Born-Oppenheimer Approximation in an Effective Field Theory framework

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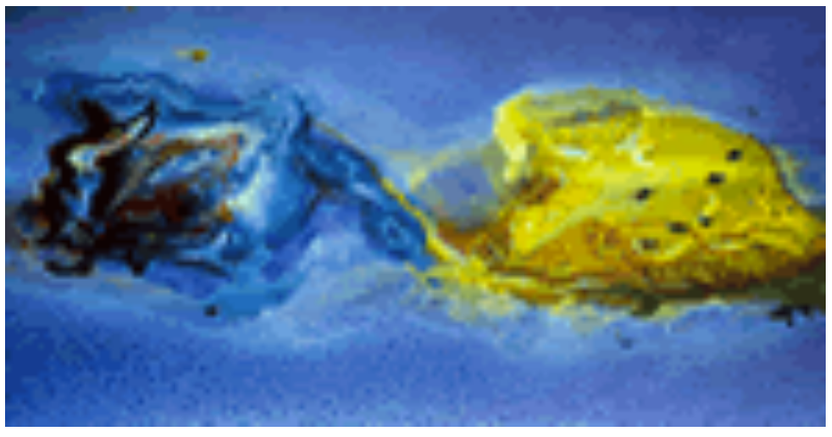
SPRACE



XIIIth Quark Confinement and the Hadron Spectrum

31 July to 6 August 2018
Maynooth University

Based on:

N. Brambilla, G.K, J. Tarrús-Castellà, A. Vairo
Phys. Rev. D 97, 016016 (2018)

XYZ mesons

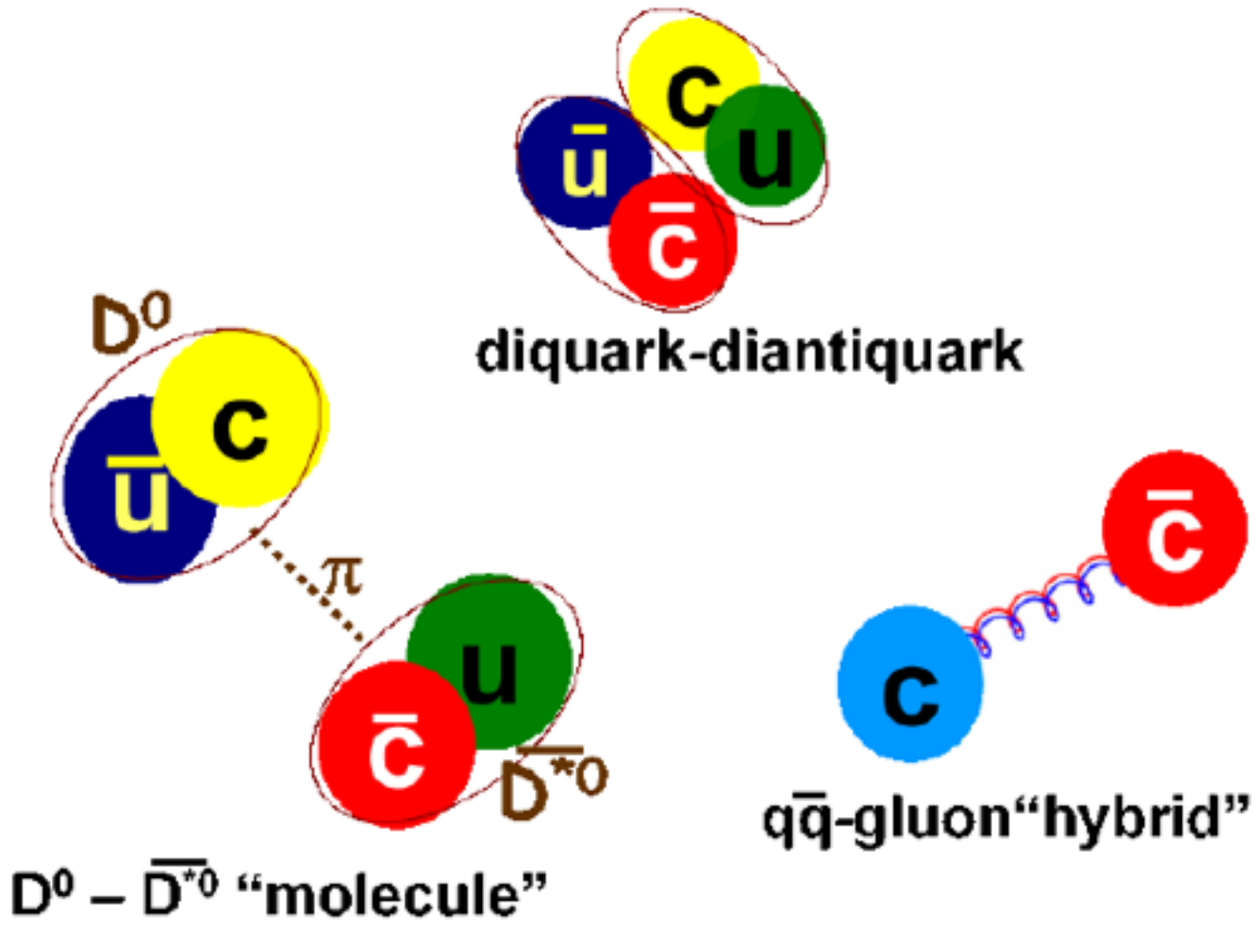


Figure 2: Cartoon representations of molecular states, diquark-diantiquark tetraquark mesons and quark-antiquark-gluon hybrids.

