Baryonic vs. Quarkyonic Matter

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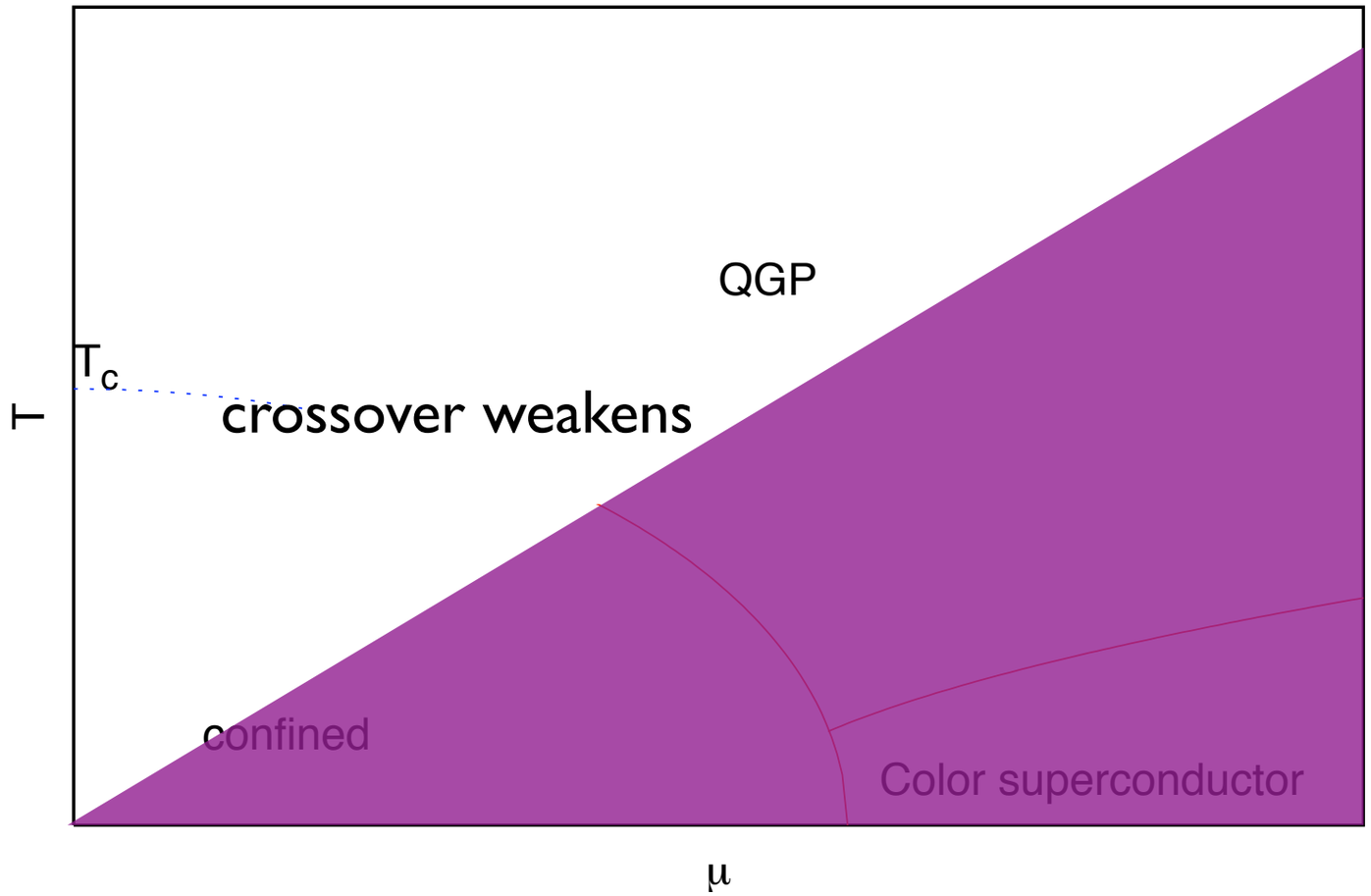
- Review: effective lattice theory for finite density QCD
- The nuclear liquid gas transition
- What happens at large Nc

HIC for FAIR Helmholtz International Center

NIC

CRC-TR 211 Strong-interaction matter under extreme conditions

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond

Effective lattice theory for heavy dense QCD

O.P. with Fromm, Langelage, Lottini, Neuman, Glesaaen

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

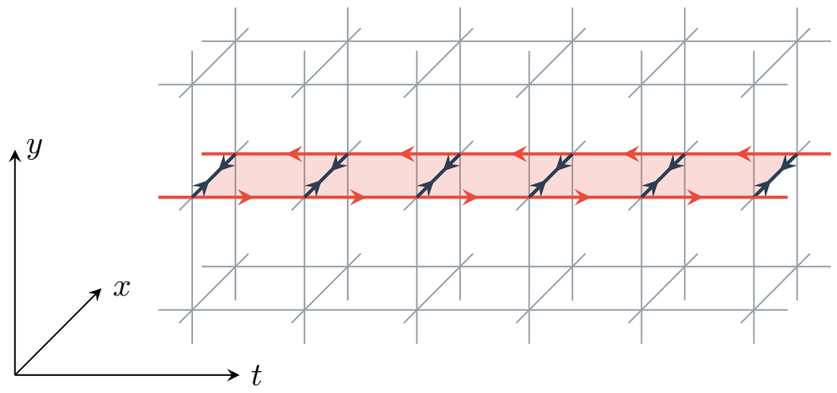
$$Z = \int DU_0 DU_i \ \det Q \ e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \ e^{-S_{eff}[L]}$$

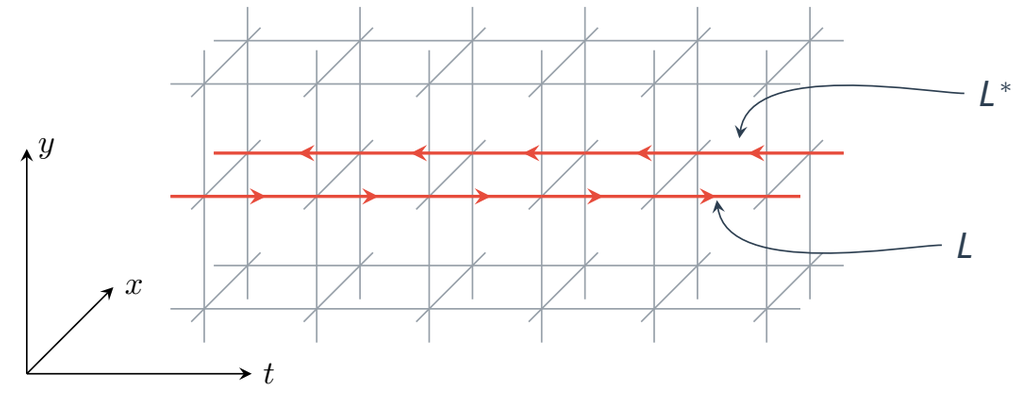
  Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$

- Step II.: mild sign problem of effective theory

- Analytic solution by linked cluster expansion

LO 3d effective theory for lattice YM





Integrate over all spatial gauge links

What remains is an interaction between Polyakov Loops

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \,\mathrm{tr}\, W_j$$

Polonyi, Szachlanyi 82

Svetitsky-Yaffe conjecture

Character expansion: $u = \frac{\beta}{18} + O(\beta^2) < 1 \qquad \beta = \frac{2N}{g^2}$

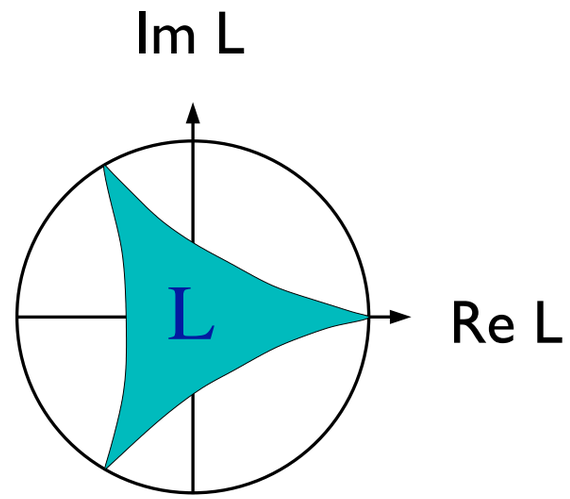
- larger distances between loops, higher powers of loops
- higher representations of loops
- decorations of LO graphs by additional plaquettes

Effective one-coupling theory for SU(3) YM

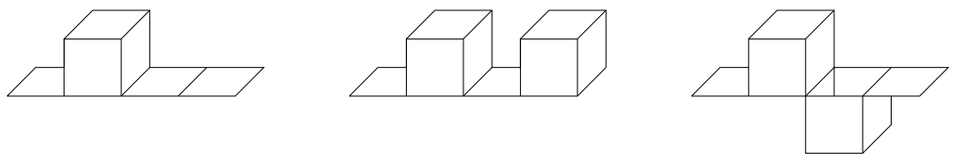
Langelage, Lottini, O.P. 10

(L= Tr W)

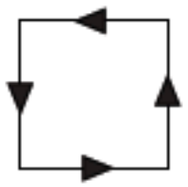
$$
\begin{aligned}
Z &= \int [dL] \exp\left[-S_1 + V_{SU(3)}\right] \\
&= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] * \\
&\quad * \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}
\end{aligned}
$$



$$
\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau \left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]
$$

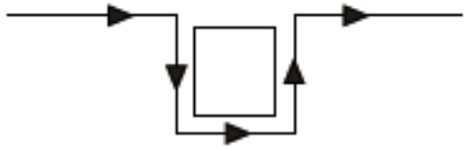


Hopping expansion: $\kappa = \dfrac{1}{2am + 8}$



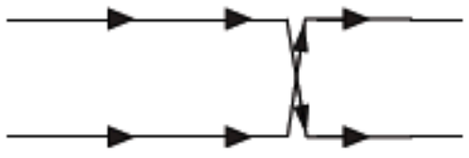
spatial: reabsorbed in gauge part: $\begin{cases} \beta \to \beta + \mathcal{O}(\kappa^4) \\ u(\beta) \to u(\beta, \kappa) \end{cases}$

LO Polyakov "magnetic" term $\sim \begin{cases} \underbrace{(2\kappa e^{+a\mu})^{N_\tau}}_{h_1} L \\ \underbrace{(2\kappa e^{-a\mu})^{N_\tau}}_{\overline{h}_1} L^* \end{cases}$



higher corrections to the above:

$$h_1 = (2\kappa e^{a\mu})^{N_\tau} \left[1 + \mathcal{O}(k^2) f(u) + \ldots\right]$$



other (suppressed) terms, such as $h_2(L_x L_{x+\hat{i}})$,

$$h_2 \sim (2\kappa e^{a\mu})^{2N_\tau} \kappa^{2N_\tau}.$$

Fromm, Langelage, Lottini, Neuman, Glesaaen, O.P. 12-15

The effective 3d theory

$$-S_{\text{eff}} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^S - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau) S_i^A + \overline{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger A} \right]$$

effective couplings

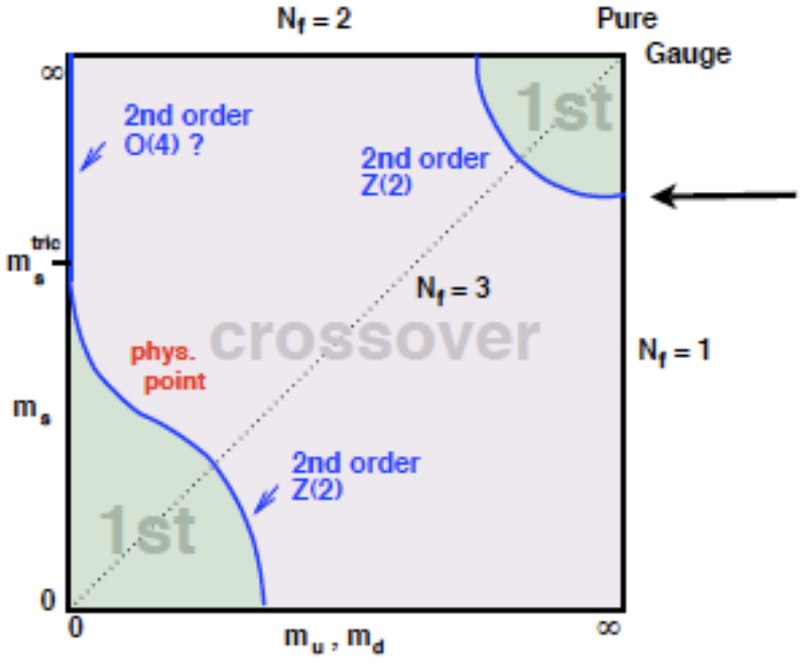
$$S_i^{A,S} = S_i^{A,S}[L, L^*]$$

This is a 3d continuous spin model!

