The mass of the QCD axion

Kalman Szabo
Forschungszentrum Jülich & University of Wuppertal

Borsanyi, Fodor, Guenther, Kampert, Katz, Kawanai, Kovacs, Mages, Pasztor, Pittler, Redondo, Ringwald
Calculation of the axion mass based on high-temperature lattice QCD
Nature 539 69 (2016)

Outline

1. Introduction

axion= hypothetical elementary particle to solve the strong CP problem *and* dark matter candidate

2. Topology from lattice QCD

3. Axion strings

$\rightarrow$ prediction on m_a

Introduction

Strong CP problem

Most general $SU(3)$ symmetric Lagrangian

$$L = L_{QCD} + \theta \cdot G\widetilde{G}$$

θ could be the source of P, CP violation.

It isn't. From nEDM experiments $\rightarrow \theta < 10^{-10}$

2020 target is 10^{-11} [Roccia(PSI) Wed:1700]

Why?

A solution by Peccei-Quinn '77

Turn the parameter into
a dynamical field!

figs/thetapot/plot.gif

$$L_{QCD} + \theta \cdot G\widetilde{G} + \frac{1}{2} f_a^2 \cdot (\partial_\mu \theta)^2 + V(\theta, \partial_\mu \theta)$$

with $V(\theta, \partial_\mu \theta)$ such, that minimum stays at $\theta = 0$.

PQ: Spontaneously broken global $U(1)_{PQ}$ at scale f_a.
θ= Goldstone mode. Only derivative couplings $V = V(\partial\theta)$.

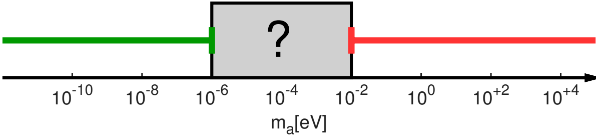
$\rightarrow$ axion pseudo-Goldstone $m_a^2 = \chi / f_a^2$ [Weinberg,Wilczek]

The axion mass window

Can't be too large → would have "seen" it, since coupling $\sim m_a$

ABRACADABRA, ADMX, ALPS, BEAST, BRASS, CAPP, CAST, CASPER, CULTASK, HAYSTAC, IAXO, LAMPOST, KLASH, MADMAX, NEWS-G, QUAX, RADES, ... [AxionWIMP '18]

Can't be too small → too much dark matter . . .



Axion is cold dark matter

[Preskill,Wilczek,Wise;Dine,Fischler;Abbott,Sikivie '83]

Potential becomes flat at QCD transition ($T_c \approx 150$MeV)

Smaller m_a gives larger Ω_a.

Assuming $\Omega_{DM} = \Omega_a(m_a) \to$ prediction for m_a. We need:

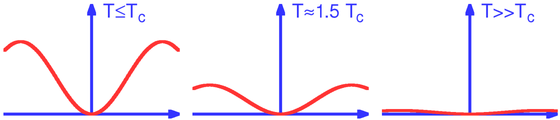
1. Axion potential $\chi(T)$

2. Solve evolution equation $\frac{d^2\theta}{dt^2} + \cdots = 0$

Axion potential from lattice QCD

Axion potential at $T > 0$

$\chi(T) = \frac{\langle Q^2 \rangle}{V} \sim$ fraction of gauge field configurations with non-trivial topology (Q)



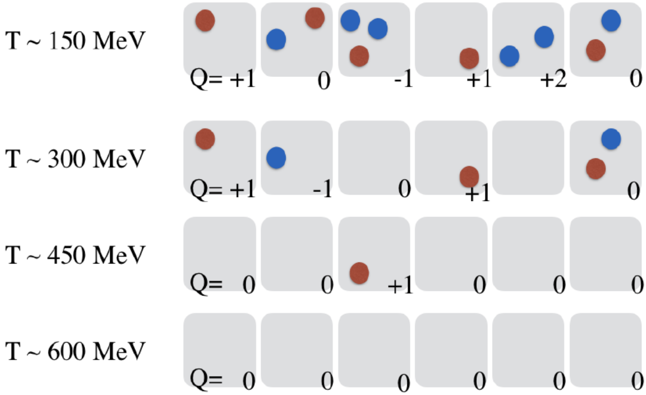
Strong suppression for high temperatures:

