



Determination of the quark-gluon string parameters from the data on pp, pA and AA collisions at wide energy range using Bayesian Gaussian Process Optimization

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XIIIth Quark Confinement and the Hadron Spectrum

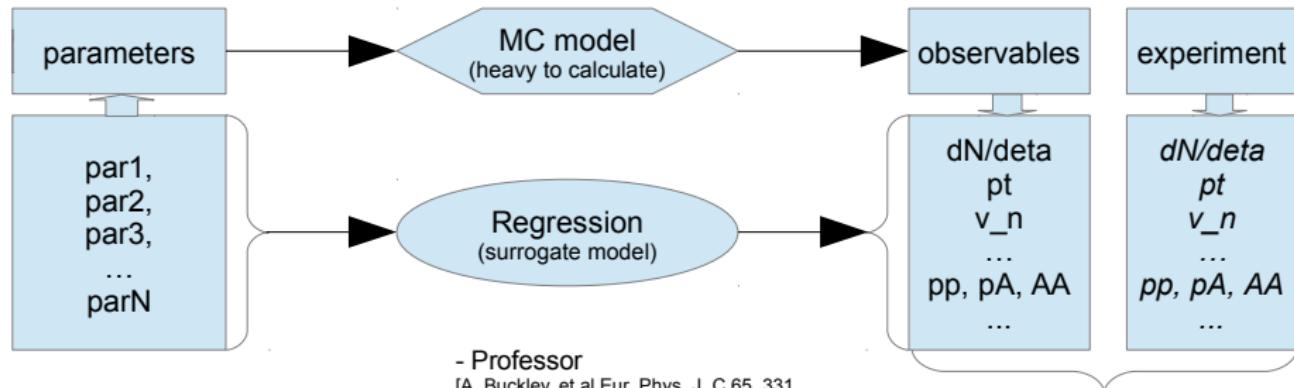
This work is supported by the Russian Science Foundation under grant 17-72-20045.

Outline

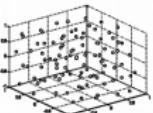
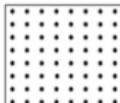
- Formulation of the problem for Bayesian Gaussian process optimization
- Description of the Non-Glauber Monte Carlo model with string fusion
- Results of Bayesian Optimization for string parameters
- Conclusions and Outlook

Parameter dependence approximation

- Monte Carlo models generally use many parameters, but can calculate many observables to compare with experiment



Sampling:
 - grid (brute force)
 - Latin hypercube
 (maximize minimum distance)



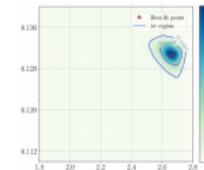
- Professor
 [A. Buckley, et al Eur. Phys. J. C 65, 331 (2010)], (new version 2.2.2 more advance)

- Gaussian Processes
 [C. E. Rasmussen C.K. I. Williams
 The MIT Press, 2006]
<http://www.gaussianprocess.org/gpml/>

Example for heavy ion:
 [Jonah E. Bernhard et al, Phys. Rev. C 94, 024907 (2016)]

Parameter estimation
+ error (Posterior)

Bayesian
parameter
estimation



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Parameter dependence approximation Gaussian processes regression

Stochastic process is Gaussian if and only if for every finite set of indices

$\mathbf{X}_{t_1, \dots, t_k} = (\mathbf{X}_{t_1}, \dots, \mathbf{X}_{t_k})$ is a multivariate Gaussian random variable.

Gaussian processes can be completely defined by covariance functions. Examples:

- Constant : $K_C(x, x') = C$
- Linear: $K_L(x, x') = x^T x'$
- Gaussian noise: $K_{GN}(x, x') = \sigma^2 \delta_{x,x'}$
- Squared exponential: $K_{SE}(x, x') = \exp\left(-\frac{\|d\|^2}{2\ell^2}\right)$

In our analysis we will use sum of Squared exponential and Gaussian noise:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{|\mathbf{x}_i - \mathbf{x}_j|^2}{2\ell^2}\right) + \sigma_n^2 \delta_{ij}$$

Software we used:

GPML Matlab Code, v. 4.1, in GNU Octave, v. 4.0.0 <http://www.gaussianprocess.org/gpml/>
Scikit-learn Gaussian Process Regressor (python 2.7.12, Sklearn 0.18.1)
http://scikit-learn.org/stable/modules/generated/sklearn.gaussian_process.GaussianProcessRegressor.html