"Duality transformation":

$$\beta = \frac{2N}{g^2} \longleftrightarrow u = \frac{\beta}{18} + O(\beta^2) < 1$$

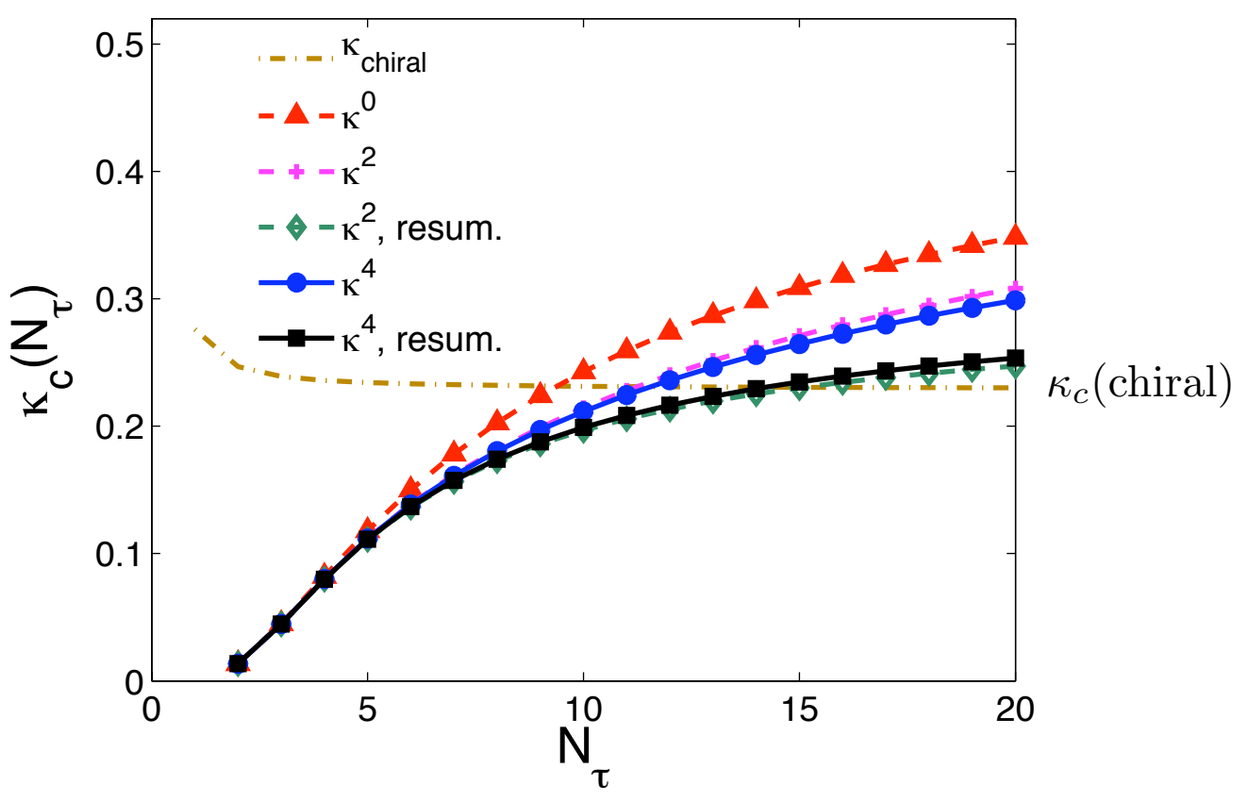
The deconfinement transition for heavy quarks



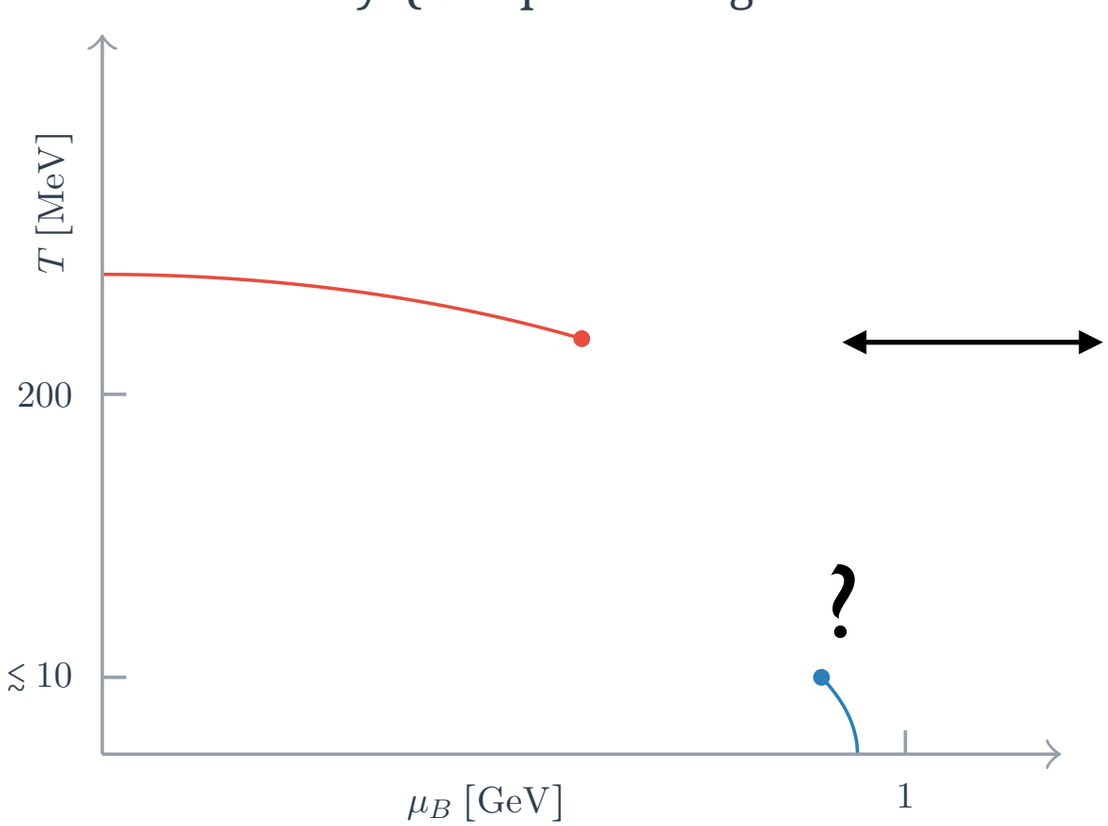
eff. theory 4d MC, WHOT 4d MC, de Forcrand et al

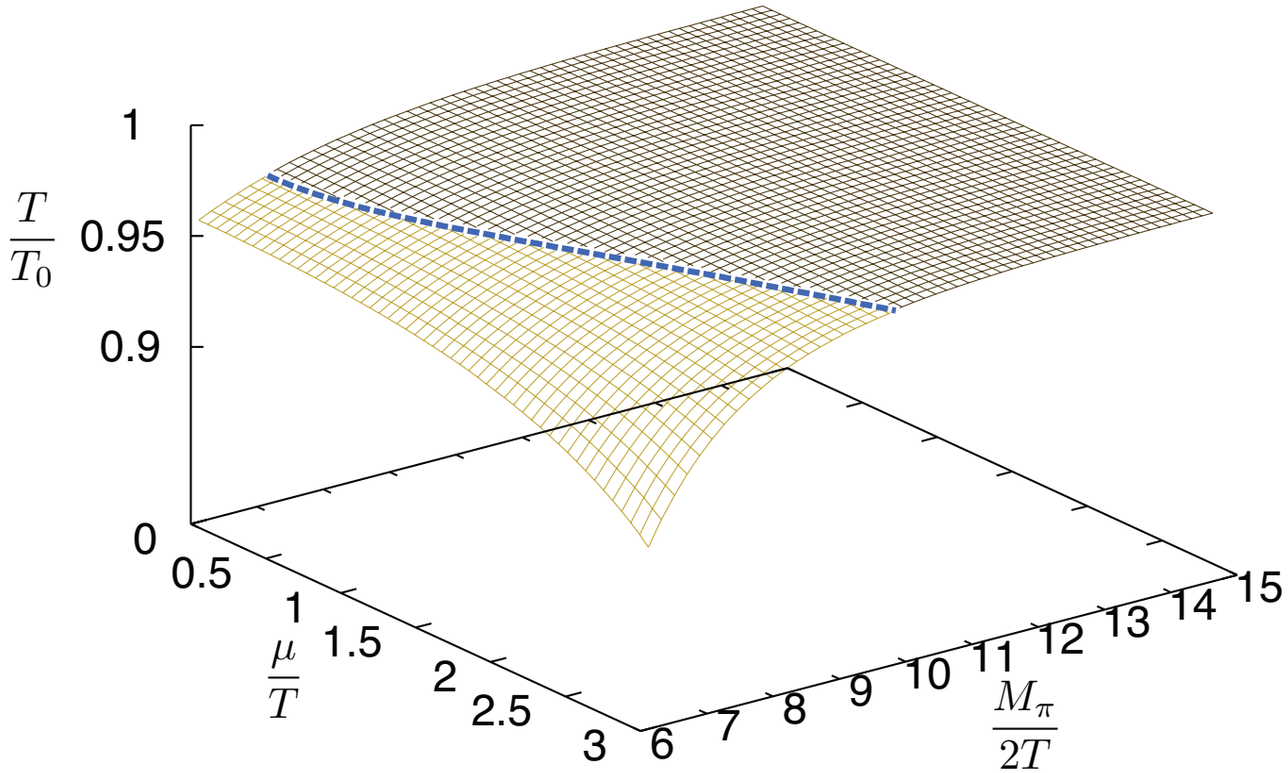
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!



The fully calculated deconfinement transition





Continuum, functional methods:

Fischer, Lücker, Pawlowski 15

Fromm, Langelage, Lottini, O.P. 11

Cold and dense: static strong coupling limit

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13

T=0: anti-fermions decouple:

$$h_1 = (2\kappa e^{a\mu})^{N_\tau} = e^{\frac{\mu-m}{T}}$$
$$\bar{h}_1 = (2\kappa e^{-a\mu})^{N_\tau} = e^{\frac{-\mu-m}{T}}$$

$$Z(\beta=0) \overset{T\to 0}{\longrightarrow} \left[\prod_f \int dW (1 + h_1 L + h_1^2 L^* + h_1^3)^2\right]^V = z_0^V$$

$$N_f = 1: \qquad z_0 = 1 + 4h_1^3 + h_1^6$$

free baryon gas

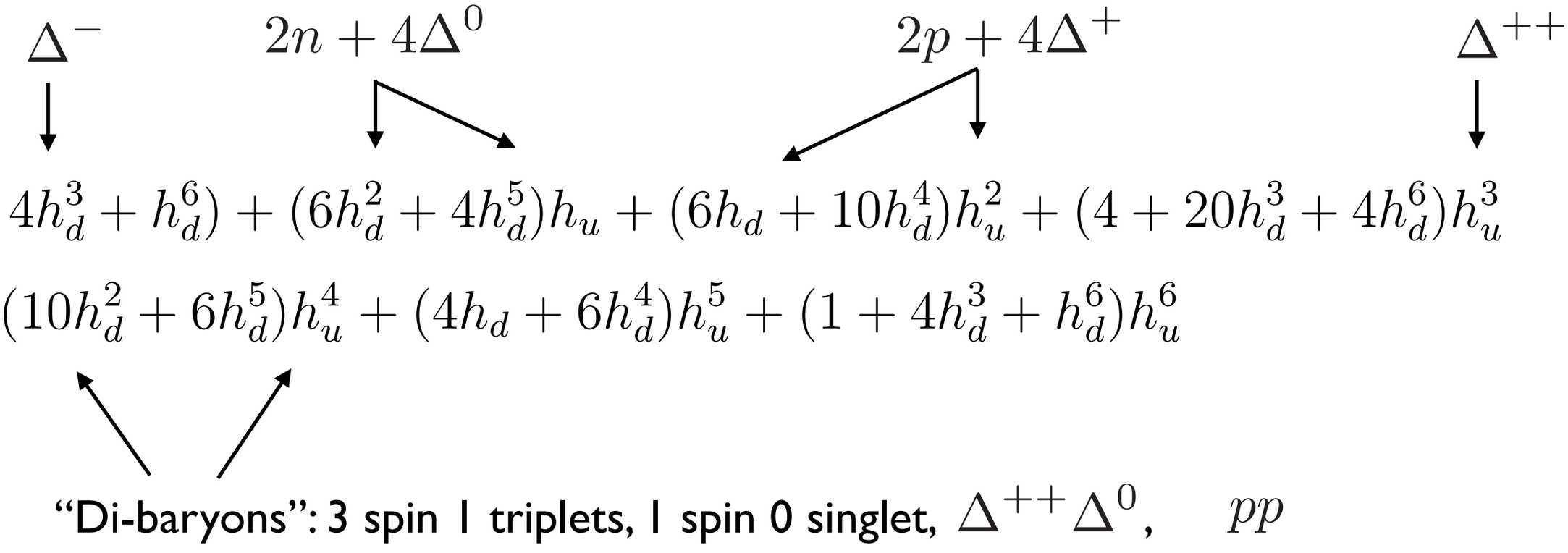
spin 3/2, 0

Silver blaze phenomenon + Pauli principle:

$$\lim_{T\to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

1st order phase transition from vacuum to saturated baryon crystal

$N_f = 2$:



"Di-baryons": 3 spin 1 triplets, 1 spin 0 singlet, $\Delta^{++}\Delta^0$, pp

Complete spin-flavour structure of baryons (mesons for isospin chemical potential)

Gauge and Lorentz symmetries!

