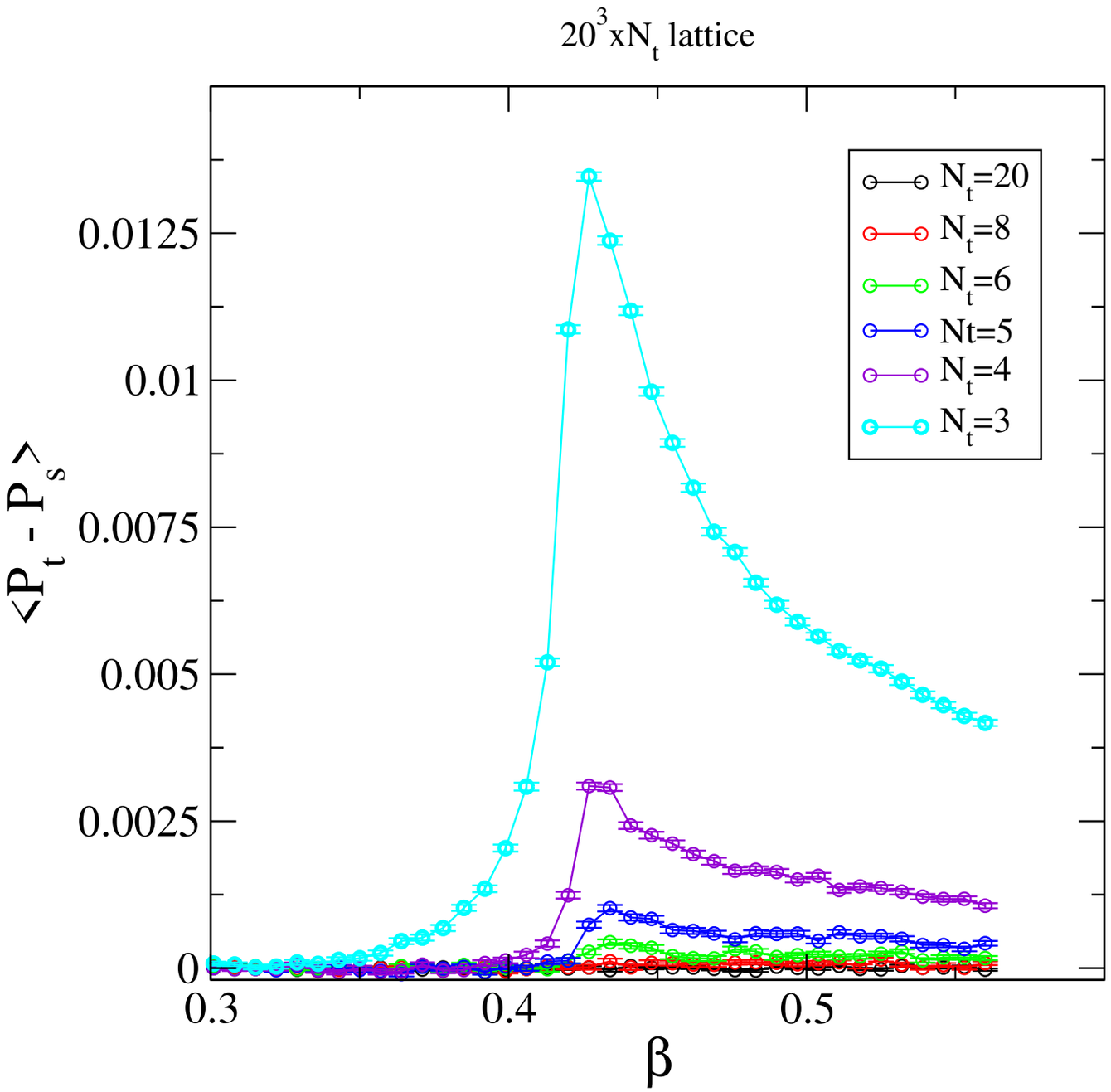


$\Omega(\beta, \kappa, \mu) > 0$



RESULTS Finite Temperatures

Study $\langle P_t - P_s \rangle$ ~ entropy density



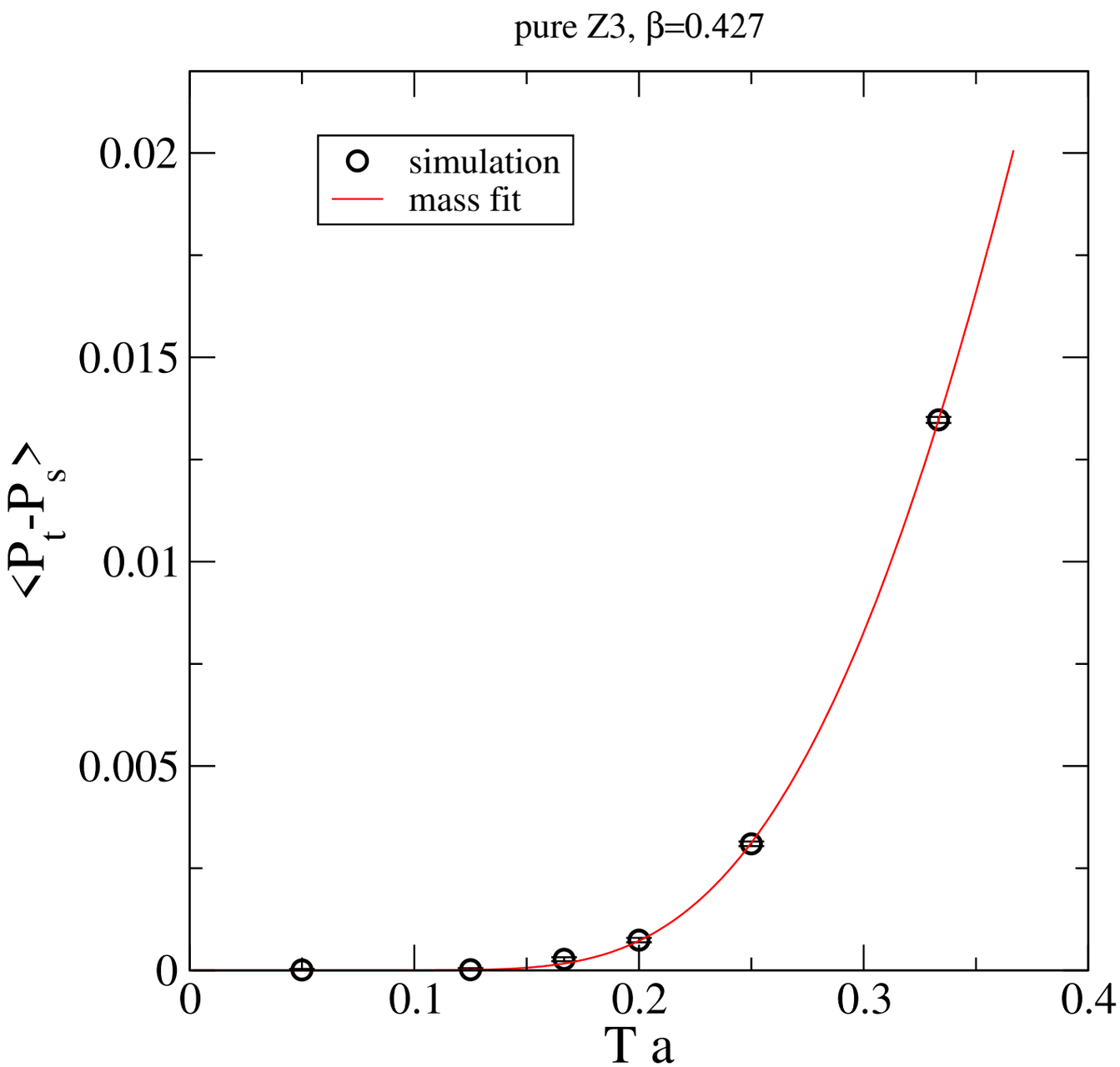
$$1/T = N_t\, a$$

Lattice spacing

RESULTS

Weak Renormalisability

Study $\langle P_t - P_s \rangle$ ~ entropy density



