Hadron gas with repulsive mean field

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XIII Quark Confinement and the Hadron Spectrum

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in collaboration with Peter Petreczky, Phys. Lett. B 777, (2018)

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Dashen-Ma-Bernstein: Phys. Rev. 187, 345 (1969)

If interactions mediated by *narrow* **resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas**

$\Rightarrow$ **Hadron resonance gas model**

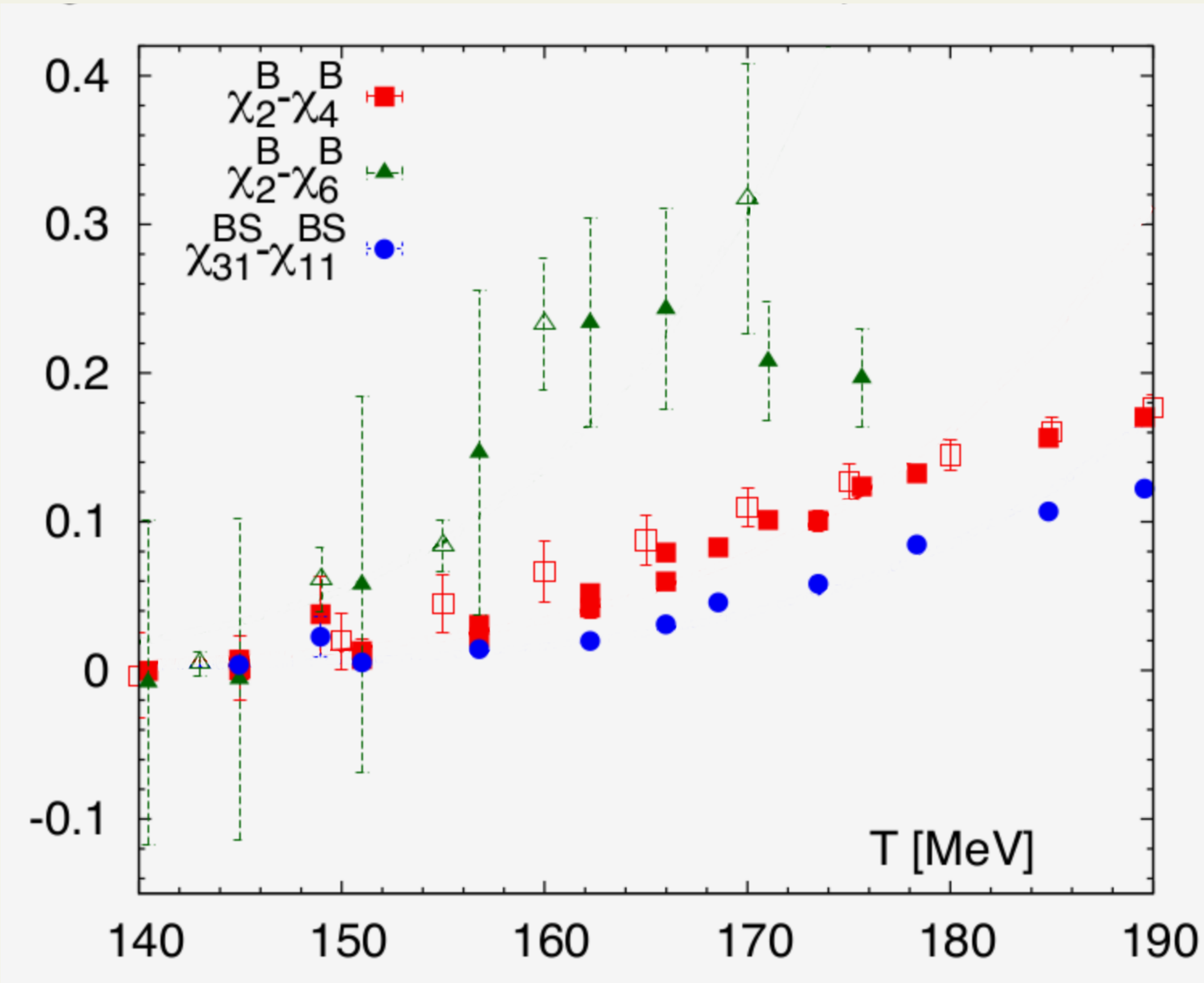
- **treat hadrons and resonances as free particles:**

$$P(T) = \sum_i \int \mathrm{d}^3p \, \frac{p^2}{3E} \, f(p,T)$$

- **Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts**
- **Gerber & Leutwyler, NPB321, 387 (1989): chiral perturbation theory**

$\Rightarrow$ **HRG good approximation at low temperatures**

Differences of fluctuations



Filled symbols: HISQ

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PRL111, 082301 (2013)

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4th order

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- **These zero in Boltzmann approximation**

Virial expansion

$$P = P^{\mathrm{ideal}} + T \sum_{ij} b_2^{ij}(T) e^{\beta\mu_i} e^{\beta\mu_j}$$

b_2^{ij} **can be related to the S-matrix of scattering of particles i and j**

- $\pi\pi$, πN, **etc. scatterings dominated by resonance formation**
- **no resonances in NN scatterings**

Virial expansion in nucleon gas

$$P(T,\mu) = P_0(T)\cosh(\beta\mu) + 2b_2(T)\,T\cosh(2\beta\mu)$$

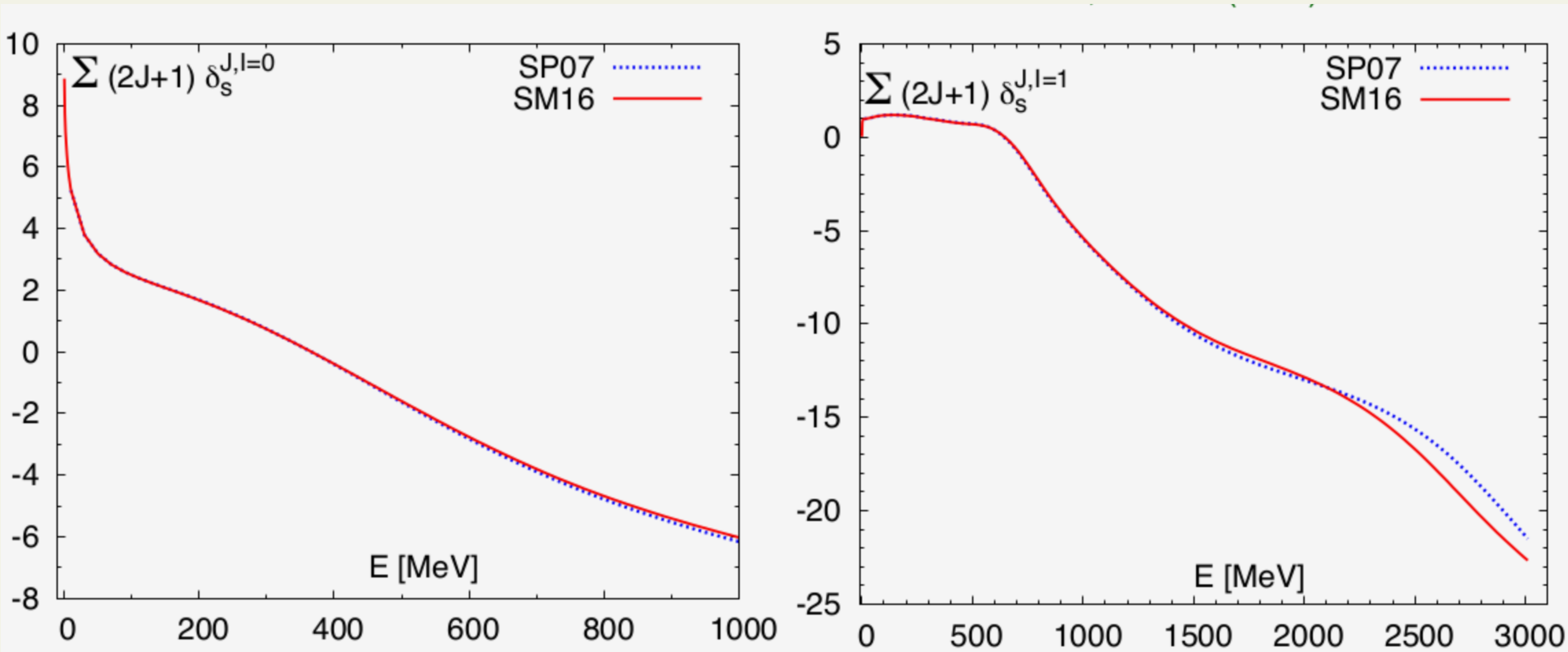
$$P_0(T) = \frac{4m^2T^2}{\pi^2}K_2(\beta m)$$

$$b_2(T) = \frac{2T}{\pi^3}\int_0^\infty \mathrm{d}E\left(\frac{mE}{2}+m^2\right)K_2\left(2\beta\sqrt{\frac{mE}{2}+m^2}\right)\frac{1}{4i}\mathrm{Tr}[S^\dagger\frac{\mathrm{d}S}{\mathrm{d}E}-\frac{\mathrm{d}S^\dagger}{\mathrm{d}E}S]$$

Virial expansion in nucleon gas

Elastic part of the S-matrix from scattering phase shift:

$$\frac{1}{4i}\mathrm{Tr}[S^\dagger \frac{\mathrm{d}S}{\mathrm{d}E} - \frac{\mathrm{d}S^\dagger}{\mathrm{d}E}S] \to \sum_s \sum_J (2J+1)\left(\frac{\mathrm{d}\delta_s^{J,I=0}}{\mathrm{d}E} + 3\frac{\mathrm{d}\delta_s^{J,I=1}}{\mathrm{d}E}\right)$$