1. path integral weight $\exp(-S_Q/g^2)$ with $g(T) \to 0$
2. fermion zero modes $\det(D+m) \sim m^{|Q|}$

Signal is small → challenges:

large statistical error and large lattice artefacts

$\chi(T)$ from standard approach

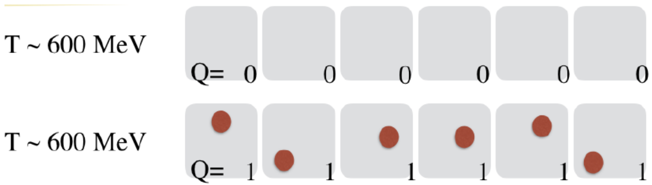


Simulate for centuries without any $Q > 0$ configurations!

$\chi(T)$ from fixed Q integral

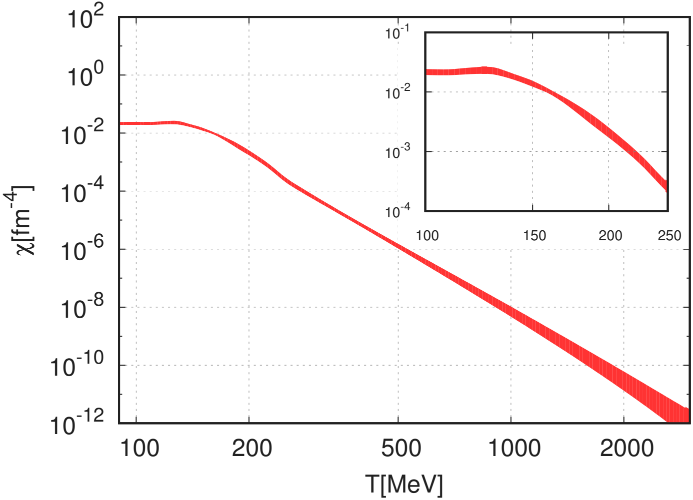
Determine slope instead of susceptibility:
see also in [Frison et al '16]

$$-\frac{d\log\chi}{d\log T} = b = 4 + \frac{d\beta}{dT}\langle S_g\rangle_{1-0} + \sum_f \frac{dm_f}{dT} m_f \langle\overline{\psi}\psi\rangle_{1-0}$$



finally perform an integral $\chi(T) = -\int d\log T\ b(T)$

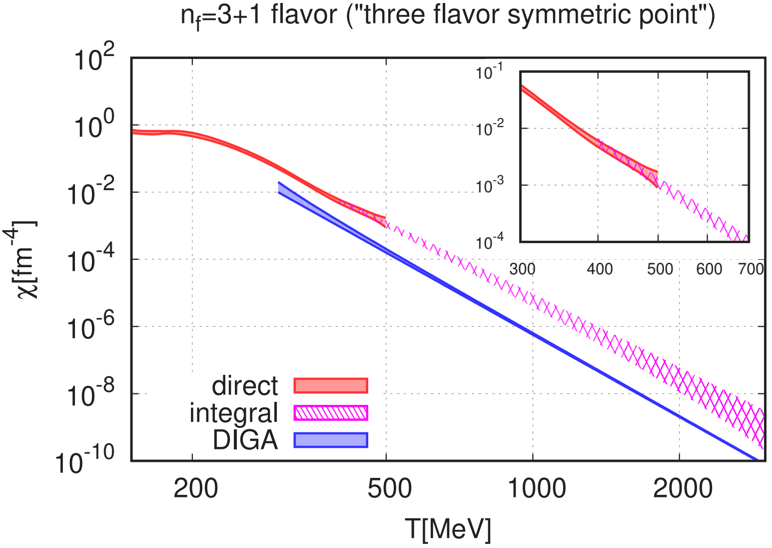
Axion potential



Exponent consistent with Dilute Instanton Gas Approximation (−8), prefactor is 5x larger.

