Ghost Sector in Minimal Linear Covariant Gauge

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Abstract

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Why study the linear covariant gauge?

- Study Green's functions in the IR limit of Yang-Mills theories in order to understand low-energy properties of the theory.
- Since they depend on the gauge, consider different gauges (Landau gauge, Coulomb gauge, λ-gauge, MAG, etc.).
- Extend the Gribov-Zwanziger approach to the linear covariant gauge.
- Linear covariant gauge, very popular in continuum studies, proved quite hostile to the lattice approach.

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- Transverse component of the gluon propagator is similar to the Landau case, with $D(0)$ decreasing when ξ increases (F. Siringo, PRD90 2014, variational method; M.A.L. Capri et al., EPJC75 2015, Gribov-Zwanziger setup; A.C. Aguilar et al., PRD91 2015, SDE of the ghost prop. $+$ Nielsen identitiies).

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- Ghost dressing function is flat in the IR limit (F. Siringo, PRD90 2014, variational method).
- Ghost dressing function in the IR limit is decreasing as ξ increases (M. Huber, PRD91 2015, coupled system of DSEs).
- Infrared-finite ghost propagator i.e. ghost dressing function goes to 0 in the IR limit, (A.C. Aguilar et al., PRD77 2008 and PRD91 2015, SDE of the ghost prop. $+$ Nielsen identitiies; J. Serreau et al., PRD89 2014, averaging over Gribov copies, quartic ghost self-interaction term; M.A.L. Capri et al., PRD93 2016, PRD93 2016, Gribov-Zwanziger setup).

Linear Covariant Gauge on the Lattice

We want to impose the gauge condition $\partial_\mu A_\mu^b(x) = \Lambda^b(x)$, for real-valued functions $\Lambda^b(x)$, generated using a Gaussian distribution with width $\sqrt{\xi}$.

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Landau gauge [$\Lambda^b(x) = 0$] is obtained on the lattice by minimizing the functional

$$\mathcal{E}_{LG}[U^g] = -\,\Re\,\mathrm{Tr}\sum_{\mu,x} g(x)U_\mu(x)g^\dagger(x+e_\mu)\;.$$

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From the second variation of $\mathcal{E}_{LG}[U^g]$ we define the symmetric, semi-positive definite Faddeev-Popov operator

$$\mathcal{M}^{bc}(x,y) \equiv \sum_{\mu=1}^{d}\Big\{\Gamma^{bc}_\mu(x)\left[\delta_{x,\,y} - \delta_{x+e_\mu,\,y}\right] + \Gamma^{bc}_\mu(x-e_\mu)\left[\delta_{x,\,y} - \delta_{x-e_\mu,\,y}\right]$$
$$- \sum_{e=1}^{N_c^2-1} f^{bec}\left[A^e_\mu(x-e_\mu/2)\,\delta_{x-e_\mu,\,y} - A^e_\mu(x+e_\mu/2)\,\delta_{x+e_\mu,\,y}\right]\Big\}.$$

The New Minimizing Functional

The lattice linear covariant gauge condition can be obtained by minimizing the functional (A.C., T. Mendes and E.M.S. Santos, PRL103 2009)

$$\mathcal{E}_{LCG}[U^g, g, \Lambda] \;=\; \mathcal{E}_{LG}[U^g] + \Re\,\mathrm{Tr} \sum_x i\, g(x)\, \Lambda(x) \;.$$

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One can interpret the Landau-gauge functional $\mathcal{E}_{LG}[U^g]$ as a spin-glass Hamiltonian for the spin variables $g(x)$ with a random interaction given by $U_\mu(x)$. Then, the extra term corresponds to a random external magnetic field $\Lambda(x)$. Note: the functional $\mathcal{E}_{LCG}[U^g, g, \Lambda]$ is linear in the gauge transformation $\{g(x)\}$.