Corrections in perturbation theory

$N_f = 1, T \to 0$: LO in effective couplings λ, h_2

$$Z = \int [dW] \prod_{<\vec{x},\vec{y}>} \left[1 + \lambda(L_{\vec{x}} L^*_{\vec{y}} + L^*_{\vec{x}} L_{\vec{y}})\right] \prod_{\vec{x}} [1 + h_1 L_{\vec{x}} + h_2^2 L^*_{\vec{x}} + h_1^3]^2$$

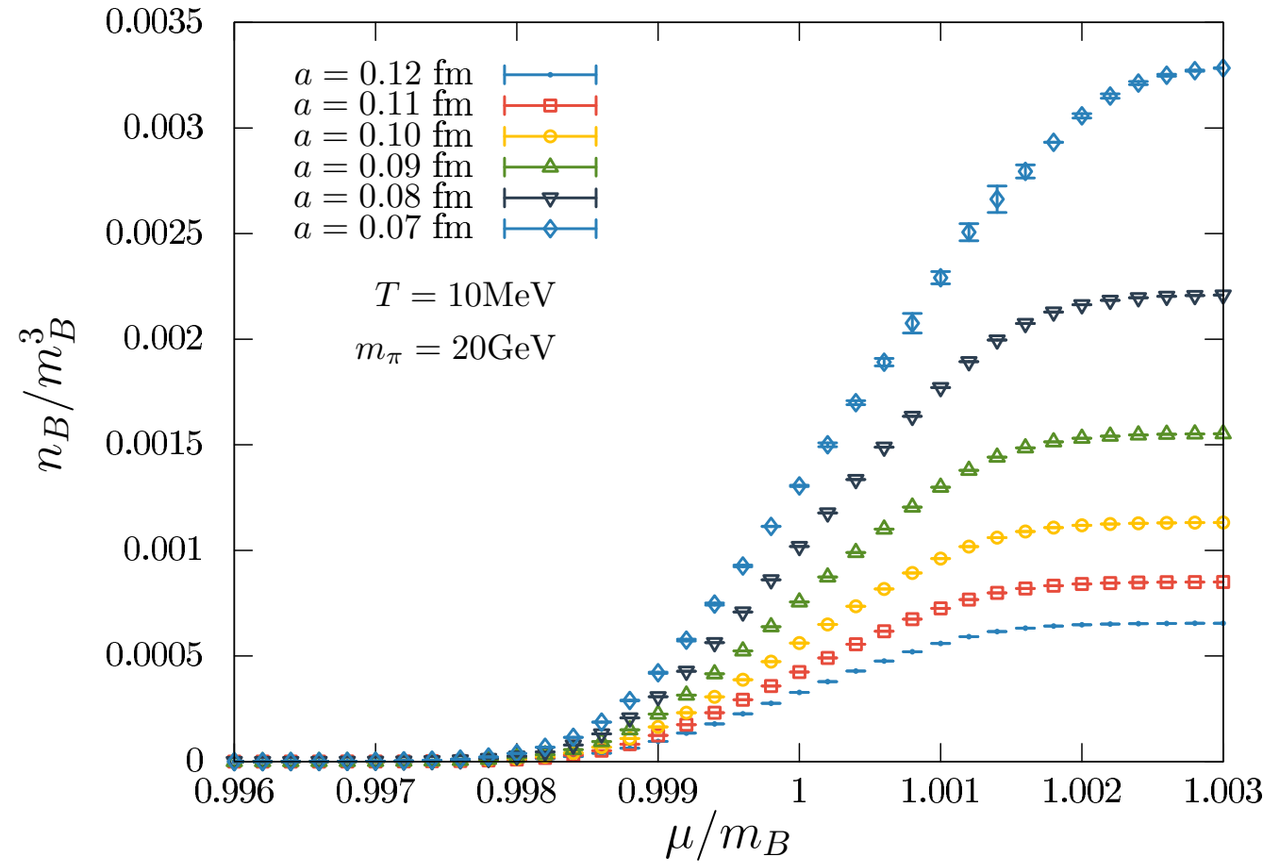
$$\times \prod_{<\vec{x},\vec{y}>} \left[1 - 2h_2 \left(\frac{h_1 L_{\vec{x}} + 2h_1^2 L^*_{\vec{x}} + 3h_1^3}{1 + h_1 L_{\vec{x}} + h_1^2 L^*_{\vec{x}} + h_1^3}\right) \left(\frac{h_1 L_{\vec{y}} + 2h_1^2 L^*_{\vec{y}} + 3h_1^3}{1 + h_1 L_{\vec{y}} + h_1^2 L^*_{\vec{y}} + h_1^3}\right)\right]$$

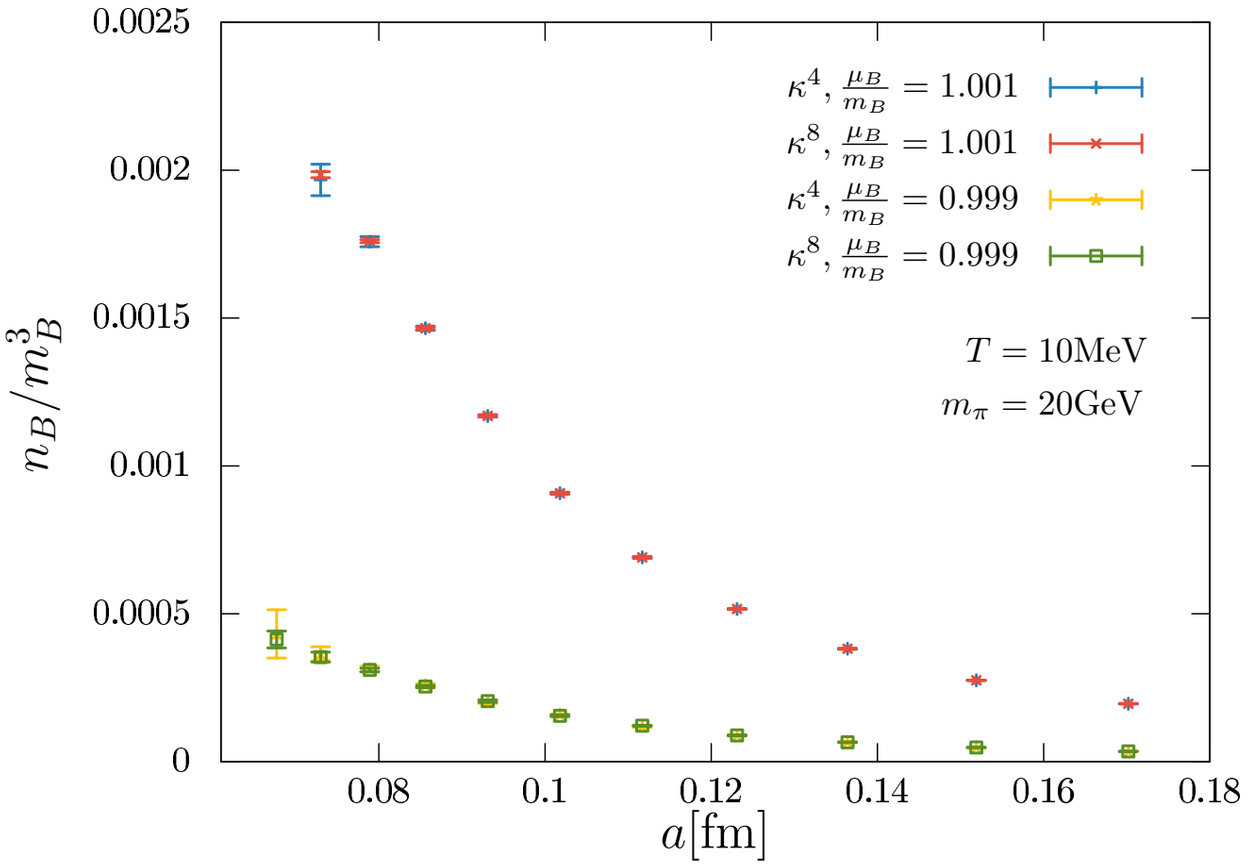
$$= z_0^{N_s^3} + 6\lambda N_s^3 z_0^{N_s^3 - 2} z_1 z_2 - 6h_2 N_s^3 z_0^{N_s^3 - 2} z_3^2$$

$$z_0 = 1 + 4h_1^3 + h_1^6 \,, \qquad z_2 = 2h_1 + 3h_1^4 \,,$$
$$z_1 = 3h_1^2 + 2h_1^5 \,, \qquad z_3 = 6h_1^3 + 3h_1^6 \,.$$

Cold and dense regime

Glesaaen, Neuman, O.P., JHEP 15





- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition: lattice saturation!
- Finer lattice necessary for larger density!

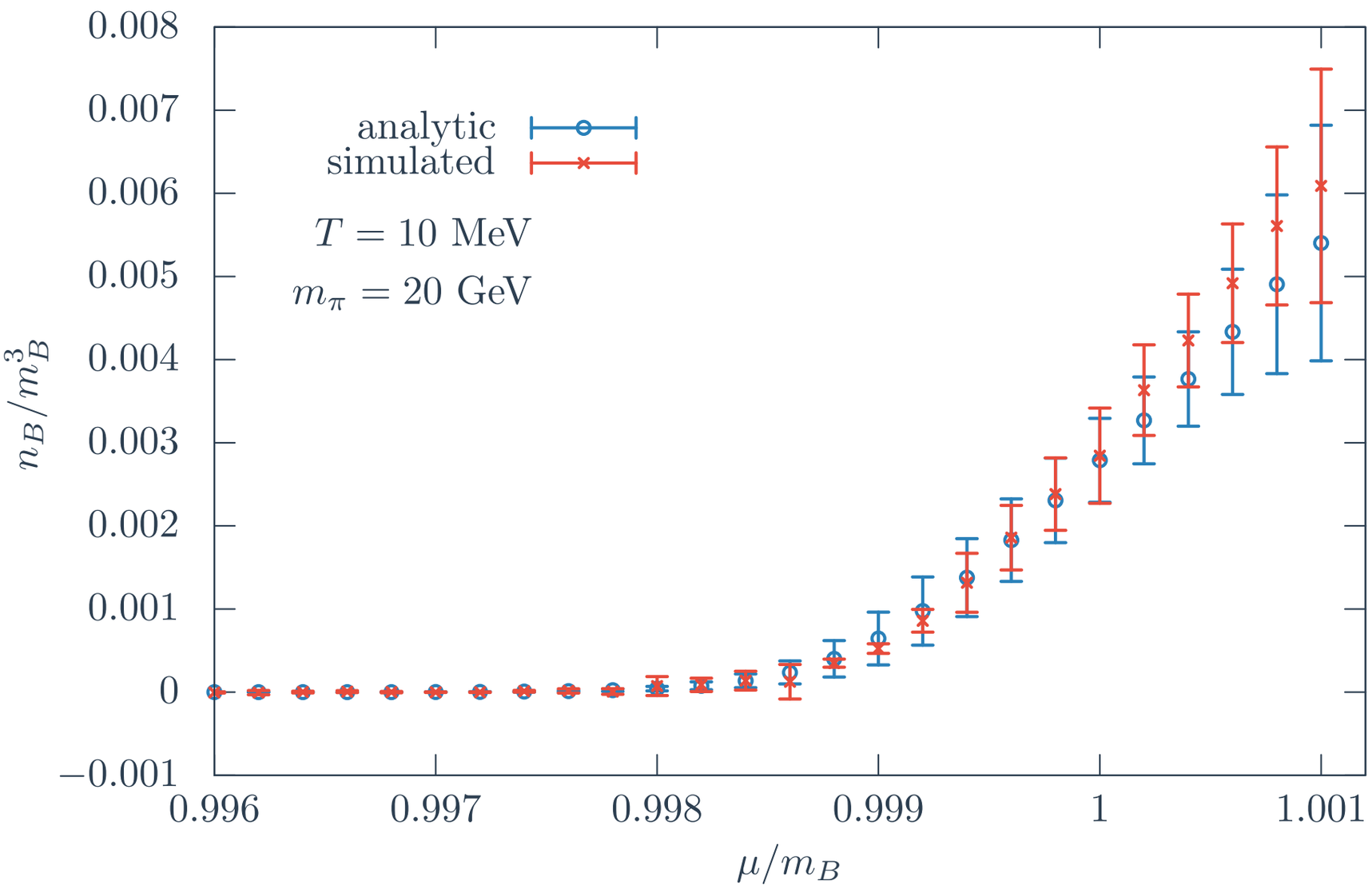
Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi\, e^{-S_0[\phi] + \frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y) + \frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z) + \ldots}$$