Workman et al., PRC94, 065203 (2016); Arndt et al., PRC76, 025209 (2007)

Repulsive mean field

Assume: interactions reduce single partice energy by $U = Kn_b$ where n_b is single nucleon density (Olive, NPB190, 483 (1981))

$$n_b = \int \frac{\mathrm{d}^3p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}$$

Small $\mu \Rightarrow \beta K n_b \ll 1$ and

$$\begin{aligned} n_b &\approx n_b^0(1 - \beta K n_b^0) \Rightarrow \\ P(T,\mu) &= T(n_b + n_{\bar{b}}) - \frac{K}{2}\left((n_b^0)^2 + (n_{\bar{b}}^0)^2\right) \end{aligned}$$

or

$$P(T,\mu) = P_0(T)\left(\cosh(\beta\mu) - \frac{Km}{\pi^2} K_2(\beta m)\cosh(2\beta\mu)\right)$$

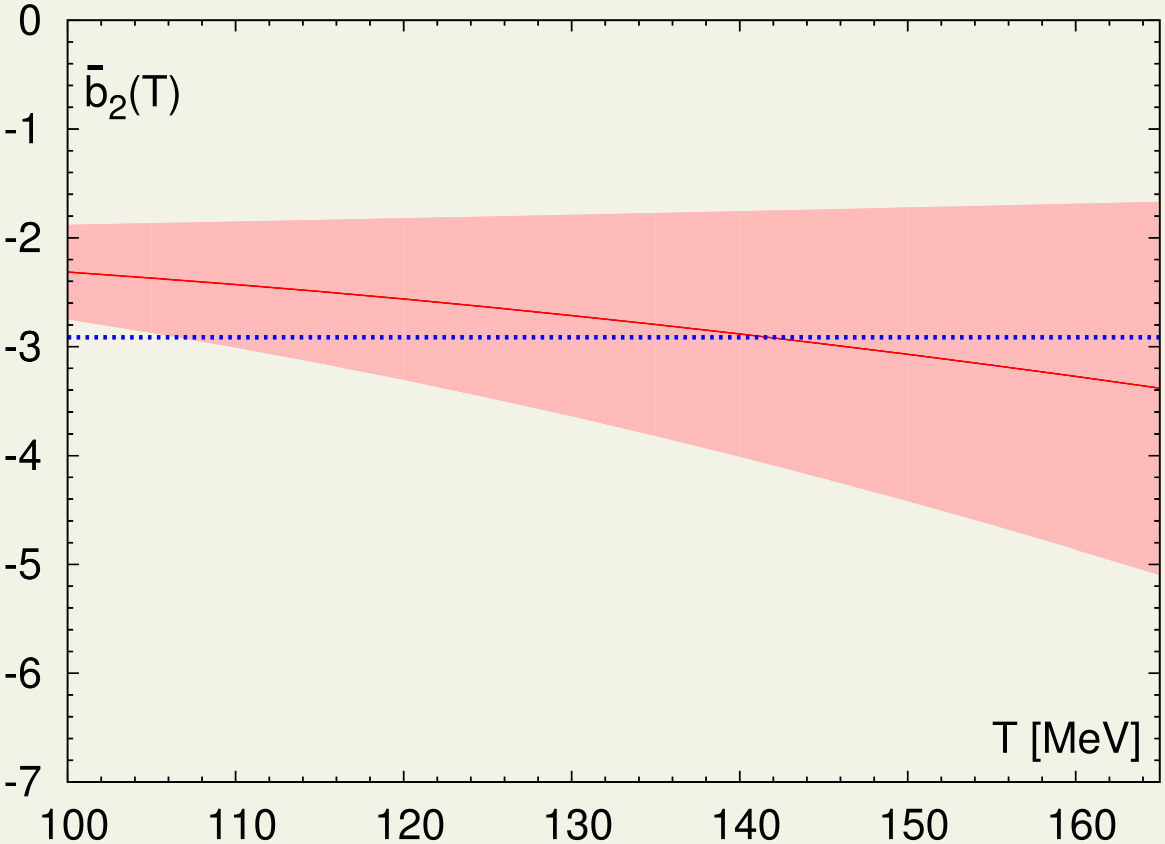
Virial expansion vs. mean field

Repulsive mean field

$P(T,\mu) = P_0(T)\times$
$\left(\cosh(\beta\mu) - \frac{Km}{\pi^2}K_2(\beta m)\cosh(2\beta\mu)\right)$

Virial expansion

$P(T,\mu) = P_0(T)\times$
$\left(\cosh(\beta\mu) - \bar{b}_2(T)K_2(\beta m)\cosh(2\beta\mu)\right)$
where $\bar{b}_2 = \frac{2Tb_2(T)}{P_0(T)K_2(\beta m)}$



- **inelastic scattering**

$\Rightarrow b_2$ **uncertain**

- **set** $Km^2/\pi^2 \approx \bar{b}_2(T)$

$\Rightarrow K = 250$ **MeV fm**^{3}

Hadron Resonance Gas with mean field

Assume:

- **only members of baryon octet and decuplet contribute to mean field**
- **heavier resonances feel the effect of mean field**
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$$u(n_b, \bar{n}_b, n_r, \bar{n}_r) = \frac{1}{2}K(n_b^2 + \bar{n}_b^2) + K(n_b n_r + \bar{n}_b \bar{n}_r)$$

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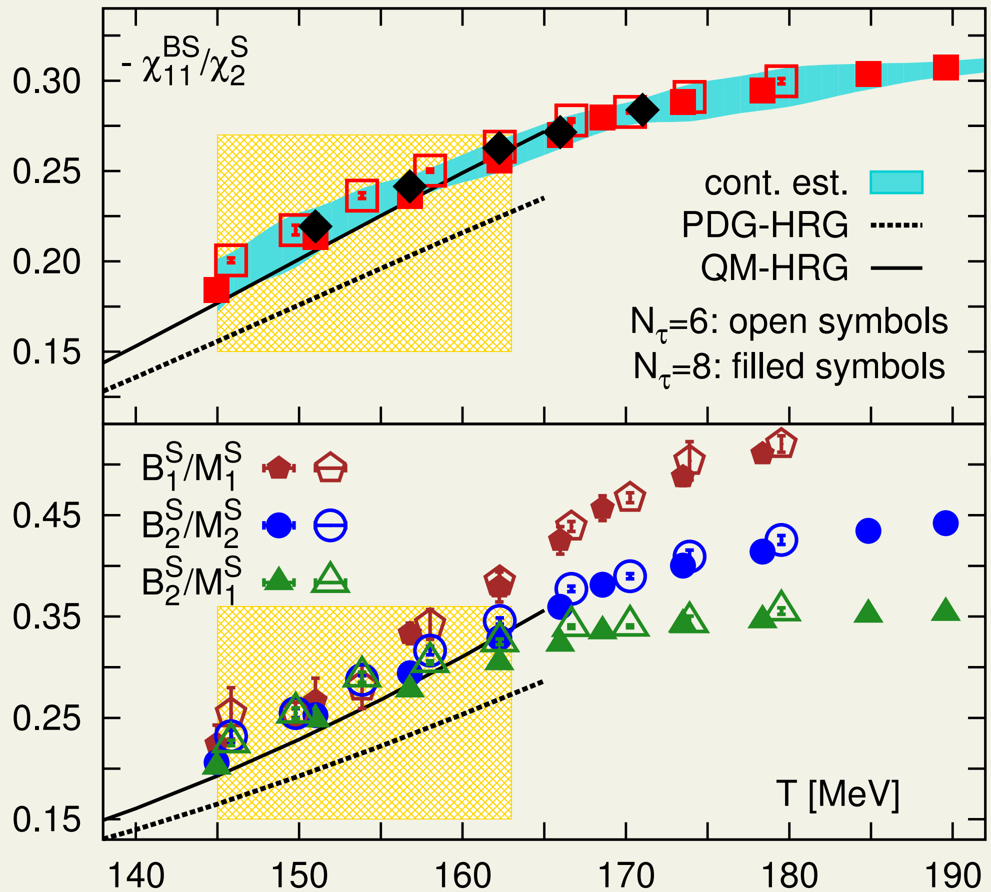
$$P(T, \{\mu_i\}) = P_0(T, \{\mu_{i,\mathrm{eff}}\})$$