Picture from:
Godfrey & Olsen, Ann.Rev.Nucl.Part.Sci. 58, 51(2008)

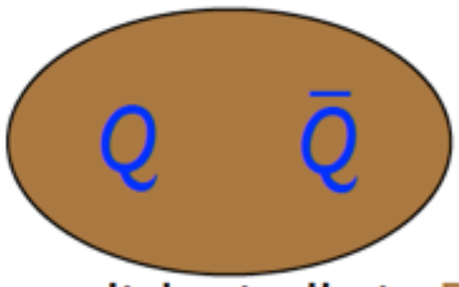
PHYSICAL REVIEW D **90,** 014044 (2014)

Born-Oppenheimer approximation for the *XYZ* mesons

Eric Braaten, Christian Langmack, and D. Hudson Smith
Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA
(Received 5 February 2014; published 24 July 2014)

Born-Oppenheimer approximation

- Born-Oppenheimer potentials $V_n(R)$
  defined by energies of gluons and light quarks
  in presence of static Q and $\bar{Q}$ sources
  separated by distance R



- Q and $\bar{Q}$ move adiabatically in B-O potential
- gluons and light quarks respond instantaneously
  to the motion of the Q and $\bar{Q}$

Braaten @ Charm 2013

VOLUME 82, NUMBER 22 PHYSICAL REVIEW LETTERS 31 MAY 1999

Ab Initio Study of Hybrid $\bar{b}gb$ Mesons

K. J. Juge
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510

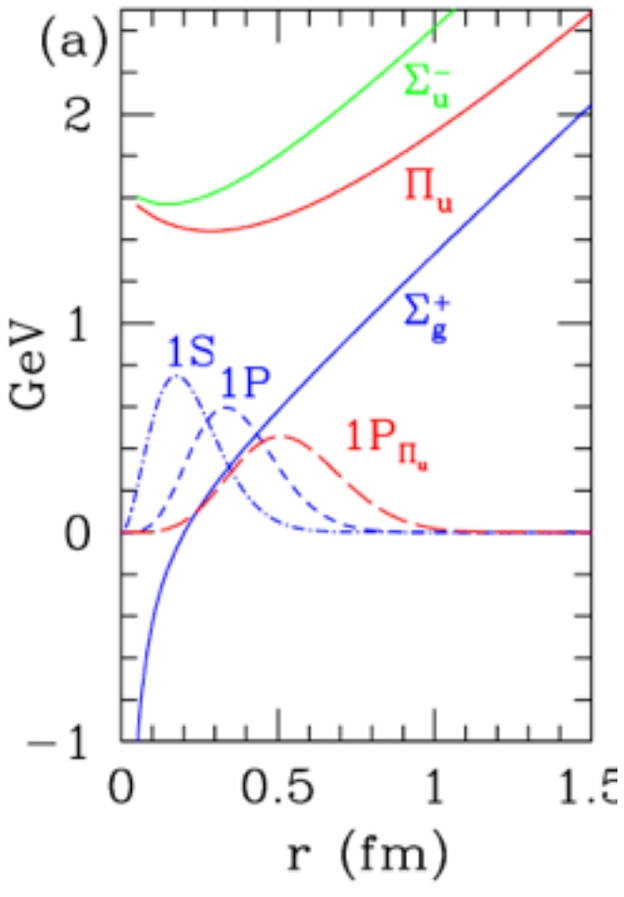
J. Kuti and C. J. Morningstar
Department of Physics, University of California at San Diego, La Jolla, California 92093-0319
(Received 16 February 1999)

Hybrid $\bar{b}gb$ molecules in which the heavy $\bar{b}b$ pair is bound together by the excited gluon field g are studied using the Born-Oppenheimer expansion and quenched numerical simulations. The consistency of results from the two approaches reveals a simple and compelling physical picture for heavy hybrid states. [S0031-9007(99)09174-7]

Born-Oppenheimer approximation for hybrids

Juge, Kuti, Morningstar 1999

solve Schroedinger equation
for Q and $\overline{Q}$ in B-O potentials
energy levels labelled by nL
radial quantum number n = 1,2,3,...
orbital angular momentum L = S,P,D,...



quarkonium hybrid multiplets
$\Pi_u(1P)$: $\{1^{--}, (0,1,2)^{-+}\}$, $\{1^{++}, (0,1,2)^{+-}\}$
$\Sigma_u^-(1S)$: $\{0^{++}, 1^{+-}\}$
$\Pi_u(2P)$: $\{1^{--}, (0,1,2)^{-+}\}$, $\{1^{++}, (0,1,2)^{+-}\}$

challenge to Lattice QCD:
calculate Born-Oppenheimer potentials with dynamical quarks

Braaten @ Charm 2013

Progress of Theoretical Physics, Vol. 82, No. 4, October 1989

Baryons with Double Charm

Sonia FLECK*) and Jean-Marc RICHARD*

Department of Physics, The University of Tokyo, Tokyo 113
**Institut des Sciences Nucléaires, Université Joseph Fourier, 38026 Grenoble*

(Received May 31, 1989)

We use potential models and several versions of the bag model to calculate the mass spectrum of baryons with two charmed quarks surrounded by an ordinary or strange quark. Such a system is conveniently described in the Born-Oppenheimer approximation, where the light quark dynamics provides an effective potential in which the two heavy quarks move non-relativistically. We survey briefly the possibilities of producing such heavy baryons and discuss their decay properties.

Table VIII. Masses of ground-state baryons with double charm, calculated either with a potential model (Eq. (25)) or with the bag model, using the parameter given in Eq. (28).

		Potential	Bag
ccq	Ξ_{cc}	3.613	3.516
	Ξ_{cc}^*	3.741	3.636
ccs	Ω_{cc}	3.703	3.657
	Ω_{cc}^*	3.835	3.805

Selex 3520 MeV
LHCb 3621 MeV

Our aim

Set up an EFT to calculate corrections to the BO approximation in a systematic and controllable manner

QED: interaction weak, can use perturbation theory

QCD: need input from lattice, large N_c, models, etc

BO EFT FOR QED MOLECULES

— Example: two heavy particles M
one light particle m

$$m << M$$

$$H_2^+$$

Why bother?