"perturbation theory" in effective couplings

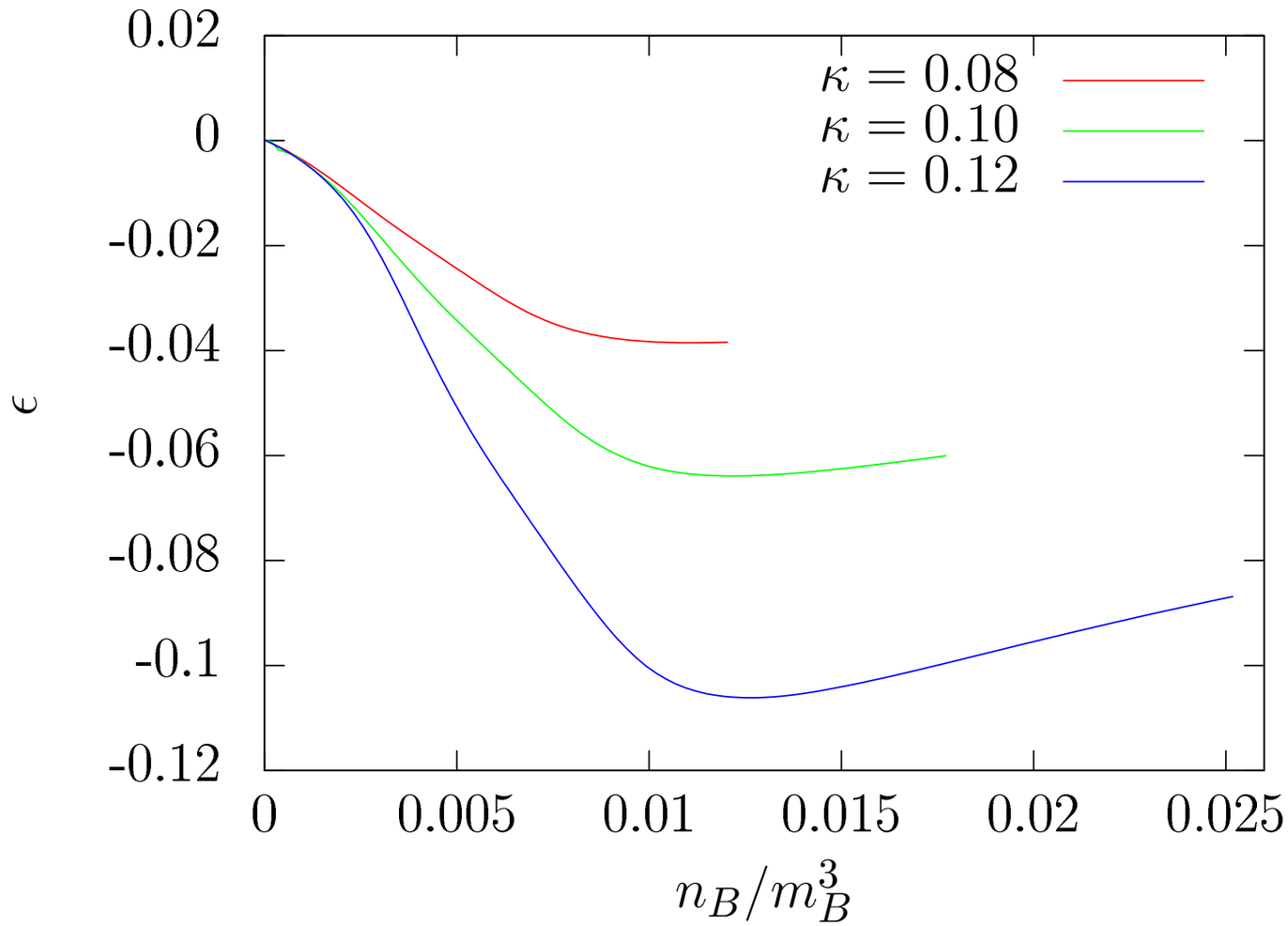
Glesaaen, Neuman, O.P. 15

through $u^5\kappa^8$

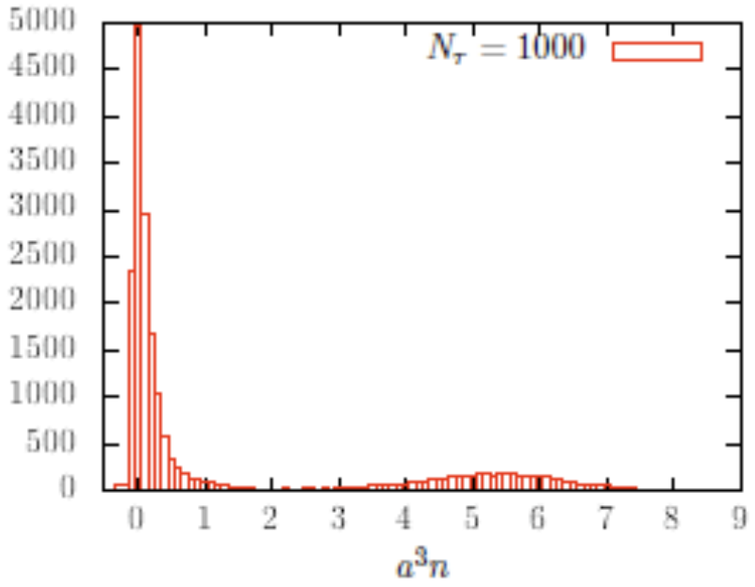


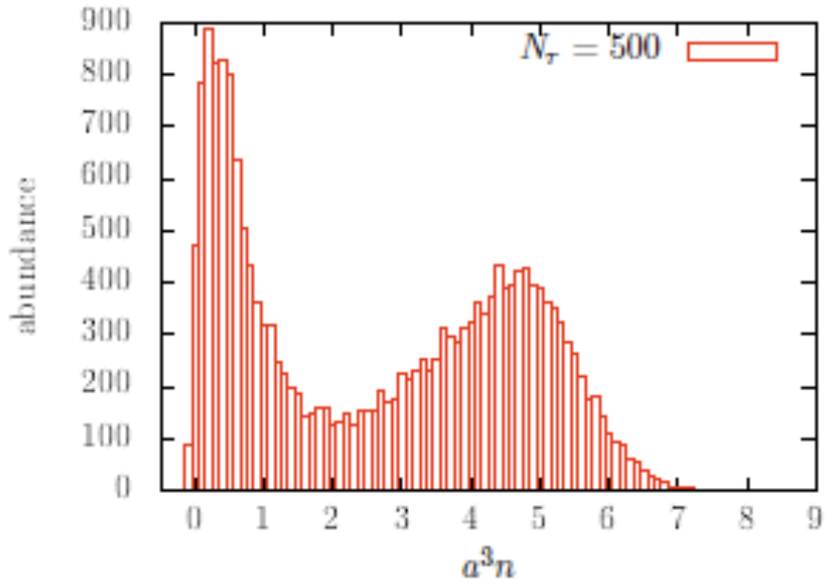
Binding energy per nucleon

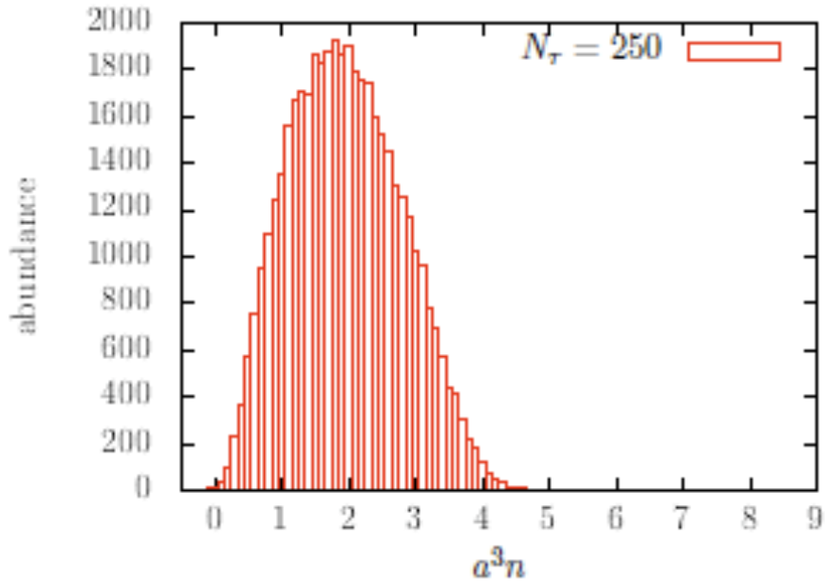
$$\epsilon \equiv \frac{e - n_B m_B}{n_B m_B} \overset{LO}{=} -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2 = -\frac{1}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 e^{-a m_M}$$