where

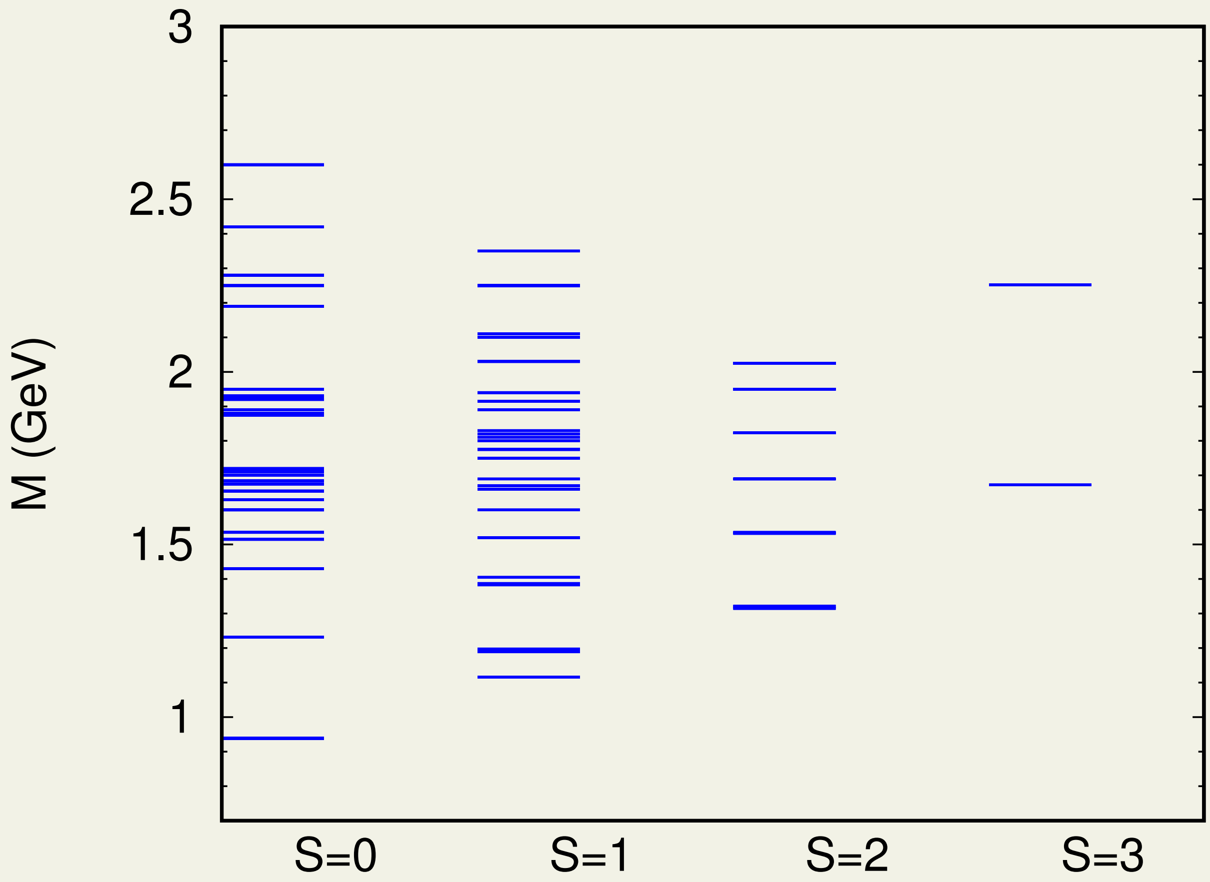
$$\mu_{i,\mathrm{eff}} = \mu_i - \frac{\partial u}{\partial n_i}$$

More resonances?



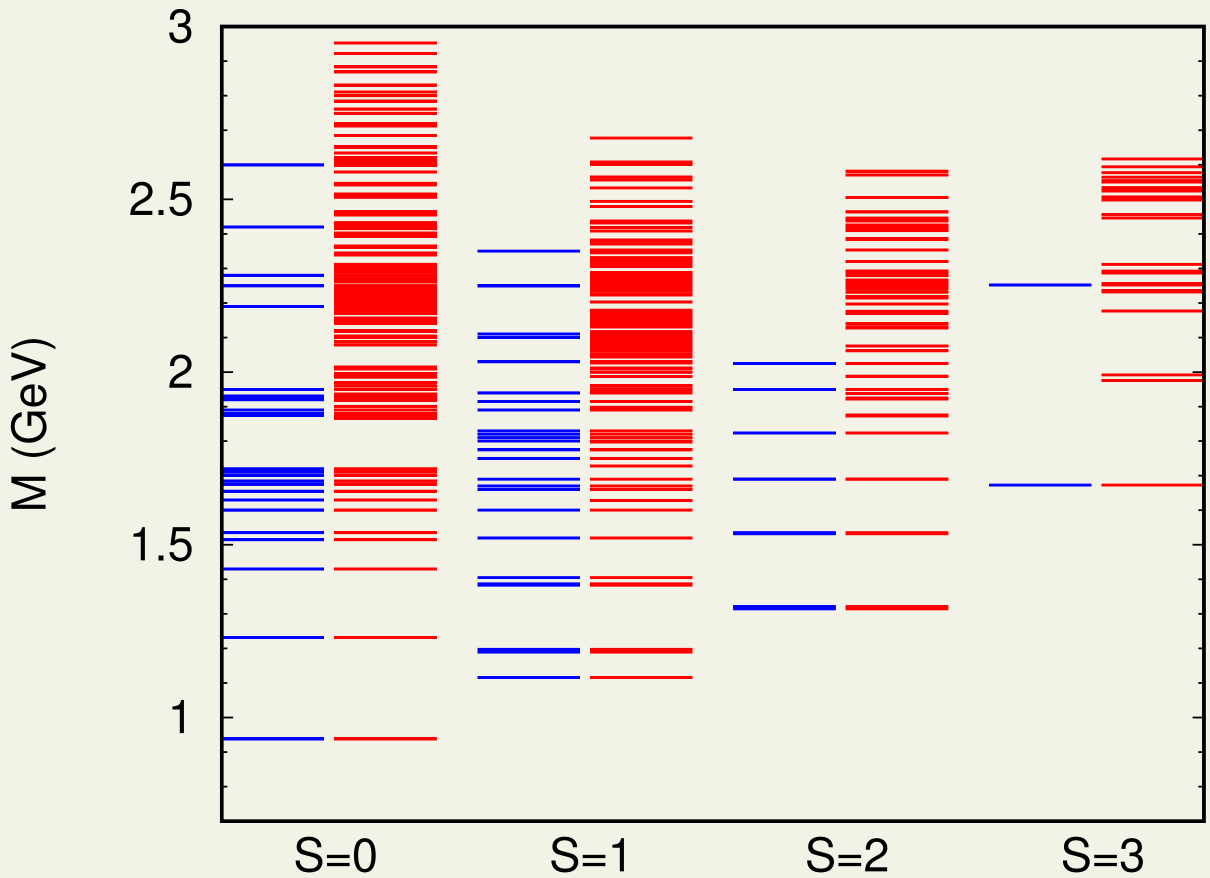
Bazavov et al., PRL113, 072001 (2014)

Baryon spectrum



Blue: Particle Data Group 2016 summary tables

Baryon spectrum



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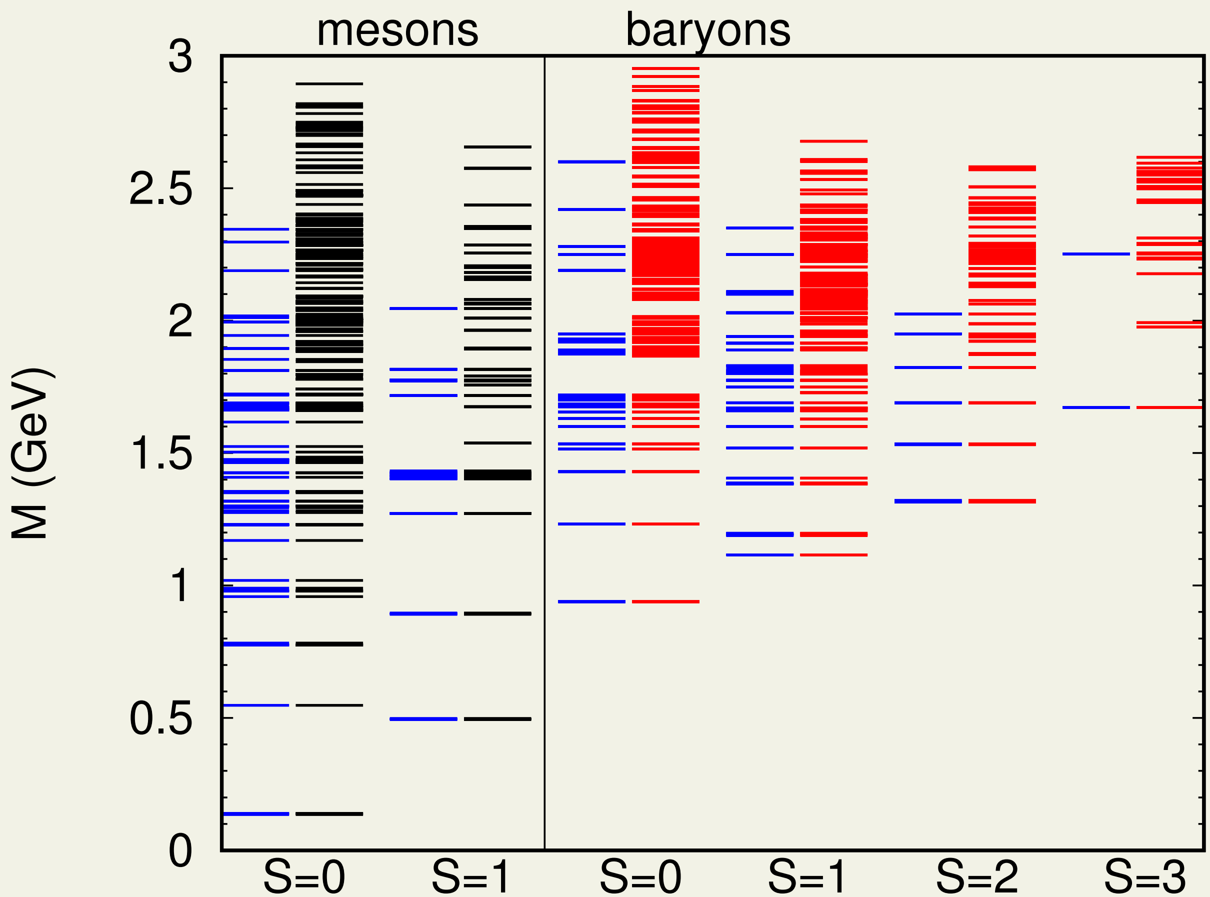
Hadron spectrum



