

Determination of the strong coupling constant from lattice QCD and EFT

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Determination of the strong coupling constant from the energy of static quark anti-quark pair (2012-2018)

Determination of the strong coupling constant from the moments of quarkonium correlators (2008-2018)

Methodology: perform LQCD calculations in the continuum limit and compare to The perturbative results in MSbar scheme; This is very similar to non-lattice methods, especially for moments of quarkonium correlators. In the case of static energy multiple energy scales are involved => EFT

Static quark anti-quark energy in perturbation theory

Potential from pNRQCD, ultrasoft logs, renormalon

$$E_0(r) = V_s(r, \nu, \mu) + \delta_{US}(r, \nu, \mu) + RS(\rho)$$

Problem : Either we have a large $\log(vr)$ or r -dependent renormalon term (large uncertainty)

Solution: First take the derivative in r (RS term is gone) the re-sum the large logarithm $v=1/r$
 \Rightarrow calculate the force:

Necco, Sommer, PLB 523 (2001) 135, Sumino, PRD 65 (2002) 054003

$$F\left(r, \frac{1}{r}\right) = \frac{C_F}{r^2} \alpha_s(1/r) \left[1 + \frac{\alpha_s(1/r)}{4\pi} \left(\tilde{a}_1 - 2\beta_0 \right) + \frac{\alpha_s^2(1/r)}{(4\pi)^2} \left(\tilde{a}_2 - 4\tilde{a}_1\beta_0 - 2\beta_1 \right) + \frac{\alpha_s^3(1/r)}{(4\pi)^3} \left(\tilde{a}_3 - 6\tilde{a}_2\beta_0 - 4\tilde{a}_1\beta_1 - 2\beta_2 \right) + \frac{\alpha_s^3(1/r)}{(4\pi)^3} a_3^L \ln \frac{C_A \alpha_s(1/r)}{2} + \mathcal{O}(\alpha_s^4, \alpha_s^5 \ln \alpha_s) \right]$$

see e.g. Garcia i Tormo, MPLA 28 133028 for a review

$$E_0(r) = \int_{r^*}^r dr' F(r') + const$$

Alternatively use renormalon subtraction scheme: \Rightarrow Less precise

Set $\nu = const \Rightarrow RS(\rho)$ is constant
 that is subtracted

$$\alpha_s(M_Z) = 0.1156(22)$$

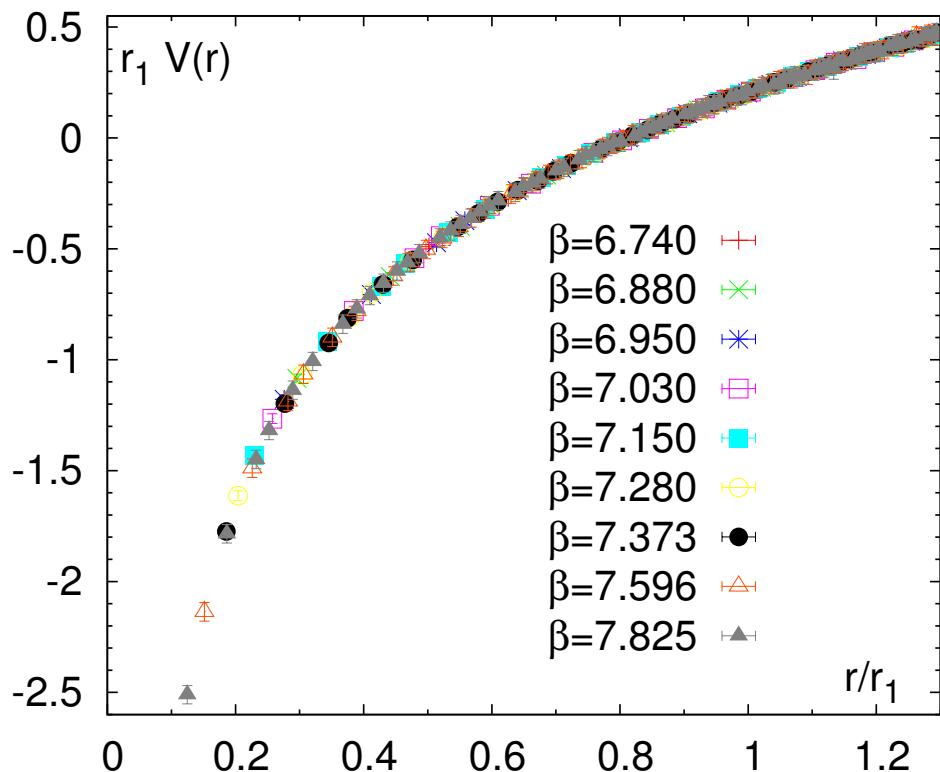
Brambilla et al, PRD86 (2012) 11403

Static quark anti-quark energy on the lattice

Set-up: 2+1 flavor HISQ (highly improved staggered quarks), $m_\pi = 161$ MeV, 10 lattice spacings in the range $a=0.04\text{--}0.11$ fm used for calculating T_c and equation of state