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By considering a one-parameter subgroup, it is easy to check that the stationarity condition implies the lattice linear covariant gauge condition $\nabla \cdot A^b(x) \,=\, \sum_\mu A^b_\mu(x + e_\mu/2) - A^b_\mu(x - e_\mu/2) \,=\, \Lambda^b(x)$.

Numerical Gauge Fixing

Conceptual problem: using the standard compact discretization, the gluon field is bounded while the four-divergence of the gluon field satisfies a Gaussian distribution, i.e. it is unbounded. This can give rise to convergence problems when a numerical implementation of the linear covariant gauge is attempted (A.C. et al., PRL103 2009, PoS QCD-TNT09, PoS FACESQCD 2010, AIP Conf.Proc.1354 2011; P. Bicudo et al., PRD92 2015, PoS LATTICE2015).

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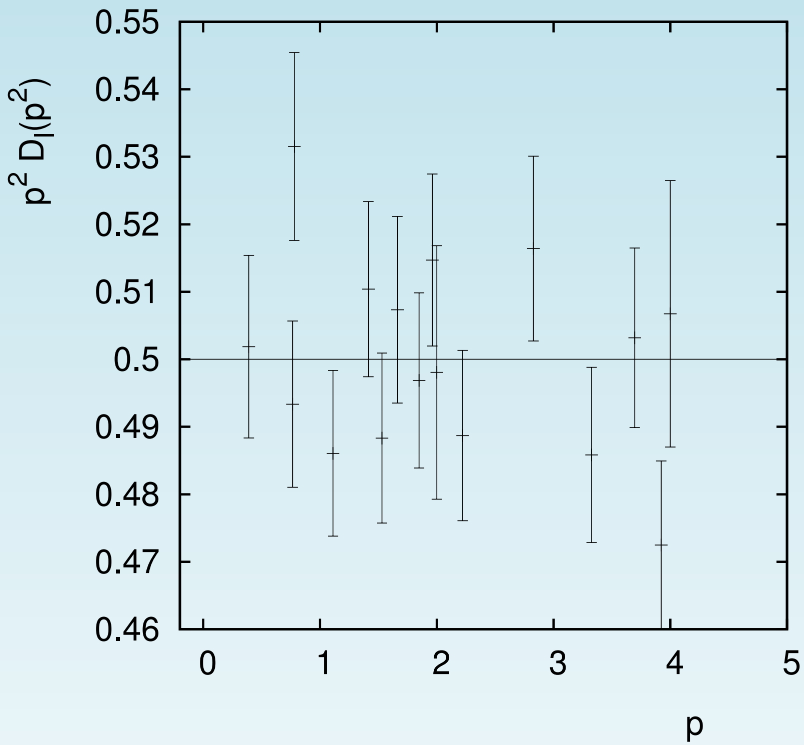
Moreover, the dimensionless gauge-fixing condition is given by $a^2 g_0 \partial_\mu A^b_\mu(x) = a^2 g_0 \Lambda^b(x)$, in a generic d-dimensional space. Since $\beta = 2N_c/(a^{4-d} g_0^2)$ in the SU(N_c) case, we have that

$$\frac{\beta/(2N_c)}{2\xi} \sum_{x,b} \left[a^2 g_0 \Lambda^b(x)\right]^2 \to \frac{1}{2\xi} \int d^d x \sum_b \left[\Lambda^b(x)\right]^2$$

in formal continuum limit $a \to 0$.