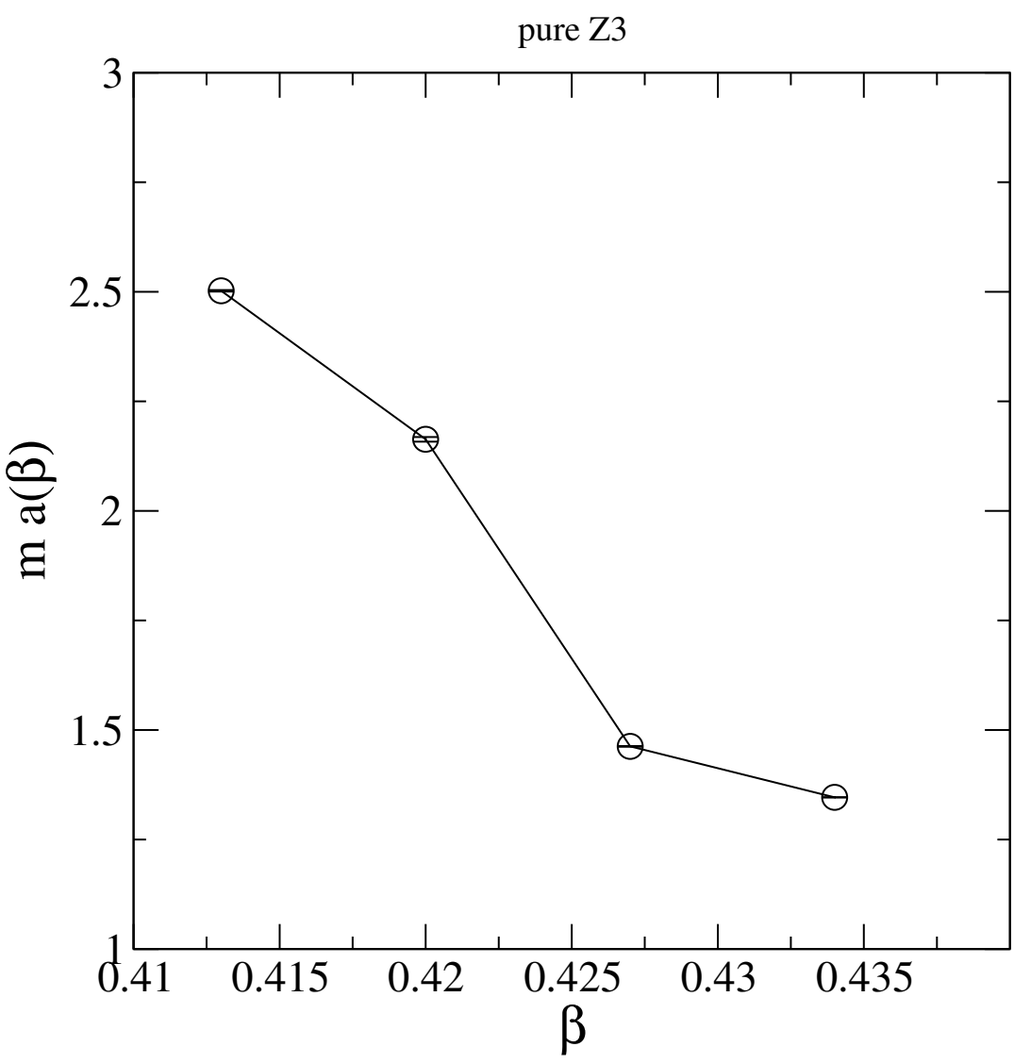
$$A \ \exp\{-m/T\}$$

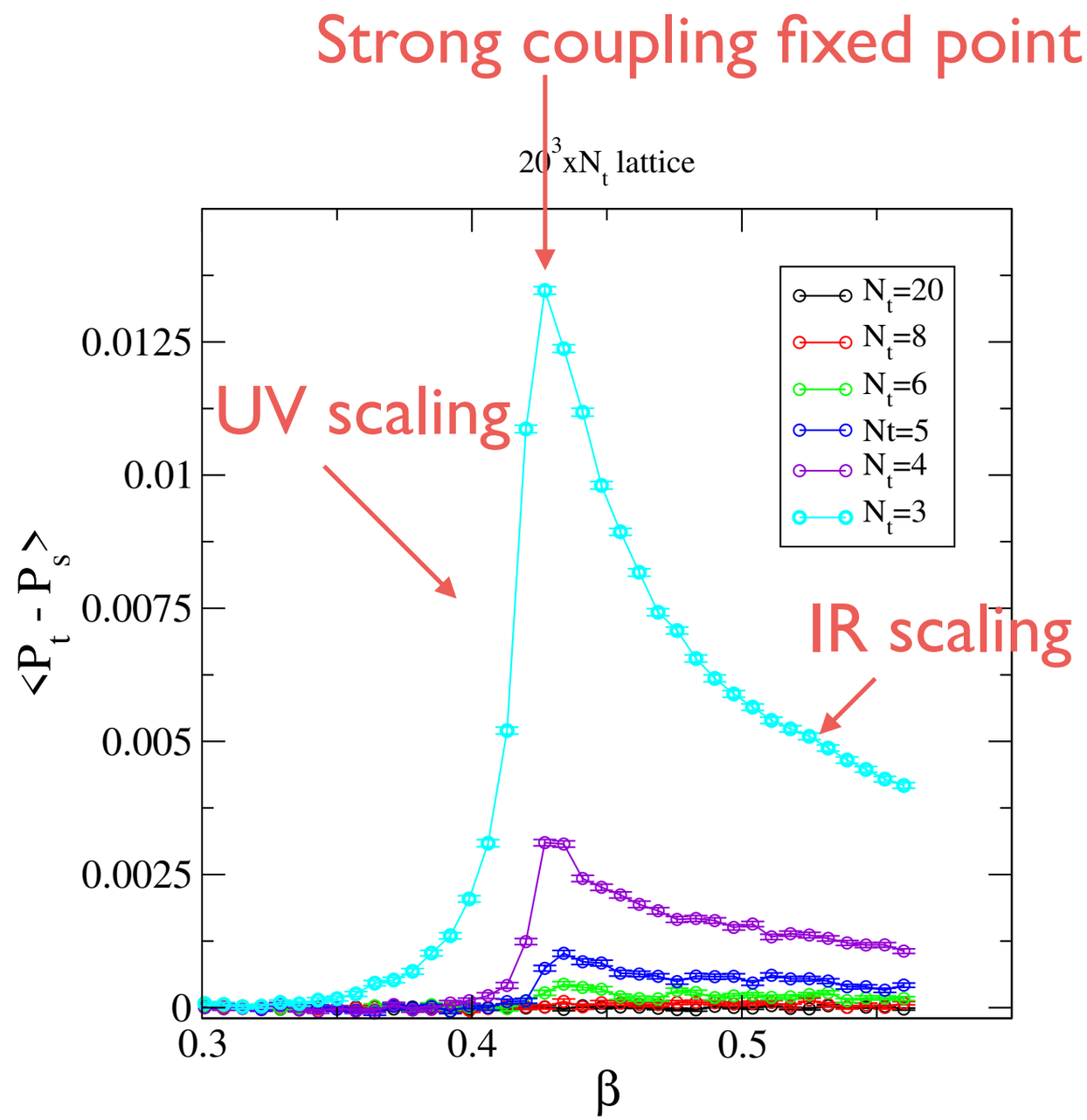
Mass Gap!

RESULTS

Weak Renormalisability

Scaling analysis $m\ a(\beta)$





Effective theory ("a" cannot shrink to zero)

Weak Renormalisability

- Effective theory is (largely) cutoff independent (if physical scales are used)
- …as long as the cut-off is smaller than a critical value

  [be aware of rotational symmetry breaking]

Examples:

- QED (merges into GUT)
- Strong coupling lattice QCD

  e.g. [Philipsen, Towards a theoretical description of dense QCD, EPJ Web Conf 137 (2017) 03016]

Let us consider dynamical matter (at finite densities)

