Nonperturbative Casimir Effects

M. N. Chernodub

- Centre national de la recherche scientifique (CNRS)
  Institut Denis Poisson, University of Tours, France
- Far East Federal University, Vladivostok, Russia

Plan of this talk:

The Casimir effect:
– tree-level phenomenon
– perturbative (radiative) corrections

Non-perturbative Casimir effects:
– Confinement
– Chiral symmetry breaking

XIIIth Quark Confinement and the Hadron Spectrum, Maynooth University, Aug. 4, 2018

The Casimir Effect

Named after Dutch physicist Hendrik Casimir
[H.B.G. Casimir, Proc. K. Ned. Acad. Wet. 51, 793 (1948)]
(2.5 page–long article).

[Source: Wikipedia]

Mathematics. — *On the attraction between two perfectly conducting plates.* By H. B. G. CASIMIR.

(Communicated at the meeting of May 29, 1948.)

The Casimir effect in its original formulation is a tree-level phenomenon.

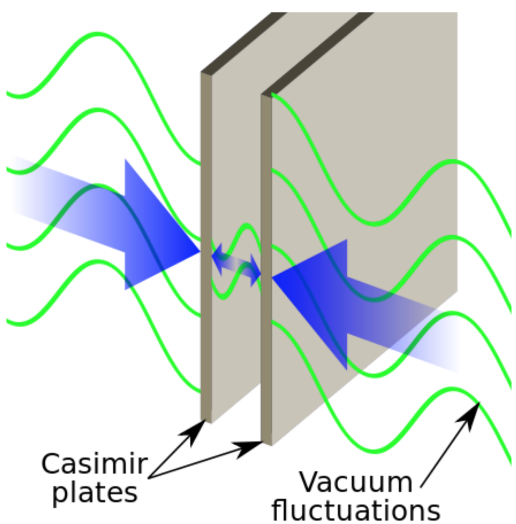
We discuss effects at

→ a “tree level” (→ while there are no radiative corrections, the effect effect is quantum, not classical)
→ with perturbative corrections (some effects of radiative corrections)
→ non-perturbative phenomena (recent results, models and lattice simulations*)

*) short review: lattice simulations of thermodynamic (Fisher–de Gennes) Casimir force by M. Hasenbusch et al. and world-line approaches (H. Gies, K. Langfeld, L. Moyaerts) cannot be covered within the given ~~proper~~ time scale.

Casimir effect

Simplest setup: two parallel perfectly conducting plates at finite distance R.



From Wikipedia

- The plates modify the energy spectrum of the electromagnetic field, and lead to a finite contribution to the vacuum energy.

$$\langle E \rangle = \frac{1}{2} \sum_n E_n$$

- The energy depends on the inter-plate distance R,

$$\frac{\langle E \rangle}{\text{Area}} = -\frac{\pi^2}{720} \frac{1}{R^3} \hbar c$$

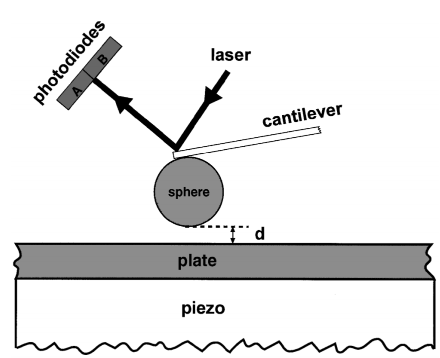
Proves existence of the zero-point energy? Questioned in [R. L. Jaffe, Phys. Rev. D72, 021301 (2005)]

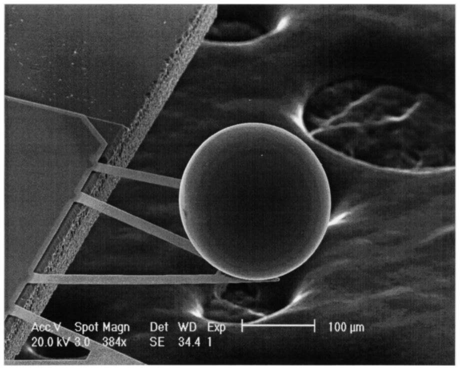
leading to an attraction between the neutral plates.

Experimentally confirmed (in plate-sphere geometries)
1% agreement with the theory

A very small force at human scales. However, at R =10 nm the pressure is about 1 atmosphere.

[S. K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997)]





From U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998), down to 100 nm scale.

Types of boundary conditions for a gauge field

- Ideal electric conductor: at the boundary
  normal magnetic field and tangential electric field are vanishing:

$$B_\perp \Big|_{x\in S} = 0\,, \qquad \boldsymbol{E}_\parallel \Big|_{x\in S} = 0$$

$\longrightarrow$ These conditions were originally considered by H. Casimir.

- Ideal magnetic conductor:
  normal electric field and tangential magnetic field are vanishing:

$$E_\perp \Big|_{x\in S} = 0\,, \qquad \boldsymbol{B}_\parallel \Big|_{x\in S} = 0$$

$\longrightarrow$ A non-Abelian version of these conditions is suitable for the MIT bag model (normal component of the classical gluon current vanishes at the boundary).

Tree-level Casimir effect: difficulties

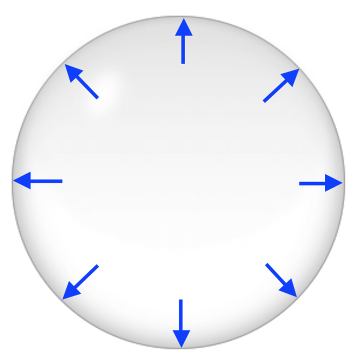
Apart from simplest geometries (plates, spheres, ...), an analytical calculation of
– the simplest, tree-level Casimir effect (→ no radiative/loop corrections)
– for simplest boundary conditions (→ ignore frequency dependent ε and μ of a real material)
is a very difficult task because it requires calculation of a full eigenspectrum of a (usually) simple operator (say, Laplacian) subjected to some boundary conditions.

And then one should calculate, regularize and find a finite part of the sum over all energies ...

This, this and this make the problem difficult.

[Marc Kac,
"Can One Hear the Shape of a Drum?"
Am. Math. Mon. 73, 1 (1966).]

- A popular explanation of the effect:
"boundaries restrict the number of virtual photons inside a cavity so that the pressure of the virtual photons from outside prevails"
is, actually, incorrect.



- For a spherical geometry the force is acting outwards:

$$\langle E \rangle_{\text{sphere}} = +\frac{0.0461765}{R}$$

[T. H. Boyer, Phys. Rev. 174, 1764 (1968)]

This property is used in bag models of hadrons.

[A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974); T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); K. A. Milton, Phys. Rev. D 22, 1441 (1980)]

The question of negative mass and negative energy

Qualitative effect:

– the Casimir energy between two perfectly conducting plates is negative

→ the gravitational mass and the inertial mass associated with the Casimir energy are equal and are negative as well:

$$M^{\rm Cas}_{\rm Inertial} = M^{\rm Cas}_{\rm Gravitational} = \frac{E}{c^2} < 0$$

[K. A. Milton et al, "How Does Casimir Energy Fall? I-IV" (2004-2007); J.Phys.A41, 164052 (2008); G. Bimonte et al., Phys.Rev. D76, 025008 (2007); V. Shevchenko, E. Shevrin, Mod.Phys.Lett. A31 (2016) no.29, 1650166]

→ A pure Casimir energy would levitate in a gravitational field due to existence of an upward "buoyant" force exerted by the outside vacuum on a "Casimir apparatus" following a quantum "Archimedes' principle".