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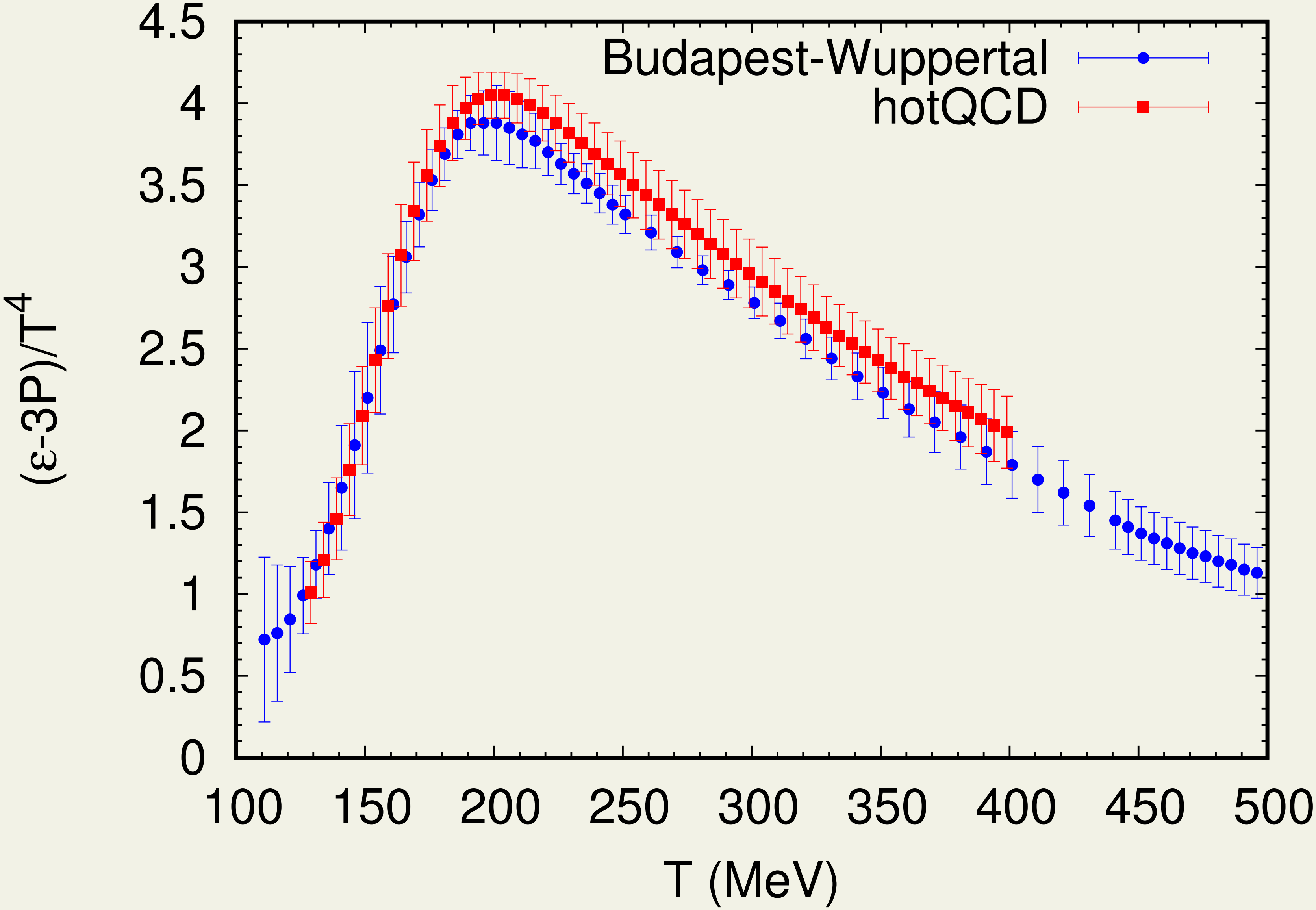
Black: PDG + Ebert et al., PRD79, 114029 (2009)

Trace anomaly



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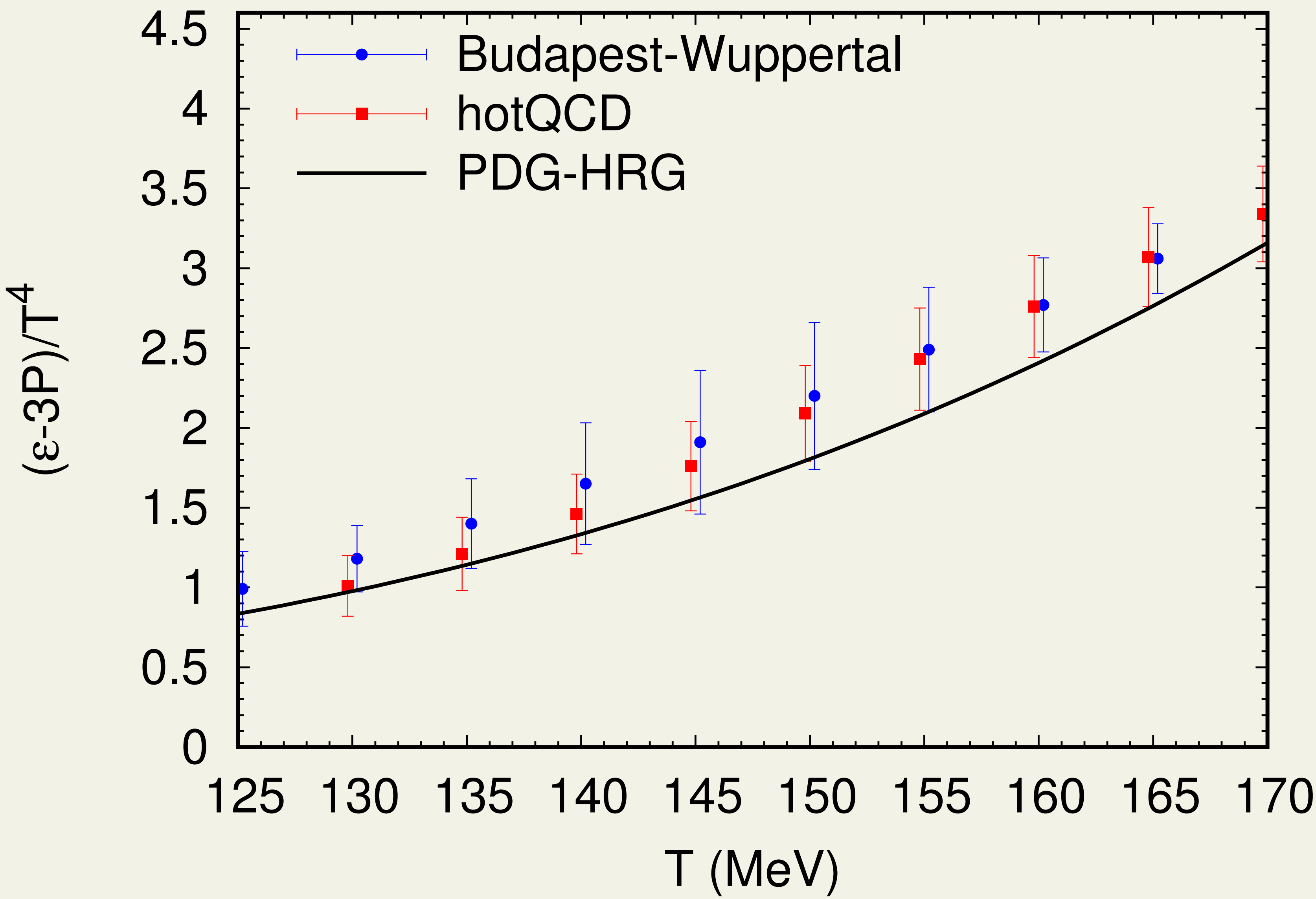