— Open problems in QED

— Techniques useful for QCD

Accurate theory of the hydrogen molecule

Krzysztof Pachucki

University of Warsaw, Poland

FFK10, Sankt Petersburg 2010

Challenges

- beyond Born-Oppenheimer: combined finite nuclear mass and (relativistic) QED corrections to rovibrational energies, shielding constant,...
- higher order QED effects $\sim \alpha^4\, Ry$
- extension of the computational technique to an arbitrary light molecule

Atomic or molecular energy

$\alpha \approx 1/137$ – fine structure constant, $\beta = m_e/\mu_n \sim 10^{-3}$ electron-nuclear mass ratio

$$
\begin{array}{ccccccc}
E(\alpha,\beta) = & E_0 & + & \alpha^2 E_{\mathrm{REL}} & + & \alpha^3 E_{\mathrm{QED}} & + \quad \alpha^4 E_{\mathrm{HQED}} + \ldots \\
& + & & + & & + & \vdots \\
& \beta E_{\mathrm{AD}} & & \alpha^2 \beta E^{\mathrm{rec}}_{\mathrm{REL}} & & \alpha^3 \beta E^{\mathrm{rec}}_{\mathrm{QED}} & \\
& + & & \vdots & & \vdots & \\
& \beta^2 E_{\mathrm{NA}} & & & & & \\
& \vdots & & & & &
\end{array}
$$

E_0 – Born-Oppenheimer energy

E_{AD} – adiabatic correction

E_{NA} – nonadiabatic correction

$\alpha^2 E_{\mathrm{REL}}$ – relatywistic correction

$\alpha^3 E_{\mathrm{QED}}$ – leading radiative (QED) correction

$\alpha^4 E_{\mathrm{HQED}}$ – higher order QED correction

Scales in Coulomb bound states

$$v/c \sim \alpha = 1/137$$

m : hard scale, QED $\longrightarrow$ NRQED

$m\alpha$: soft scale, NRQED $\longrightarrow$ pNRQED

$m\alpha^2$: ultrasoft scale, pNRQED

Interactions are weak,
matching can be done perturbatively

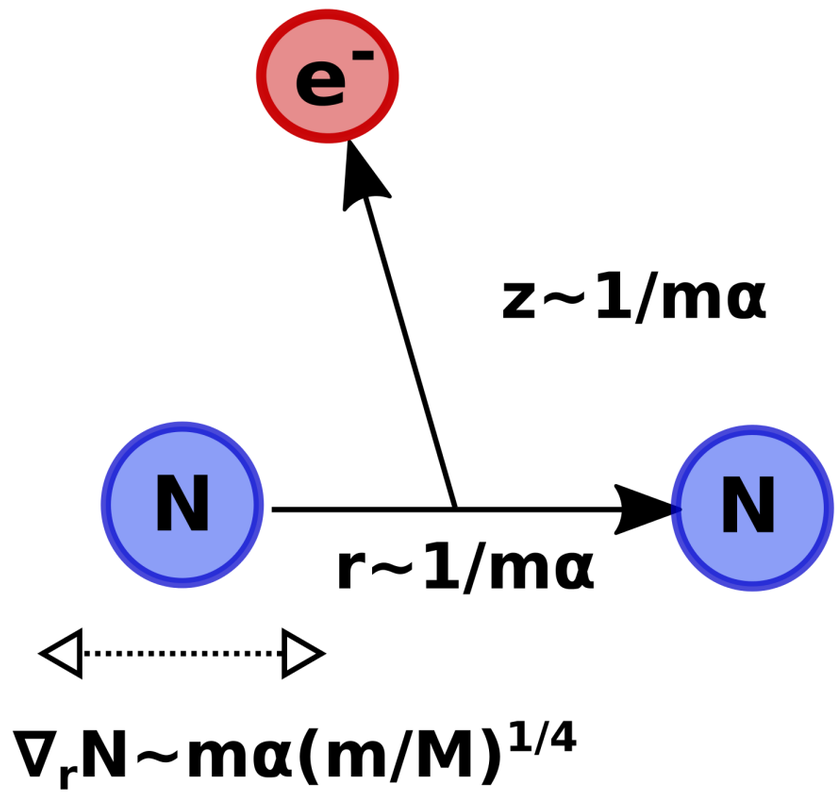
H_2^+ molecule

$$\boldsymbol{R} = \frac{m\boldsymbol{x} + M(\boldsymbol{y}_1 + \boldsymbol{y}_2)}{m + 2M}$$

$$\boldsymbol{z} = \boldsymbol{x} - \frac{\boldsymbol{y}_1 + \boldsymbol{y}_2}{2}$$

$$\boldsymbol{r} = \boldsymbol{y}_1 - \boldsymbol{y}_2$$

$E_{heavy} \sim m\alpha^2(m/M)^{1/2}$ $E_{light} \sim m\alpha^2$



$\nabla_r N \sim m\alpha(m/M)^{1/4}$

Starting point: pNRQED

Lagrangian adequate for a calculation up to $\begin{cases} \mathcal{O}(1/M) \\ \mathcal{O}(m\alpha^5) \end{cases}$

N: proton field ψ: electron field

$$
\begin{aligned}
L_{\mathrm{pNRQED}} = & -\frac{1}{4}\int d^3x F_{\mu\nu}(x)F^{\mu\nu}(x) \\
& + \int d^3x \{\psi^\dagger(t,\boldsymbol{x})[i\partial_t - h_e(t,\boldsymbol{x})]\psi(t,\boldsymbol{x}) \\
& + N^\dagger(t,\boldsymbol{x})[i\partial_t - h_{Ze}(t,\boldsymbol{x})]N(t,\boldsymbol{x})\} \\
& - \int d^3x d^3y [\psi^\dagger(t,\boldsymbol{x})\psi(t,\boldsymbol{x})V_{Ze}(\boldsymbol{x}-\boldsymbol{y},\boldsymbol{\sigma}) \\
& + N^\dagger(t,\boldsymbol{x})N(t,\boldsymbol{x})V_{ZZ}(\boldsymbol{x}-\boldsymbol{y})]N^\dagger(t,\boldsymbol{y})N(t,\boldsymbol{y})
\end{aligned}
$$