Bazavov et al (HotQCD), PRD85 (2012) 054503

Bazavov et al (HotQCD), D90 (2014) 094503



Cutoff effects in the potential are small even at short distances

Lattice spacing is determined from the r_1 scale

$$\left. r^2 \frac{dV}{dr} \right|_{r=r_1} = 1$$
$$r_1 = 0.3106(18) \text{ fm}$$

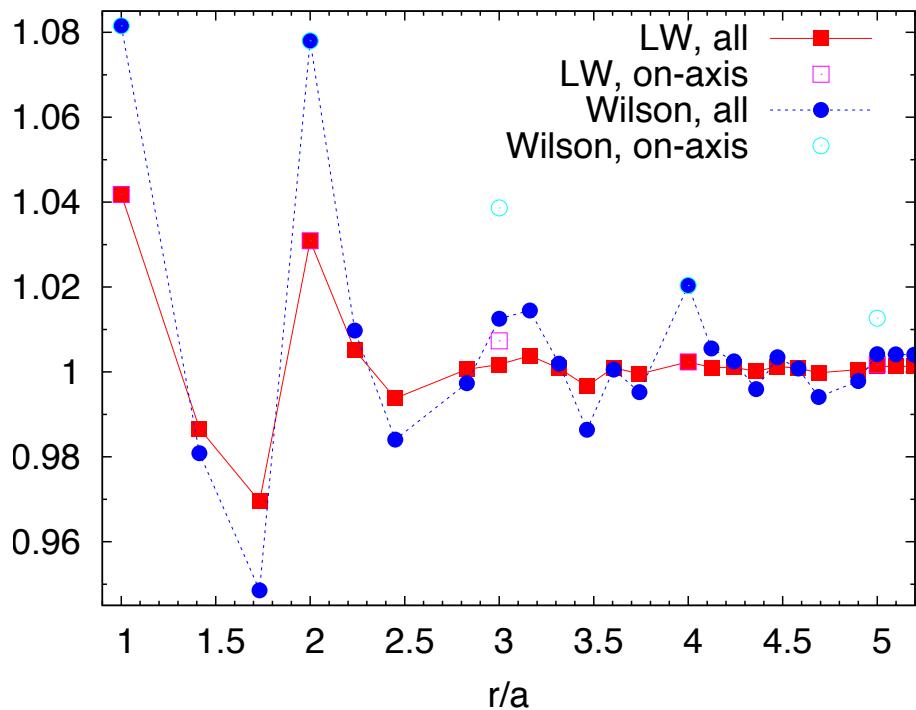
Bazavov et al (MILC), PoS (Lattice 2010) 074

2018: additional 3 lattice spacings
 $a=0.025, 0.03$ and 0.035 fm from
equation of state calculations at very high
temperatures:

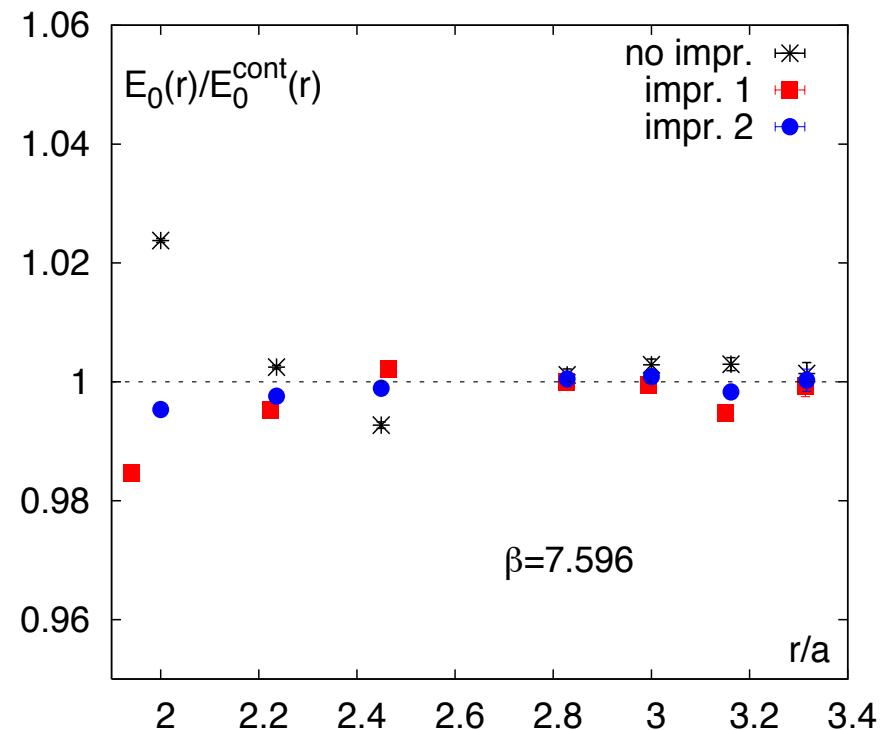
Bazavov, PP, Weber, PRD97 (2018) 014510

Static quark anti-quark energy on the lattice (cont'd)

$E_0(r)/E_0^{\text{cont}}(r)$ at tree level (free theory)



$\beta = 7.596$



Use the lattice data at the highest beta and for $r/a > 2.6$ to provide a continuum estimate for the static energy $E_0^{\text{cont}}(r)$