DIGA [Gross,Pisarski,Yaffe '81]

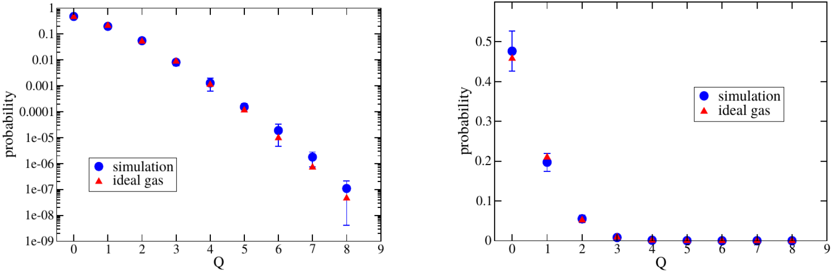
$$f(\theta) = \chi_{1loop}(T) \cdot (1 - \cos\theta)$$



Ideal gas

$f(\theta) = \chi(T) \cdot (1 - \cos\theta)$ already at T=180 MeV

physical point $L = 6.6$ fm:

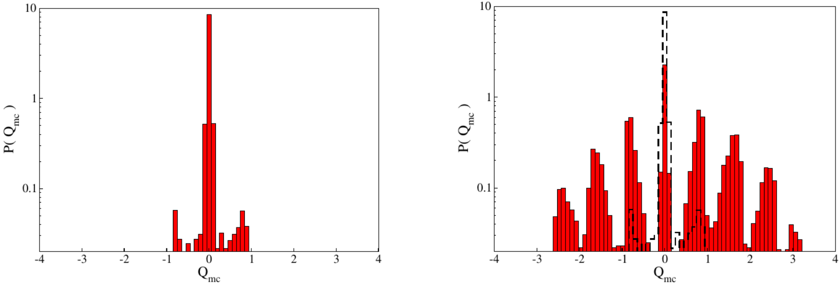


$\chi(T)$ from multicanonical

Add a bias potential $V(Q)$ to the simulation and weight it away

$$\langle O \rangle = \frac{\langle O \exp[V(Q)] \rangle_V}{\langle \exp[V(Q)] \rangle_V}$$

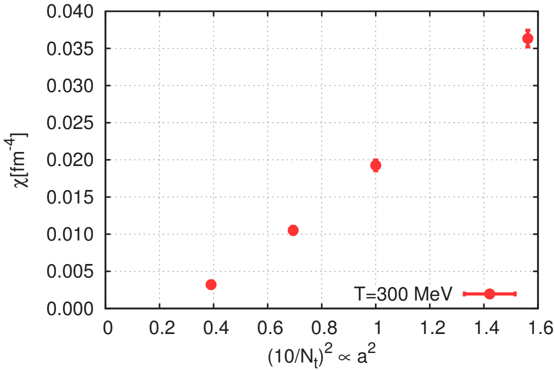
Metadynamics by [Laio et al '16; Bonati et al '18]



Multicanonical by [Jahn et al '18] in pure YM.

Lattice artefacts

Difficult continuum extrapolation



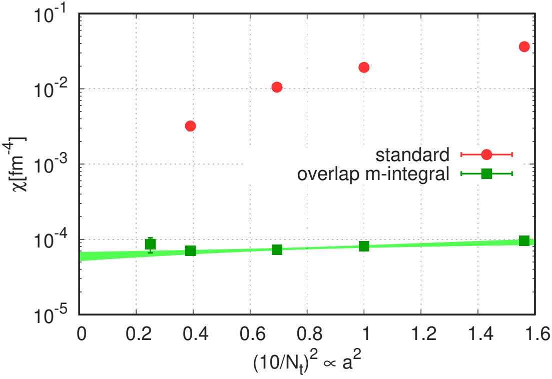
Non-chiral fermions have no exact fermion zero modes

$\det(D+m) \sim (m+\lambda_0)^{|Q|}$ with $\lambda_0 \neq 0$ on the lattice

$\rightarrow$ Too large χ, too small slope!

Continuum extrapolation

Doing full simulation with chiral fermions is too expensive.

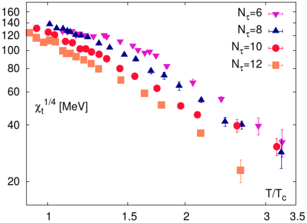


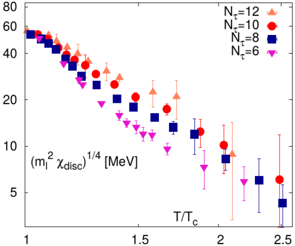
1. Simulate at large mass ($30 \cdot m_{ud}^{phys}$), continuum extrapolation behaves much better.
2. Calculate difference to m_{ud}^{phys} by integrating in m using fermion with exact chiral symmetry.