A. Pineda and J. Soto, Nucl. Phys. B, Proc. Suppl. 64, 428 (1998)
N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nucl. Phys. B 566, 275 (2000)

Kinetic terms

$$h_e(t,\boldsymbol{x}) = eA_0(t,\boldsymbol{x}) - \frac{\boldsymbol{D}^2_{-e}(t,\boldsymbol{x})}{2m} - \frac{\boldsymbol{D}^4_{-e}(t,\boldsymbol{x})}{8m^3}$$

$$h_{Ze}(t,\boldsymbol{x}) = -ZeA_0(t,\boldsymbol{x}) - \frac{\boldsymbol{D}^2_{Ze}(t,\boldsymbol{x})}{2M}$$

$$i\boldsymbol{D}_q(t,\boldsymbol{x}) = i\boldsymbol{\nabla}_x - q\boldsymbol{A}(t,\boldsymbol{x})$$

$$c_D = 1 + \frac{\alpha}{\pi}\left(\frac{8}{3}\log\frac{m}{\mu}\right)$$

$$c_S = 1 + \frac{\alpha}{\pi}$$

$$d_2 = \frac{\alpha}{60\pi}$$

Potential terms

$$V^{\mathrm{LO}}_{Ze}(\mathbf{x},\sigma) = V^{\mathrm{C}}_{Ze}(\mathbf{x},\sigma) = -\frac{Z\alpha}{|\mathbf{x}|}$$

$$V^{\mathrm{LO}}_{ZZ}(\mathbf{x}) = \frac{Z^2\alpha}{|\mathbf{x}|}$$

$$V^{\mathrm{NLO}}_{Ze}(\mathbf{x},\sigma) = V^{\mathrm{ct}}_{Ze}(\mathbf{x}) + V^{\mathrm{SO}}_{Ze}(\mathbf{x},\sigma)$$

$$\begin{cases} V^{ct}_{Ze}(\mathbf{x}) = \dfrac{Z\alpha}{m^2}\left(-\dfrac{c_D}{8} + 4d_2\right)4\pi\delta(\mathbf{x}) \\ V^{\mathrm{SO}}_{Ze}(\mathbf{x}) = ic_S\dfrac{Z\alpha}{4m^2}\boldsymbol{\sigma}_e\cdot\left(\dfrac{\mathbf{x}}{|\mathbf{x}|^3}\times\nabla_x\right) \end{cases}$$

BO EFT

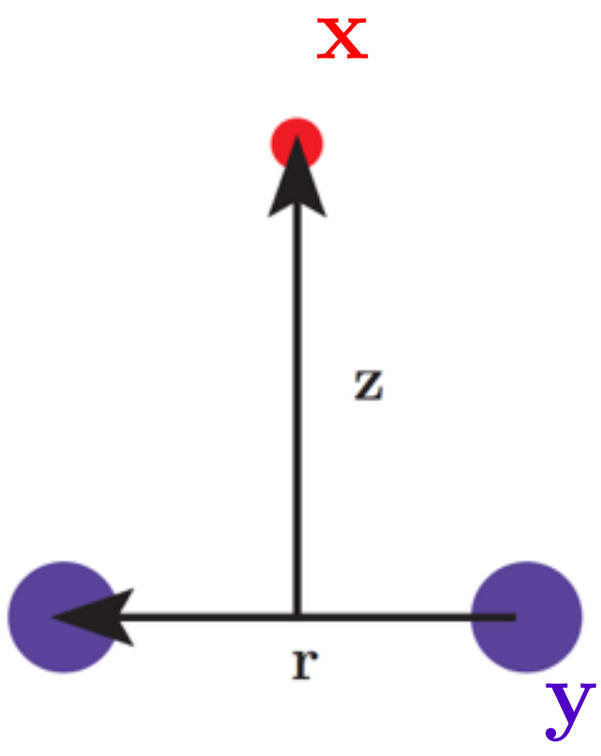
— integrate out the light particle (electron)

1) Projection on subspace of

2 heavy + 1 light + ultrasoft photons

$$|\varphi(t)\rangle = \int d^3x d^3y_1 d^3y_2 \varphi(t, \boldsymbol{x}, \boldsymbol{y}_1, \boldsymbol{y}_2) \times \psi^\dagger(t, \boldsymbol{x}) N^\dagger(t, \boldsymbol{y}_1) N^\dagger(t, \boldsymbol{y}_2) |\mathrm{US}\rangle$$

2) Multipole-expand (ultrasoft) photon fields



$$\mathbf{R} = \frac{m\mathbf{x} + M(\mathbf{y}_1 + \mathbf{y}_2)}{m + 2M}$$

$$\mathbf{z} = \mathbf{x} - \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}$$

$$\mathbf{r} = \mathbf{y}_1 - \mathbf{y}_2$$

$$\begin{aligned}
A_\mu(t, \mathbf{x}) &\simeq A_\mu(t, \mathbf{R}) + \frac{2M}{\mathcal{M}}\, \mathbf{z} \cdot \nabla_R A_\mu(t, \mathbf{R}) \\
A_\mu(t, \mathbf{y}_1) &\simeq A_\mu(t, \mathbf{R}) + \left(\mathbf{r} - \frac{m}{\mathcal{M}}\, \mathbf{z}\right) \cdot \nabla_R A_\mu(t, \mathbf{R}) \\
A_\mu(t, \mathbf{y}_2) &\simeq A_\mu(t, \mathbf{R}) + \left(-\mathbf{r} - \frac{m}{\mathcal{M}}\, \mathbf{z}\right) \cdot \nabla_R A_\mu(t, \mathbf{R})
\end{aligned}$$

$$\mathcal{M} = m + 2M$$

3) To have manifest (ultrasoft) gauge invariance

$$\varphi(\boldsymbol{x},\boldsymbol{y}_1,\boldsymbol{y}_2,t) = U_{-e}(\boldsymbol{x},\boldsymbol{R},t)U_{+Ze}(\boldsymbol{y}_1,\boldsymbol{R},t) \times U_{+Ze}(\boldsymbol{y}_2,\boldsymbol{R},t)S(t,\boldsymbol{x},\boldsymbol{y}_1,\boldsymbol{y}_2,t)$$

$$U_q(\boldsymbol{x},\boldsymbol{R},t) = e^{iq\int_{\boldsymbol{x}}^{\boldsymbol{R}} d\boldsymbol{x}'\cdot\boldsymbol{A}(\boldsymbol{x}',t)}$$