Longitudinal Gluon Propagator



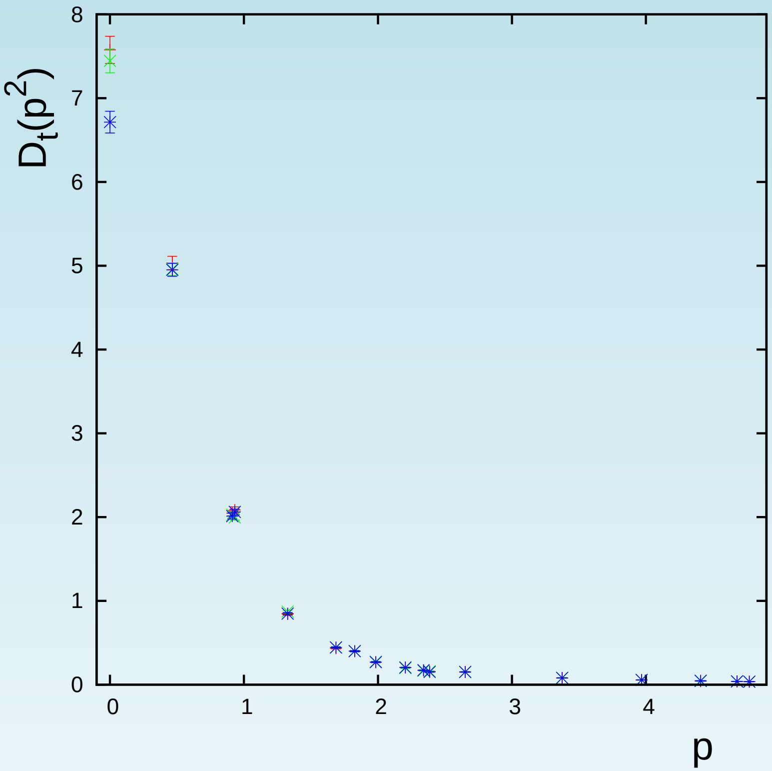
We have checked that

$$p^2 D_l(p^2) = \xi$$

as predicted by perturbation theory. In the SU(2) case, for $V = 16^4$, $\beta = 4$ and $\xi = 0.5$ a fit a/p^b for $D_l(p^2)$ gives $a = 0.502(5)$ and $b = 2.01(1)$ with a $\chi^2/dof = 1.1$.

(A.C. et al., PRL103 2009, PoS QCD-TNT09; in agreement with P. Bicudo et al., PRD92 2015, PoS LATTICE2015).

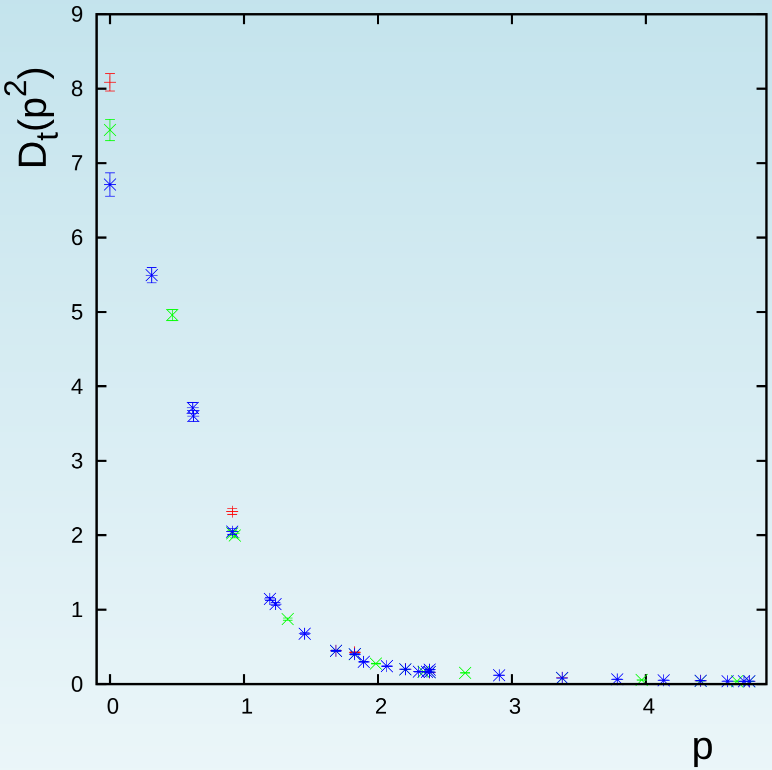
Transverse Gluon Propagator (I)



Transverse gluon propagator $D_t(p^2)$ [using the stereographic projection in the SU(2) case] as a function of the momentum p (both in physical units) for the lattice volume $V = 16^4$, $\beta = 2.3$ and $\xi = 0\,(+), 0.05\,(\times),\ 0.1\,(*)$ (A.C. et al., PRL103 2009, PoS QCD-TNT09).

