Hadron gas with repulsive mean field

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XIII Quark Confinement and the Hadron Spectrum

August 3, 2018, Maynooth University, Ireland


The speaker has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 665778 via the National Science Center, Poland, under grant Polonez DEC-2015/19/P/ST2/03333

If interactions mediated by narrow resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas

⇒ Hadron resonance gas model

- treat hadrons and resonances as free particles:

\[ P(T) = \sum_i \int d^3 p \frac{p^2}{3E} f(p, T) \]

- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts

⇒ HRG good approximation at low temperatures
Differences of fluctuations

Filled symbols: HISQ
Bazavov et al.,
PRL111, 082301 (2013)
PRD95, 054504 (2017)

Open symbols: stout
4th order
Bellwied et al.,
PRD92, 114505 (2015)
6th order
D’Elia et al.,
PRD95, 094503 (2017)

• These zero in Boltzmann approximation
Virial expansion

\[ P = P^\text{ideal} + T \sum_{ij} b_{ij}^2(T)e^{\beta \mu_i}e^{\beta \mu_j} \]

\( b_{ij}^2 \) can be related to the S-matrix of scattering of particles \( i \) and \( j \)

- \( \pi\pi, \pi N, \) etc. scatterings dominated by resonance formation
- no resonances in \( NN \) scatterings
Virial expansion in nucleon gas

\[ P(T, \mu) = P_0(T) \cosh(\beta \mu) + 2b_2(T) T \cosh(2\beta \mu) \]

\[ P_0(T) = \frac{4m^2 T^2}{\pi^2} K_2(\beta m) \]

\[ b_2(T) = \frac{2T}{\pi^3} \int_0^\infty dE \left( \frac{mE}{2} + m^2 \right) K_2 \left( 2\beta \sqrt{\frac{mE}{2} + m^2} \right) \frac{1}{4i} \text{Tr}[S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S] \]
Virial expansion in nucleon gas

Elastic part of the S-matrix from scattering phase shift:

\[
\frac{1}{4i} \text{Tr} \left[ S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S \right] \rightarrow \sum_s \sum_J (2J + 1) \left( \frac{d\delta_{s, I=0}^J}{dE} + 3 \frac{d\delta_{s, I=1}^J}{dE} \right)
\]

Workman et al., PRC94, 065203 (2016); Arndt et al., PRC76, 025209 (2007)
Repulsive mean field

Assume: interactions reduce single particle energy by $U = Kn_b$ where $n_b$ is single nucleon density (Olive, NPB190, 483 (1981))

$$n_b = \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p-\mu+U)}$$

Small $\mu \Rightarrow \beta Kn_b \ll 1$ and

$$n_b \approx n_b^0(1 - \beta Kn_b) \Rightarrow$$

$$P(T, \mu) = T(n_b + n_b) - \frac{K}{2} [(n_b^0)^2 + (n_b^0]^2)$$

or

$$P(T, \mu) = P_0(T) \left( \cosh(\beta \mu) - \frac{Km}{\pi^2} K_2(\beta m) \cosh(2\beta \mu) \right)$$
Virial expansion vs. mean field

**Repulsive mean field**

\[
P(T, \mu) = P_0(T) \times 
(cosh(\beta\mu) - \frac{Km}{\pi^2} K_2(\beta m) \cosh(2\beta\mu))
\]

**Virial expansion**

\[
P(T, \mu) = P_0(T) \times 
(cosh(\beta\mu) - \bar{b}_2(T) K_2(\beta m) \cosh(2\beta\mu))
\]

where \( \bar{b}_2 = \frac{2Tb_2(T)}{P_0(T)K_2(\beta m)} \)

- inelastic scattering
  \( \Rightarrow b_2 \) uncertain
- set \( Km^2/\pi^2 \approx \bar{b}_2(T) \)
  \( \Rightarrow K = 250 \text{ MeV fm}^3 \)
Hadron Resonance Gas with mean field

Assume:

- only members of baryon octet and decuplet contribute to mean field
- heavier resonances feel the effect of mean field
- baryons and antibaryons do not interact
Hadron Resonance Gas with mean field

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\[ u(n_b, \bar{n}_b, n_r, \bar{n}_r) = \frac{1}{2}K(n_b^2 + \bar{n}_b^2) + K(n_b n_r + \bar{n}_b \bar{n}_r) \]
Hadron Resonance Gas with mean field

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\]

\[
P(T, \{\mu_i\}) = P_0(T, \{\mu_{i,\text{eff}}\})
\]

where

\[
\mu_{i,\text{eff}} = \mu_i - \frac{\partial u}{\partial n_i}
\]
More resonances?

Bazavov et al., PRL113, 072001 (2014)
Baryon spectrum

Blue: Particle Data Group 2016 summary tables
Baryon spectrum

Blue: Particle Data Group 2016 summary tables
Hadron spectrum

Blue: Particle Data Group 2016 summary tables
Black: PDG + Ebert et al., PRD79, 114029 (2009)
Trace anomaly

Budapest-Wuppertal hotQCD

\[
\frac{\epsilon - 3P}{T^4}
\]

\( T \text{ (MeV)} \)
Trace anomaly

\[ \frac{(\varepsilon - 3P)}{T^4} \]

- Budapest-Wuppertal
- hotQCD
- PDG-HRG

\( T \) (MeV)
Trace anomaly

\[ \frac{(\varepsilon - 3P)}{T^4} \]

\( T \) (MeV)

- Budapest-Wuppertal
- hotQCD
- PDG-HRG
- PDG-HRG-MF
Trace anomaly

\[(\varepsilon - 3P)/T^4\]

- Budapest-Wuppertal
- hotQCD
- PDG-HRG
- PDG-HRG-MF
- QM-HRG
- QM-HRG-MF

P. Huovinen @ Confinement XIII, August 3, 2018
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- These zero in Boltzmann approximation
- Repulsive interactions create similar differences
Summary

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- strength of repulsion fixed by experimental phase shifts
Summary

- repulsive mean field improves fit to fluctuations
- strength of repulsion fixed by experimental phase shifts
- lattice QCD indicates there are more resonances than observed
  - inclusion of quark model states improves the fit to some, and weakens the fit to some observables