Gauge transformations of S:

$$S(t,\boldsymbol{R},\boldsymbol{r},z) \to e^{-ie_{\text{tot}}\theta(t,\boldsymbol{R})}S(t,\boldsymbol{R},\boldsymbol{r},z)$$

$$A_0(t,\boldsymbol{R}) \to A_0(t,\boldsymbol{R}) - \partial_t\theta(t,\boldsymbol{R}) \qquad e_{\text{tot}} = -e(1-2Z)$$

$$\boldsymbol{A}(t,\boldsymbol{R}) \to \boldsymbol{A}(t,\boldsymbol{R}) + \boldsymbol{\nabla}_R\theta(t,\boldsymbol{R})$$

4) The effective Lagrangian becomes:

$$L_{\mathrm{pNRQED}} = -\frac{1}{4}\int d^3x F_{\mu\nu}(x)F^{\mu\nu}(x)$$
$$+\int d^3R d^3r d^3z S^\dagger(t,\boldsymbol{R},\boldsymbol{r},\boldsymbol{z})\left[i\partial_t + \frac{\boldsymbol{\nabla}_R^2}{2M_{\mathrm{tot}}} + e_{\mathrm{tot}}A_0(t,\boldsymbol{R}) + e_{\mathrm{eff}}\boldsymbol{z}\cdot\boldsymbol{E}(t,\boldsymbol{R})\right.$$
$$-h_0(\boldsymbol{r},\boldsymbol{z}) + \frac{\boldsymbol{\nabla}_r^2}{M} - V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + \frac{\boldsymbol{\nabla}_z^2}{4M} + \frac{\boldsymbol{\nabla}_z^4}{8m^3}$$
$$\left. - V_{Ze}^{\mathrm{NLO}}(\boldsymbol{z}+\boldsymbol{r}/2,\boldsymbol{\sigma}) - V_{Ze}^{\mathrm{NLO}}(\boldsymbol{z}-\boldsymbol{r}/2,\boldsymbol{\sigma})\right] S(t,\boldsymbol{R},\boldsymbol{r},\boldsymbol{z})$$

$$\boldsymbol{E}(t,\boldsymbol{R}) = -\partial_t \boldsymbol{A}(t,\boldsymbol{R}) - \boldsymbol{\nabla}_R A_0(t,\boldsymbol{R})$$

$$h_0(\boldsymbol{r},\boldsymbol{z}) = -\frac{\boldsymbol{\nabla}_z^2}{2m} + V_{Ze}^{\mathrm{LO}}(\boldsymbol{z}+\boldsymbol{r}/2) + V_{Ze}^{\mathrm{LO}}(\boldsymbol{z}-\boldsymbol{r}/2)$$

$$M_{\mathrm{tot}} = m + 2M$$

$$e_{\mathrm{tot}} = -e(1-2Z)$$

$$e_{\mathrm{eff}} = 2e\frac{M+Zm}{m+2M} = e + \mathcal{O}(\alpha^2)$$

5) Integrate out light particle (electron)

$$S(t,\mathbf{r},\mathbf{z}) = \sum_{\kappa} \Psi_{\kappa}(t,\mathbf{r})\,\phi_{\kappa}(\mathbf{r},\mathbf{z})$$

where

$$\hat{h}_0(\mathbf{r},\mathbf{z})\phi_{\kappa}(\mathbf{r},\mathbf{z}) = V_{\kappa}^{\mathrm{light}}(\mathbf{r})\,\phi_{\kappa}(\mathbf{r},\mathbf{z})$$

$$\int d^3\mathbf{z}\,\phi^*_{\kappa'}(\mathbf{r},\mathbf{z})\phi_{\kappa}(\mathbf{r},\mathbf{z}) = \delta_{\kappa'\kappa}$$

Example: $\kappa = {}^1\Sigma_g^+$

Ang. mom.	$\Sigma, \Pi, \Delta, \cdots$
Spin	${}^{2S+1}\Sigma$
Parity	$g = +1 \;\; u = -1$

$V_{\kappa}^{\mathrm{light}}(\mathbf{r})$: eigenvalue, light-particle energy,
(adiabatic)potential for the heavy particles

Born-Oppenheimer APPROXIMATION

— is truncation of the sum in the expansion

$$S(t, \mathbf{r}, \mathbf{z}) = \sum_{\kappa} \Psi_{\kappa}(t, \mathbf{r})\, \phi_{\kappa}(\mathbf{r}, \mathbf{z})$$