$\chi(T)$ from HISQ fermions [Petreczky et al '16]

Use two different definitions for topological charge (gluonic and fermionic).

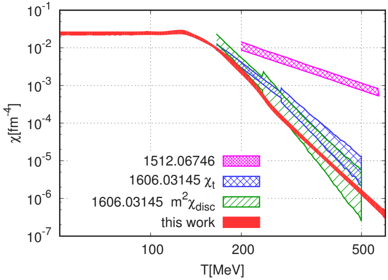
Both have sizeable discretization errors but approach the continuum limit from different directions.





Comparison to others

- [Bonati et al '15] lattice artefacts halve the fall-off exponent



- [Petreczky et al '16] "the dependence is found to be consistent with dilute instanton gas approximation"
- [Taniguchi et al '17] "a decrease in T which is consistent with the predicted $\chi(T) \propto T^{-8}$"
- [Lombardo et al '18]: "with an exponent close to the one predicted by the DIGA"
- [Bonati et al '18] "The continuum extrapolation is in agreeement with previous lattice determinations"

The simplest estimate

Assuming

1. all DM is axion $\Omega_{DM} = \Omega_a(m_a)$
2. axion field is spatially constant in very large domains
3. there are many domains with random initial value of the field (θ_0)

Evolution equations are simple to solve.

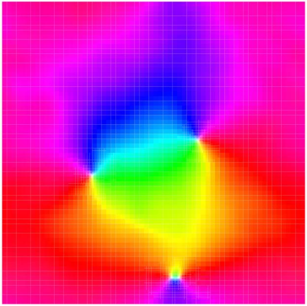
$$\to m_a = 28(1)\mu\text{eV}$$

Howto improve: take into account spatial dependence $\theta(\vec{x})$ and take θ_0 from PQ transition

Axion strings

Axion strings [Vilenkin,Everett]

θ_0 can be undefined $\equiv$ axion string.



What is their effect on axion production? Vastly different estimates.

Proper way: classical field theory simulation,

but extreme demanding: f_a, H differ by factor 10^{30}!

Heavy string simulation [Moore, Klaer '17]

Problem: coarse lattice does not resolve string core $\rightarrow$ too small string tension.

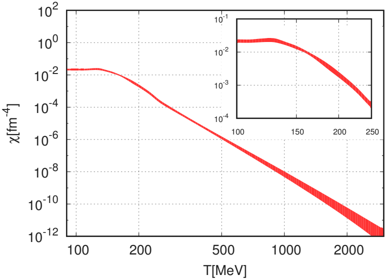
Idea: make string cores artifically heavier, while not changing long distance properties. Attach a local string to each global string.

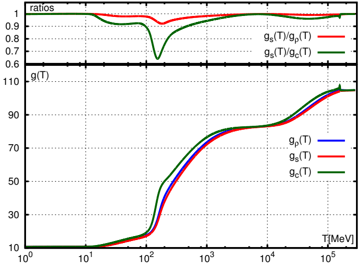
Surprise: less axions in the presence of strings.

$$\rightarrow m_a = 26.2(3.4)\mu\text{eV}$$

Summary

Lattice QCD has made a good progress in calculating the necessary inputs for axion cosmology.





Several algorithmic developments were necessary.

Still not calculated: axion potential beyond leading order b_2

Still not well understood: global string dynamics, simulations with large string tension is already possible

On good way to a solid theory prediction!