- Hopping parameter regulates (matter) mass:

$$\kappa \;\to\; \kappa_c \qquad\qquad m \;\to\; 0$$

- Access to matter density via: $\rho(\mu) \;=\; \dfrac{d}{d\mu}\,\ln Z(\mu)$

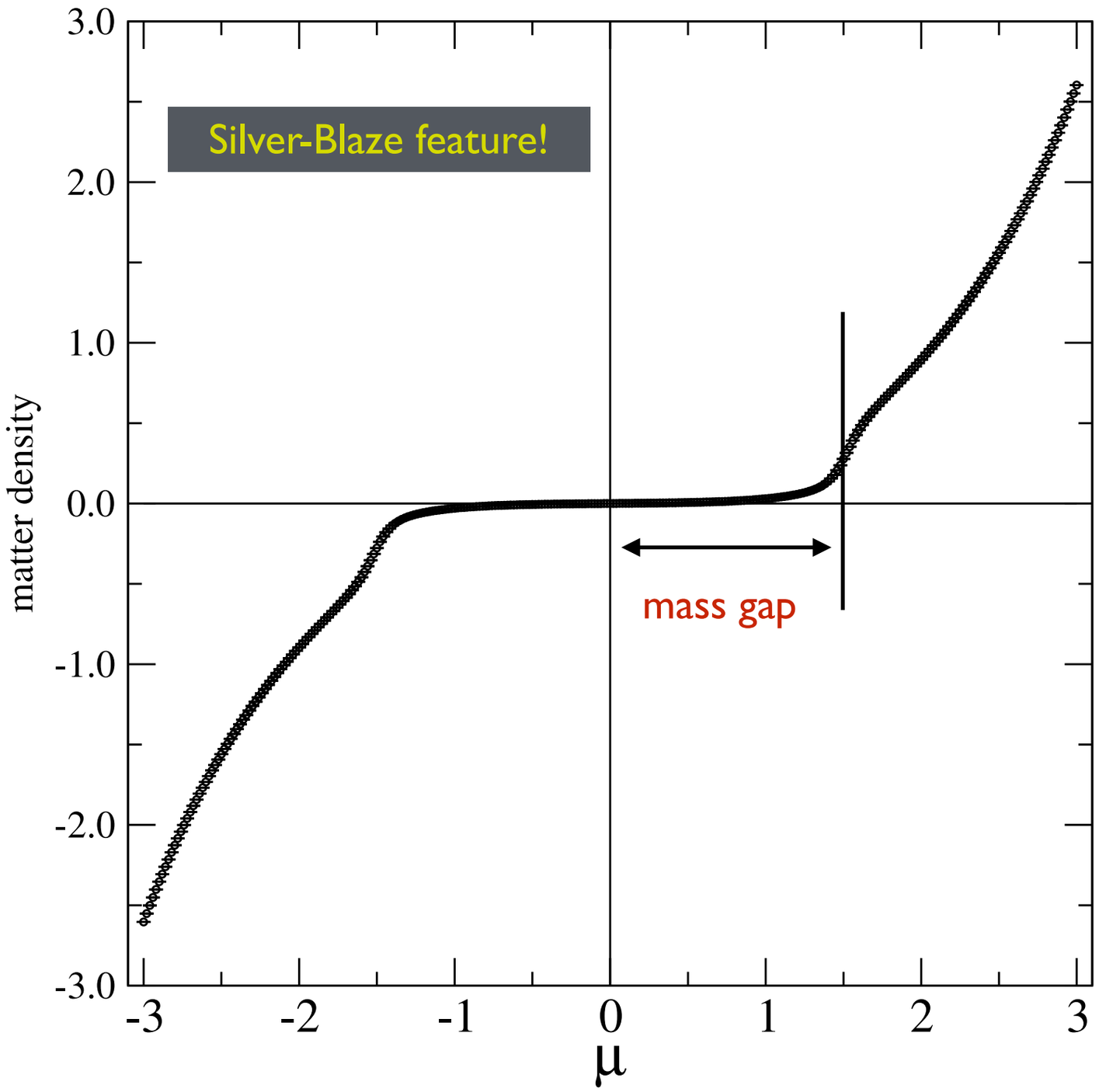
$$Z(\beta,\kappa,\mu) = 3^{N_\ell+V}\, c(\beta)^{N_p}\, c_f(\kappa,\mu)^{N_\ell} \sum_{n_p,k_\ell} t^{S(n_p,k_\ell)}\, t_f^{L(k_\ell)}\, \Omega^{t_+(k_\ell)-t_-(k_\ell)}$$

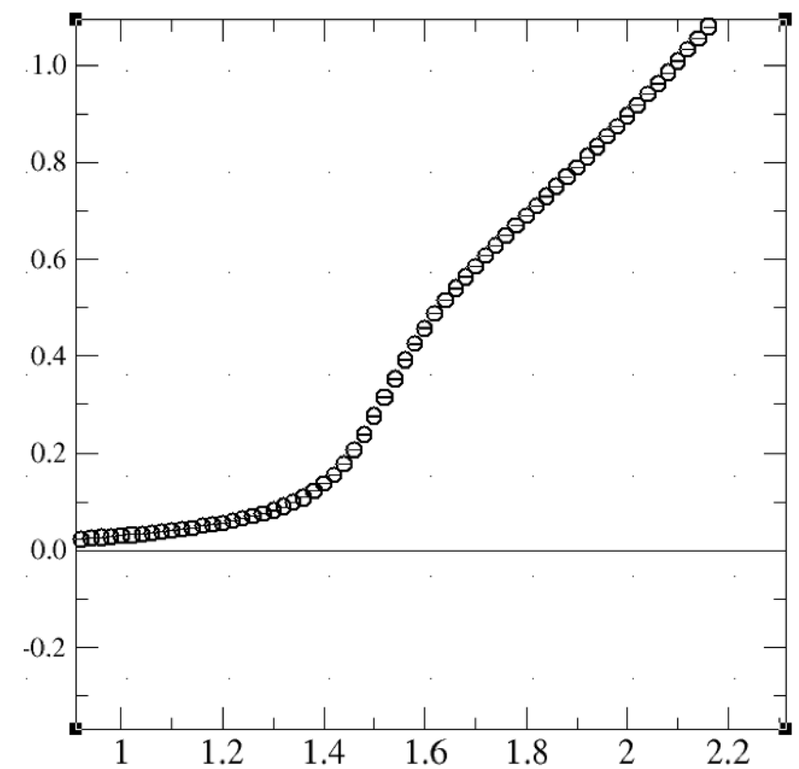
μ dependence

"Silver-Blaze" feature would result from cancellations!