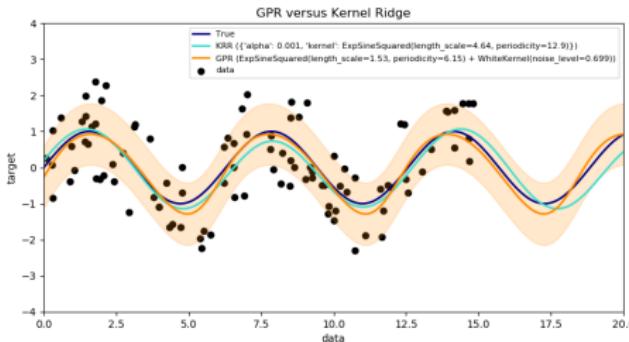
Parameter dependence approximation Gaussian processes regression

Advantages of Gaussian processes:

- The prediction interpolates the observations (at least for regular kernels).
- The prediction is probabilistic (Gaussian), one can compute empirical confidence intervals - can be used for online fitting, adaptive refitting

Disadvantages of Gaussian processes:

- They are not sparse, i.e., they use the whole samples/features information to perform the prediction.
- They lose efficiency in high dimensional spaces – namely when the number of features exceeds a few dozens.



Bayesian parameters optimization

Bayes' theorem

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}, \quad p(x|\mathbf{y}, X) = \frac{p(\mathbf{y}|X, x)p(x)}{p(\mathbf{y}|X)}$$

Prior – we take uniform (if the model good in constraining the data, the posterior will slightly depend on prior)

$$P(\mathbf{x}) \propto \begin{cases} 1 & \text{if } \min(x_i) \leq x_i \leq \max(x_i) \text{ for all } i \\ 0 & \text{else.} \end{cases}$$

Marginal likelihood – normalization constant

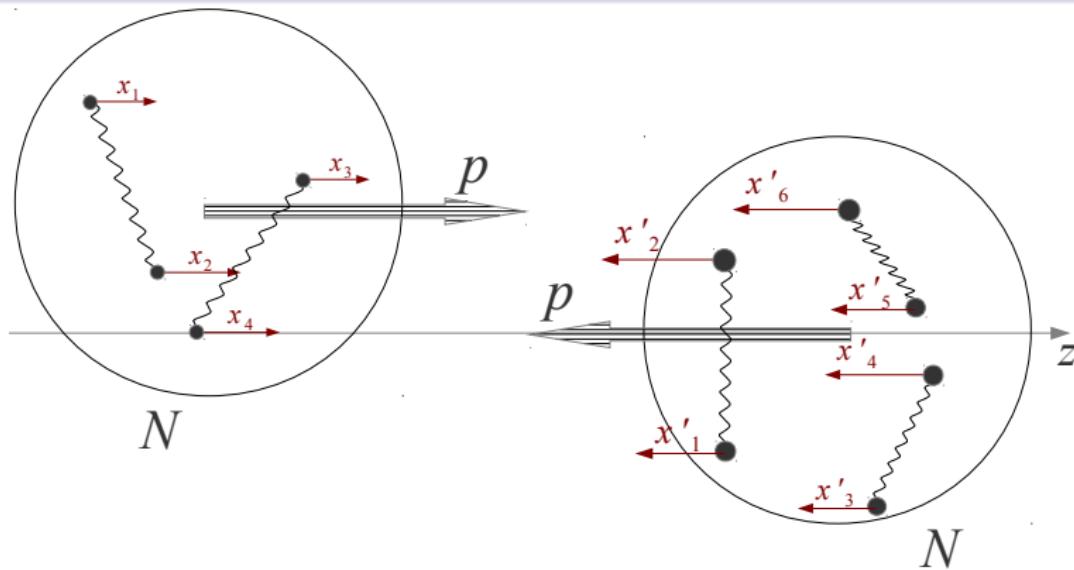
Likelihood (determined from the Gaussian Processes)

$$\ln P(x|y, X) = -\frac{1}{2} \frac{(y - y_{\text{exp}})^2}{\sigma_{\text{exp}}^2} + \text{const}$$

$$\text{Posterior } P(x|y, X) \propto e^{-\frac{1}{2} \frac{(y - y_{\text{exp}})^2}{\sigma_{\text{exp}}^2}}$$

For several observables we take independent Gaussian Process for each one and then combine in the Likelihood