However:

1. The buoyant force will be extremely small;
2. "... the mass energy of the cavity structure necessary to enforce the boundary conditions must exceed the magnitude of the negative vacuum energy, so that all systems of the type envisaged necessarily have positive mass energy." [J.D. Bekenstein, PRD 88, 125005 (2013)]

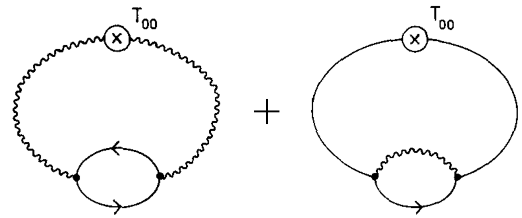
→ The Casimir apparatus will anyway be drown in the gravitational field.

Perturbative Casimir effect

The case of QED:

$\rightarrow$ perturbative corrections are very small: $\langle \mathcal{E} \rangle = -\frac{\pi^2}{720}\frac{\hbar c}{R^3}\left(1 - \frac{9\alpha\hbar}{32 m_e c}\frac{1}{R}\right)$

For the ideal plates separated at optimistic R =10 nm the radiative correction is 10^{-7} of the leading term.



$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

[M. Bordag, D. Robaschik, E. Wieczorek, Ann. Phys. 165, 192 (1985)]

$\rightarrow$ Unexpected qualitative phenomenon: between the plates, light travels faster than light outside the plates (the Scharnhorst effect).

[G. Barton, K. Scharnhorst, J. Phys. A 26, 2037 (1993); K. Scharnhorst, Annalen Phys. 7, 700 (1998)]

- Despite the Scharnhorst effect formally implies “faster-than-light travel” it cannot be used to create causal paradoxes.

[S. Liberati; S. Sonego, M. Visser, Ann. Phys. 298, 167 (2002); J.-P. Bruneton, Phys. Rev. D, 75, 085013 (2007)]

- The excess of c_{cavity} over the usual c is tremendously small, given by a two-loop radiative contribution to a refractive index in between plates.

$$\delta c = +\frac{11\pi^2}{90^2}\alpha^2\left(\frac{\hbar}{m_e c}\frac{1}{R}\right)^4$$

The Scharnhorst effect gives a 10^{-24} correction to c at optimistic R =10 nm.

Casimir effect and confinement

CP(N-1) model in (1+1)d

CP(N-1) model is a toy model for QCD because of
– asymptotic freedom (via the dimensional transmutation)
– mass gap generation (dynamical)
– topological defects (instantons)
– "confining" and non-confining ("Higgs") phases

[A. D'Adda, M. Lüscher, P. Di Vecchia, Nucl.Phys. B146, 63 (1978); E. Witten, Nucl.Phys. B149, 285 (1979)]

Action:
$$S = \int dxdt\,\left((D_\mu n_i)^* D^\mu n_i - \lambda(n_i^* n_i - r)\right)$$

Size of the CP(N-1) manifold, $r = \frac{4\pi}{g^2}$, related to the coupling g.

– N complex scalar fields, n_i, with U(1) gauge freedom, $n_i \to e^{i\alpha} n_i$ (we consider $N \to \infty$)
– the gauge field A_μ with the covariant derivative $D_\mu = \partial_\mu - igA_\mu$
(no kinetic term for A_μ, so that it may effectively be integrated out)
– λ is a Lagrange multiplier which enforces the classical constraint $n_i^* n_i = r$

In infinite space one has a confining phase with scale-dependent renormalized coupling

$$r(\mu) = \frac{4\pi}{g(\mu)^2} \simeq \frac{N}{2\pi}\log\left(\frac{\mu}{\Lambda}\right) \qquad \text{confinement phase}$$

and the mass gap generation determined by the dynamical scale $m = \Lambda$ (and $\lambda = m^2$).

Casimir effect in the CP(N-1) model

What are the properties of this model in a finite space interval?*

– Boundary conditions for the $\boldsymbol{n}$ field:

0 L

Dirichlet–Dirichlet: $n_1(0) = n_1(L) = \sqrt{r}\,, \qquad n_i(0) = n_i(L) = 0\,, \quad i > 1$

Neumann–Neumann: $\partial_x n_i(0) = \partial_x n_i(L) = 0$

– The $\boldsymbol{n}$ field is separated into two components, classical $\sigma \equiv n_1$ and quantum (other n_i).

– Total energy:
$$E = N \sum_n \omega_n + \int_0^L \left((\partial_x \sigma)^2 + \lambda(\sigma^2 - r) \right) dx$$

Casimir energy

usual kinetic + potential energies

– Casimir energy is determined by the eigenenergies ω_n of the eigensystem:

$$\left(-\partial_x^2 + \lambda(x) \right) f_n(x) = \omega_n^2 \; f_n(x)$$

– Two possible phases: “confinement phase” ($\lambda \neq 0$, $\sigma = 0$),
“Higgs phase” ($\lambda = 0$, $\sigma \neq 0$).

* Review in a historic order.

The Casimir effect beyond the uniform approximation

The issues of self-consistency: in a finite-geometry system the fermionic condensate may (and, logically should be) a coordinate–dependent quantity.

A difficult problem:

1. the mass gap (related to a condensate) is determined by the minimum of the free energy;
2. the free energy includes the Casimir energy;
3. the Casimir energy depends on the spectrum of quantum fluctuations;
4. the spectrum of quantum fluctuations depends on the mass gap;

It is difficult to find a self-consistent solution satisfying all the requirements from 1 to 4, given the existence of 5^{th} very natural property:

5. the mass gap is a function of the distance(s) to the plates (not translationally invariant).

In the free energy, one could have an interplay between
– an attractive force from the Casimir effect (quantum fluctuations);
– a repulsive force from a condensate.

Casimir effect in the CP(N-1) model: uniform case

Solving the model in a translationally invariant way:
– assume a translationally invariant anzats $\lambda \equiv m^2$=const, σ=const;
– solve gap equations (in a $N \to \infty$ limit):

$$g^2\mathrm{Tr}\frac{1}{(-\partial_\mu)^2 - m^2 + i\epsilon} + i\int\left(1 - \frac{g^2\sigma^2}{N}\right)\mathrm{d}^2\mathrm{x} = 0 \qquad \text{and} \qquad m^2\sigma = 0$$

[A. Milekhin, Phys.Rev. D86, 105002 (2012)]

One finds two phases:
– the "Higgs phase" at small lengths of interval, $L < L_{\mathrm{crit}}$
– the "confinement phase" at large lengths, $L > L_{\mathrm{crit}}$

where the critical distance is

$$L_{\mathrm{crit}} \sim 1/\Lambda = 1/m$$

*) No violation of the Mermin-Wagner-Coleman theorem: finite geometry + boundary conditions break SU(N) → SU(N-1), and SU(N-1) is always unbroken.

→ however, the translationally invariant anzats is too much restrictive as the fields may acquire certain space dependence.