The cutoff effects in the free theory and QCD are quite similar

No cutoff effects visible in the data for $r/a > 2.6$ within statistical errors

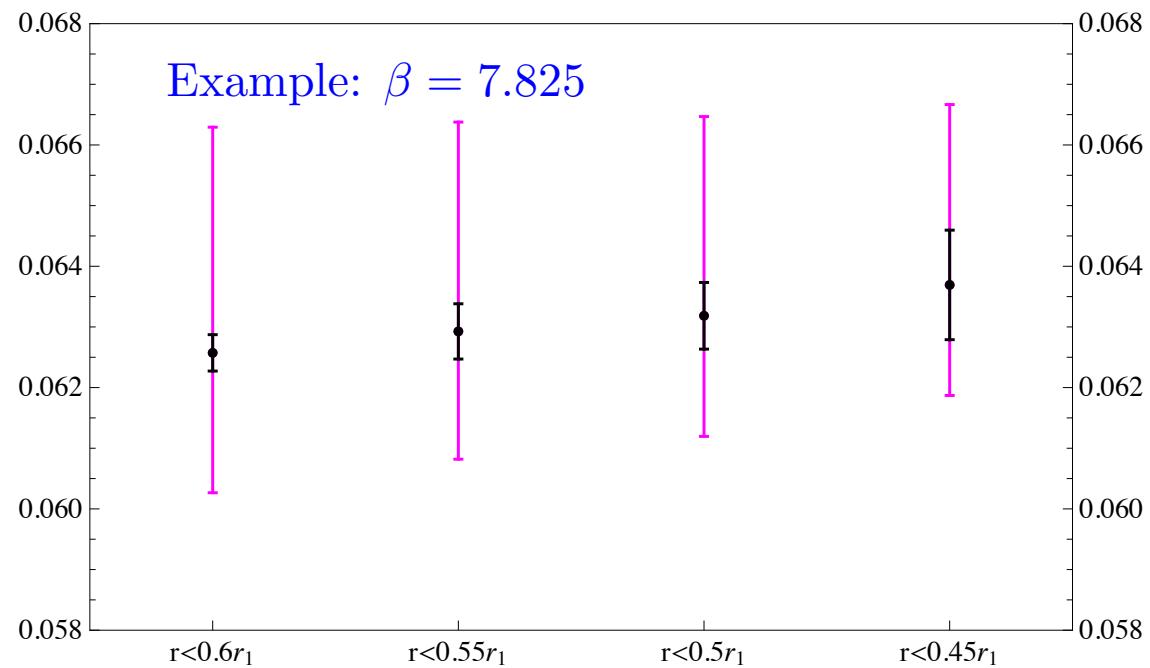
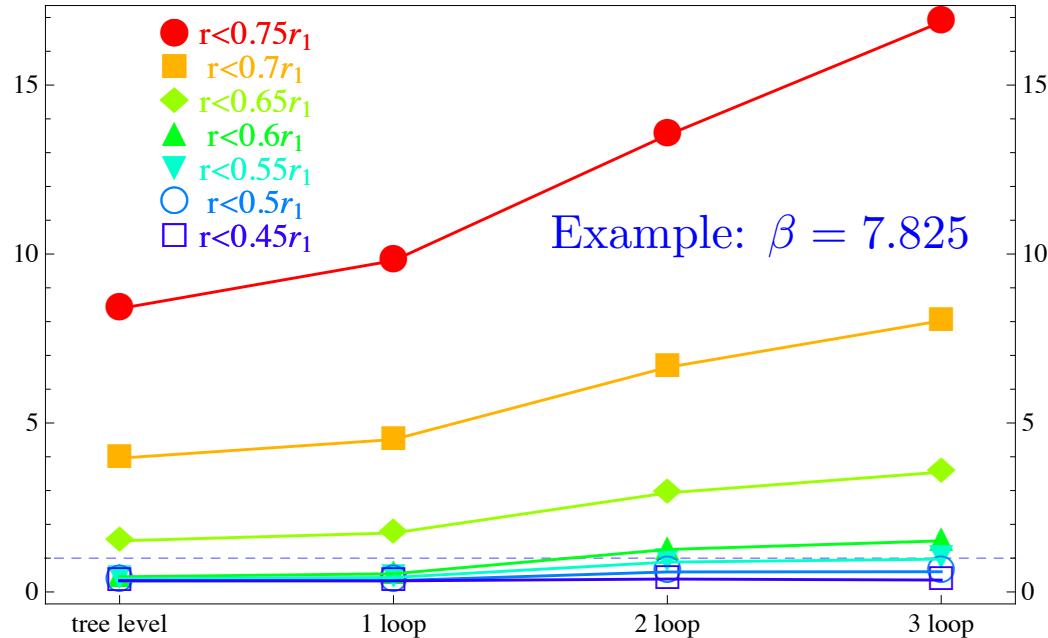
Very little cutoff dependence even for $r/a < 2.6$ if tree level improvement is used $r \rightarrow r_I = C_{\text{latt}}^{-1}$

Use the values of $E_0(r)/E_0^{\text{cont}}(r)$ to correct for the residual cutoff effects

Fitting the lattice results on the static energy

One can fit the data with tree-level 1-loop, 2-loop or 3-loop expressions for $r < 0.5 r_1$ up to an additive constant specified by matching the perturbative potential to the lattice one at $(r/a)^2 = N_{ref}$

Bazavov, Brambilla, Gacia i Tormo, PP,
Soto, Vairo, PRD 90 (2014) 074038



Obtain $a\Lambda_{\overline{MS}}$ or $r_1\Lambda_{\overline{MS}}$ from the fit

Perturbative errors
(from scale variations
 $\nu = [1/(\sqrt{2}r), \sqrt{2}/r]$ or
higher orders $\pm \alpha_s^4/r^2$)
dominate:

$$3\text{-loop: } r_1\Lambda_{\overline{MS}} = 0.486^{+0.028}_{-0.018}$$

$$\alpha_s(M_Z) = 0.1166^{+0.0012}_{-0.0008}$$

Going to shorter distances with the static energy

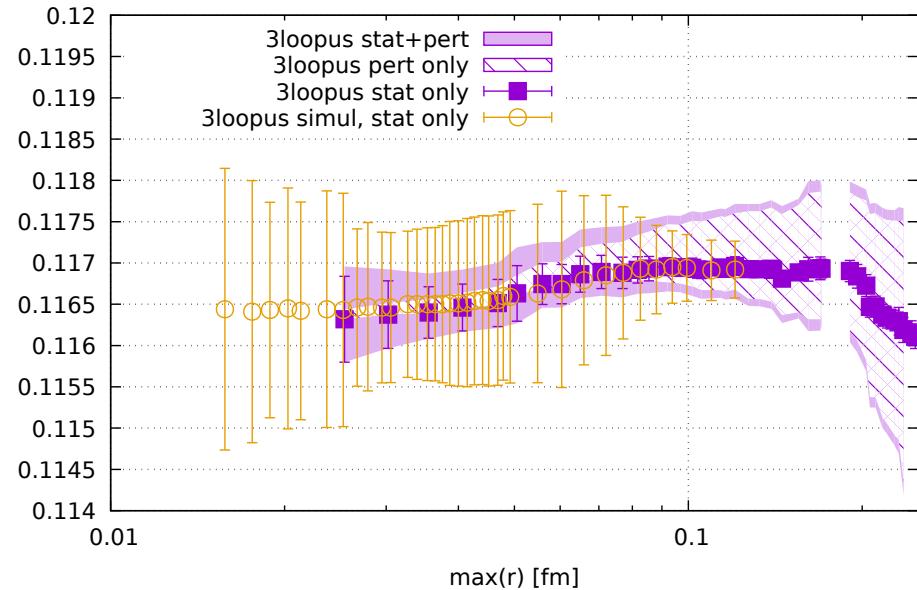
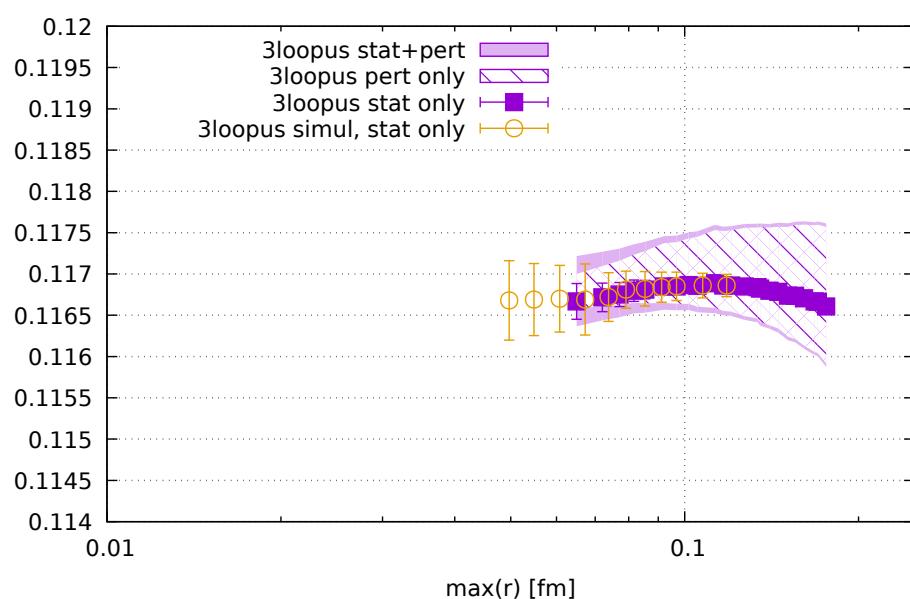
2014: smallest distance available 0.04 fm , dominant errors from perturbation theory

2018: additional 3 lattice spacings $a=0.025, 0.03$ and 0.035 fm for the static energy

Bazavov, PP, Weber, PRD97 (2018) 014510

Singlet free energy of quark anti-quark pair approaches the static energy at short distances, Bazavov et al (TUMQCD), arXiv:1804.10600

=> use the singlet free energy for $rT < 0.25$ as proxy for the static energy => access to very short distance down to $a=0.01 \text{ fm}$



$$T=0 \text{ result at } r_{\text{max}}=0.05 \text{ fm}: \quad \alpha_s(M_Z, n_f = 5) = 0.1167 \pm 0.008$$

Moments of charmonium correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G(t) = a^6 m_{c0}^2 \sum_{\mathbf{x}} \langle j_5(\mathbf{x}, t) j_5(0, 0) \rangle, \quad j_5 = \bar{\psi}_c \gamma_5 \psi_c$$

$$G_n = \sum_t (t/a)^n G(t)$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), m_c(\mu))}{m_c^{n-4}(\mu)}, \quad g_n = \sum_j g_{nj}(m_c, \mu) \alpha_s^j(\mu)$$