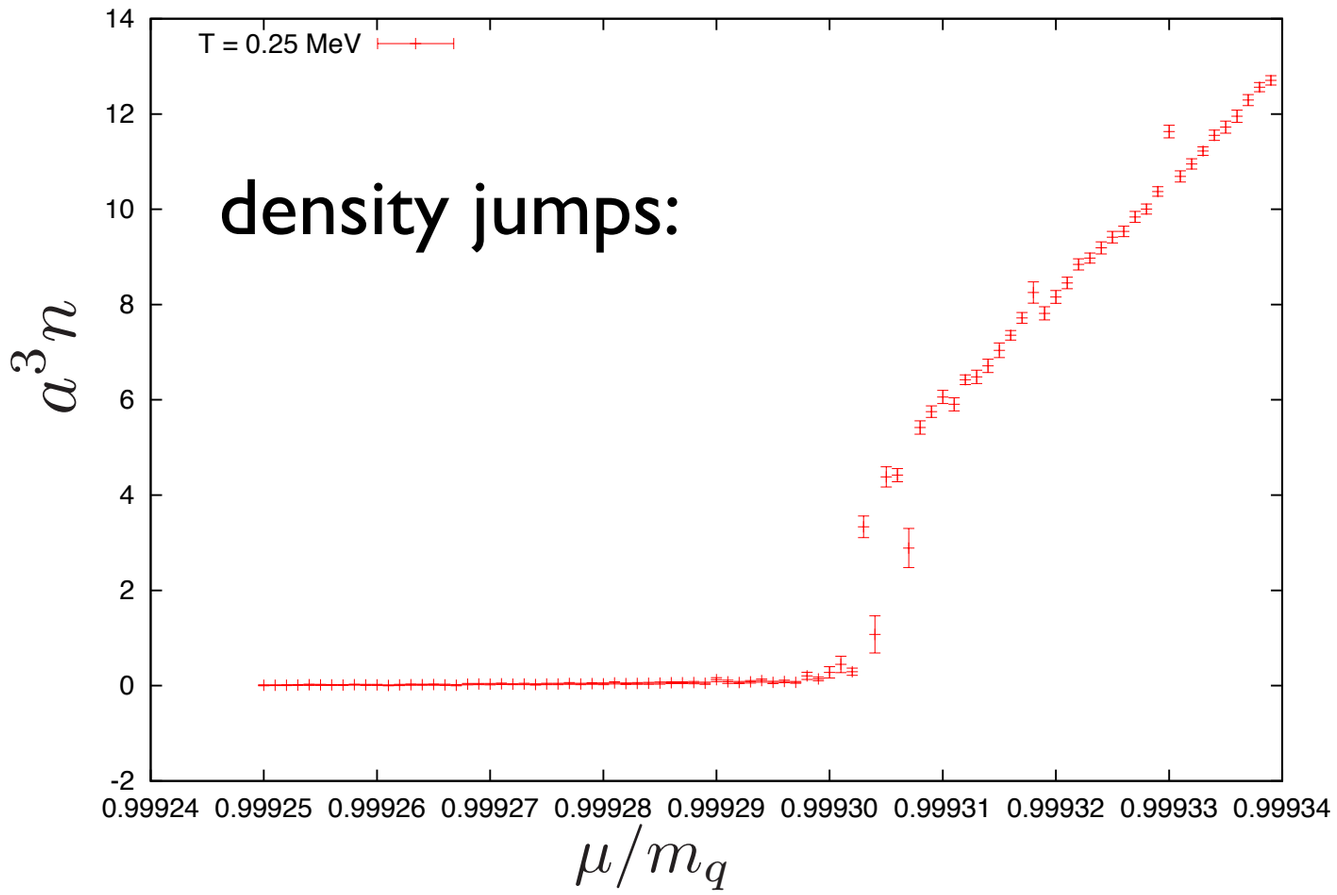


Light quarks: first order transition + endpoint



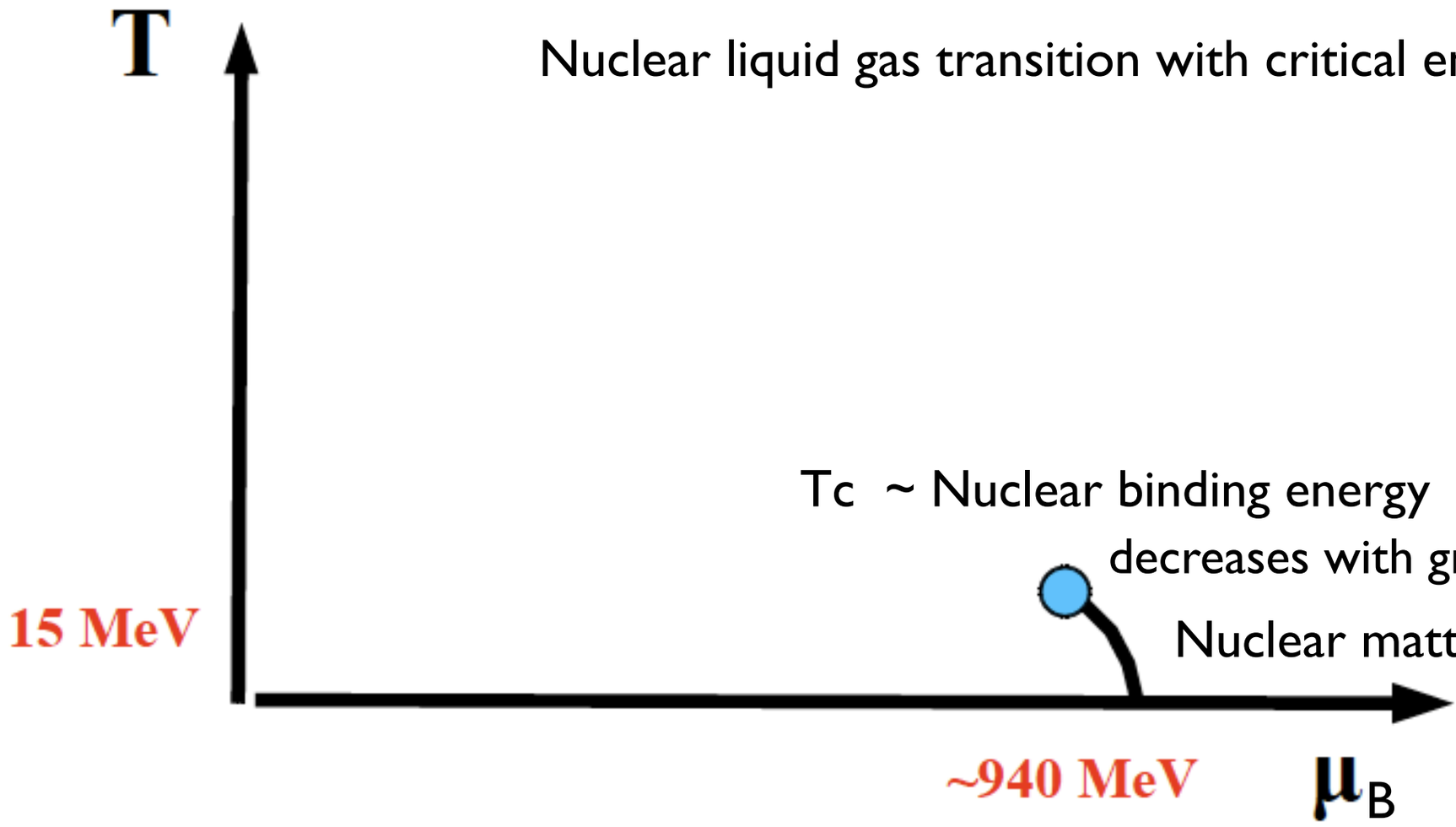






- phase coexistence: first order
- for higher $T = \frac{1}{aN_\tau}$ crossover
- nuclear liquid gas transition!

Nuclear liquid gas transition with critical end point

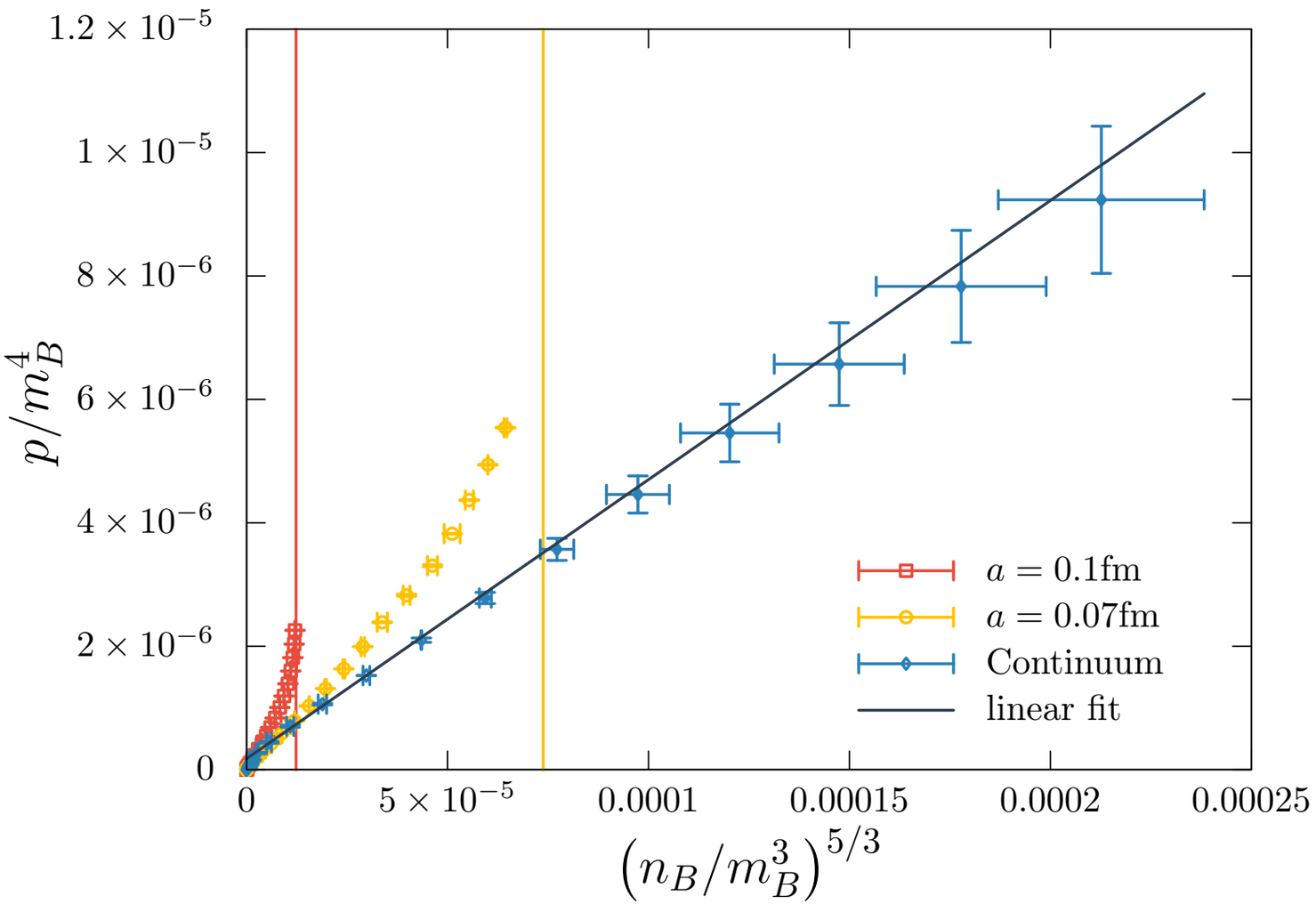


Tc ~ Nuclear binding energy

decreases with growing quark mass

Nuclear matter

Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…

QCD at large N_c

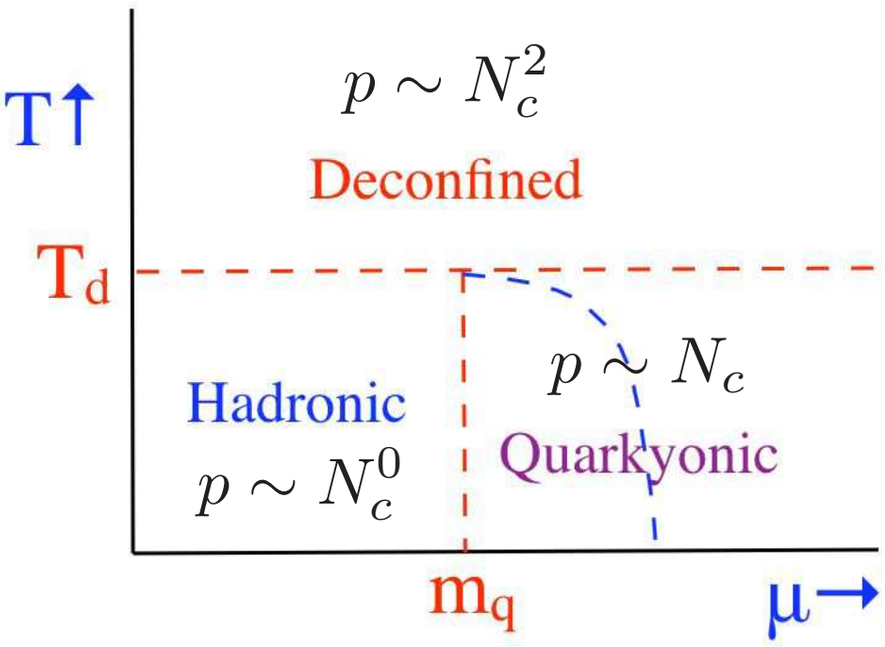
Definition, 't Hooft 1974 : $\qquad N_c \longrightarrow \infty, \quad g^2 N_c = const.$

- suppresses quark loops in Feynman diagrams
- mesons are free;
  corrections: cubic interactions $\sim 1/\sqrt{N_c}$, quartic int. $\sim 1/N_c$
- meson masses $\sim \Lambda_{QCD}$
- baryons: N_c quarks, baryon masses $\sim N_c \Lambda_{QCD}$
- baryon interactions: $\sim N_c$ Witten 1979

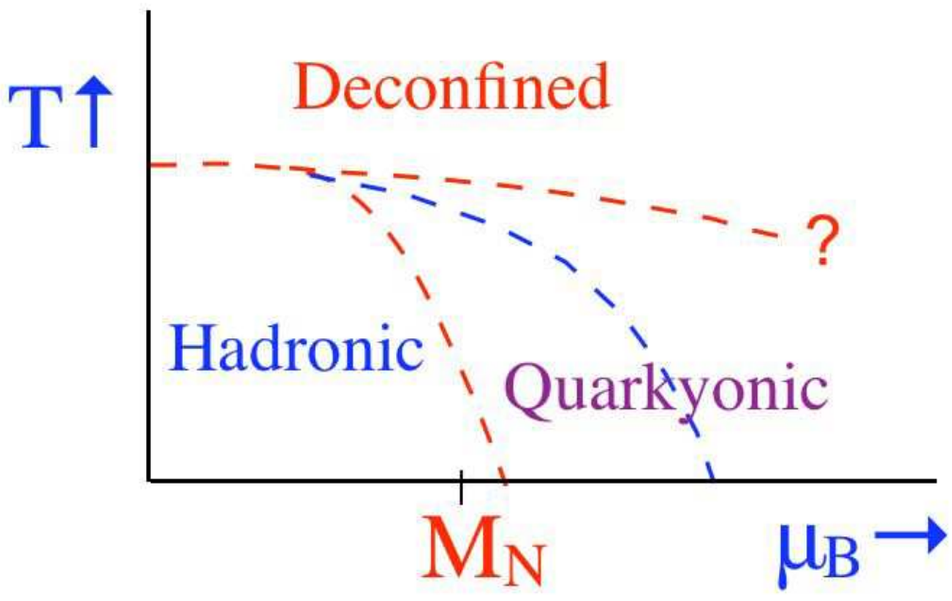
Implications on the phase diagram

McLerran, Pisarski 07: from Nucl. Phys. A 796 (2007)

large Nc



QCD, conjectured



Deconfinement different
from chiral symmetry restoration

The effective theory for large $N_c(=N)$

Disclaimer: here we consider strong coupling limit, cannot yet keep $g^2 N_c = const$

1st step: recalculate previous results for general N_c

Static determinant:

$$\int\limits_{SU(N)} \mathrm{d}U \det(1+h_1 U)^{2N_f} = \sum_{p=0}^{N_f} \left(\prod_{i=1}^{p} \frac{(i-1+2N_f-p+N)^{\underline{2N_f-p}}}{(i-1+2N_f-p)^{\underline{2N_f-p}}} \right) \left(h_1^{pN} + h_1^{(2N_f-p)N} \right) \left(1 - \frac{\delta_{p,N_f}}{2} \right)$$

And corrections: $$\int\limits_{SU(N)} \mathrm{d}U \det(1+h_1 U)^{2N_f} \operatorname{tr}\left(\frac{(h_1 U)^n}{(1+h_1 U)^m} \right)$$

$$= h_1^{N(2N_f+1)} \sum_{r=\max(0,N-m)}^{2N_f+N-m} (-1)^{r+N+1} \binom{N+r-1}{r} (r+m-1)^{\underline{N-1}} \frac{(2N_f)^{\underline{2N_f+1-r-m}}}{(N+2N_f-r-m)}$$

$$+ \sum_{p=0}^{2N_f} h_1^{Np} \det_{1\le i,j\le N} \left[\binom{2N_f}{i-j+p} \right] \sum_{\mu=1}^{N} \sum_{r=\max(0,\mu-m)}^{\mu+p-m} (-1)^r \binom{r+n-1}{r}$$

$$\times \frac{(-1)^{\mu+1}}{r+m} \frac{(r+m+N-\mu)^{\underline{r+m}}}{(r+m-\mu)!(\mu-1)!} \frac{(\mu+p-1)^{\underline{r+m}}}{(N+2N_f-p+r+m-\mu)^{\underline{r+m}}}.$$

Results for $N_f = 2$:

Static determinant:

$$z_0 = 1 + \frac{1}{6}(h_1^N + h_1^{3N})(N+3)(N+2)(N+1) + \frac{1}{12}h_1^{2N}(N+3)(N+2)^2(N+1) + h_1^{4N}$$

Curious: spin degeneracy of a baryon determined by N!