to a single κ

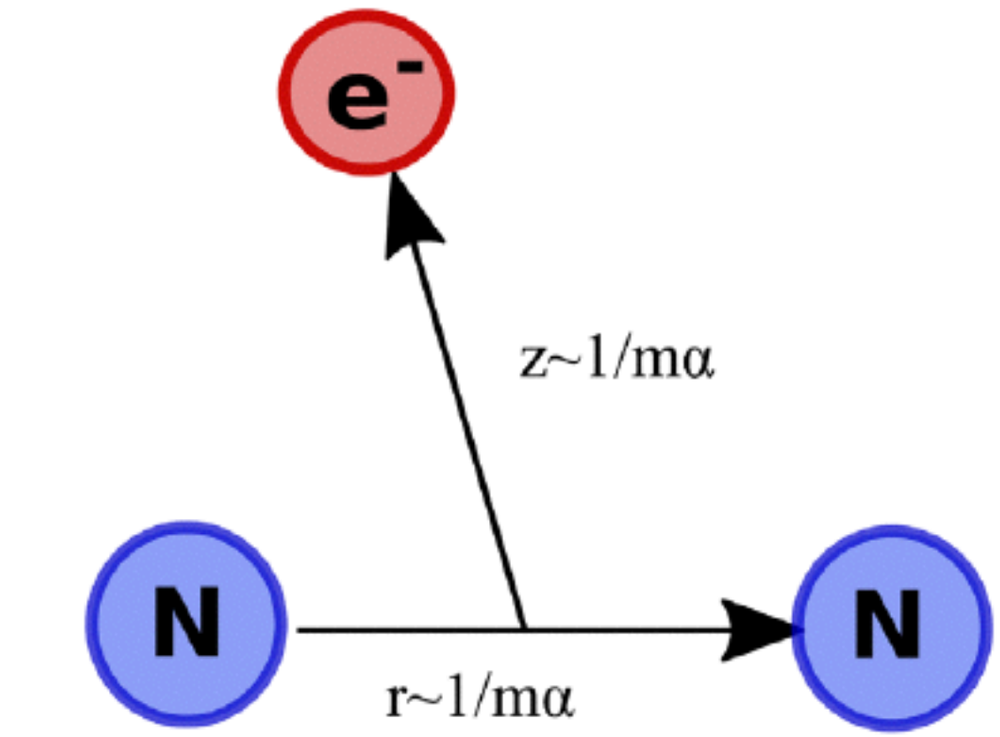
6) Lagrangian for the heavy particles, no truncation

Replace in the projected pNRQCD Lagrangian

$$S(t,\mathbf{r},\mathbf{z}) = \sum_{\kappa} \Psi_{\kappa}(t,\mathbf{r})\,\phi_{\kappa}(\mathbf{r},\mathbf{z})$$

$$\begin{aligned} L_{\mathrm{BOEFT}} = & -\frac{1}{4}\int d^3x F_{\mu\nu}(x)F^{\mu\nu}(x) \\ & + \int d^3r \sum_{\kappa\kappa'} \Psi^{\dagger}_{\kappa}(t,\boldsymbol{r})\{[i\partial_t + e_{\mathrm{tot}}A_0(t,\mathbf{0}) \\ & - H^{(0)}_{\kappa}(\boldsymbol{r}) - \delta E_{\kappa}(\boldsymbol{r})]\delta_{\kappa\kappa'} - C^{\mathrm{nad}}_{\kappa\kappa'}(\boldsymbol{r})\}\Psi_{\kappa'}(t,\boldsymbol{r}) \end{aligned}$$

a sum over pieces of well-defined sizes:



$$\frac{m}{M} \sim \alpha^{3/2}$$

$$\nabla_r \Psi \sim m\alpha \left(\frac{m}{M}\right)^{1/4} \sim m\alpha^{5/8}$$

$$\nabla_r \phi \sim m\alpha$$

$$H_{\kappa}^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_{\kappa}^{\mathrm{light}}(\boldsymbol{r})$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

$$\delta E_\kappa(\boldsymbol{r}) = \delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{rec},2} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{NLO}} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{US}} E_\kappa(\boldsymbol{r})$$

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$$\delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) = \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \left(-\frac{\boldsymbol{\nabla}_z^2}{4M}\right) \phi_\kappa(\boldsymbol{r};\boldsymbol{z})$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

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$$\sim (m\alpha)^2 \frac{1}{M} = m\alpha^2 \frac{m}{M} \sim m\alpha^{7/2}$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

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$$\begin{aligned}\delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) &= \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \left(-\frac{\boldsymbol{\nabla}_z^2}{4M}\right) \phi_\kappa(\boldsymbol{r};\boldsymbol{z}) \\ &= \langle \boldsymbol{r}, \kappa | (-\boldsymbol{\nabla}_z^2)/(4M) | \boldsymbol{r}, \kappa \rangle\end{aligned} \qquad \sim (m\alpha)^2 \frac{1}{M} = m\alpha^2 \frac{m}{M} \sim m\alpha^{7/2}$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\rm LO}(\boldsymbol{r}) + V_\kappa^{\rm light}(\boldsymbol{r}) \qquad \sim m\alpha^2\left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

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$$\delta^{\rm NLO} E_\kappa(\boldsymbol{r}) = \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z})\left[V_{Ze}^{\rm NLO}(\boldsymbol{z}+\boldsymbol{r}/2,\sigma) + V_{Ze}^{\rm NLO}(\boldsymbol{z}-\boldsymbol{r}/2,\sigma) - \frac{\nabla_z^4}{8m^3}\right]\phi_\kappa(\boldsymbol{r};\boldsymbol{z})$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\rm LO}(\boldsymbol{r}) + V_\kappa^{\rm light}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