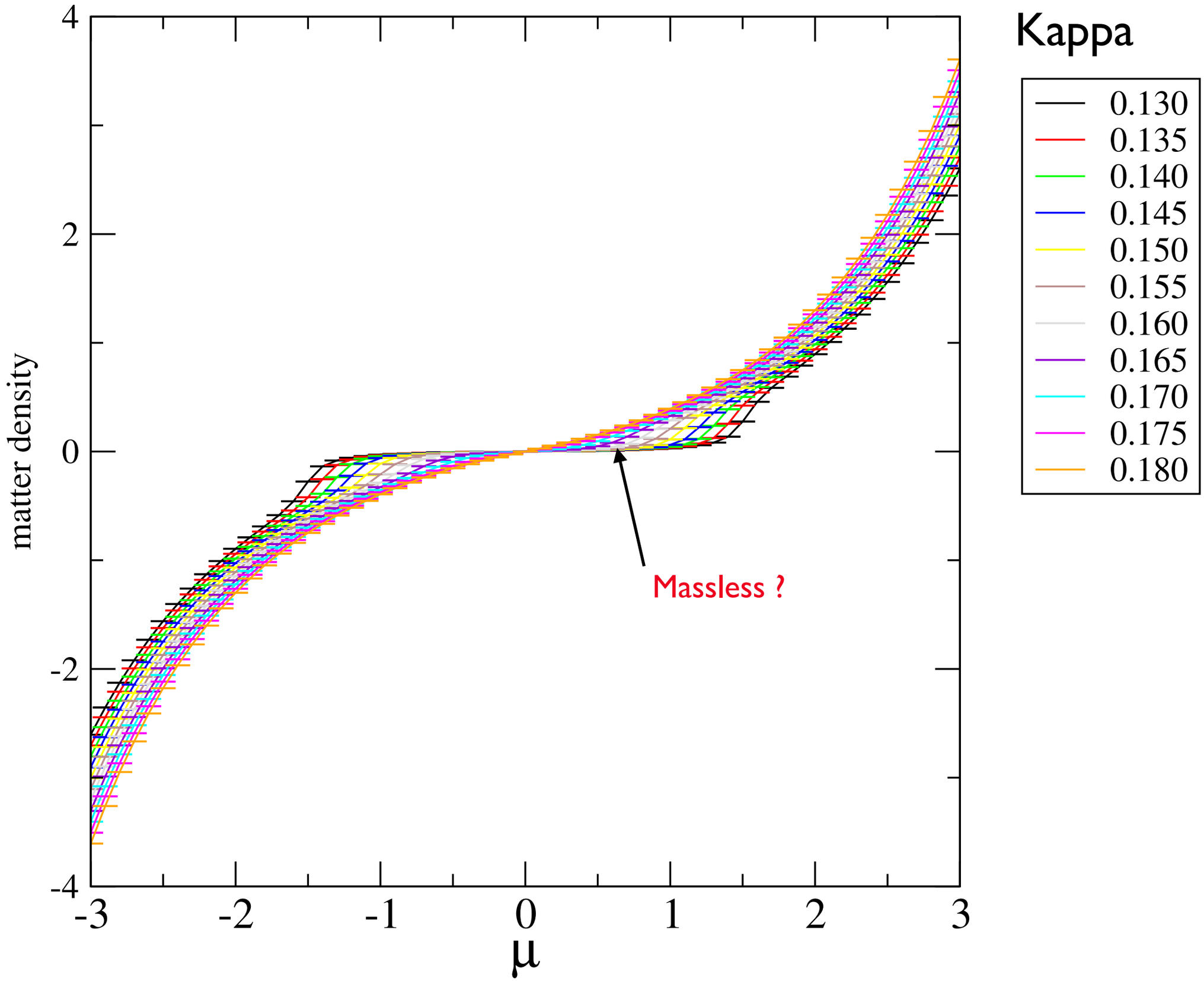
Dense & cold matter:

$\beta = 0.43 \quad 20^4 \text{ lattice} \quad \kappa = 0.130$

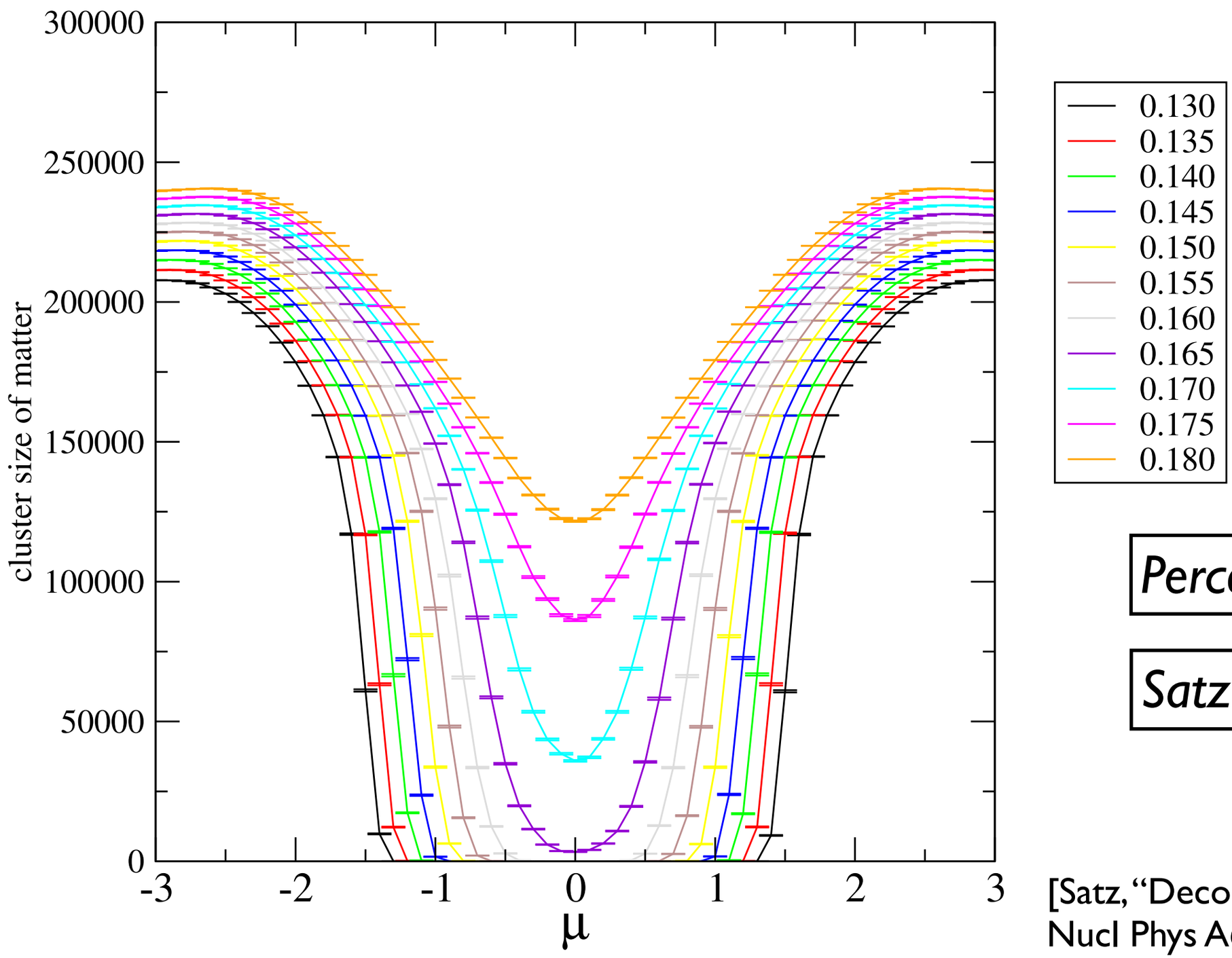




Bose-Einstein condensation?


Kappa
0.130
0.135
0.140
0.145
0.150
0.155
0.160
0.165
0.170
0.175
0.180
Massless ?
Massless ?
matter density
μ
4
2
0
-2
-4
-3
-2
-1
0
1
2
3

Average cluster size to which a matter link belongs



Massless ?

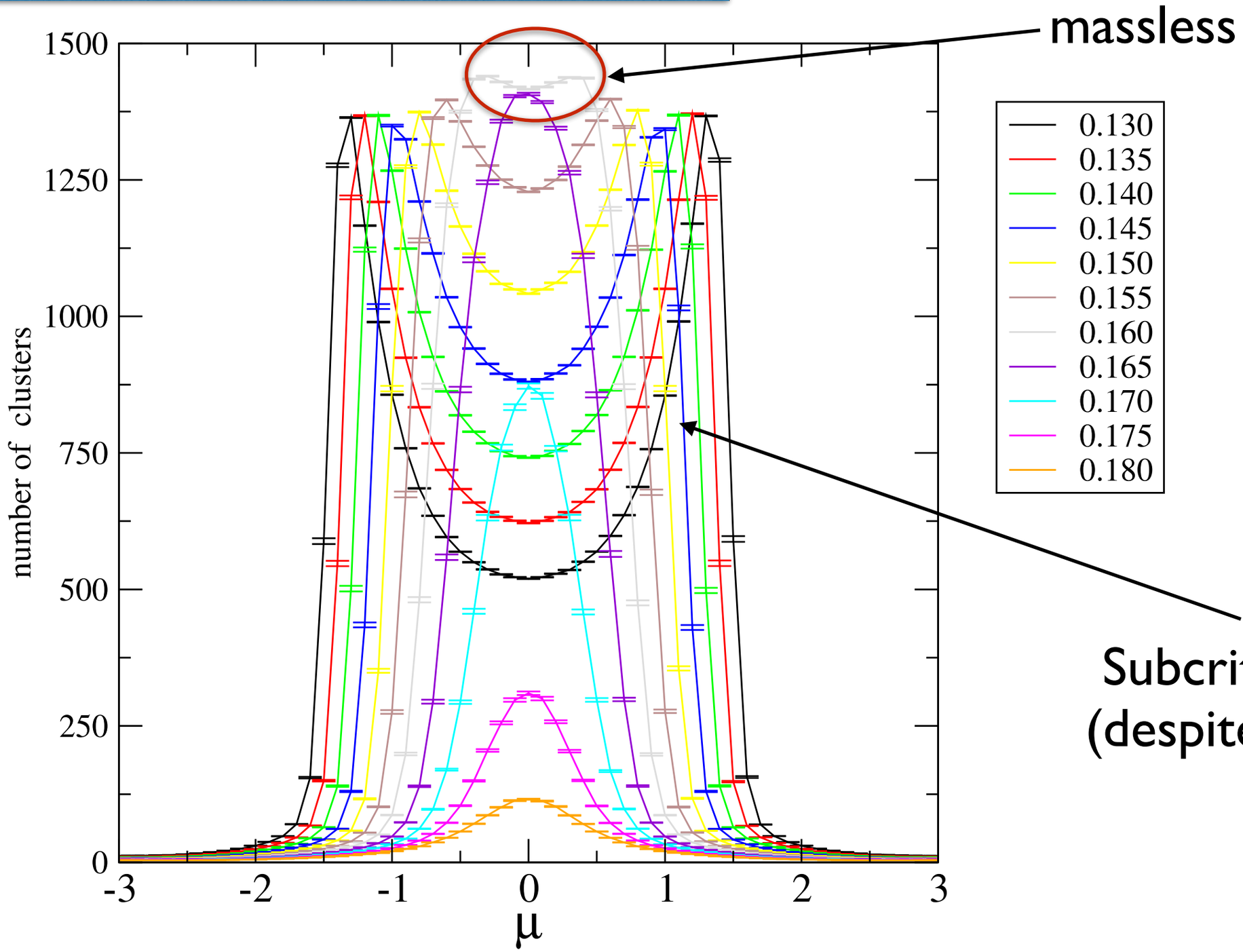
Percolation transition!