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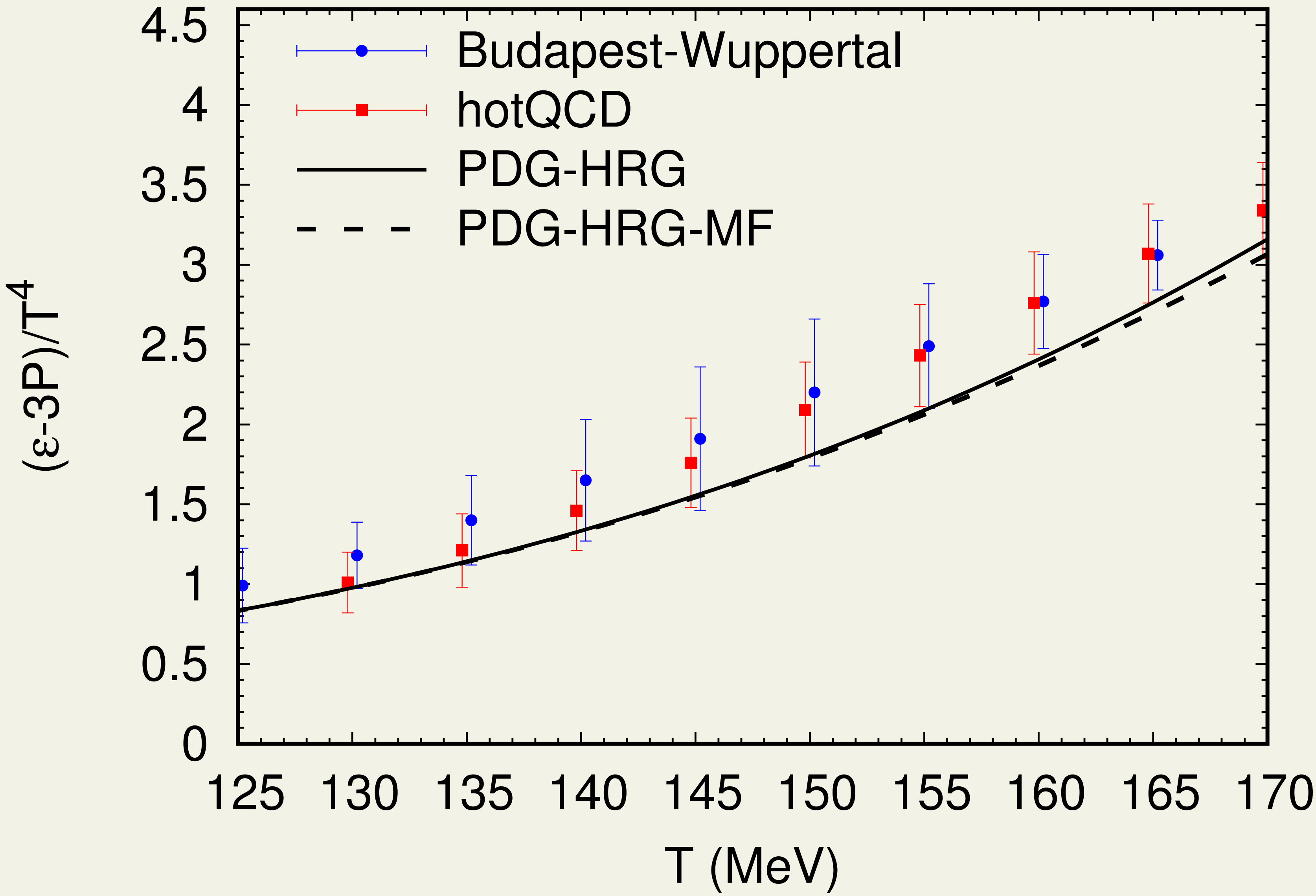

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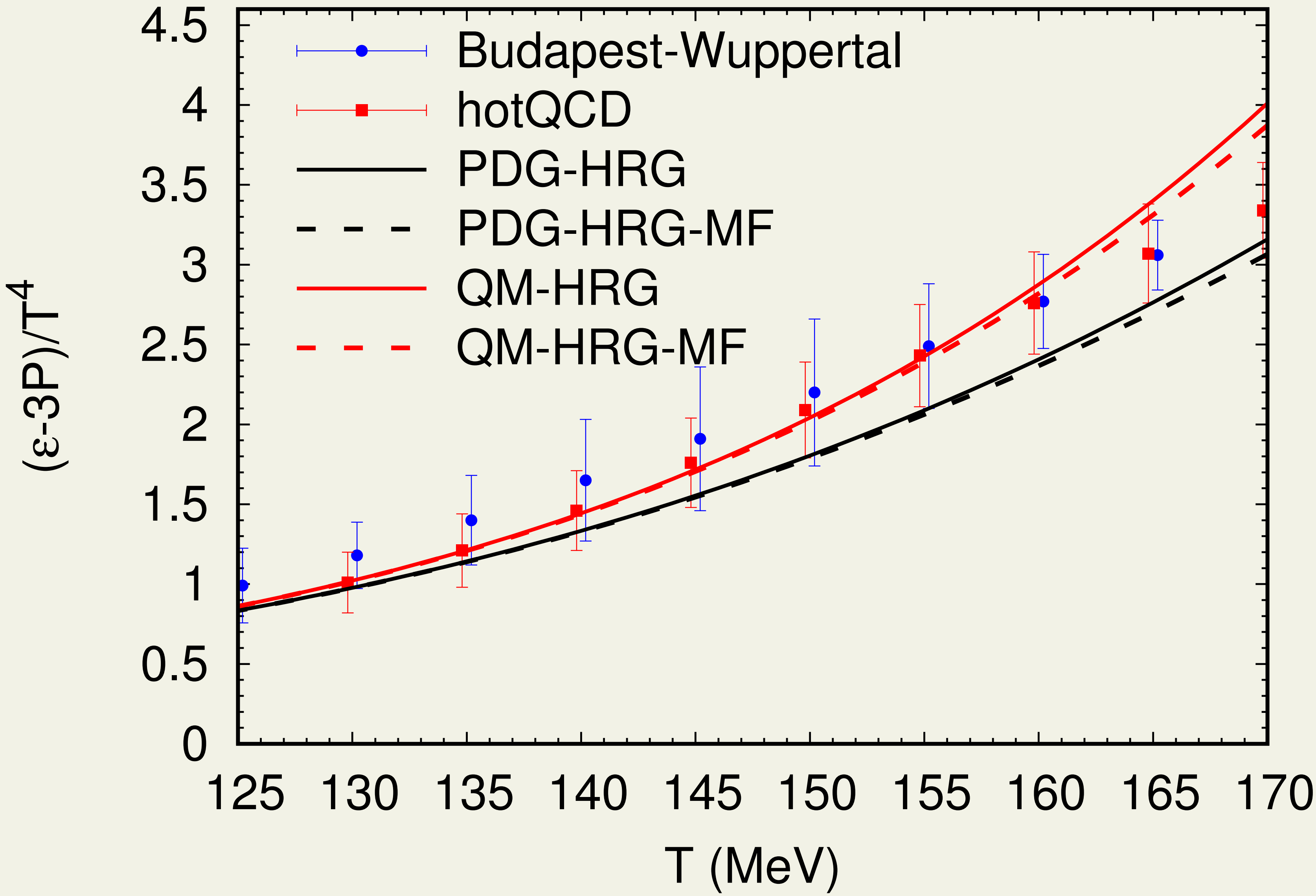
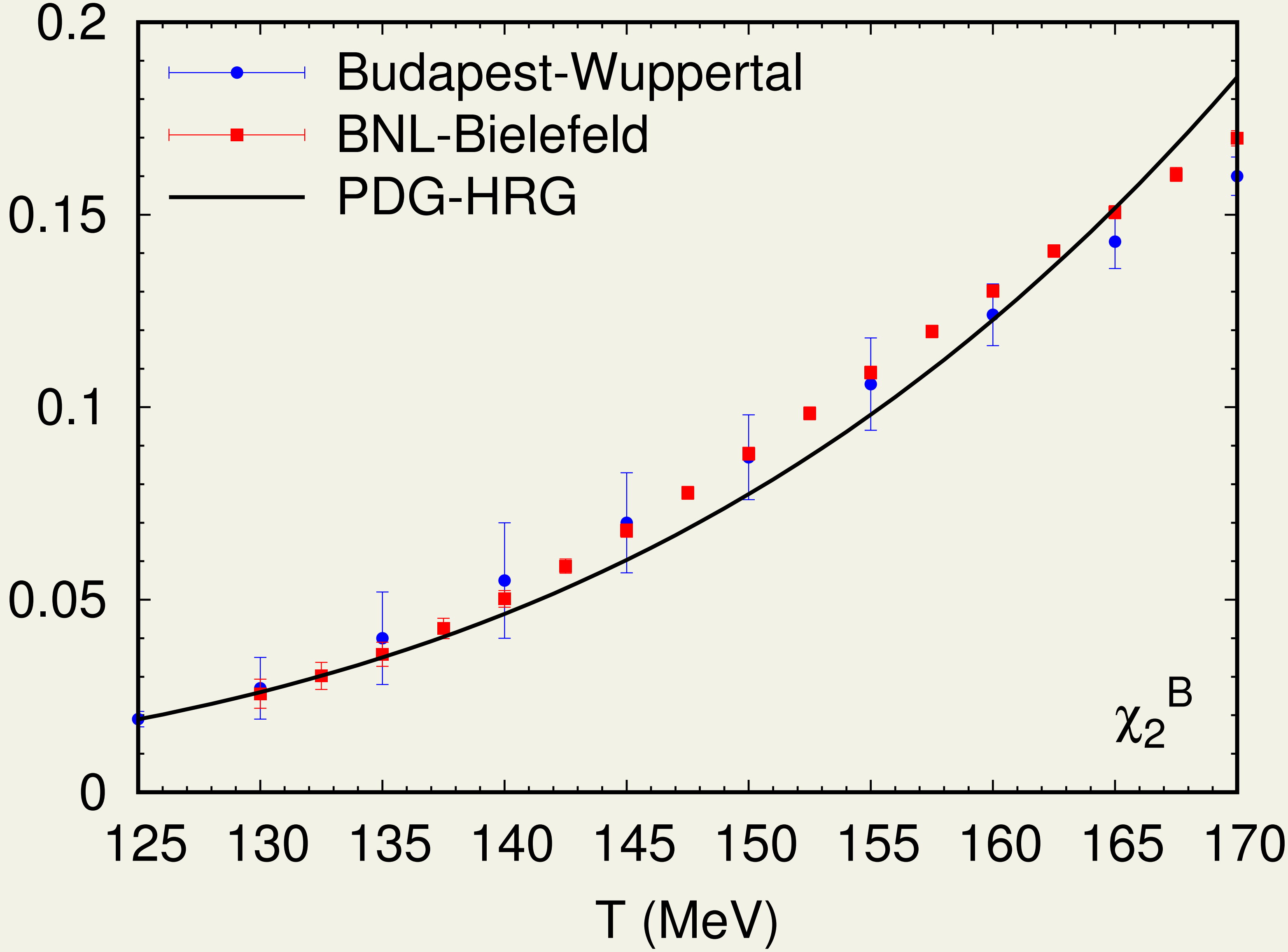
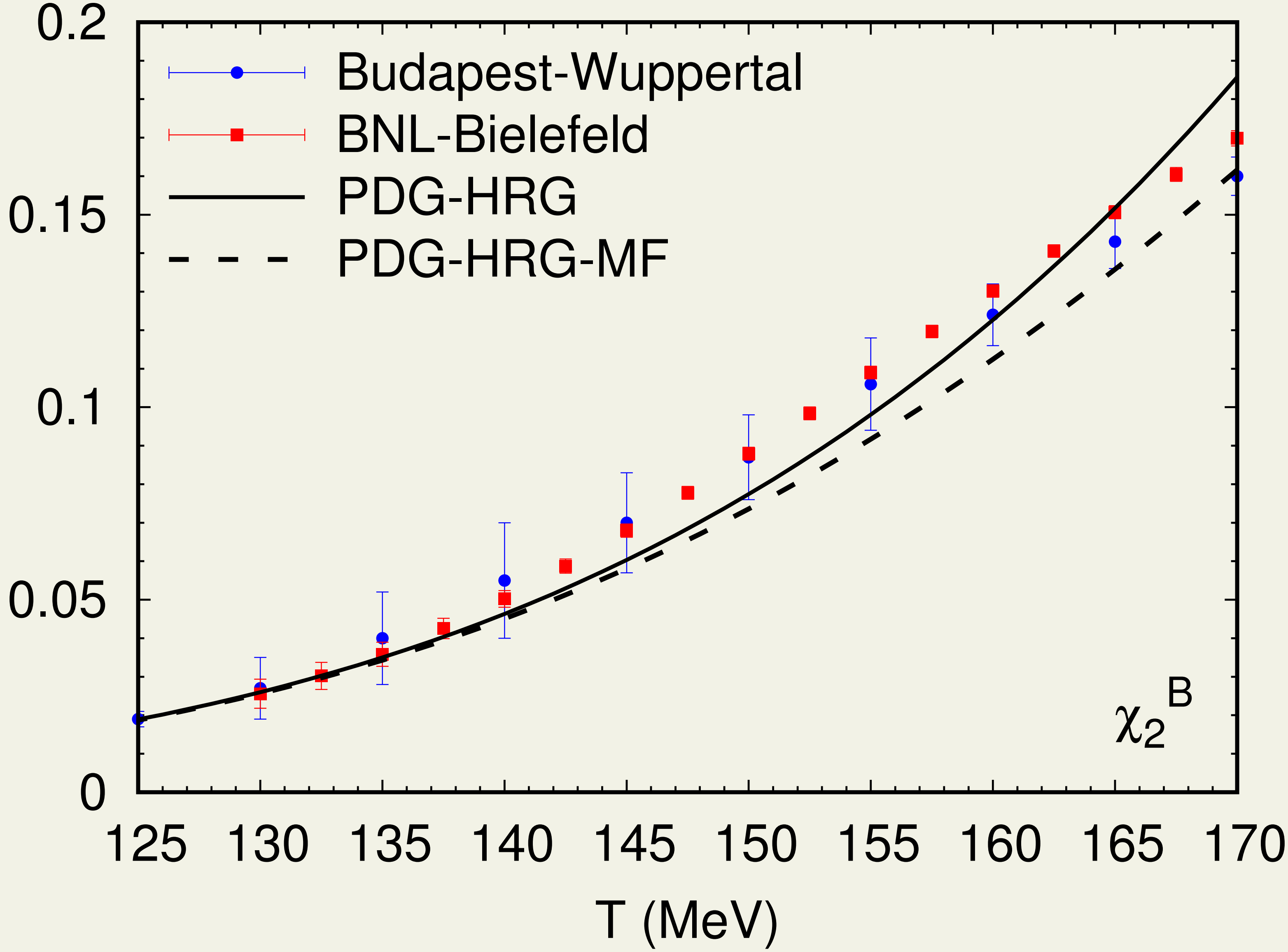