To cancel lattice effects consider the reduced moments

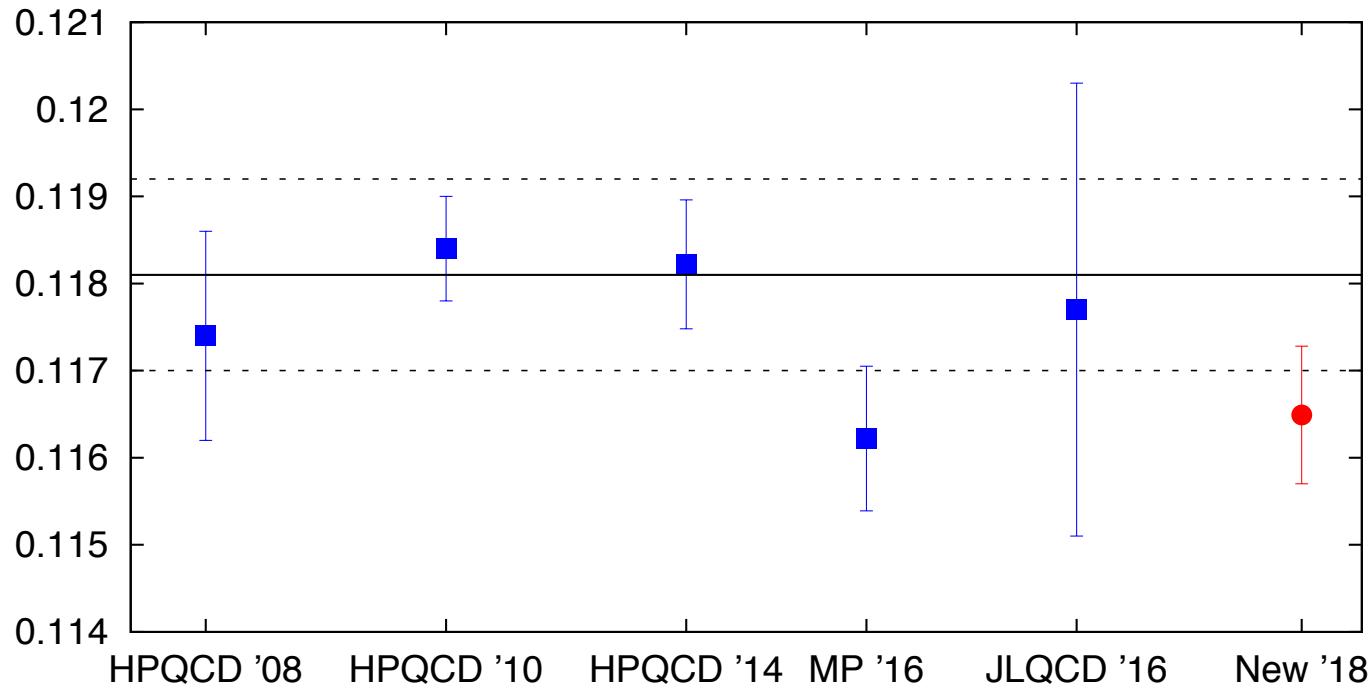
$$R_n = \left(\frac{G_n}{G_n^0} \right)^{1/(n-4)}$$

and similarly on the weak coupling side:

$$r_n = \sum_j r_{nj}(m_c, \mu) \alpha_s^j(\mu)$$

Allison et al (HPQCD), PRD78 (2008) 054513

Summary of lattice results on the moments



HPQCD 2008: Allison et al , PRD 78 (2008) 054513

HPQCD 2010: McNeil et al, PRD 82 (2010) 034512

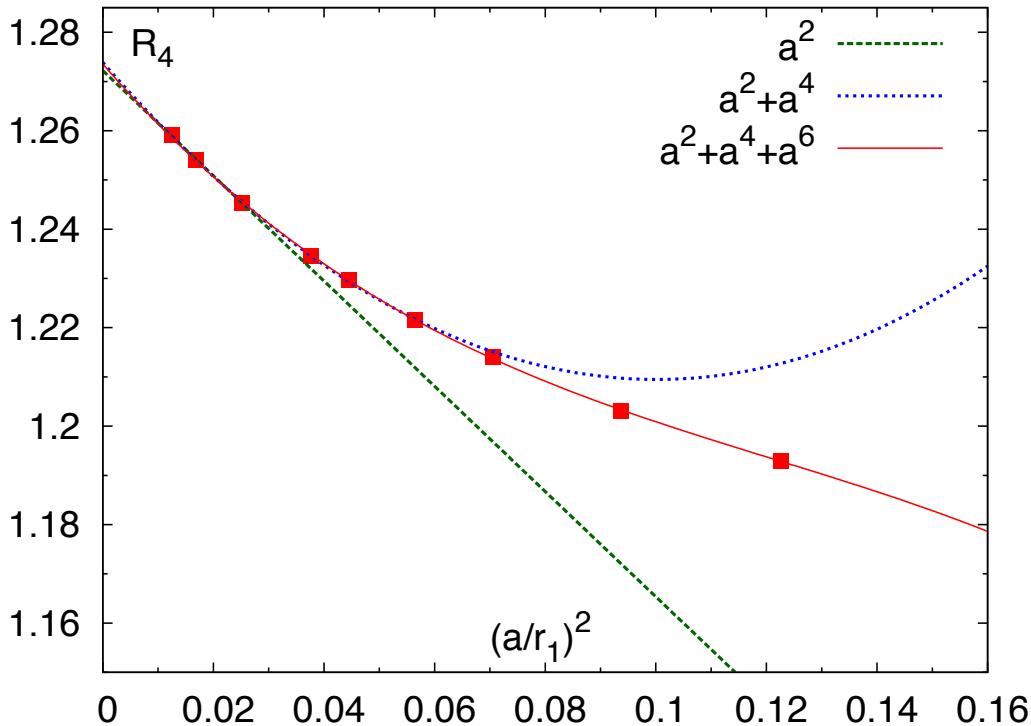
HPQCD 2014: Chakraborty et al, PRD 91 (2015) 054508

MP2016: Maezawa, PP, PRD 94 (2016) 034507

JLQCD 2016: Nakayama et al, PRD 94 (2016) 054507

New 2018: Weber, PP, preliminary results from R_4 and R_6

Moment of quarkonium and coupling constant



$\mu = m_c \rightarrow$ No large logs in $r_4 \rightarrow$

$$\alpha_s(m_c) \rightarrow m_c = 1.2662(91)\text{GeV}$$

Lat.	Pert.	Cond.	$\alpha_s^{n_f=5}(M_Z)$
\downarrow	\downarrow	\downarrow	$\rightarrow 0.11646(39)$
$\alpha_s(m_c) = 0.3729(28)(33)(17)$			$\rightarrow 0.11623(63)$
$\alpha_s(1.5m_c) = 0.2914(41)(12)(3)$			$\rightarrow 0.11748(109)$
$\alpha_s(2m_c) = 0.2610(55)(8)(1)$			