**) This anzats is appropriate for the CP(N-1) model compactified to a circle → finite-T-like case, modes of a closed non–Abelian vortex string

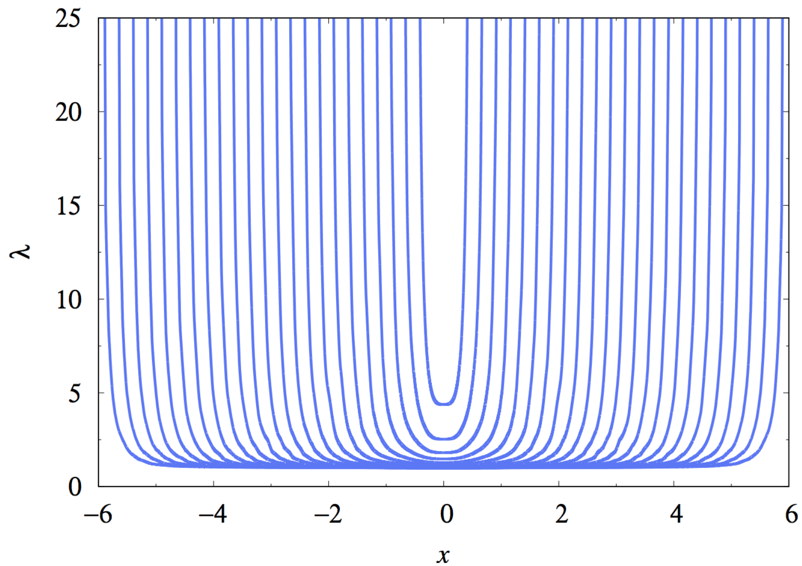
[R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi, A. Yung, Nucl. Phys. B 673, 187 (2003);
M. Shifman and A. Yung, PRD 70, 045004 (2004)]

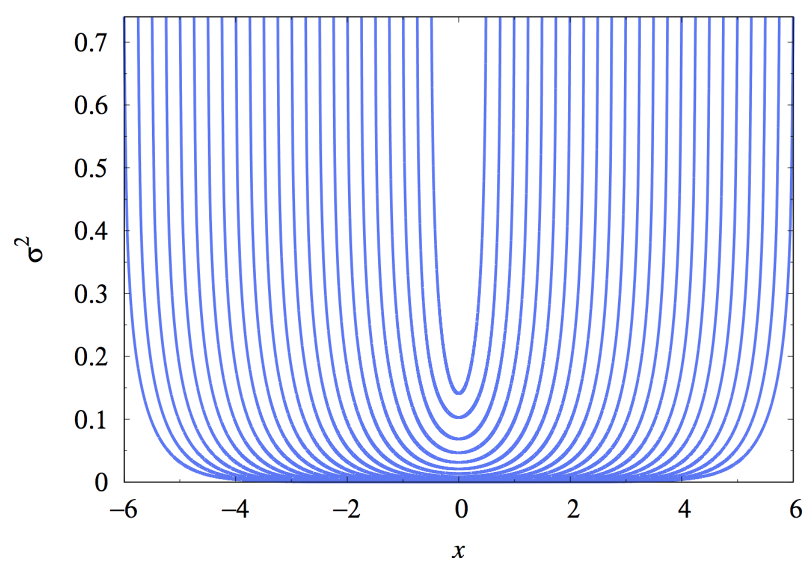
Casimir effect in the CP(N-1) model: non-uniform case (I)

– A more detailed analysis show that a constant mass m does not represent a quantum saddle point of the CP(N-1) model in a finite interval.

[S. Bolognesi, K. Konishi, K. Ohashi, JHEP 1610, 073 (2016)]

– The fields λ and σ are lively functions of the spatial coordinate x:





– plots are in units of $\Lambda = 1$ for a set of lengths of the interval $L = 1 \sim 12$

Near the boundaries:

$$\sigma^2 \simeq \frac{N}{2\pi} \log \frac{1}{|x \pm L/2|} \; ; \qquad \lambda(x) \simeq \frac{1}{2\,(x \pm L/2)^2 \log 1/|x \pm L/2|}$$

[A. Betti, S. Bolognesi, S. B. Gudnason, K. Konishi, K. Ohashi, JHEP 1801, 106 (2018)]

Casimir effect in the CP(N-1) model: conclusions (I)

Conclusions I:

– One (confining) phase for all lengths L of the interval.

- short interval: small but nonzero mass gap, attractive Casimir force, with the free energy density (basically, N free massless fields):

$$F \simeq \frac{N\pi}{6L^2}$$

- long interval: nonzero mass gap $m = \Lambda$, no Casimir force, constant free energy (= "string tension"):

$$F \simeq \frac{N\Lambda^2}{4\pi}$$

[A. Betti, S. Bolognesi, S. B. Gudnason, K. Konishi, K. Ohashi, JHEP 1801, 106 (2018), S. Bolognesi, S. B. Gudnason, K. Konishi, K. Ohashi, JHEP 1806, 064 (2018)]

Casimir effect in the CP(N-1) model: non-uniform case (II)

Use a mapping between CP(N-1) and Gross-Neveu models

$$\lambda = \Delta^2 + \partial_x \Delta \qquad \text{and} \qquad \sigma = A \exp[\textstyle\int_0^x \Delta(y) dy]$$

where Δ is a gap function satisfying the Bogoliubov-de Gennes equation:

$$\begin{pmatrix} 0 & \partial_x + \Delta \\ -\partial_x + \Delta & 0 \end{pmatrix} \begin{pmatrix} f_n \\ g_n \end{pmatrix} = \omega_n \begin{pmatrix} f_n \\ g_n \end{pmatrix}$$

and the gap equation:

$$\Delta = \frac{N}{2r} \sum_{\omega_n \geq 0} f_n g_n$$

[M. Nitta, R. Yoshii, JHEP 12, 145 (2017)]

Self-consistent analytical solutions:

unbroken–confining phase

$$\lambda_{\mathrm{Conf}} = 4\kappa^2 \frac{1 - \mathrm{cn}(2\kappa x, \nu)\mathrm{dn}(2\kappa x, \nu)}{\mathrm{sn}^2(2\kappa x, \nu)}$$

$$\sigma_{\mathrm{Conf}} = A_{\mathrm{conf}} \frac{\mathrm{dn}(2\kappa x, \nu) - \mathrm{cn}(2\kappa x, \nu)}{\mathrm{sn}(2\kappa x, \nu)}$$

broken–Higgs phase

$$\lambda_{\mathrm{Higgs}} = \kappa^2 \frac{\mathrm{cn}^2(\kappa x + \mathbf{K}, \nu) + \mathrm{dn}(\kappa x + \mathbf{K}, \nu)}{\mathrm{sn}^2(\kappa x + \mathbf{K}, \nu)}$$

$$\sigma_{\mathrm{Higgs}} = A_{\mathrm{Higgs}} \frac{(1 - \sqrt{1 - \nu})\mathrm{sn}(\kappa x + \mathbf{K}, \nu)}{1 - \mathrm{dn}(\kappa x + \mathbf{K}, \nu)}$$