Correction:

$$z_{11} = \frac{1}{24}h_1^N(N+3)(N+2)(N+1)N + \frac{1}{24}h_1^{2N}(N+3)(N+2)^2(N+1)N$$
$$+ \frac{1}{8}h_1^{3N}(N+3)(N+2)(N+1)N + h_1^{4N}N$$

Results for observables

$$\mu < m, h_1 < 1: \qquad a^4 p_0 \sim \frac{1}{6N_t} N^3 h_1^N \qquad a^4 p_1 \sim -\frac{1}{48} N^7 h_1^{2N}$$

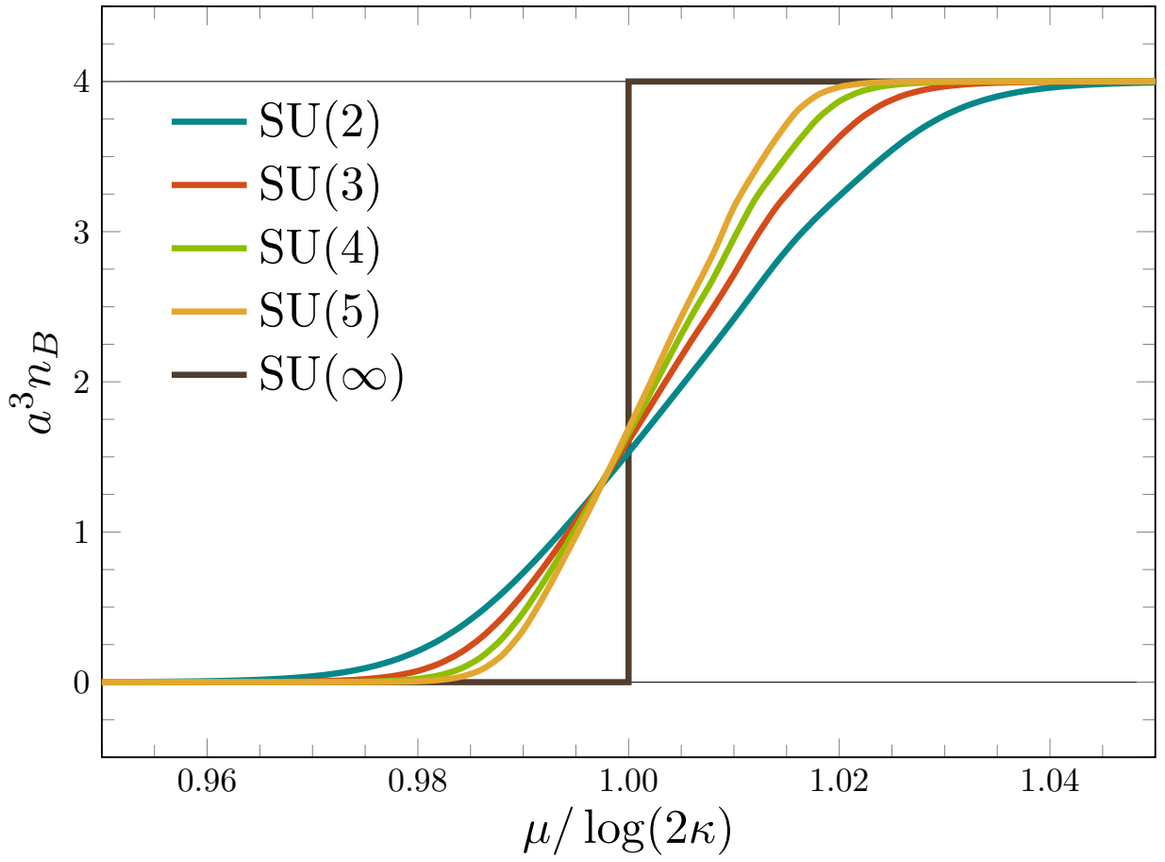
$$a^3 n_0 \sim \frac{1}{6} N^4 h_1^N \qquad a^3 n_1 \sim \frac{N_t}{24} N^9 h_1^{2N}$$

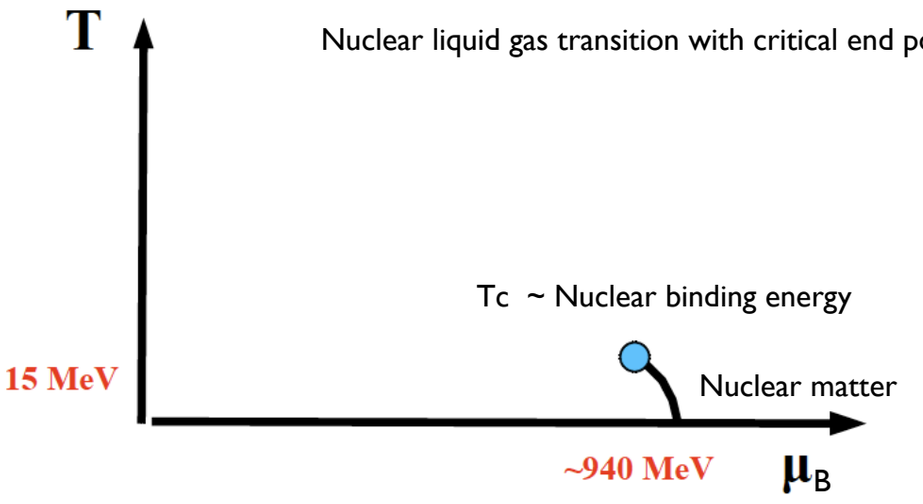
$$\mu > m, h_1 > 1: \qquad a^4 p_0 \sim \frac{4\log(h_1)}{N_t} N \qquad a^4 p_1 \sim -12N$$

$$a^3 n_0 \sim 4N \qquad a^3 n_1 \sim -288 N_t \frac{N^5}{h_1^N}$$

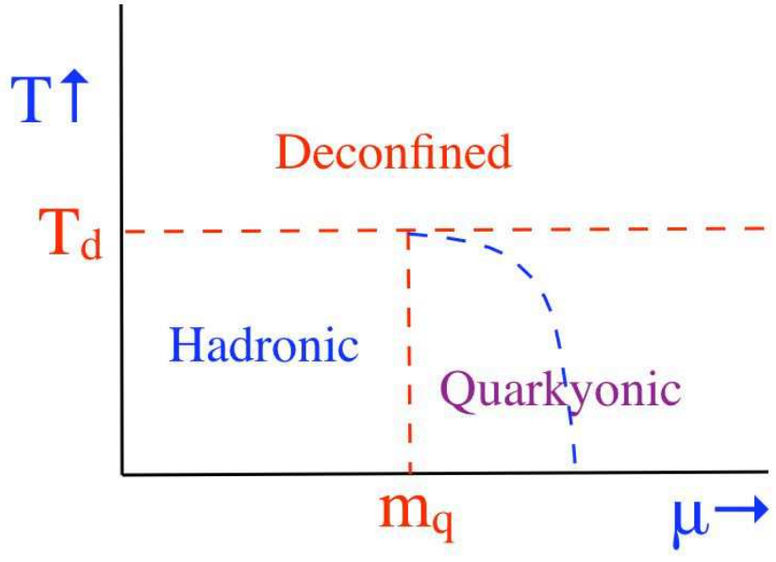
Quarkyonic?

The liquid gas transition for large N





$\underrightarrow{\text{large } N_c}$



Conclusions

- Sign problem beaten by effective lattice theory for heavy quarks
- Nuclear liquid gas transition and equation of state calculable in the heavy mass region
- Varying Nc: is quarkyonic matter the same as baryonic matter?

Backup slides

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

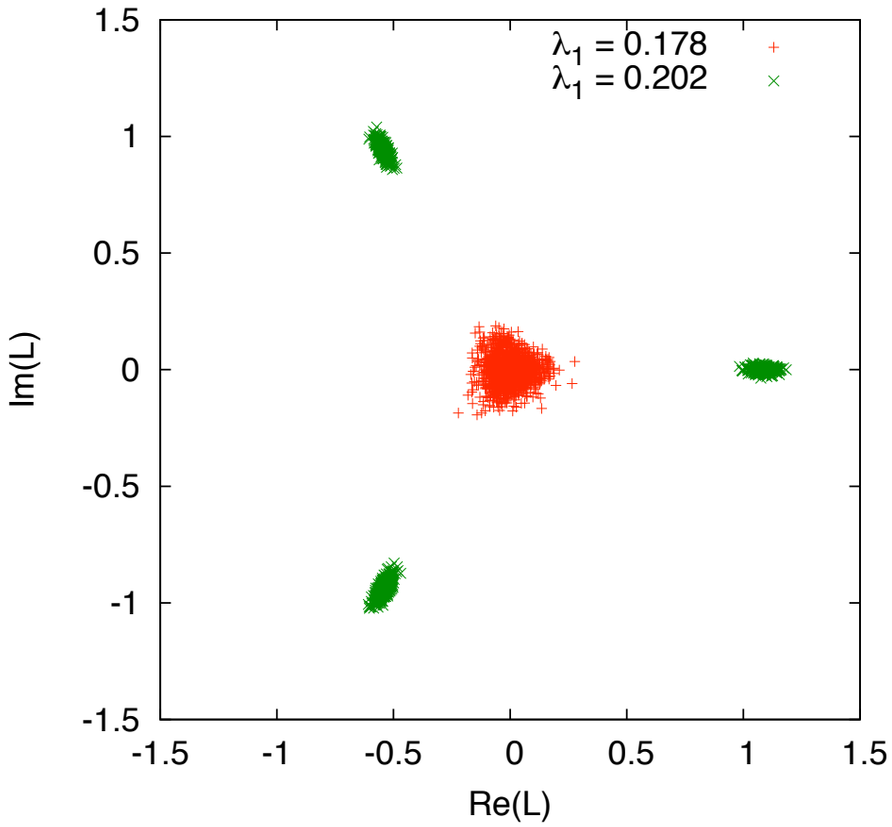
$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}{}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}{}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