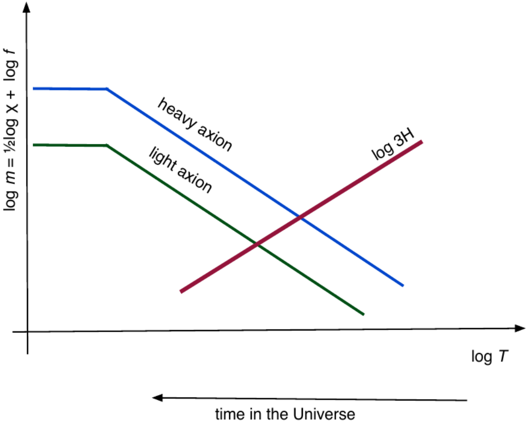
Backup

Lighter mass more axions

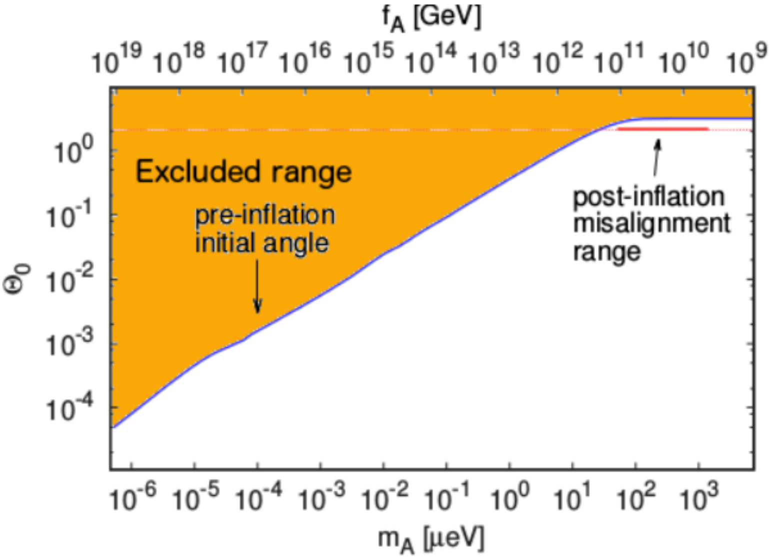
Have to solve

$$\frac{d^2\theta}{dt^2} + 3H(T)\frac{d\theta}{dt} + \frac{\chi(T)}{f_a^2}\sin\theta = 0$$

Rolling starts when $3H(T) \approx \sqrt{\chi(T)}/f_a$

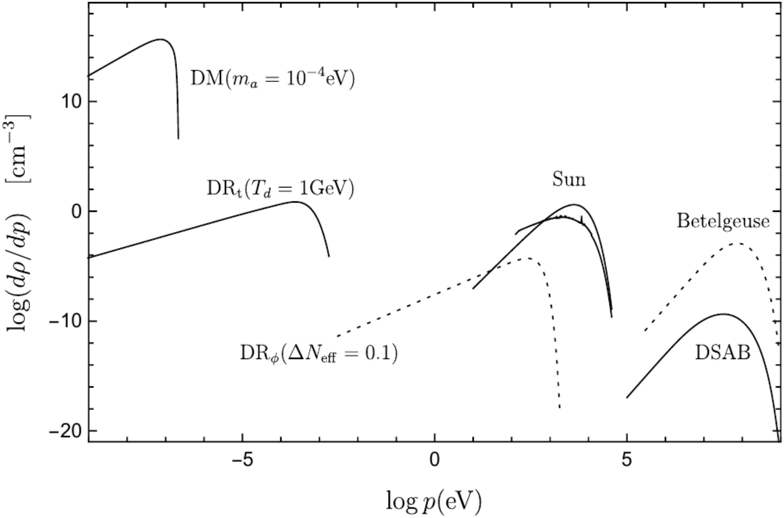


Axion mass and initial angle



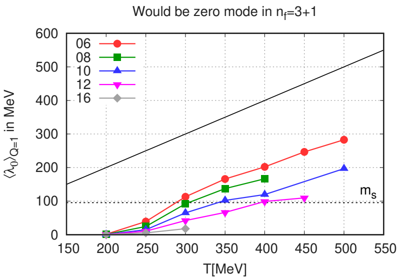
Sources of axions

[Irastorza,Redondo '18]



Continuum instanton and zero mode

Lattice instanton and zero mode

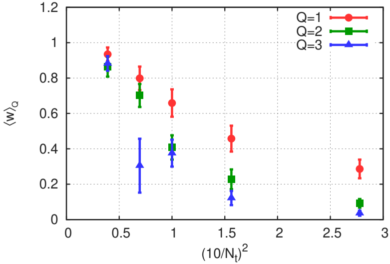


Reweighting

Problem: In continuum weight is m, on the lattice $m + \lambda_0[U]$.