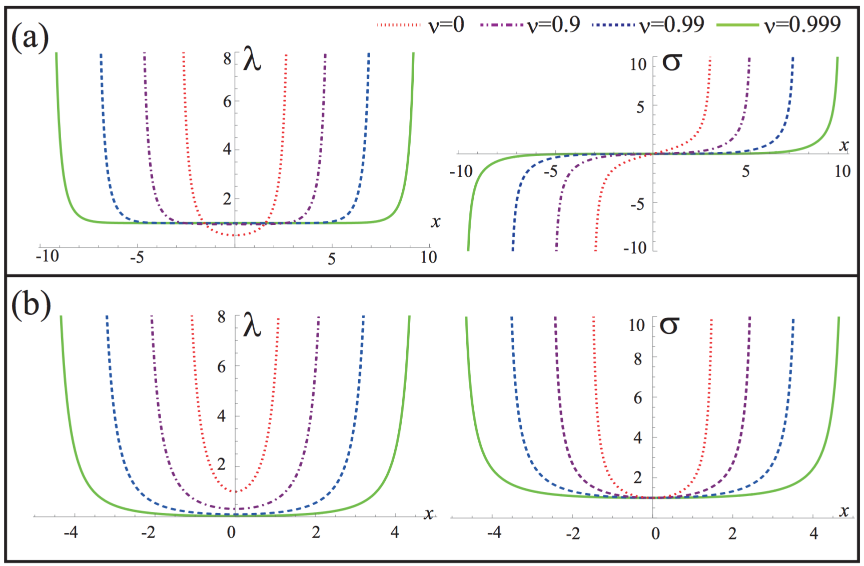
in terms of the Jacobi elliptic functions cn(x), dn(x) and sn(x) with elliptic parameter ν.
The size of the system enters via a complete elliptic integral of the first kind $2\mathbf{K}(\nu)/L \equiv \kappa$.

[A. Flachi, M. Nitta, S. Takada, R. Yoshii, arXiv:1708.08807]

Dynamical mass scale $m \equiv \Lambda$ does not enter the solution (a "semi-classical approach").

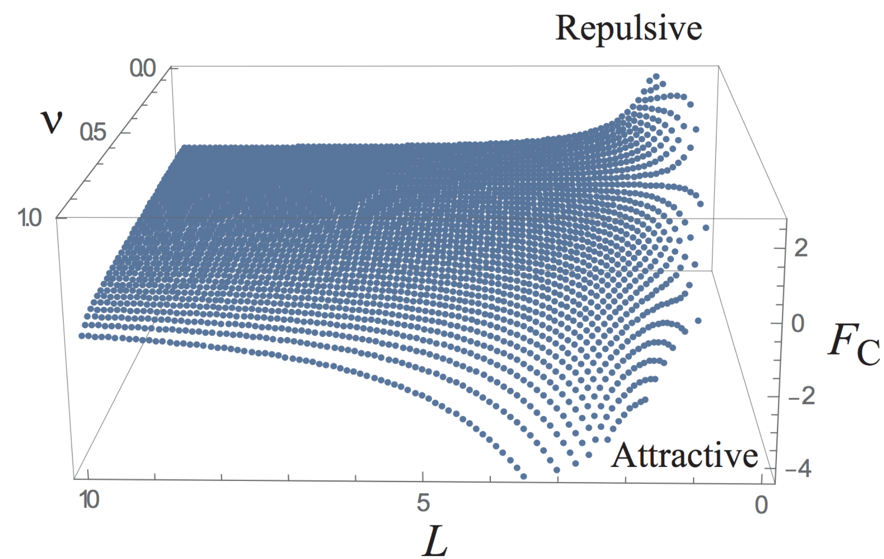
Casimir effect in the CP(N-1) model: conclusions (II)

– The fields λ and σ are lively functions of the spatial coordinate x:



confining phase

broken phase



Casimir force in the confining phase

Conclusions II:

– two classes of solutions:

- an unbroken confining phase (larger size L)
- a broken-Higgs phase (smaller sizes of the system L)

– Casimir force:

- repulsive in the Higgs phase;
- may change from repulsive to attractive in the confining phase.

[A. Flachi, M. Nitta, S. Takada, R. Yoshii, arXiv:1708.08807]

Toy model: (2+1)d compact U(1) gauge theory (cQED)

The compact QED (cQED) is a toy model for QCD:

– exhibits both confinement and mass gap generation at $T = 0$

– possesses a deconfinement phase transition at $T > 0$

– can be treated analytically and, of course, numerically.

[A. M. Polyakov, Nucl.Phys. B120, 429 (1977)]

Simple Lagrangian: $$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2$$

Field strength: $$F_{\mu\nu} = F_{\mu\nu}^{\mathrm{ph}} + F_{\mu\nu}^{\mathrm{mon}}$$

photons monopoles

$$g_{\mathrm{mon}} = \frac{2\pi}{g}$$

Photon field strength:

$$F_{\mu\nu}^{\mathrm{ph}}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

monopole density and charge

$$\rho(\boldsymbol{x}) = \sum_a q_a \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_a)$$

Field strength of the monopoles:

$$F_{\mu\nu}^{\mathrm{mon}}(\boldsymbol{x}) = -g_{\mathrm{mon}} \epsilon_{\mu\nu\alpha} \partial_\alpha \int d^3 y D(\boldsymbol{x} - \boldsymbol{y}) \rho(\boldsymbol{y})$$

Basic non-perturbative properties of cQED (at $T = 0$)

– The cQED may be mapped into the sine-Gordon model:

$$\mathcal{L}_s = \frac{1}{2 g_{\mathrm{mon}}^2} \left(\partial_\mu \chi\right)^2 - 2\zeta \cos \chi$$

χ is the scalar real-valued field. The mean density of monopoles

$$\varrho_{\mathrm{mon}} = 2\zeta$$

is controlled by the fugacity parameter ζ. The monopoles lead to

– Mass gap generation (photon mass / Debye screening):

$$m_{\mathrm{ph}} = g_{\mathrm{mon}} \sqrt{2\zeta} \equiv \frac{2\pi}{g} \sqrt{\varrho_{\mathrm{mon}}}$$

– Confinement of charges:

$$V(R) = \sigma R \quad \text{at} \quad R \to \infty$$

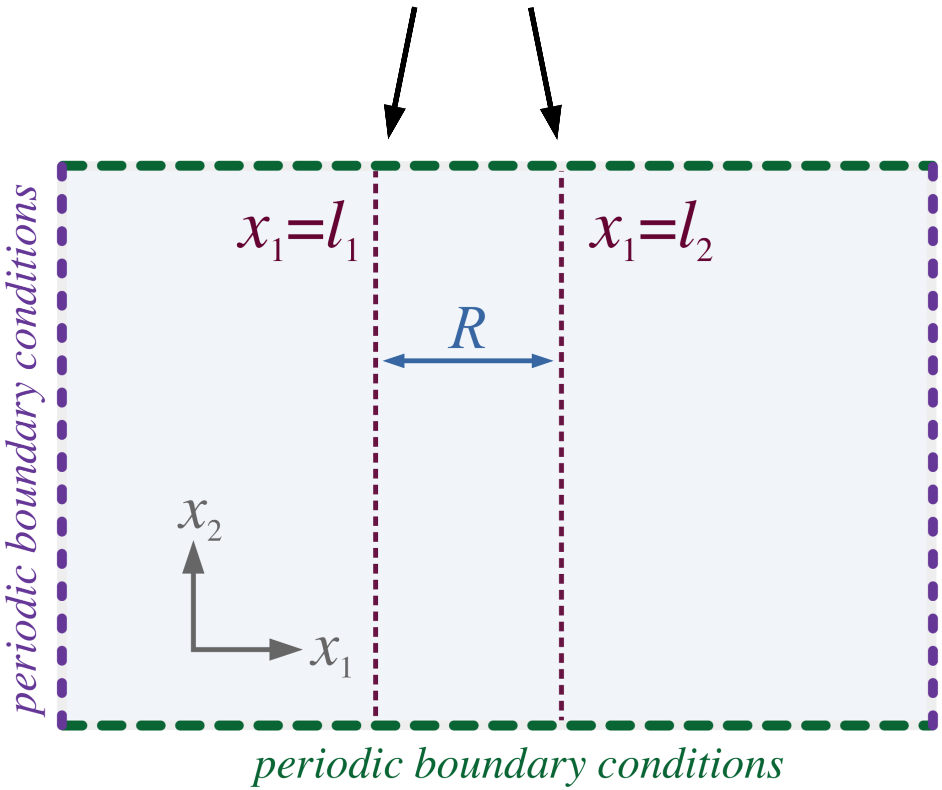
String tension:

$$\sigma = \frac{8\sqrt{2\zeta}}{g_{\mathrm{mon}}} \equiv \frac{4 g \sqrt{\varrho_{\mathrm{mon}}}}{\pi}$$