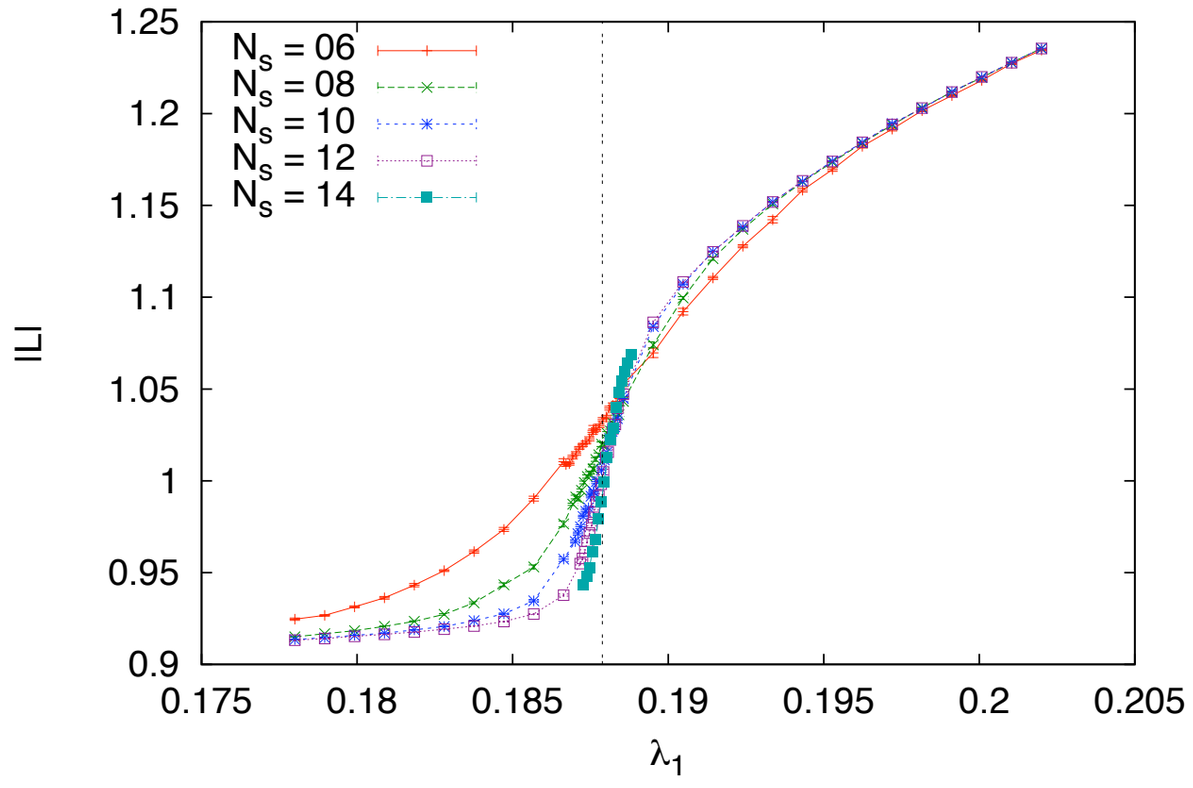
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

Numerical results for SU(3), one coupling

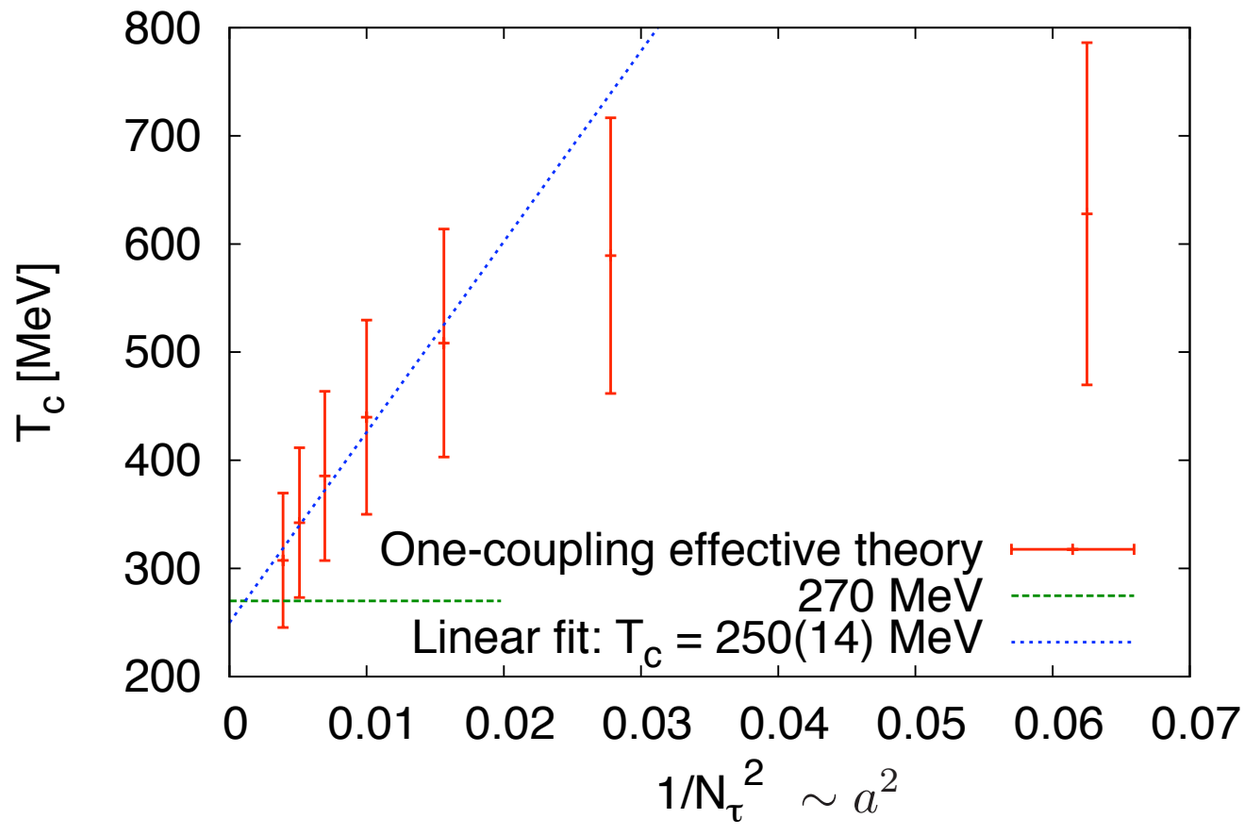


Order-disorder transition
=Z(3) breaking



Mapping back to 4d Yang-Mills

$$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$$



-error bars: difference between last two orders in strong coupling exp.

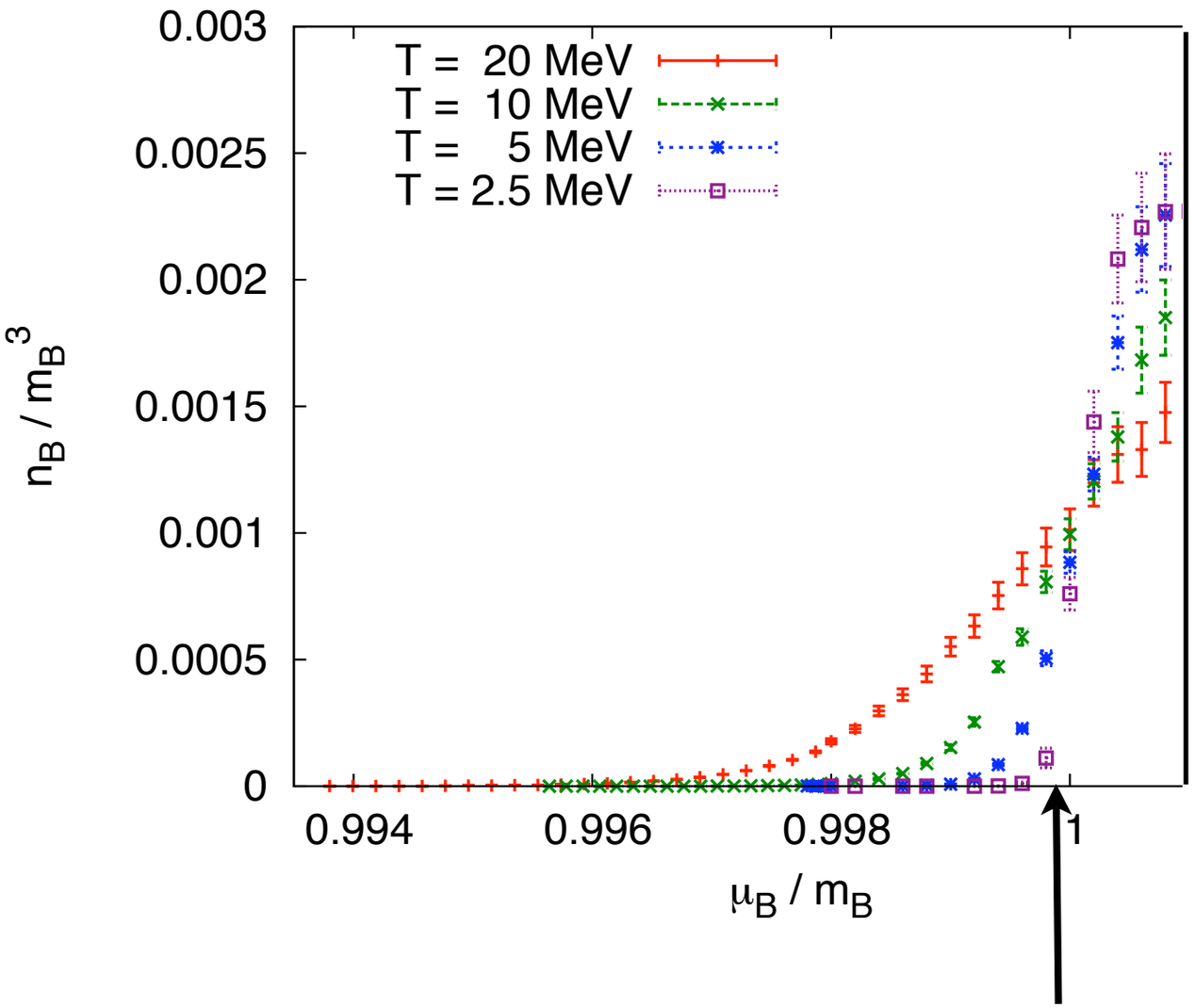
-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$m_\pi = 20$ GeV



Effect of binding between baryons: $\mu_c < m_B$

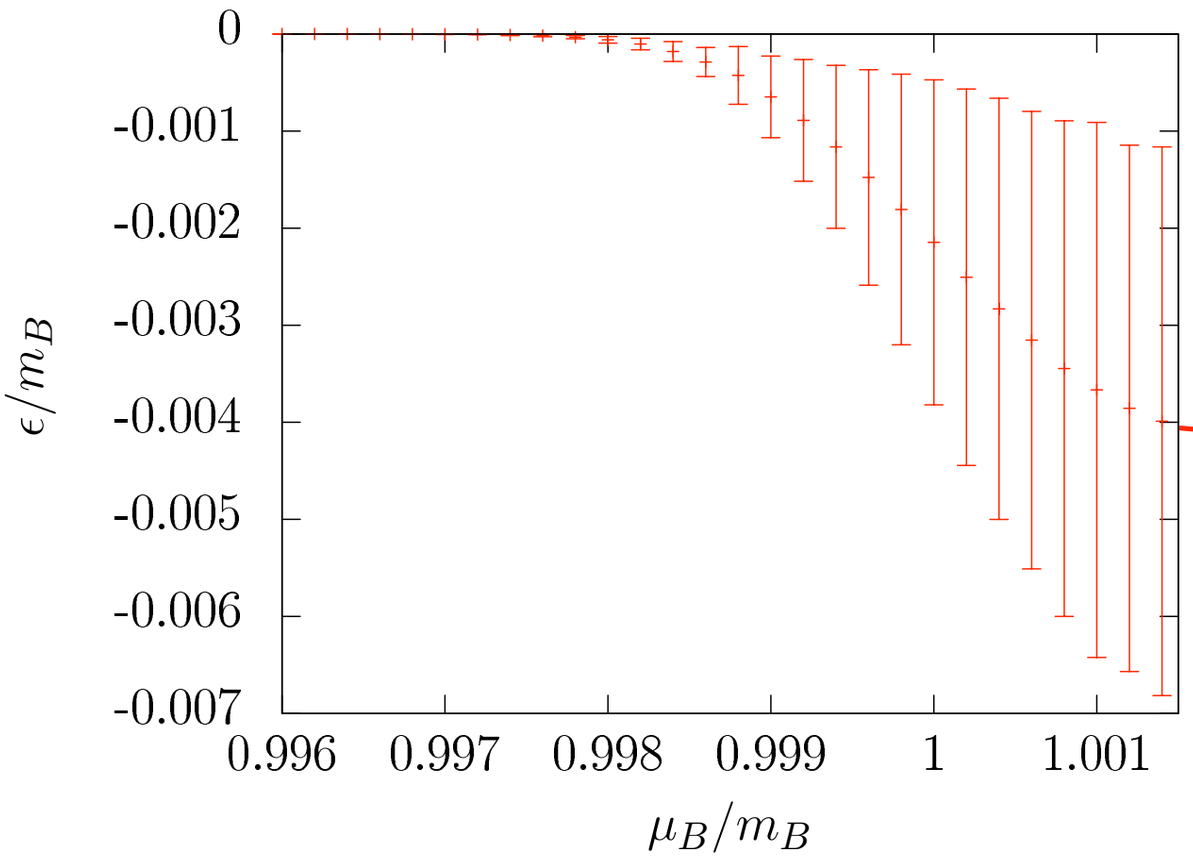
Binding energy per nucleon: $\epsilon = \dfrac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

$\dfrac{\mu}{T} \sim 4000$

Binding energy per nucleon

$$T \to 0: \quad \epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1 \overset{LO}{\sim} \kappa^2 \sim e^{-a m_{\rm meson}}$$



... to be continued...