Solution: change weight of configuration by $w[U] \equiv \frac{m}{m+\lambda_0[U]}$

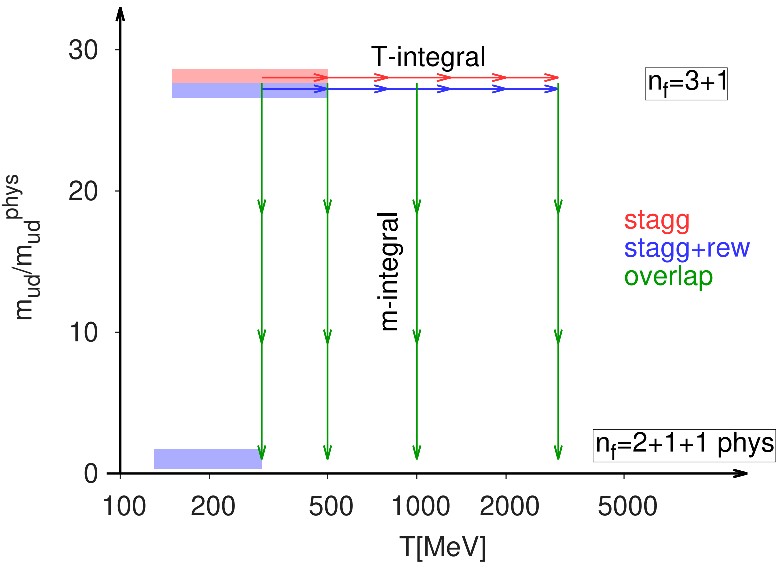
$\langle w \rangle_Q$ must approach 1 in the continuum limit.



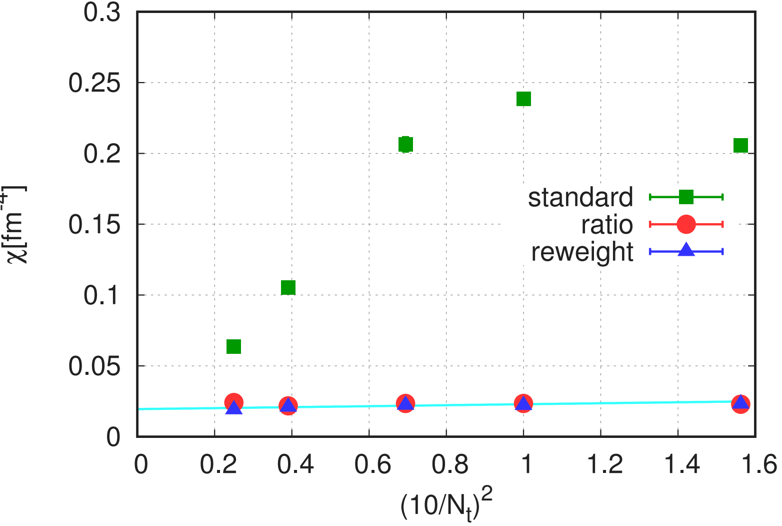
Improves the observable without changing the action.

$$\chi = \frac{\sum Q^2 \; Z_Q}{\sum Z_Q} \to \chi_{\text{rew}} = \frac{\sum Q^2 \; \langle w \rangle_Q \; Z_Q}{\sum \langle w \rangle_Q \; Z_Q}$$