(all formulae are written in a dilute gas approximation)

Casimir effect in cQED: formulation

Two parallel metallic wires in two spatial dimensions



Boundary conditions:

– tangential electric field vanishes at any point of each wire

$$E_{||}(\boldsymbol{x}) = 0$$

– there is no true magnetic field (a pseudo-scalar B in 2+1d), so one has no condition on B.

Relativistically invariant formulation: $F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x}) = 0$

The world-surface S of the wires is parameterized by a vector $\bar{\boldsymbol{x}} = \bar{\boldsymbol{x}}(\tau, \xi)$

Characteristic function of S : $$s_{\mu\nu}(\boldsymbol{x}) = \int d\tau \int d\xi \, \frac{\partial \bar{x}_{[\mu,}}{\partial \tau} \frac{\partial \bar{x}_{\nu]}}{\partial \xi} \delta^{(3)}\left(\boldsymbol{x} - \bar{\boldsymbol{x}}(\tau, \xi)\right)$$

Casimir effect in cQED and partition function

Path integral formulation:

$$Z = \int \mathcal{D}A \sum\!\!\!\!\!\!\!\int_{\mathrm{mon}} e^{-S[A,\rho]}$$

Integral over monopole configurations

$$\sum\!\!\!\!\!\!\!\int_{\mathrm{mon}} = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{a=1}^{N} \left(\sum_{q_a=\pm 1} \zeta \int d^3 x_a \right)$$

The ideal-metal boundary condition $F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x}) = 0$

corresponds to a δ function $\delta_S[F] = \prod_{\boldsymbol{x}} \delta\Big(F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x})\Big)$

which restricts the fields at the (hyper)surfaces S:

$$Z_S = \int DA \sum\!\!\!\!\!\!\!\int_{\mathrm{mon}} e^{-S[A,\rho]} \delta_S[F]$$

It may be realized with the help of Lagrange multiplier field λ:

$$\delta_S[F] = \int \mathcal{D}\lambda \exp\left[\frac{i}{2}\int d^3x\, \lambda(\boldsymbol{x}) F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x})\right] \equiv \int \mathcal{D}\lambda \exp\left[\frac{i}{2}\int d^3x\, F^{\mu\nu}(\boldsymbol{x}) J_{\mu\nu}(\boldsymbol{x};\lambda)\right]$$

with the “surface tensor field” $J_{\mu\nu}(\boldsymbol{x};\lambda) = \lambda(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x})$

Analytical calculation of Casimir energy in cQED

The Casimir energy (potential) is then calculated as a properly normalized 00-component of the energy-momentum tensor in the presence of the boundaries. In an Abelian gauge theory

$$T^{\mu\nu} = -\frac{1}{g^2} F^{\mu\alpha} F^{\nu}_{\alpha} + \frac{1}{4g^2} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

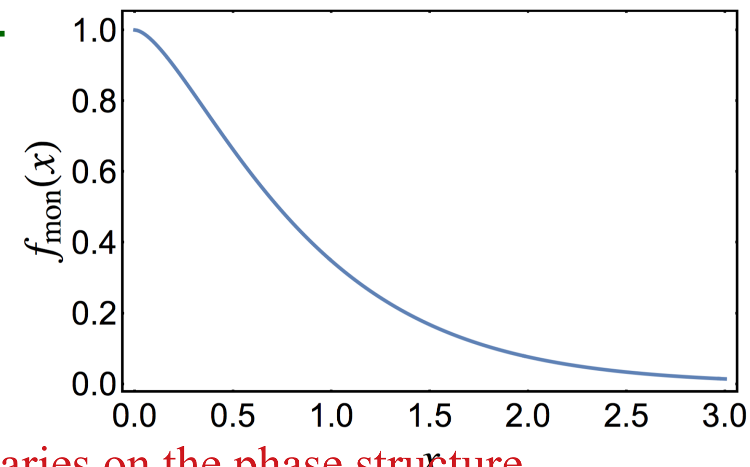
For (2+1)d cQED with two parallel wires at the distance R

$$V_{\mathrm{Cas}}(R) = -\underbrace{\frac{\zeta(3)}{16\pi}\frac{1}{R^2}}_{} \underbrace{f_{\mathrm{mon}}\left(m_{\mathrm{ph}} R\right)}_{}$$

Non-perturbative photon mass $m_{\mathrm{ph}} = \frac{2\pi}{g}\sqrt{\varrho_{\mathrm{mon}}}$

Usual tree-level term

Nonperturbative term due to monopoles
(appears basically due to the Debye screening)

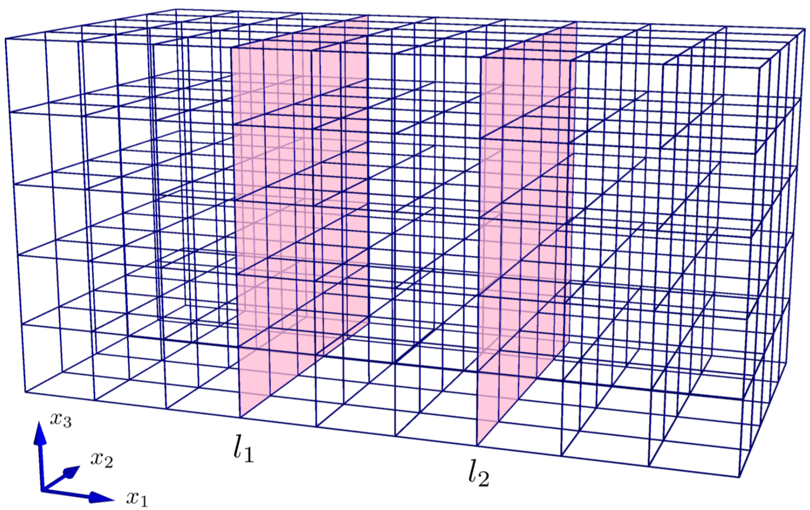


*) Dilute gas approximation, assumes no effect of boundaries on the phase structure.
**) Translationally-invariant anzats for the monopole density.