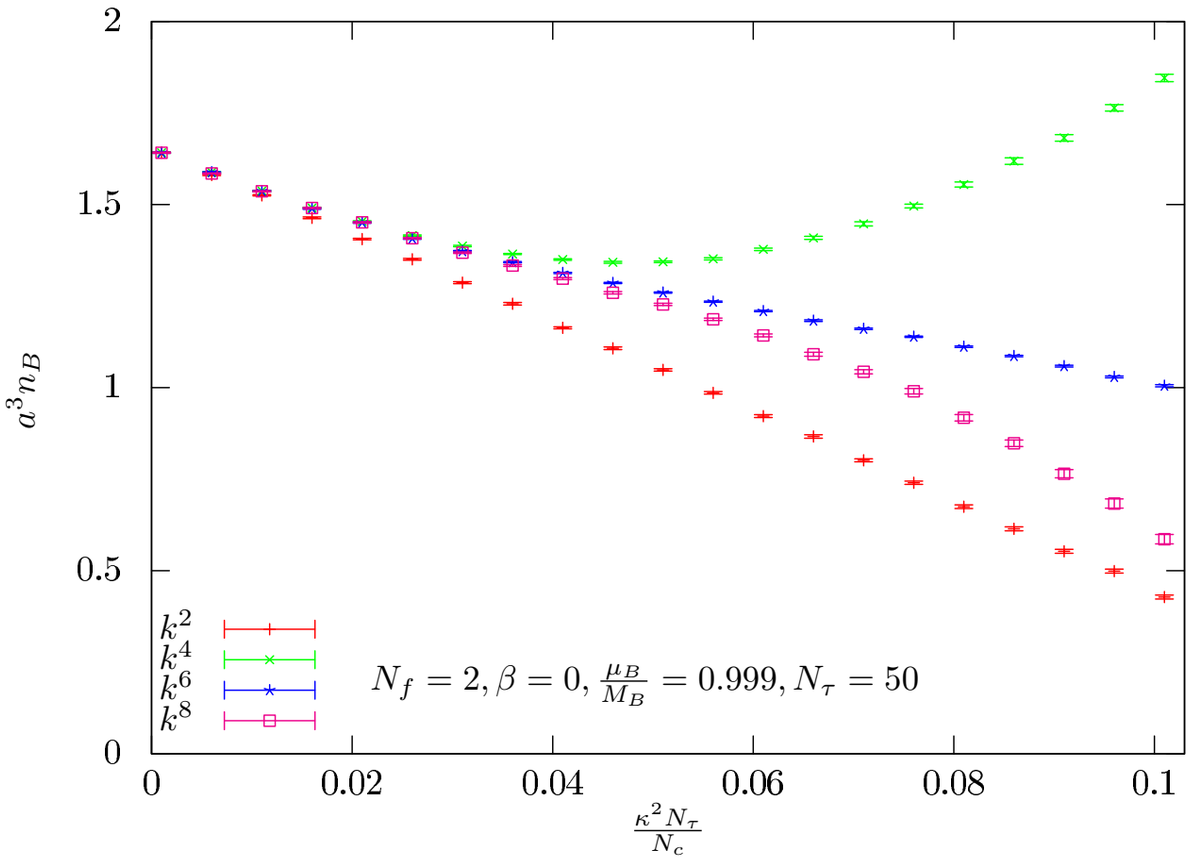
Error bars systematic:
difference between orders
in effective action

Minimum: access to nucl. binding energy, nucl. saturation density!

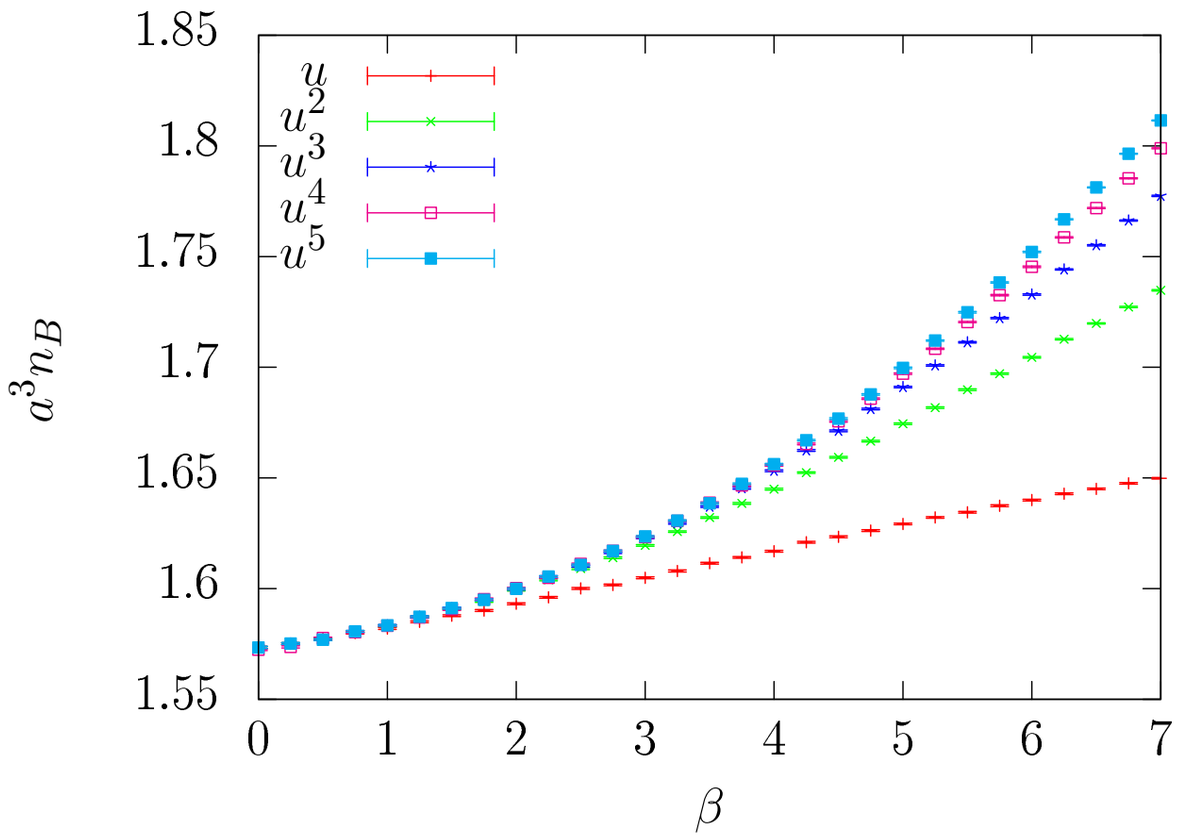
$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition , heavy quarks

$\sim 10^{-2}$ in nature

Convergence of the effective theory

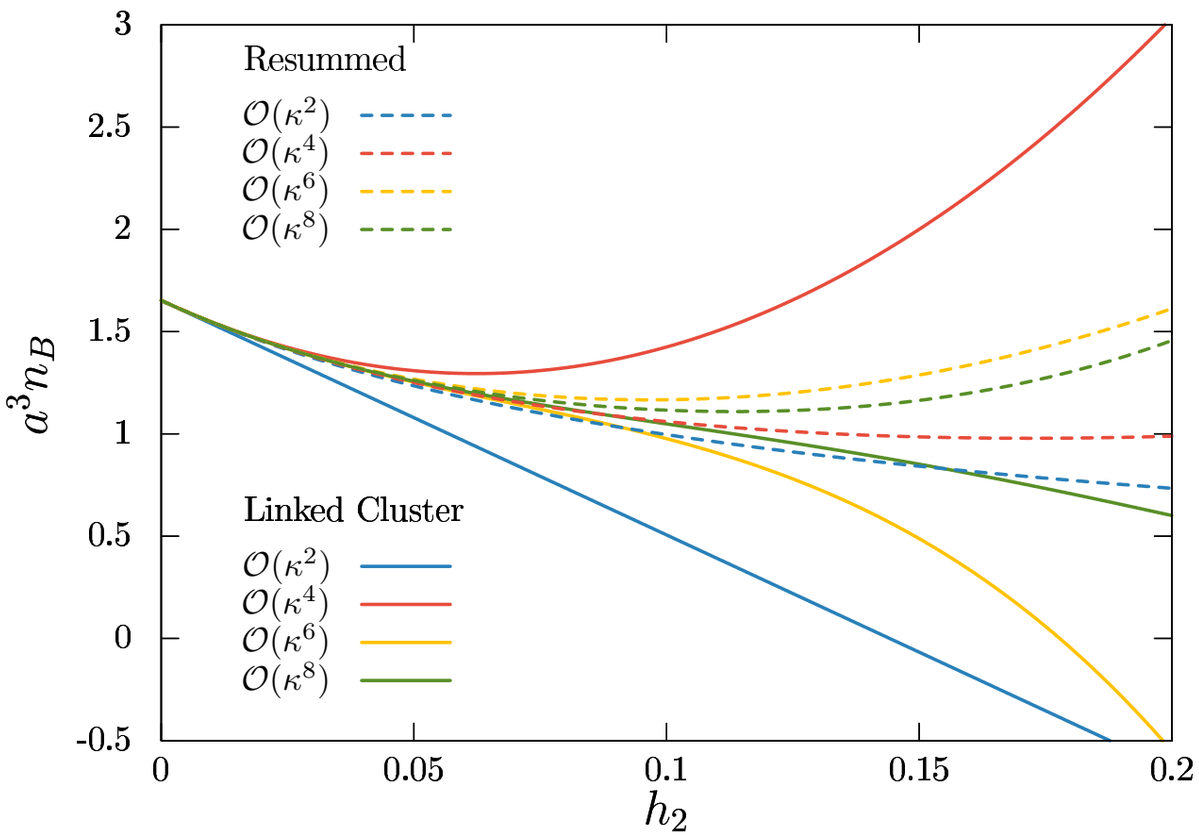


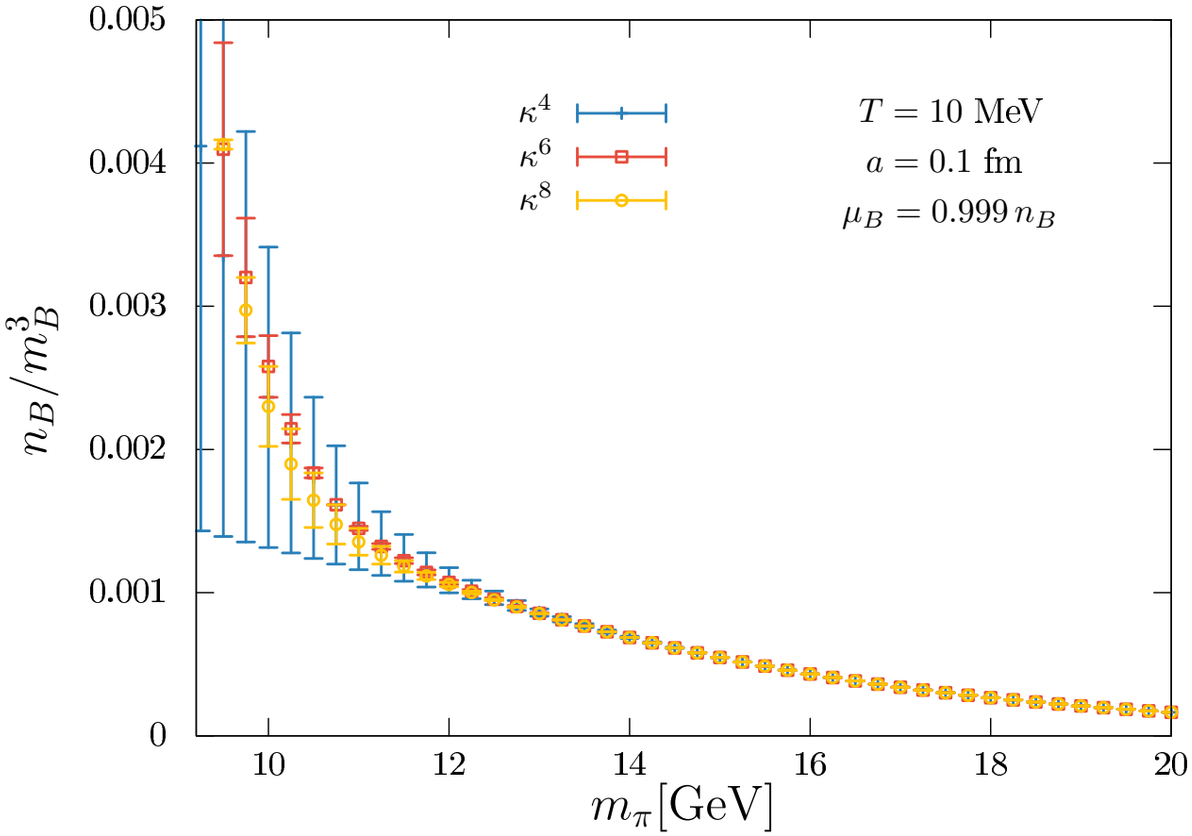
hopping expansion in strong coupling limit



strong coupling expansion at κ^8

Resummations + reach in mass range





Resumming long range non-overlapping chains, gain in mass range "sobering"