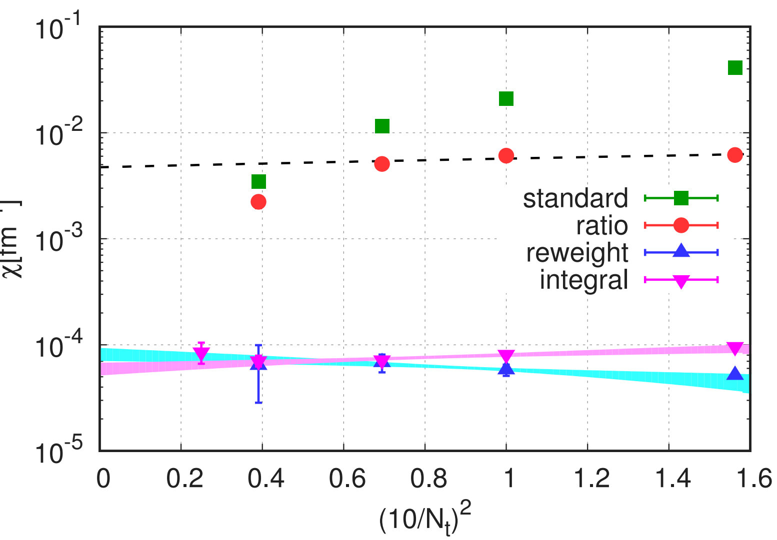
Map of simulations



Continuum extrapolation at T=150 MeV

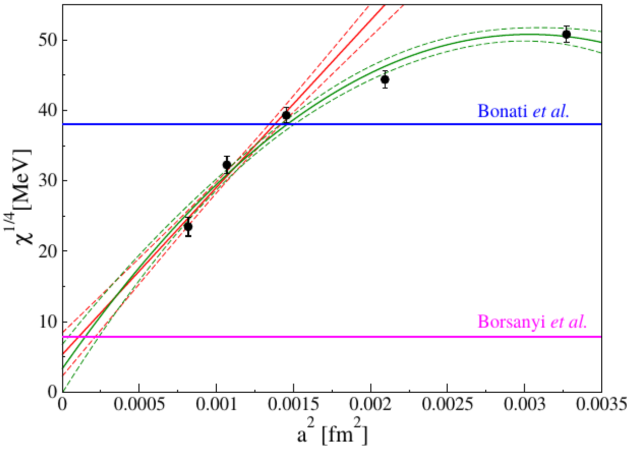


Continuum extrapolation at T=300 MeV

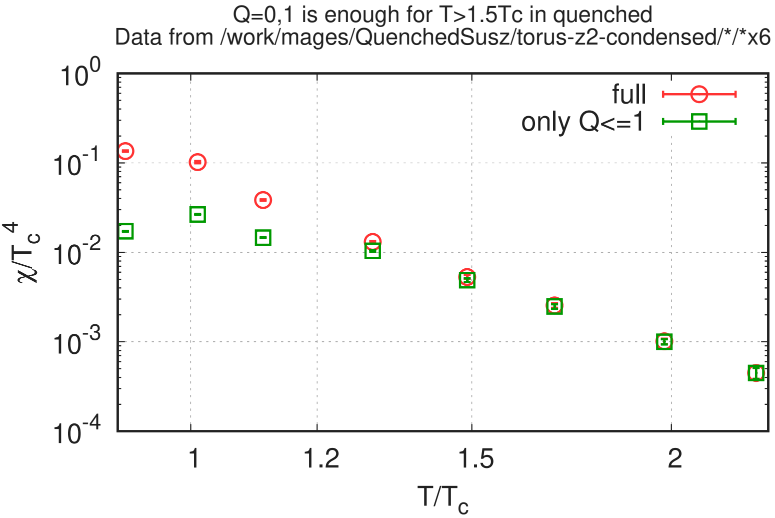


Continuum extrapolation at T=430 MeV

[Bonati et al ’18]

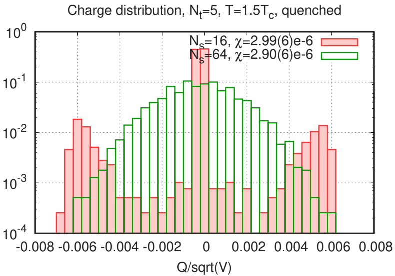


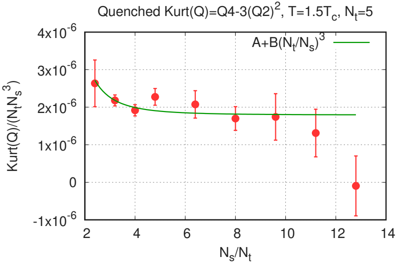
Contribution from $Q = 0, \pm 1$



Volume dependence illustration

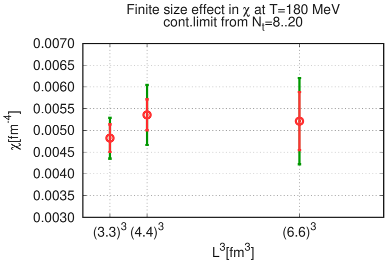
Q distribution depends (extensive quantity)



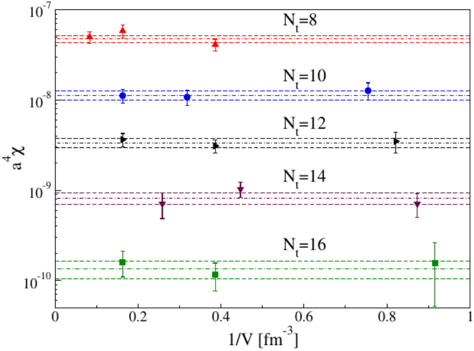


susceptibility, kurtosis (intensive quantitites) not

Volume (in)dependence at the physical point



[WB’16]



[Bonati et al ’18]