Satz type deconfinement

[Satz, "Deconfinement and Percolation", Nucl Phys A642 (1998)130]

Number of matter clusters

$\beta = 0.43$ 20^4 lattice



Subcritical mu effect!
(despite of SB feature)

Conclusions:

- Studied Z3 gauge theory with dynamical Z3 matter at finite chemical potential

  → Shares vortex confinement mechanism with QCD (including linear confinement of static charges)

  → Aim: (weakly renormalisable) theory to study the (de-)confinement transition at finite densities exactly

- **Results:**

  [preliminary]

  - Mass gap in pure gauge theory
  - Matter mass tuneable
  - Sign-problem solvable by dualisation (2-brane formulation)
  - Silver-Blaze feature
  - Satz mechanism for deconfinement at finite densities

- *Excellent theory for informing ideas re the QCD phase diagram!*

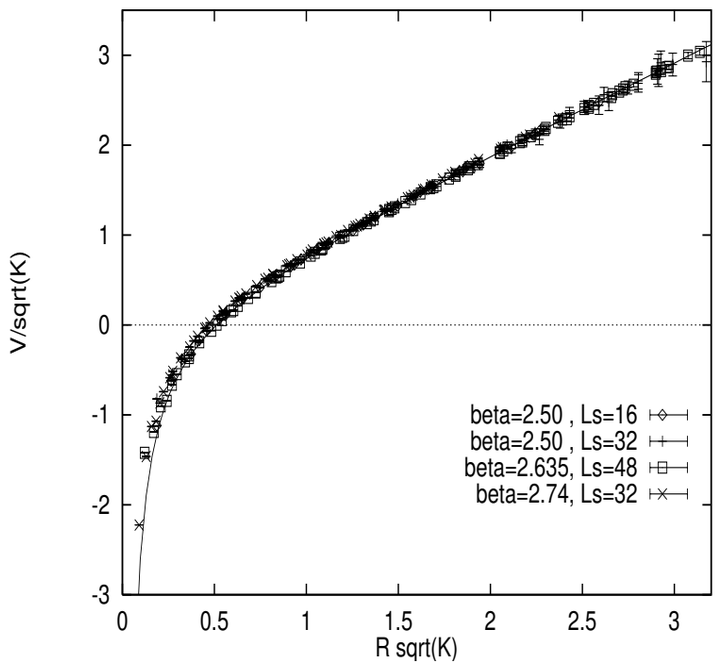
Outlook:

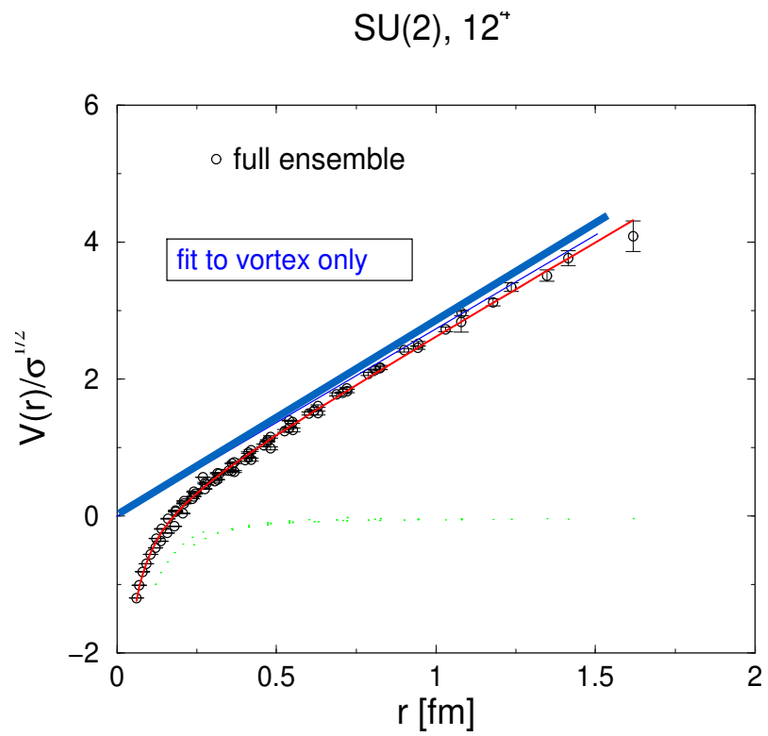
- Quantify the weakly renormalisability
- Derive the thermodynamics of the (pure) gauge theory
- Study the mu-T phase diagram
- Is there an endpoint?
- Exploring the Satz mechanism: study size of cluster container
- Investigate the "Sc confinement criterion" [Jeff Greensite, Wednesday talk]