[V.A. Goy, A.V. Molochkov, M.Ch., Phys.Rev. D94, 094504 (2016)]

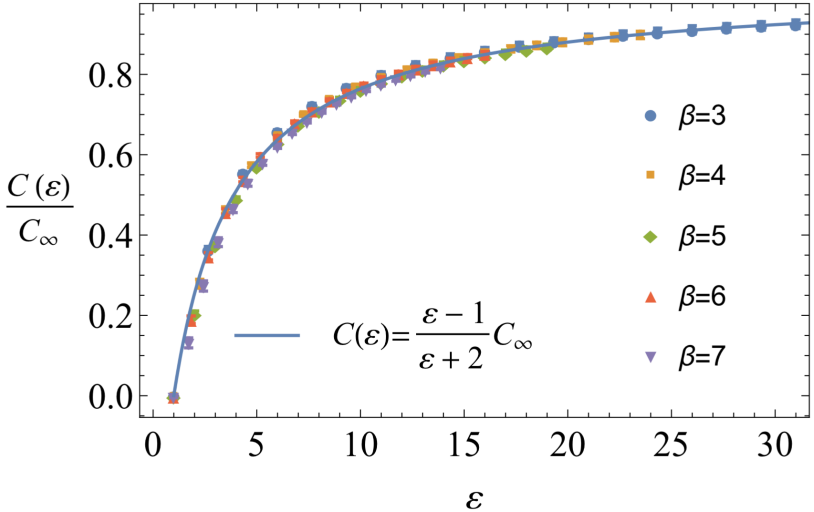
A good setup for first-principle lattice simulations

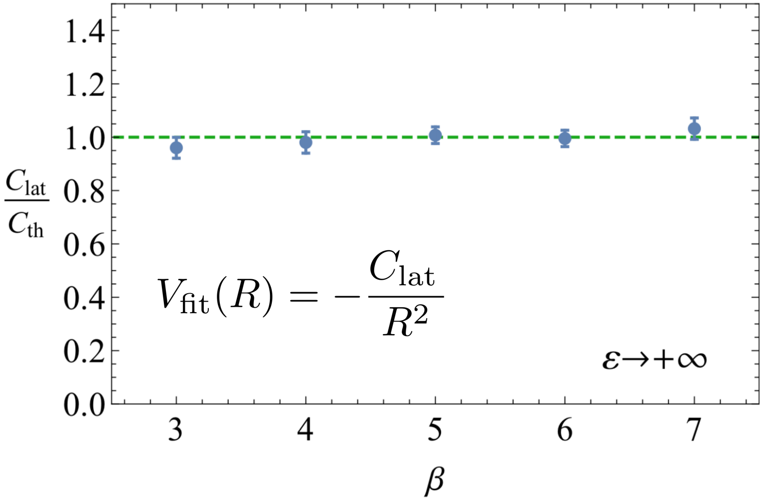
Take a lattice formulation of the theory and impose the appropriate conditions via the Lagrange multipliers at the boundaries.



Check of the approach in a free theory (no monopoles, weak coupling regime):

Casimir energy for finite static permittivity ε





Perfectly conducting wires

[= infinite static permittivity ε in (2+1)d]

[V.A. Goy, A.V. Molochkov, M.Ch., Phys.Rev. D94, 094504 (2016)]

Phase structure: deconfinement transition at $T = 0$

Electric charges exhibit a linear confinement in a Coulomb gas of monopoles.

If the wires are close enough, then
 → between the wires, the dynamics of monopoles is dimensionally reduced;
 → the inter-monopole potential becomes log-confining;

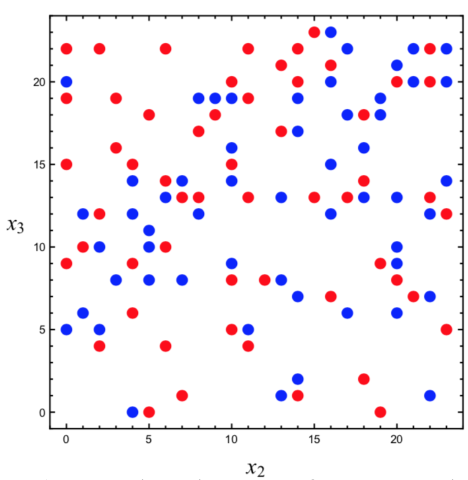
$$D_{3D}(\boldsymbol{x}) = -\frac{1}{4\pi|\boldsymbol{x}|} \to D_{2D}(\boldsymbol{x}) = \frac{2}{R} \ln \frac{|\boldsymbol{x}|}{R}$$

A very smooth transition,
a BKT type or crossover?

→ the monopoles form magnetic-dipole pairs (and are suppressed);
→ the confinement of electric charges disappears (a deconfining transition).

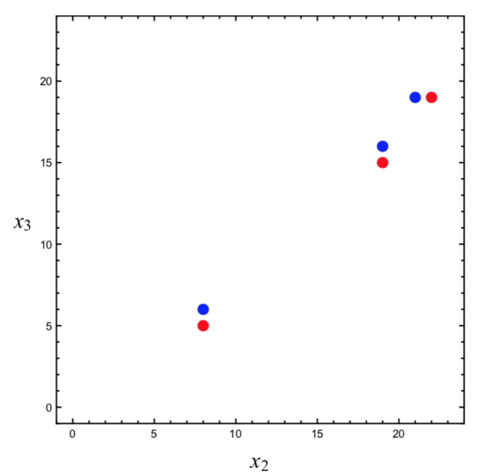
Examples of (anti-)monopole configurations

widely-spaced wires



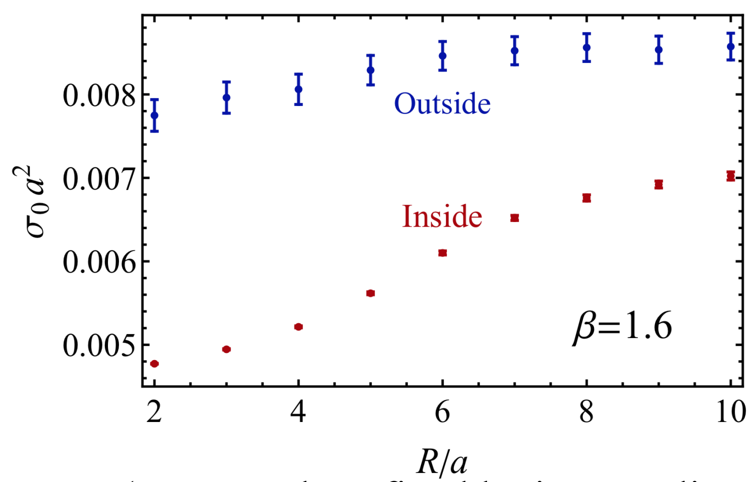
(a Coulomb gas of monopoles)

narrowly-spaced wires



(a dilute gas of magnetic dipoles)

String tension inside and outside wires



(an example at fixed lattice coupling β)

[V.A. Goy, A.V. Molochkov, M.Ch., Phys.Rev. D95, 074511 (2017); Phys.Rev. D96, 094507 (2017)]