χ_B^2


0.2
0.15
0.1
0.05
0
Budapest-Wuppertal
BNL-Bielefeld
PDG-HRG
χ2B
125
130
135
140
145
150
155
160
165
170
T (MeV)

χ_B^2



Budapest-Wuppertal
BNL-Bielefeld
PDG-HRG
PDG-HRG-MF
0.2
0.15
0.1
0.05
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125
130
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155
160
165
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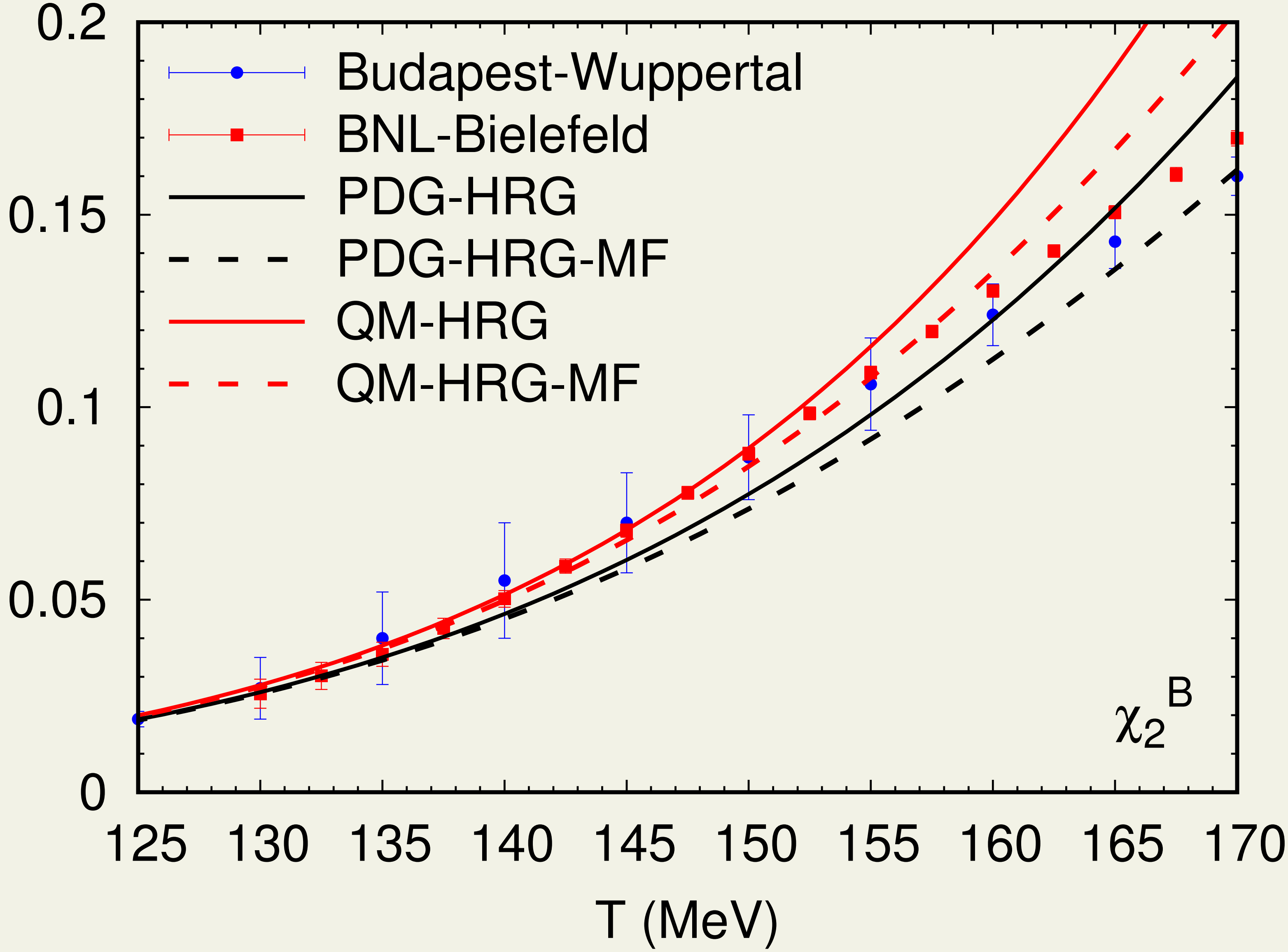
T (MeV)
χ_2^B