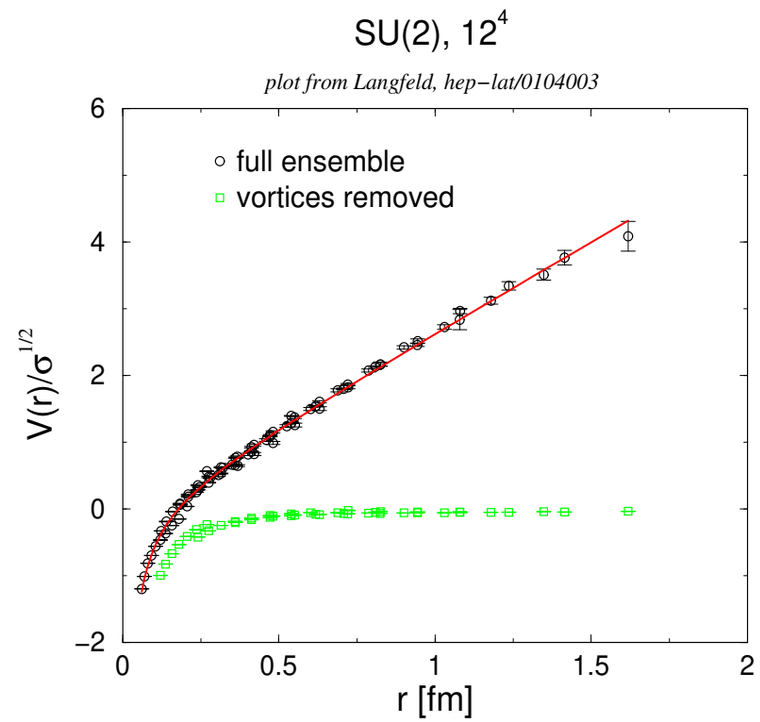
Could also fix the gauge and study:

- Fix the gauge and study the mu-dependence of propagators (e.g. in the confinement phase)
- How does the Silver Blaze feature manifest itself in propagators?

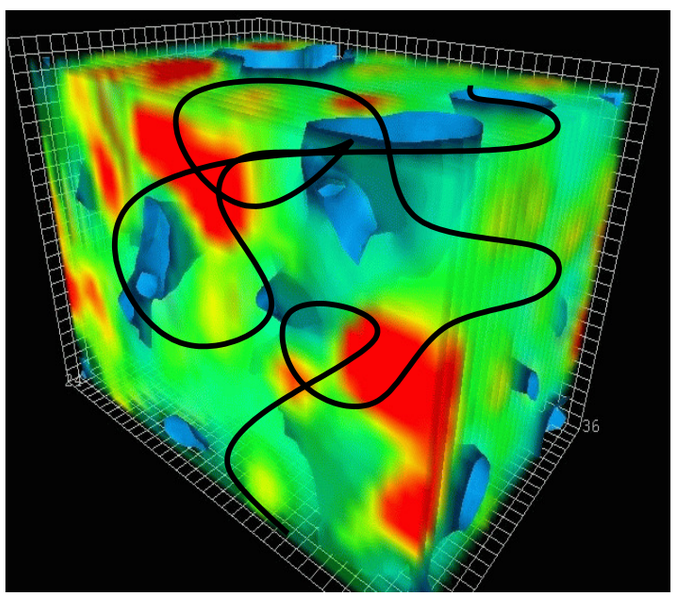
Thank you!



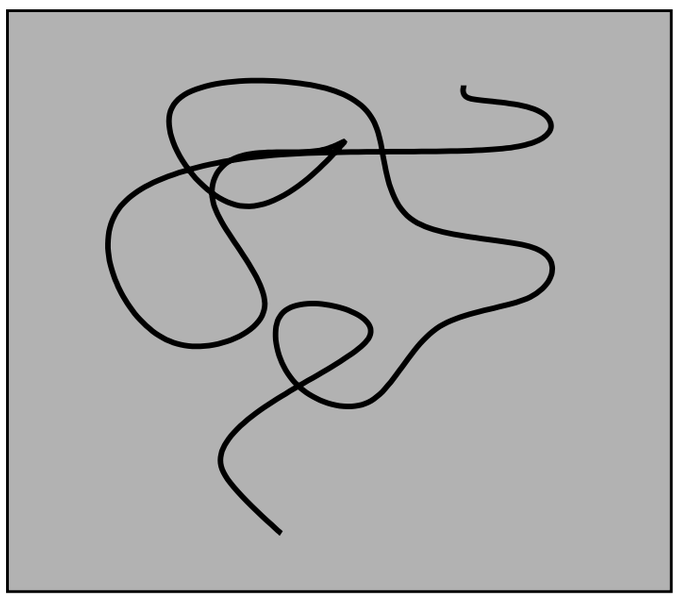




full SU(2)



vortices only



vortices removed