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$$\begin{aligned}\delta^{\rm NLO} E_\kappa(\boldsymbol{r}) = \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \Big[& V_{Ze}^{\rm NLO}(\boldsymbol{z} + \boldsymbol{r}/2, \sigma) \\ & + V_{Ze}^{\rm NLO}(\boldsymbol{z} - \boldsymbol{r}/2, \sigma) - \frac{\nabla_z^4}{8m^3} \Big] \phi_\kappa(\boldsymbol{r};\boldsymbol{z}) \end{aligned} \qquad \sim m\alpha^4$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

$$\delta E_\kappa(\boldsymbol{r}) = \delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{rec},2} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{NLO}} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{US}} E_\kappa(\boldsymbol{r})$$

$$\begin{aligned}\delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) &= \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \left(-\frac{\boldsymbol{\nabla}_z^2}{4M}\right) \phi_\kappa(\boldsymbol{r};\boldsymbol{z}) \\ &= \langle \boldsymbol{r}, \kappa | (-\boldsymbol{\nabla}_z^2)/(4M) | \boldsymbol{r}, \kappa \rangle\end{aligned} \qquad \sim (m\alpha)^2 \frac{1}{M} = m\alpha^2 \frac{m}{M} \sim m\alpha^{7/2}$$

$$\begin{aligned}\delta^{\mathrm{NLO}} E_\kappa(\boldsymbol{r}) = \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \Big[& V_{Ze}^{\mathrm{NLO}}(\boldsymbol{z} + \boldsymbol{r}/2, \sigma) \\ & + V_{Ze}^{\mathrm{NLO}}(\boldsymbol{z} - \boldsymbol{r}/2, \sigma) - \frac{\nabla_z^4}{8m^3} \Big] \phi_\kappa(\boldsymbol{r};\boldsymbol{z})\end{aligned} \qquad \sim m\alpha^4$$

$$\delta^{\mathrm{rec},2} E_\kappa(\boldsymbol{r}) = \sum_{\bar{\kappa} \neq \kappa} \frac{|\langle \boldsymbol{r}, \kappa | (-\boldsymbol{\nabla}_z^2)/(4M) | \boldsymbol{r}, \bar{\kappa} \rangle|^2}{V_\kappa^{\mathrm{light}}(\boldsymbol{r}) - V_{\bar{\kappa}}^{\mathrm{light}}(\boldsymbol{r})}$$

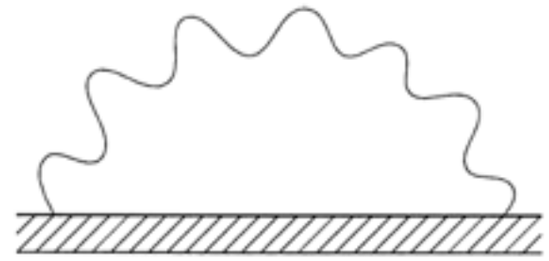
$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r}) \qquad \sim m\alpha^2 \left(\frac{m}{M}\right)^{1/2} \sim m\alpha^{11/4}$$

$$\delta E_\kappa(\boldsymbol{r}) = \delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{rec},2} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{NLO}} E_\kappa(\boldsymbol{r}) + \delta^{\mathrm{US}} E_\kappa(\boldsymbol{r})$$

$$\begin{aligned}\delta^{\mathrm{rec}} E_\kappa(\boldsymbol{r}) &= \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \left(-\frac{\boldsymbol{\nabla}_z^2}{4M}\right) \phi_\kappa(\boldsymbol{r};\boldsymbol{z}) \\ &= \langle \boldsymbol{r}, \kappa | (-\boldsymbol{\nabla}_z^2)/(4M) | \boldsymbol{r}, \kappa \rangle\end{aligned} \qquad \sim (m\alpha)^2 \frac{1}{M} = m\alpha^2 \frac{m}{M} \sim m\alpha^{7/2}$$

$$\begin{aligned}\delta^{\mathrm{NLO}} E_\kappa(\boldsymbol{r}) = \int d^3z \phi_\kappa^*(\boldsymbol{r};\boldsymbol{z}) \Big[& V_{Ze}^{\mathrm{NLO}}(\boldsymbol{z} + \boldsymbol{r}/2, \sigma) \\ & + V_{Ze}^{\mathrm{NLO}}(\boldsymbol{z} - \boldsymbol{r}/2, \sigma) - \frac{\nabla_z^4}{8m^3} \Big] \phi_\kappa(\boldsymbol{r};\boldsymbol{z})\end{aligned} \qquad \sim m\alpha^4$$

$$\delta^{\mathrm{rec},2} E_\kappa(\boldsymbol{r}) = \sum_{\bar{\kappa} \neq \kappa} \frac{|\langle \boldsymbol{r}, \kappa | (-\boldsymbol{\nabla}_z^2)/(4M) | \boldsymbol{r}, \bar{\kappa} \rangle|^2}{V_\kappa^{\mathrm{light}}(\boldsymbol{r}) - V_{\bar{\kappa}}^{\mathrm{light}}(\boldsymbol{r})} \qquad \sim (m\alpha^{7/2})^2 \frac{1}{m\alpha^2} = m\alpha^5$$



$$\delta^{\mathrm{US}} E_\kappa(\boldsymbol{r}) = -\frac{e^2}{6\pi^2}\left\{-\frac{Ze^2}{2m^2}\left[\log\left(\frac{\mu}{m}\right)+\frac{5}{6}-\log(2)\right]\rho_\kappa(\boldsymbol{r})\right.$$
$$\left.+\sum_{\bar\kappa\neq\kappa}|\langle \boldsymbol{r},\kappa|\boldsymbol{v}_z|\boldsymbol{r},\bar\kappa\rangle|^2(V_\kappa^{\mathrm{light}}(\boldsymbol{r})-V_{\bar\kappa}^{\mathrm{light}}(\boldsymbol{r}))\log\left(\frac{m}{|V_\kappa^{\mathrm{light}}(\boldsymbol{r})-V_{\bar\kappa}^{\mathrm{light}}(\boldsymbol{r})|}\right)\right\}$$

$$\langle \boldsymbol{r},\kappa|\boldsymbol{v}_z|\boldsymbol{r},\bar\kappa\rangle = \int d^3z\,\phi_\kappa^*(\boldsymbol{r};\boldsymbol{z})\boldsymbol{v}_z\phi_{\bar\kappa}(\boldsymbol{r};\boldsymbol{z}) \qquad \boldsymbol{v}_z = -i\boldsymbol{\nabla}_z/m$$

$$\rho_\kappa(\boldsymbol{r}) = |\phi_\kappa(\boldsymbol{r};\boldsymbol{z}=\boldsymbol{r}/2)|^2 + |\phi_\kappa(\boldsymbol{r};\boldsymbol{z}=-\boldsymbol{r}/2)|^2$$

$$\delta^{\mathrm{US}} E_\kappa(\boldsymbol{r}) \sim m\alpha^5\log\alpha \quad \text{and} \quad m\alpha^5$$