χ_B^2



Budapest-Wuppertal
BNL-Bielefeld
PDG-HRG
PDG-HRG-MF
QM-HRG
QM-HRG-MF
$\chi_2{}^B$
0.2
0.15
0.1
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0
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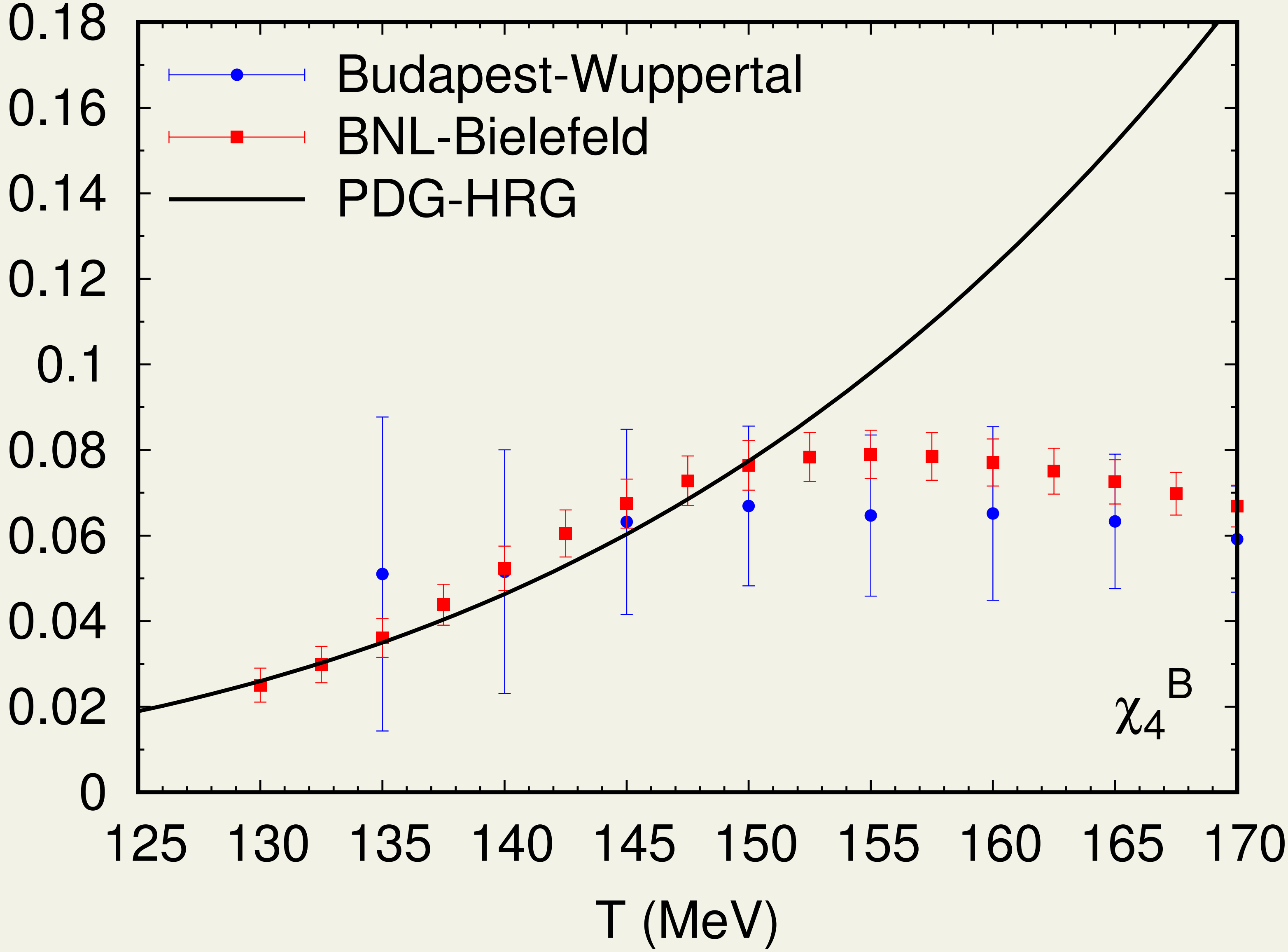
150
155
160
165
170
T (MeV)

χ^4_B


Budapest-Wuppertal
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χ4B
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
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145
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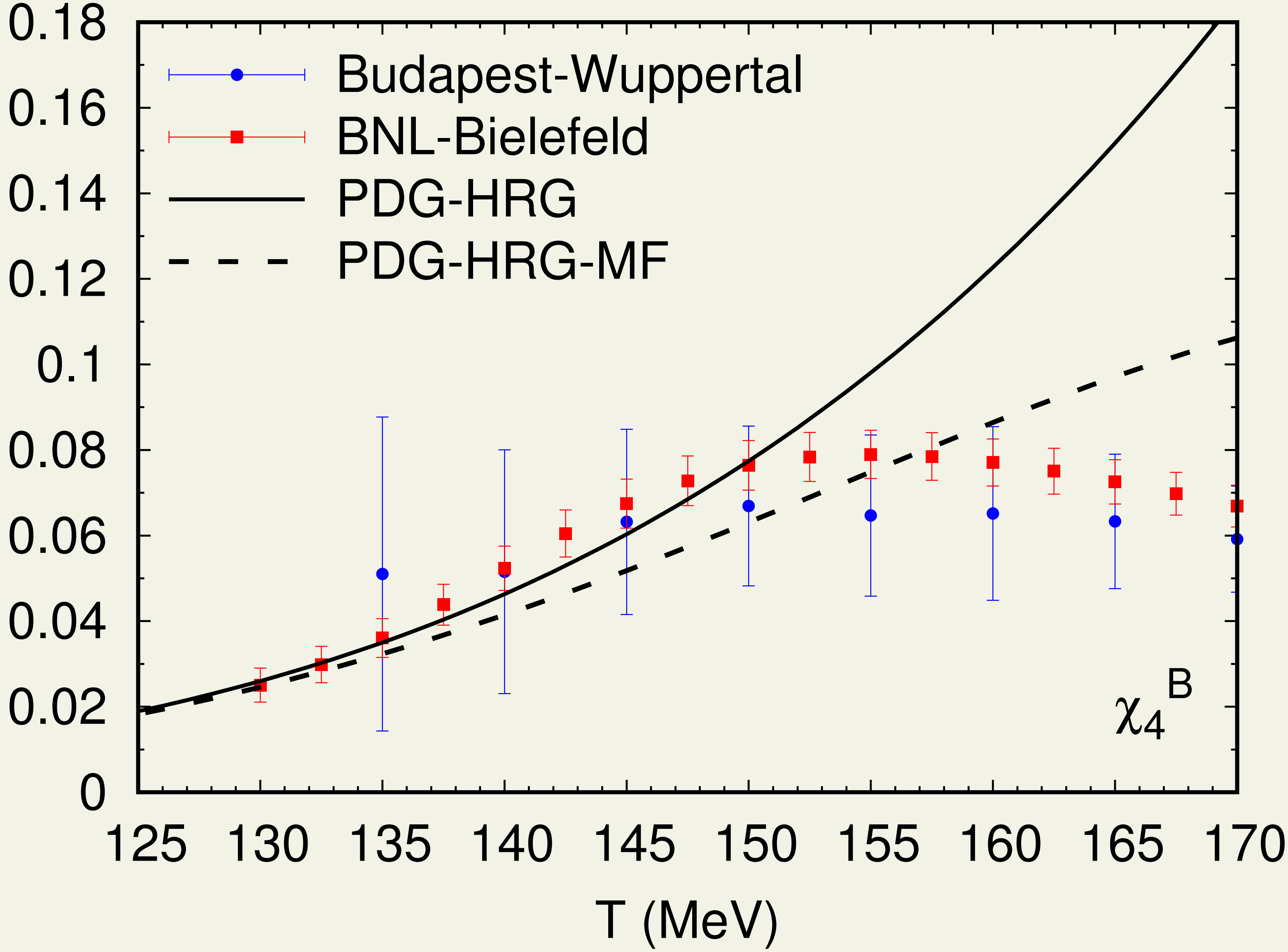
155
160
165
170
T (MeV)

χ_B^4



0.18
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T (MeV)
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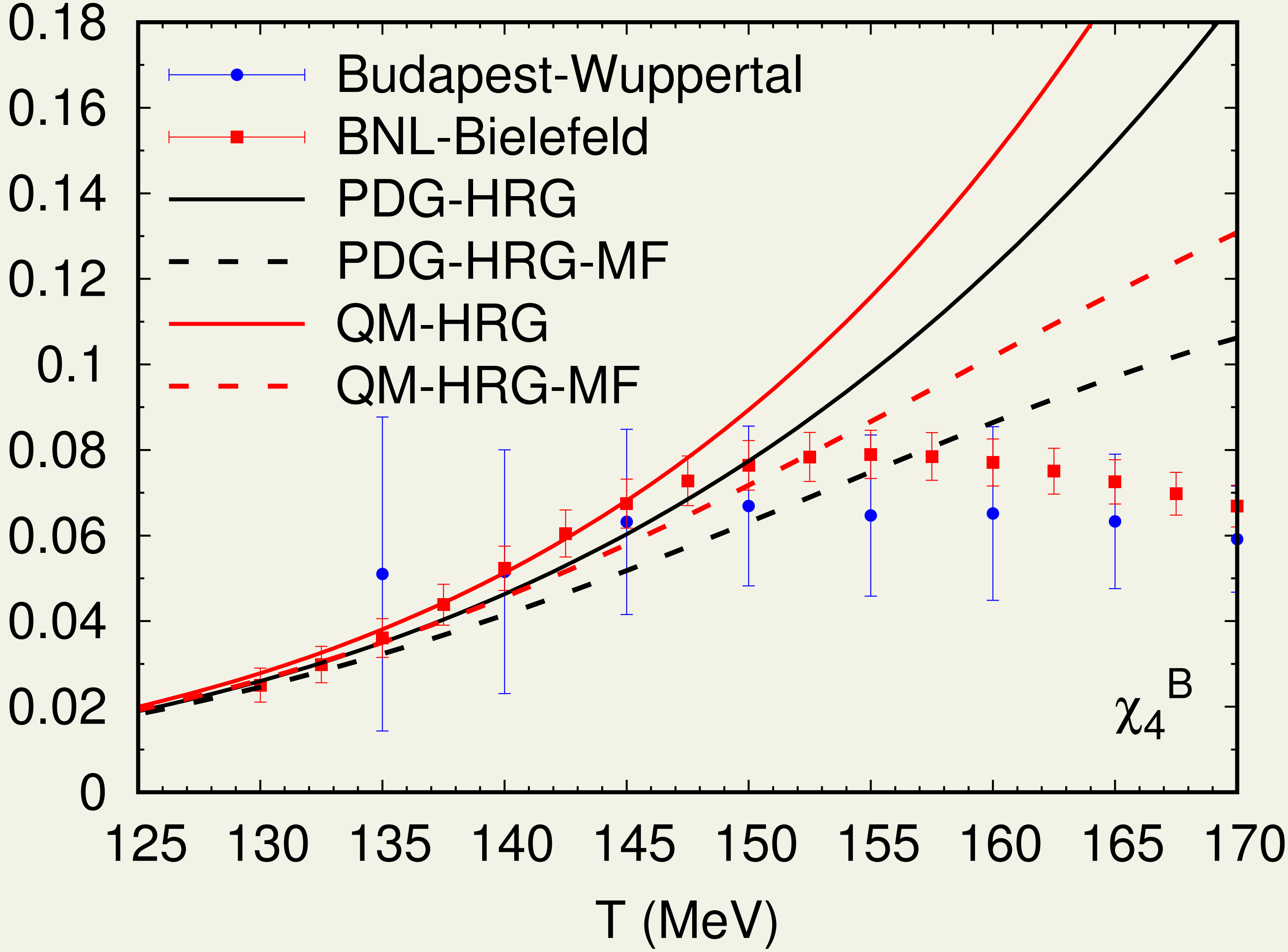
χ4B