$D_t(0)$ decreases as ξ increases (in agreement with L. Giusti et al. NP Proc.Supp.94 2001 and P. Bicudo et al., PRD92 2015, PoS LATTICE2015).

Transverse Gluon Propagator (II)



Transverse gluon propagator $D_t(p^2)$ [using the stereographic projection in the SU(2) case] as a function of the momentum p (both in physical units) for $\xi = 0.05$, $\beta = 2.3$, and the lattice volumes $V = 8^4\,(+), 16^4\,(\times)$, $24^4\,(*)$ (A.C. et al., PRL103 2009, PoS QCD-TNT09).

$D_t(0)$ decreases as V increases (as in Landau gauge).

Ghost Sector (I)

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In the continuum we have three possible setups:

1) complex ghost fields $\overline{c} = c^\dagger$: the FP matrix $-\partial \cdot D^{ab}$ and the Lagrangian density are not Hermitian;

2) complex ghost fields $\overline{c} = c^\dagger$: a symmetric FP matrix $-(\partial \cdot D^{ab} + D^{ab} \cdot \partial)/2$, plus a quartic ghost self-interaction term;

3) real independent ghost/anti-ghost fields u, iv: the "effective" FP matrix $\frac{i}{2} \begin{pmatrix} 0 & -\partial \cdot D^{bc} \\ D^{bc} \cdot \partial & 0 \end{pmatrix}$ is Hermitian.

Second Variation of $\mathcal{E}_{LCG}[U^g, g, \Lambda]$

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One can write the lattice Landau FP matrix as

$$\mathcal{M} \,=\, -\,\frac{1}{2}\left[\nabla_{\mu}^{(-)}\, D_{\mu} \,+\, D_{\mu}^{T}\,\left(\nabla_{\mu}^{(-)}\right)^{T}\right]\;.$$

This would correspond, in the continuum, to case 2).

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But, how do we simulate the quartic ghost self-interaction term?

Second Variation of $\mathcal{E}_{LCG}[U^g, g, \Lambda]$

Can we obtain the continuum case 1)? The matrix

$$\mathcal{M}_+^{bc}(x,y) \equiv \mathcal{M}^{bc}(x,y) + \sum_{e=1}^{N_c^2-1} f^{bec}\,\Lambda^e(x)\,\delta_{x,\,y}$$

is a lattice discretization of the continuum operator $-\partial \cdot D^{bc}$.

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The extra term is skew-symmetric, under the simultaneous exchanges $b \leftrightarrow c$ and $x \leftrightarrow y$, and it cannot be obtained from a second variation!

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Equivalently, we could add to the minimizing functional $\mathcal{E}_{\mathrm{LCG}}[U;\Lambda;h]$, the null term $-\Re\,\mathrm{Tr}\,\sum_x\, i\,[\,g(x),\,\Lambda(x)\,]\;g(x)^\dagger$. Indeed, by expanding to second order the above expression, we find (by a convenient reordering of the null terms) the term $f^{bec}\,\Lambda^e(x)\,\delta_{x,\,y}$.

Second Variation of $\mathcal{E}_{LCG}[U^g, g, \Lambda]$

Can we obtain the continuum case 3)? The "effective" FP matrix (without the factor i) $\frac{1}{2}\begin{pmatrix} 0 & -\partial \cdot D^{bc} \\ D^{bc} \cdot \partial & 0 \end{pmatrix}$ is skew-symmetric and it cannot be obtained from a second variation!