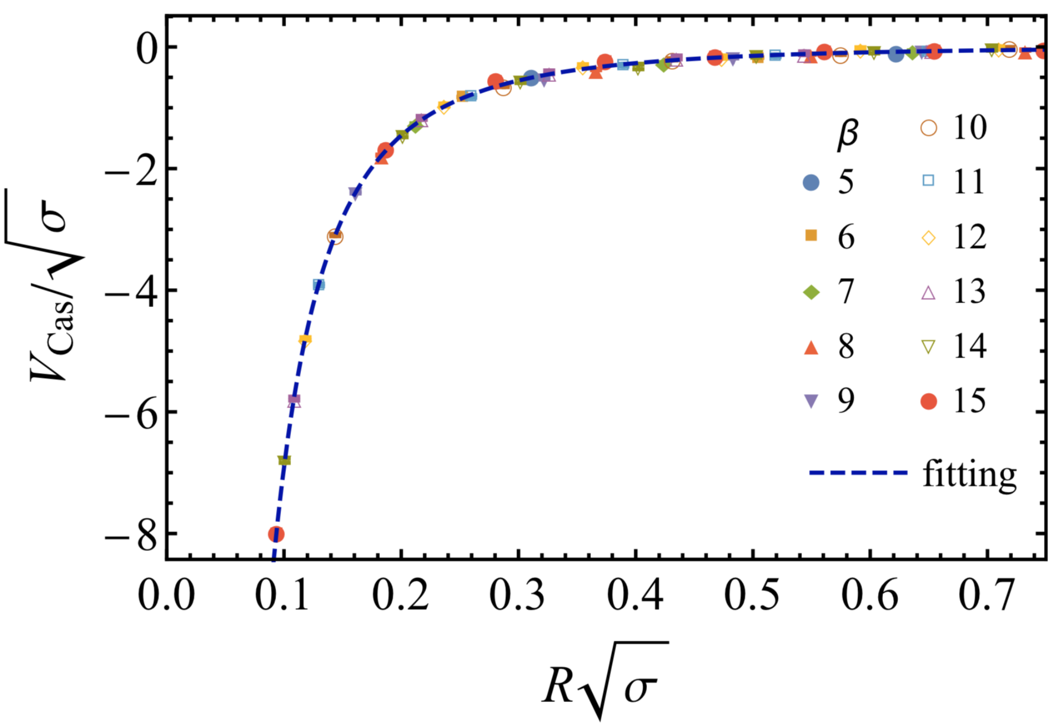
Non-Abelian Casimir effect: Casimir energy

The approach is easily generalizable to non-Abelian gauge groups.

Conditions at an ideal "chromo–metallic" boundary:

$$SU(N_c): \qquad F^{\mu\nu,a}(x)s_{\mu\nu}(x) = 0, \quad a = 1, \dots N_c^2 - 1$$

The Casimir potential in (2+1)d for SU(2) gauge theory at $T = 0$:



σ is the fundamental string tension at $T = 0$

R is the distance between the wires (plates)

[V.A. Goy, A.V. Molochkov, M.Ch., arXiv:1805.11887]

Features:

- excellent scaling
- may be described by the function

$$V_{\rm Cas}(R) = 3\underbrace{\frac{\zeta(3)}{16\pi}\frac{1}{R^2}}\underbrace{\frac{1}{(\sqrt{\sigma}R)^\nu}}\underbrace{e^{-M_{\rm Cas}R}}$$

- Tree-level contribution

- (Perturbative) anomalous dimension

$$\nu = 0.05(2)$$

- Nonperturbative Casimir mass

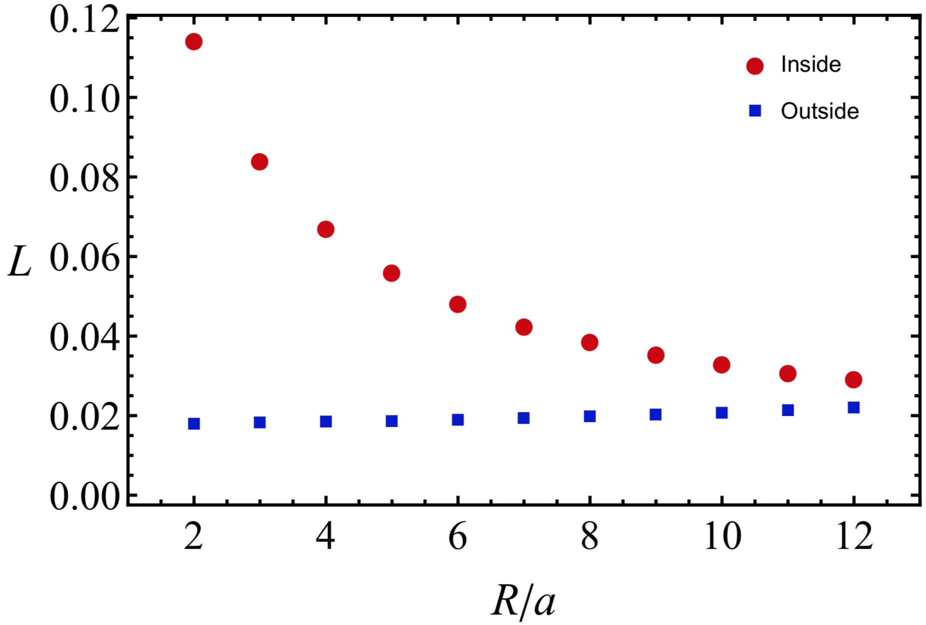
$$M_{\rm Cas} = 1.38(3)\sqrt{\sigma}$$

(*cf.* the glueball mass $M_{0^{++}} \approx 4.7\sqrt{\sigma}$)

[M. J. Teper, Phys.Rev. D59, 014512 (1999)]

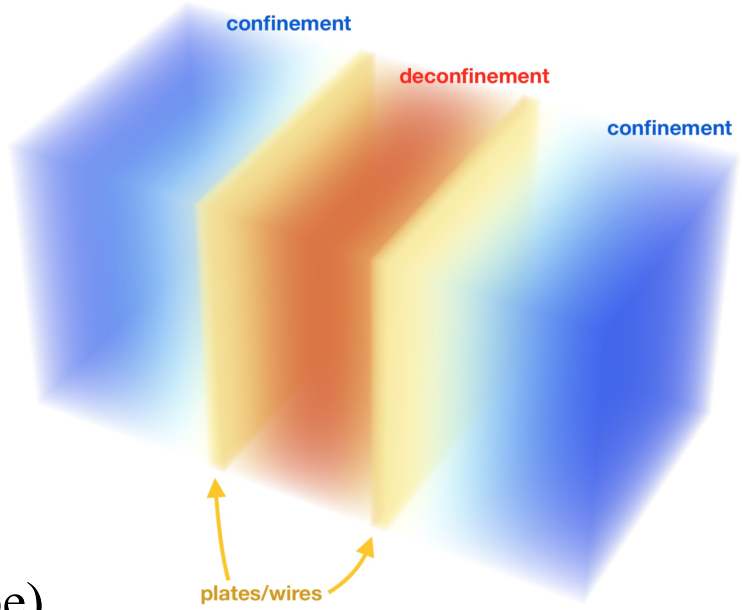
Non-Abelian Casimir effect: phase structure

- The expectation value of the Polyakov line indicates deconfinement in between the plates (wires).



- No signal for a phase transition at finite R. (an infinite-order BKT–type transition or a crossover?)

The finite Casimir geometry leads to a very smooth deconfinement transition in between the plates. The absence of a thermodynamic transition marks the difference with the finite temperature case.



In a finite-temperature SU(2) gauge theory the phase transition is of the second order (Ising-type) [M. Teper, Phys.Lett. B313,417 (1993)].

[V.A. Goy, A.V. Molochkov, M.Ch., arXiv:1805.11887]

Casimir effect and chiral symmetry breaking

The Casimir effect in interacting fermionic systems

How a finite geometry influences a chiral phase transition?

The toy model:

$$\mathcal{L} = \bar{\psi} i \gamma_\mu \partial^\mu \psi + \frac{g}{2N} (\bar{\psi} \psi)^2$$

Discrete flavor symmetry: $\psi \to \gamma^5 \psi$

Condensate of fermions: $\sigma = -g \langle \bar{\psi} \psi \rangle / N$

g is the coupling constant
N is the number of flavors

Work in $N \to \infty$ limit
in a mean-field approximation.