χ_B^4



0.18
0.16
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T (MeV)
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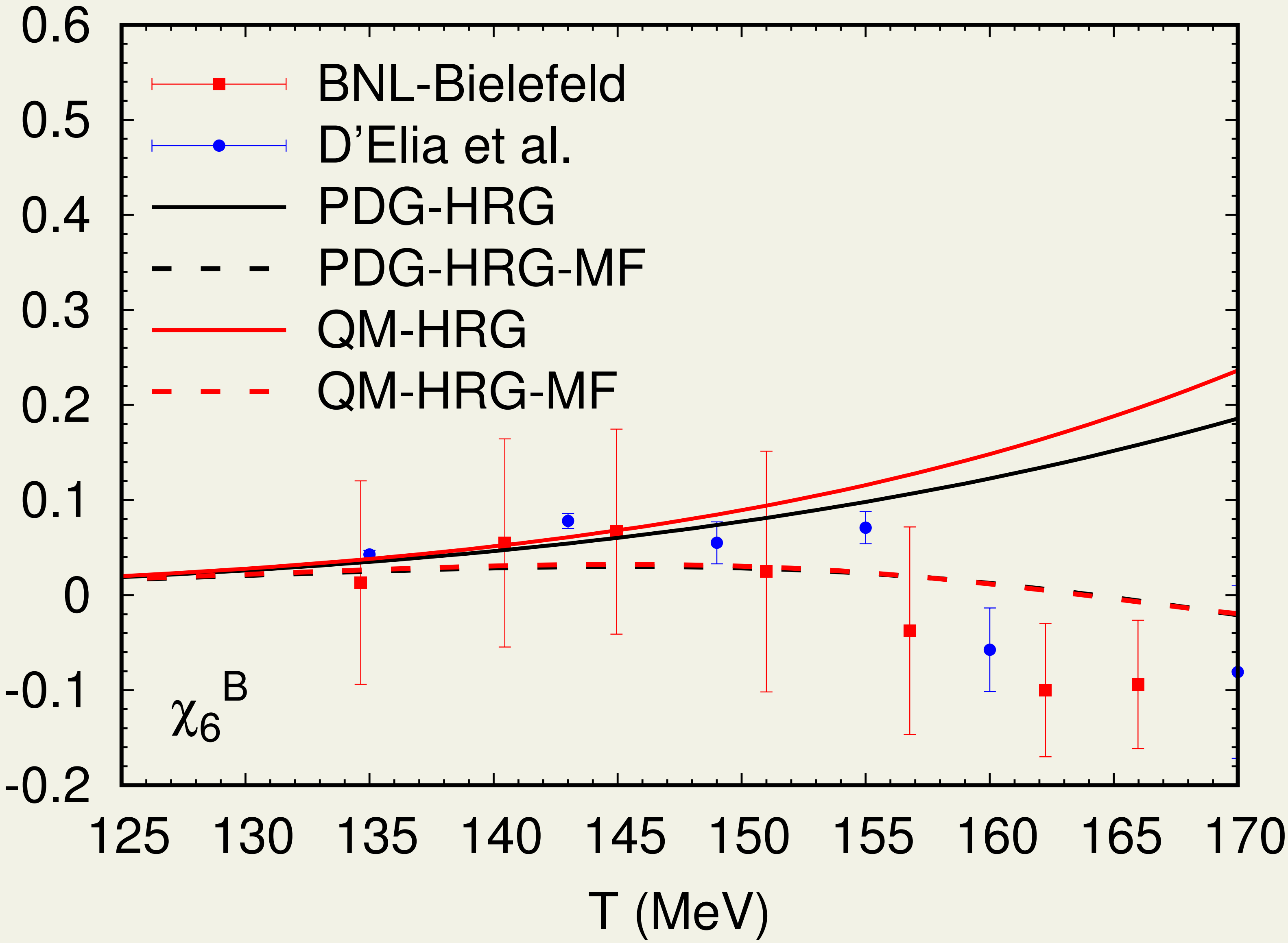
χ4 B

χ_B^6



0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
BNL-Bielefeld
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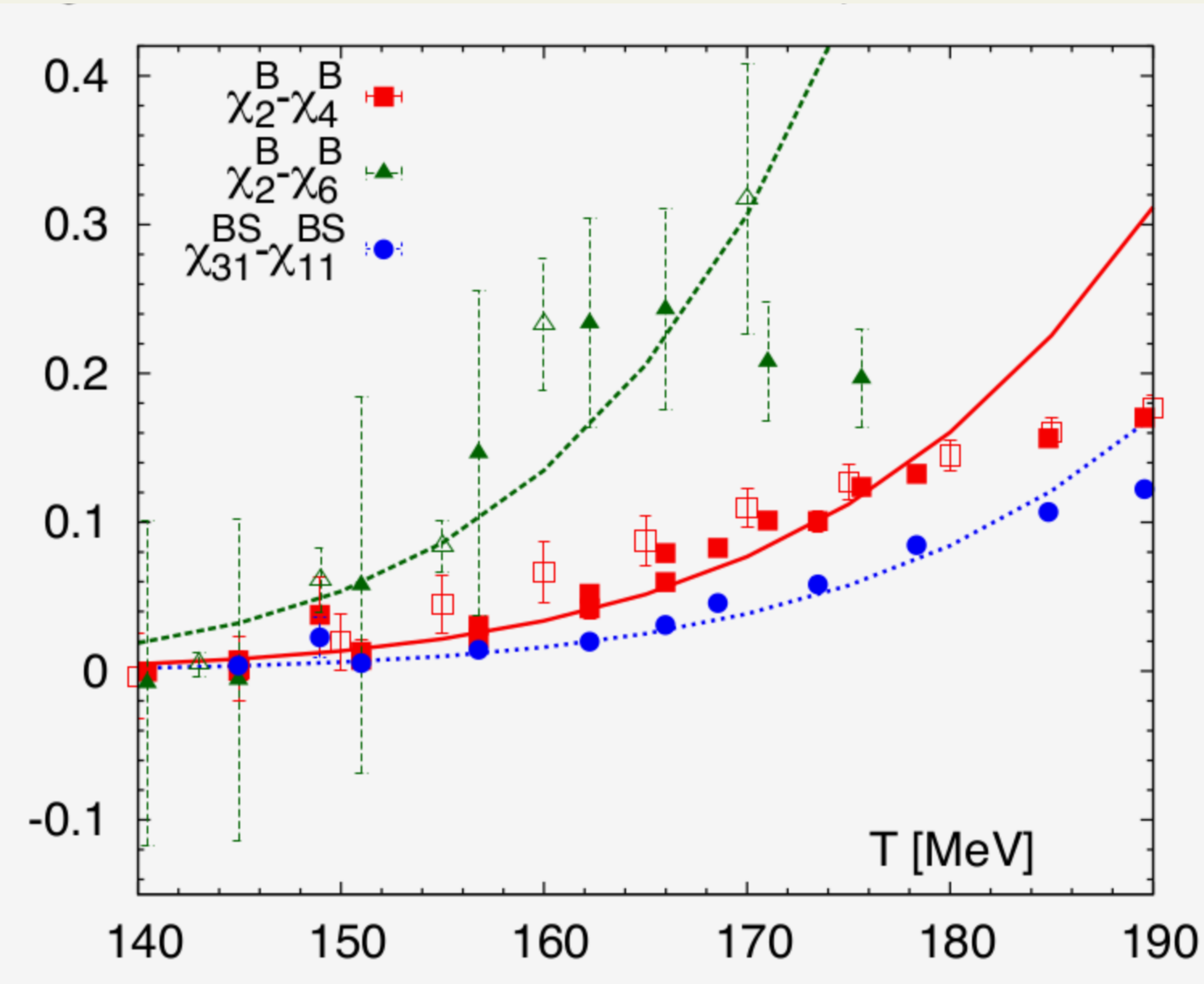
Differences of fluctuations



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- **These zero in Boltzmann approximation**
- **Repulsive interactions create similar differences**

Summary

- **repulsive mean field improves fit to fluctuations**

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- **strength of repulsion fixed by experimental phase shifts**

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- repulsive mean field improves fit to fluctuations
- strength of repulsion fixed by experimental phase shifts
- lattice QCD indicates there are more resonances than observed
  - inclusion fo quark model states improves the fit to some, and weakens the fit to some observables