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We can also consider the bilinear form $\sum_{b,c,x,y} \gamma_1^b(x) \mathcal{M}^{bc}(x,y) \gamma_2^c(y)$ and extend it to the complex case [$\gamma_1^b(x), \gamma_2^b(x) \in \mathbb{C}$]. Then, the corresponding sesquilinear form is a positive semi-definite Hermitian form. Moreover, its imaginary part is skew-symmetric and gives us a natural way of obtaining the above FP matrix.

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2) The FP matrix $\mathcal{M}$ has real non-negative eigenvalues and real eigenvectors.

3) Since the FP matrix $\frac{1}{2}\begin{pmatrix} 0 & -\partial \cdot D^{bc} \\ D^{bc} \cdot \partial & 0 \end{pmatrix}$ is skew-symmetric, its eigenvalues are complex-conjugate and purely imaginary, and they are related to the SVD of $\mathcal{M}_+$, i.e. to the eigenvalues of $\mathcal{M}_+^T \mathcal{M}_+$.

Numerical Simulations: Ghost Propagator

We have done some preliminary tests, evaluating the ghost propagator for the continuum case 1), i.e. with the FP matrix $\mathcal{M}_{+}^{bc}(x,y) = \mathcal{M}^{bc}(x,y) + \sum_e f^{bec} \Lambda^e(x) \delta_{x,y}$.

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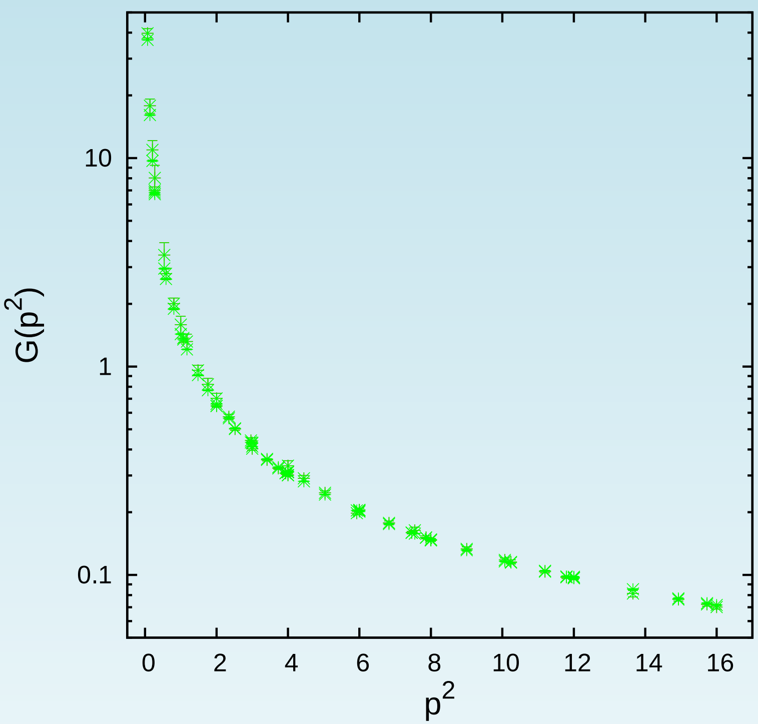
For the moment, $SU(2)$ gauge group, $\beta = 2.4469$, corresponding to $a \approx 0.1$ fm [and $\beta = 6.0$ in the SU(3) case], with $\xi \approx 0.163472$, corresponding to $\xi = 0.1$ in the continuum, using a point source for the inversion.

Ghost Propagator (Real Part)



Ghost propagator $G(p^2)$, in the SU(2) case, as a function of the square of the momentum p^2 (both in lattice units) for the lattice volume $V = 24^4$, $\beta = 2.4469$ and $\xi \approx 0.163472$, using 60 configurations and 20 sets of $\{\Lambda(x)\}$ for each configuration: comparison of Landau gauge ($+$) with Linear Covariant gauge ($*$).

Similar results for SU(3) by P. Silva.

Here, $p_{min} \approx 500$ MeV and $p_{max} \approx 7.8$ GeV.

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