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PHYSICS LETTERS B

The Lamb shift in dimensional regularisation

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Hydrogen atom

$$\delta^{US} E_n = \frac{2}{3}\frac{\alpha}{\pi}\left(\left(\log\frac{\mu}{m}+\frac{5}{6}-\log 2\right)\left(\frac{Ze^2}{2}\right)\frac{|\phi_n(\mathbf{0})|^2}{m^2}\right.$$

$$\left. -\sum_{m\neq n}|\langle n|\boldsymbol{v}|m\rangle|^2\,(E_n-E_m)\log\frac{m}{|E_n-E_m|}\right)$$

Corrections to adiabatic motion:

$$C^{\mathrm{nad}}_{\kappa\kappa'}(\boldsymbol{r}) = \int d^3z \phi^*_\kappa(\boldsymbol{r};\boldsymbol{z})[-\boldsymbol{\nabla}^2_r/M, \phi_{\kappa'}(\boldsymbol{r};\boldsymbol{z})]$$

$$= \int d^3z \phi^*_\kappa(\boldsymbol{r};\boldsymbol{z}) \left(-\frac{\boldsymbol{\nabla}^2_r}{M} \phi_{\kappa'}(\boldsymbol{r};\boldsymbol{z}) \right) \sim (m\alpha)^2 \frac{1}{M} = m\alpha^2 \frac{m}{M} \sim m\alpha^{7/2}$$

$$+ \frac{2}{M} \int d^3z \phi^*_\kappa(\boldsymbol{r};\boldsymbol{z})(-i\boldsymbol{\nabla}_r \phi_{\kappa'}(\boldsymbol{r};\boldsymbol{z}))(-i\boldsymbol{\nabla}_r)$$

$$\sim \frac{1}{M} m\alpha \left(\frac{M}{m}\right)^{1/4} m\alpha = m\alpha^2 \left(\frac{m}{M}\right)^{3/4} \sim m\alpha^{25/8}$$

BO approximation

— neglect the non adiabatic term

$$i\partial_t \Psi_\kappa(t,\boldsymbol{r}) = H_\kappa^{(0)}(\boldsymbol{r})\Psi_\kappa(t,\boldsymbol{r})$$

$$H_\kappa^{(0)}(\boldsymbol{r}) = -\frac{\boldsymbol{\nabla}_r^2}{M} + V_{ZZ}^{\mathrm{LO}}(\boldsymbol{r}) + V_\kappa^{\mathrm{light}}(\boldsymbol{r})$$

Adiabatic approximation

— include diagonal part the nonadiabatic term

$$i\partial_t \Psi_\kappa(t,\boldsymbol{r}) = [H_\kappa^{(0)}(\boldsymbol{r}) + C_{\kappa\kappa}^{\mathrm{nad}}(\boldsymbol{r})]\Psi_\kappa(t,\boldsymbol{r})$$

Nondiagonal terms?

— perturbation theory

Hard problem

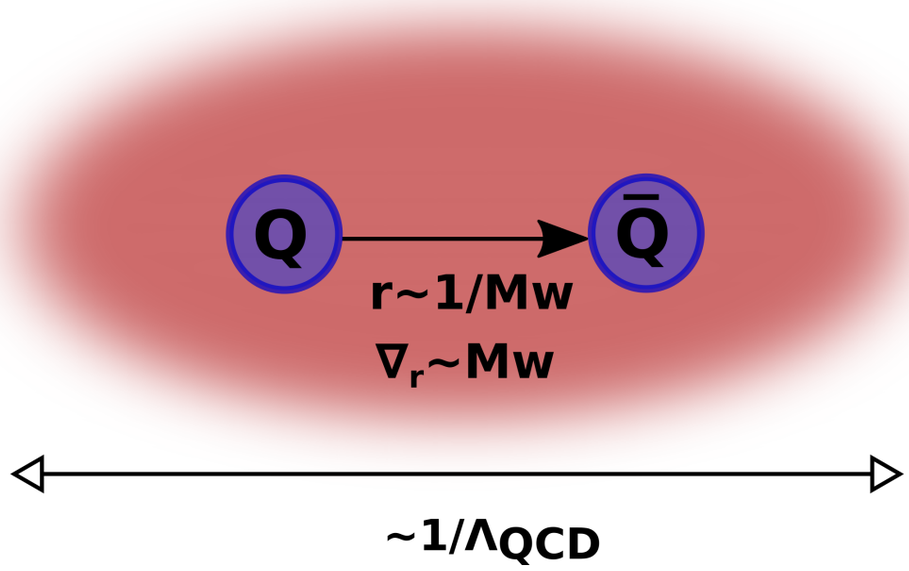
Obtain the adiabatic potentials (two-center problem)

$$\hat{h}_0(\mathbf{r},\mathbf{z})\phi_\kappa(\mathbf{r},\mathbf{z}) = V_\kappa^{\text{light}}(\mathbf{r})\,\phi_\kappa(\mathbf{r},\mathbf{z})$$

QCD

See talk by J. Soto

$E_{heavy} \sim Mw^2$ $E_{light} \sim \Lambda_{QCD}$



Hybrids

M. Berwein, N. Brambilla, J. Tarrús-Castellà and A. Vairo, PRD 92, 114019 (2015)
N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús-Castellà and A. Vairo, arXiv: 1805:07713

Conclusions & Perspectives

— QED molecules: systematic and controllable corrections to Born-Oppenheimer approximation

— QCD molecules:

- Need nonperturbative input (lattice, …)
- Framework for modelling

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