Combined result:

$$\alpha_s(M_Z, n_f = 5) = 0.11649 \pm 0.0063 \pm 0.0016(\text{scale})$$

$$R_4 \rightarrow \alpha_s$$

$$m_c : R_4 = 1.272(2)$$

$$\text{HPQCD: } R_4 = 1.272(5) \text{ [2008]}$$

$$R_4 = 1.282(4) \text{ [2010]}$$

$$1.5m_c : R_4 = 1.213(3)$$

$$2m_c : R_4 = 1.191(4)$$

R_6 charm quark mass:

$$R_6/m_{c0} = 1.0214(22)$$

Summary

The strong coupling constant has been determined using 2+1 flavor HISQ lattices provided by HotQCD and TUMQCD Collaboration from:

Moments of the pseudo-scalar quarkonium correlators

2016:

$$\alpha_s^{n_f=5}(M_Z) = 0.11622(84)$$

2018 (preliminary):

$$\alpha_s^{n_f=5}(M_Z) = 0.11649(79)$$

Static quark anti-quark energy

2016:

$$\alpha_s^{n_f=5}(M_Z) = 0.1166^{+0.0012}_{-0.0008}$$

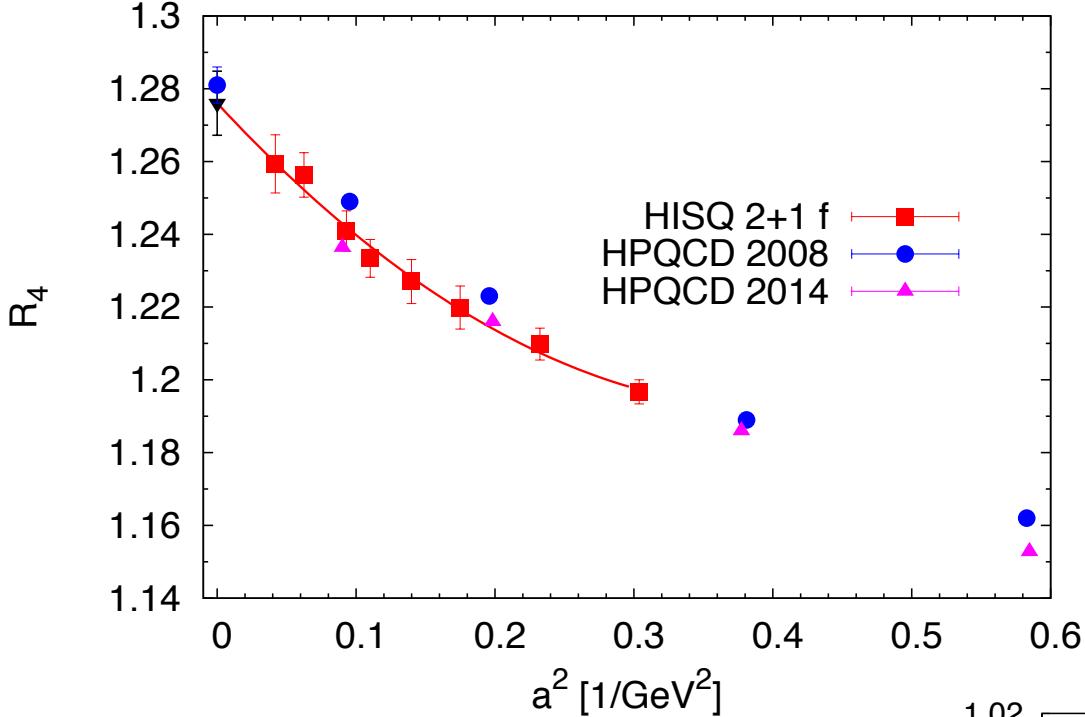
2018 (preliminary):

$$\alpha_s^{n_f=5}(M_Z) = 0.1167 \pm 0.008$$

The two results are consistent but lower than many lattice determinations and the FLAG average

HPQCD results from the moments of quarkonium correlators are consistent with PDG

Back-up slides:

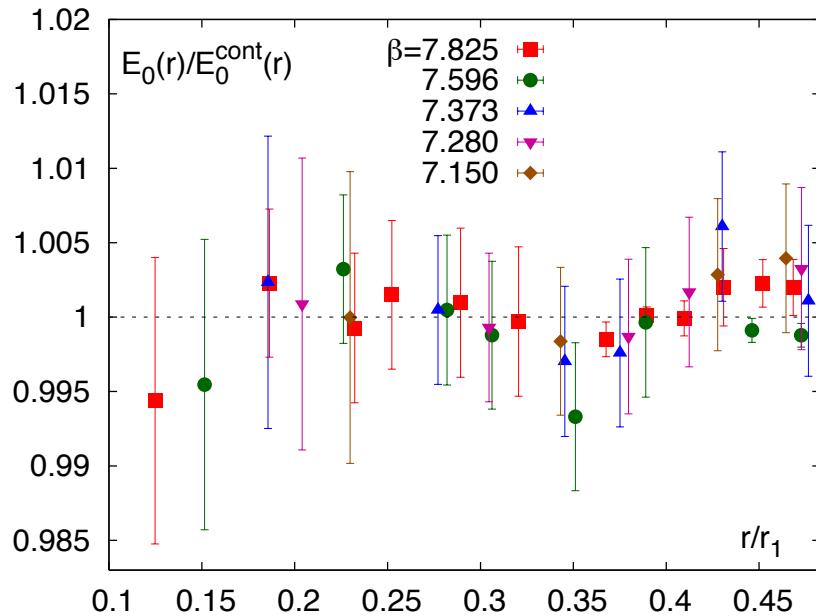


In the continuum limit
the results for the 4th moment
agree; our central
value is slightly smaller