Two phases:
- a symmetric phase with a zero fermionic condensate $\sigma = 0$ (high T)
- a dynamically broken phase with a nonzero condensate $\sigma \neq 0$ (low T)

→ a model of the chiral sector of QCD (albeit with the discrete chiral symmetry)

→ may be related to superconductivity in bilayer graphene.

[A. Flachi, PRL 110, 060401 (2013)]

The Casimir effect in interacting fermionic systems

The fermionic vacuum in between two parallel plates at $z = 0, L$.

The MIT boundary conditions for fermions:
(normal component of the fermionic current vanishes)

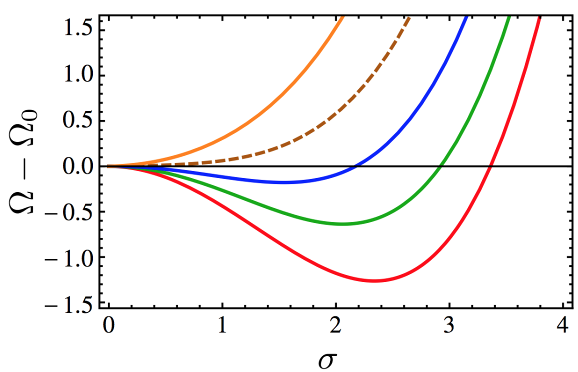
$$\left(1 + i\gamma^z\right)\psi\Big|_{z=0,L} = 0$$

A uniform condensate is assumed, σ=const.

MIT conditions break chiral symmetry

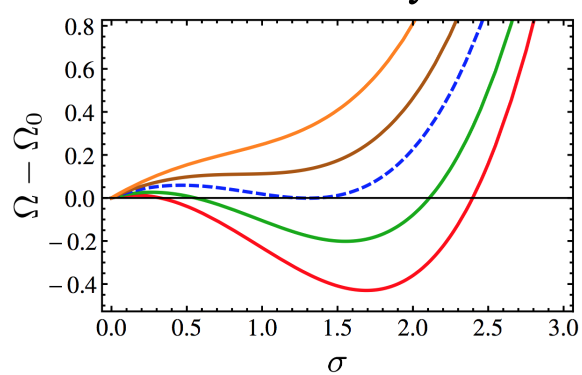
– The effective potential

Infinite volume



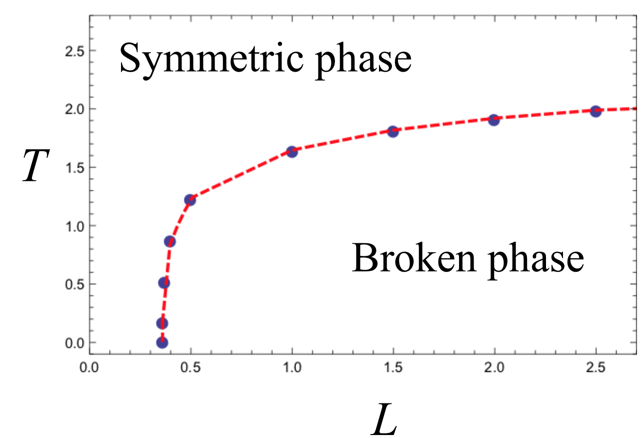
Second order phase transition

between the plates separated at the distance by R=0.6/T_c



First order phase transition

Phase diagram in L-T plane



→ As the distance between the plates gets smaller the chiral phase transition get stronger.
→ At sufficiently small inter-plate separation the broken phase disappears.

[A. Flachi, PRL 110, 060401 (2013)], further development in
[A. Flachi, K. Fukushima, V. Vitagliano, PRL 114, 181601 (2015)]

Beyond the uniform approximation: a (1+1)d model

Consider the chiral Gross–Neveu model [the Nambu–Jona-Lasinio in (1+1)d]:

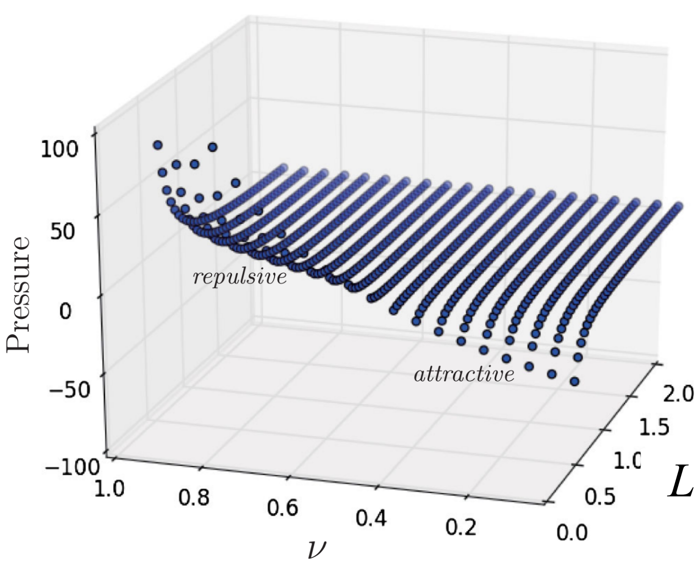
$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + \frac{g}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right]$$

at an of the length interval R at zero temperature $T = 0$.

The solution of the condensate depends on the spatial coordinate x:

$$\Delta = Ae^{-\{\zeta[\alpha\mathbf{K}(\nu)+i\theta/2]-ik\}Ax}\frac{\sigma[Ax+\alpha\mathbf{K}(\nu)+i\theta/2]}{\sigma(Ax)\sigma[\alpha\mathbf{K}(\nu)+i\theta/2]}$$

$\boldsymbol{K}(\nu)$ is the complete elliptic integral of the 1st kind, ζ and σ are the Weierstrass elliptic functions, θ, A, ν, k are constant parameters.



Interactions generate a sign flip in the Casimir force:

– The parameter ν is a monotonic function of the coupling g:

→ At weak coupling the Casimir energy is negative (attraction).

→ At strong coupling the Casimir energy is positive (repulsion).

[A. Flachi, M. Nitta, S. Takada, R. Yoshii, PRL 119, 031601 (2017)]

Conclusions

- Nonperturbative Casimir effect:
  – Finite geometry has a profound effect on phases and condensates.
  [note: apart from (1+1)d, the Casimir effect is an infinite-volume phenomenon].

- **Confinement** in zero-temperature theories:
  – toy models: CP(N-1) in (1+1)d and compact QED in (2+1)d
  – Yang-Mills theory in (2+1)d

  ► confinement and dynamical mass gap are gradually lost as one of the space dimensions shrinks
  $\longrightarrow$ similarity to the effect of finite temperature

  ► no phase transition, one common phase [note: two opinions on CP(N-1) in (1+1)d, lattice is already mobilized]
  $\longrightarrow$ difference from the effect of finite temperature (BKT–type of transition?)

  ► new mass scale in Yang-Mills theory? [the Casimir mass is three times lighter than lowest glueball]

- **Chiral symmetry breaking**:
  – in a Nambu–Jona-Lasinio–*like* model with a discrete chiral symmetry in (3+1)d at $T \neq 0$:
  ► enhancement of the chiral phase transition at small distances between plates
  ► the broken phase disappears at sufficiently close plates.

  – in the chiral Gross–Neveu model in (1+1)d:
  ► a sign flip of the Casimir force at the certain distance L is predicted;
  ► fate of phase transition is unclear at the moment.