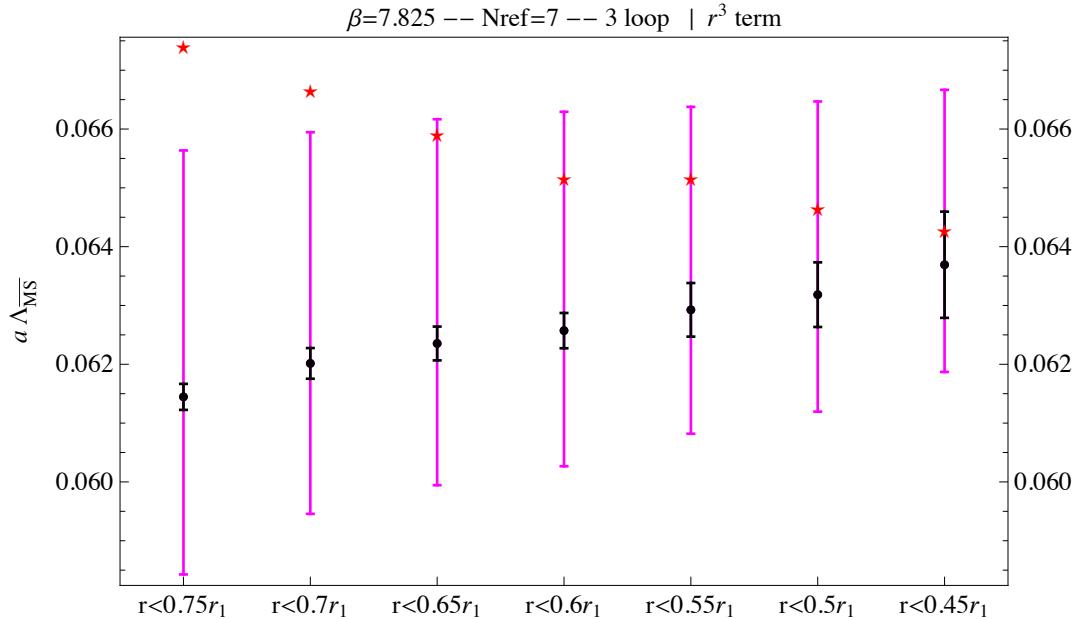
HPQCD 2008: Allison et al,
PRD78 (2008) 054513

HPQCD 2014: Chakraborty et al,
PRD 91 (2015) 054508

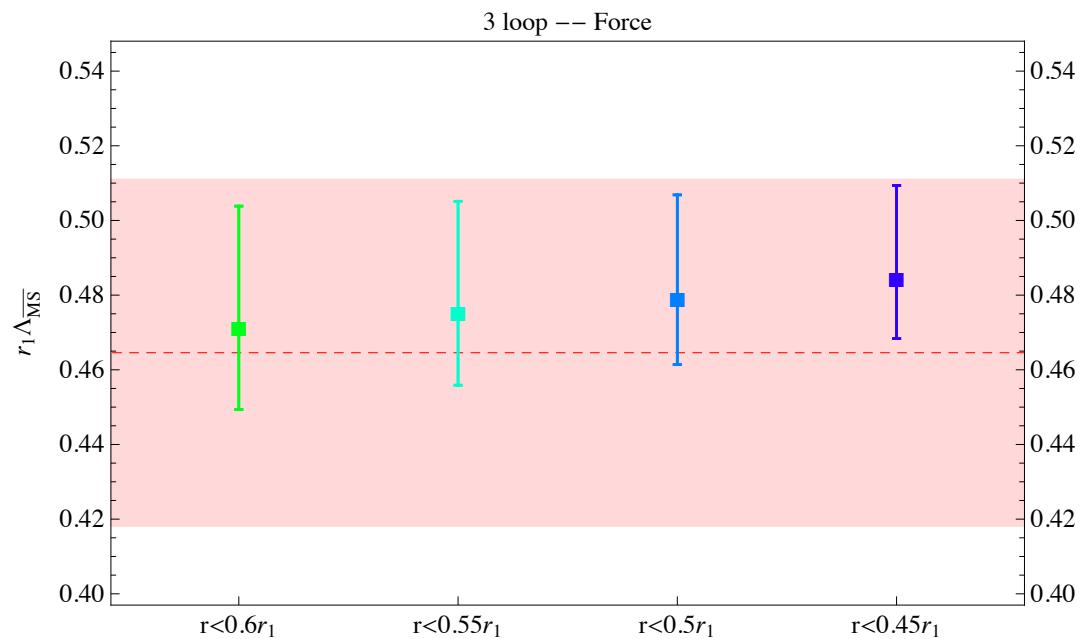
Data after corrections



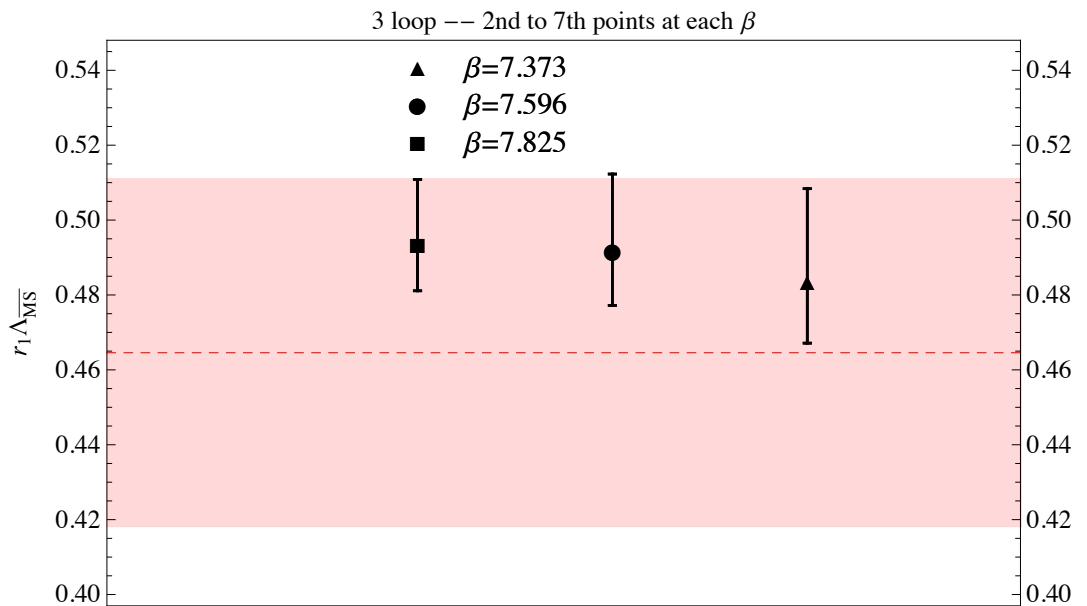
Correction from condensate:



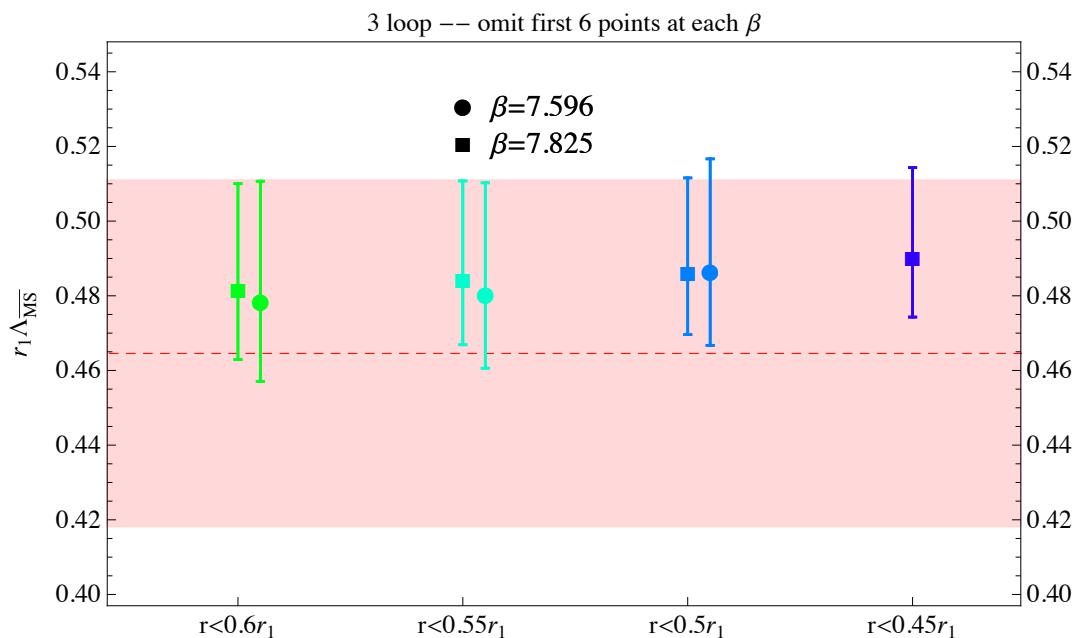
Using the force:



Using only $1 < (r/a)^2 < 7$:



Using only $(r/a)^2 > 6$:



Fitting the lattice results on the static energy (cont'd)

For the final result we also included the leading ultra-soft log with $\mu = 1.26/r_1 \sim 0.8$ GeV
 Results at different lattice spacings (β) are similar :

	$a\Lambda_{\overline{MS}}$; $N_{\text{ref}} = 7$	$a\Lambda_{\overline{MS}}$; $N_{\text{ref}} = 9$	$a\Lambda_{\overline{MS}}$; range	$r_1\Lambda_{\overline{MS}}$; range
$\beta = 7.373$	$0.0957^{+0.0046}_{-0.0028} \pm 0.0017$	$0.0957^{+0.0046}_{-0.0028} \pm 0.0017$	$0.0957^{+0.0046}_{-0.0028} \pm 0.0017$	$0.4949^{+0.0240}_{-0.0144} \pm 0.0086 \pm 0.0025$ $= 0.4949^{+0.0256}_{-0.0170}$
$\beta = 7.596$	$0.0781^{+0.0046}_{-0.0029} \pm 0.0007$	$0.0785^{+0.0046}_{-0.0029} \pm 0.0007$	$0.0783^{+0.0048}_{-0.0031} \pm 0.0010$	$0.4961^{+0.0303+0.0066}_{-0.0197-0.0061} \pm 0.0044$ $= 0.4961^{+0.0313}_{-0.0211}$
$\beta = 7.825$	$0.0644^{+0.0032}_{-0.0019} \pm 0.0006$	$0.0643^{+0.0032}_{-0.0020} \pm 0.0008$	$0.0643^{+0.0033}_{-0.0021} \pm 0.0008$	$0.4944^{+0.0256}_{-0.0159} \pm 0.0065 \pm 0.0037$ $= 0.4944^{+0.0267}_{-0.0175}$
Average	$r_1\Lambda_{\overline{MS}} = 0.495^{+0.028}_{-0.018}$			



$$\Lambda_{\overline{MS}} = 314.5^{+17.6}_{-11.7} \pm 1.7 \text{ MeV} = 315^{+18}_{-12} \text{ MeV}$$



$$\alpha_s(1.5 \text{ GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$



$$\alpha_s(M_Z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$