Model description: Color dipoles inside a nucleon



$$\sum_i x_i p = p$$

$$\sum_i x_i = 1$$

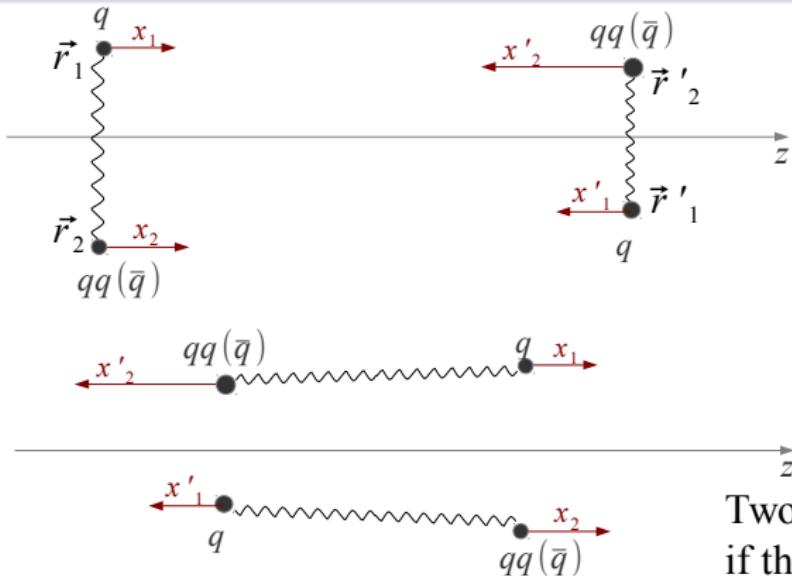
$$\sum_i x'_i p' = p$$

$$\sum_i x'_i = 1$$

Distribution in the impact parameter plane

- Exclusive distribution in the impact parameter plane is constructed from the following suppositions:
 - 1 Centre of mass is fixed: $\sum_{j=1}^N \vec{r}_j \cdot \vec{x}_j = 0$.
 - 2 Inclusive distribution of each parton is the 2-dimentional Gaussian distribution.
 - 3 Normalization condition $\langle r^2 \rangle = \langle \frac{1}{N} \sum_{j=1}^N r_j^2 \rangle = r_0^2$.
- The parameter r_0^2 is connected with the mean square radius of the proton by the formula: $\langle r_N^2 \rangle = \frac{3}{2} r_0^2$.

Monte Carlo model: Color dipoles



Interaction probability amplitude [1, 2]:

Two dipoles interact more probably, if the ends are close to each other, and (others equal) if they are wide.

$$f = \frac{\alpha_s^2}{2} \left[K_0\left(\frac{|\vec{r}_1 - \vec{r}'_1|}{r_{max}}\right) + K_0\left(\frac{|\vec{r}_2 - \vec{r}'_2|}{r_{max}}\right) - K_0\left(\frac{|\vec{r}_1 - \vec{r}_2|}{r_{max}}\right) - K_0\left(\frac{|\vec{r}_2 - \vec{r}'_1|}{r_{max}}\right) \right]^2$$

[1] G. Gustafson, Acta Phys. Polon. B40, 1981 (2009)

[2] C. Flensburg, G. Gustafson, and L. Lonnblad, Eur. Phys. J. (C) 60, 233 (2009)

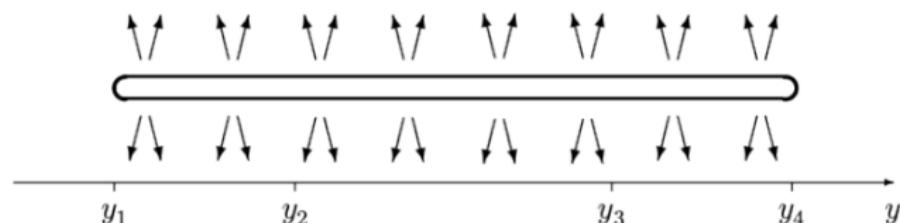
- Multiplicity is calculated in the framework of colour strings, stretched between colliding partons; x_i determine rapidity ends of strings.



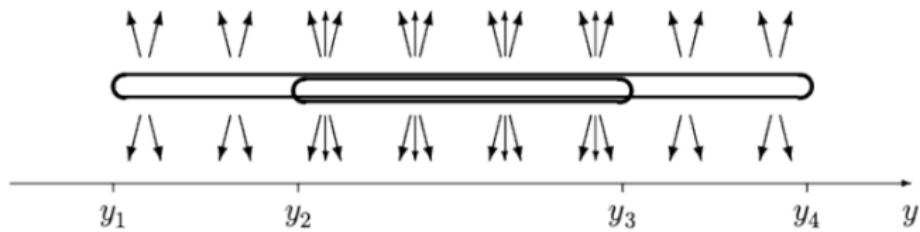
y_{\min} and y_{\max} are calculated supposing that a string fragments into only two particles with masses 0.15 GeV (for pion) and 0.94 GeV for proton and transverse momentum of 0.3 GeV (and higher at LHC)

- dN/dy from one string is supposed to be constant μ_0 .
- String fusion effects considered

- Uniform and independent distribution of particles on rapidity from y_{\min} to y_{\max}



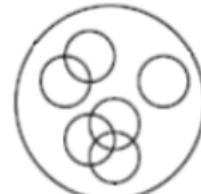
- Can study string overlaps:



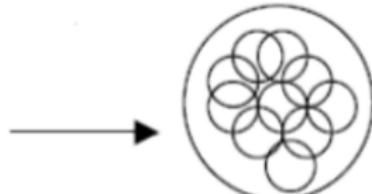
Multi-parton interactions



heavy ions



-->>> $\text{sqrt}(s)$ increases -->>>

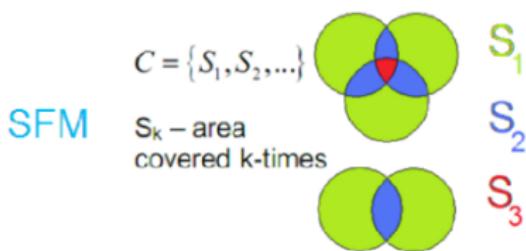


-->>>

$$Q^2(n) = \left(\sum_{i=1}^n \vec{Q}_i(1) \right)^2 = \sum_{i=1}^n Q_i^2(1) + \sum_{i \neq j} \vec{Q}_i(1) \cdot \vec{Q}_j(1)$$

$$\langle Q^2(n) \rangle = n Q^2(1)$$

overlaps



$$\langle \mu \rangle_k = \mu_0 \sqrt{k} \frac{S_k}{\sigma_0}$$

$$\langle p_t^2 \rangle_k = p_0^2 \sqrt{k}$$

$$\langle p_t \rangle_k = p_0 \sqrt[4]{k}$$

String fusion mechanism predicts (agrees with experiment):

- decrease of multiplicity
- increase of p_T
- growth of p_T with multiplicity in pp, pA and AA collisions
- growth of strange particle yields

Key parameter – transverse radius of the string r_{str} : larger string area – bigger overlapping

$r_{str} = 0$ - no fusion;

S_k – area, where k strings are overlapping, σ_0 single string transverse area, μ_0 and p_0 – mean multiplicity and transverse momentum from one string

M. A. Braun, C. Pajares, Nucl. Phys. B 390 (1993) 542.

M. A. Braun, R. S. Kolevatov, C. Pajares, V. V. Vechernin, Eur. Phys. J. C 32 (2004) 535.

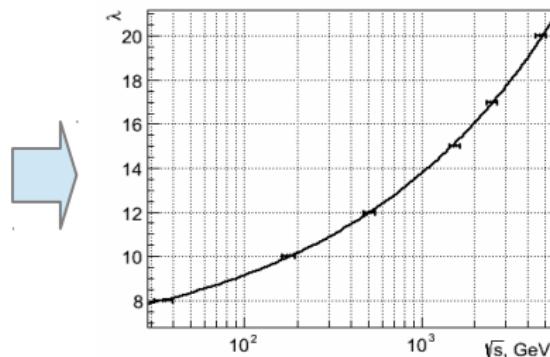
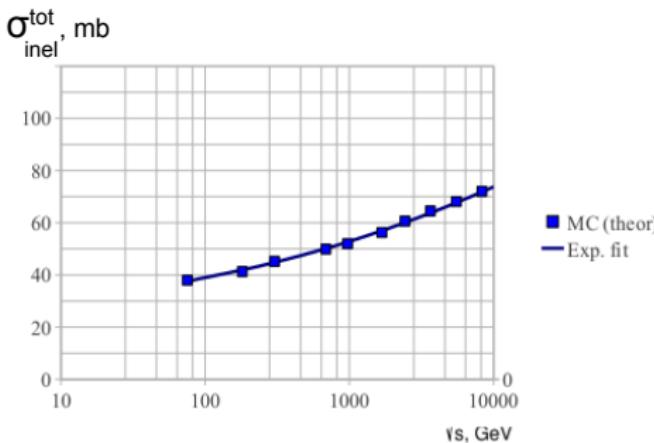
N.S. Amelin, N. Armesto, C. Pajares, D. Sousa, Eur.Phys.J.C22:149-163 (2001), arXiv:hep-ph/0103060

G. Ferreiro and C Pajares J. Phys. G: Nucl. Part. Phys. 23 1961 (1997)

p-p interaction: parameter fixing

Strategy for parameters fixing:

- Correspondence of mean number of dipoles λ and energy is obtained using data on total inelastic cross section
- Performed for each parameters combination and tabulated



p-p interaction: parameter fixing

Strategy for parameters fixing:

- Mean multiplicity per rapidity from one string μ_0 is fixed once at intermediate LHC energy (2.36 TeV)
- Data on energy dependence of multiplicity in pp collisions is used to constrain the rest of parameters
- p-Pb at 5.02 GeV minimum bias:
 $\langle dN/d\eta \rangle = 16.81 \pm 0.71$ [10]
- Look at PbPb collisions

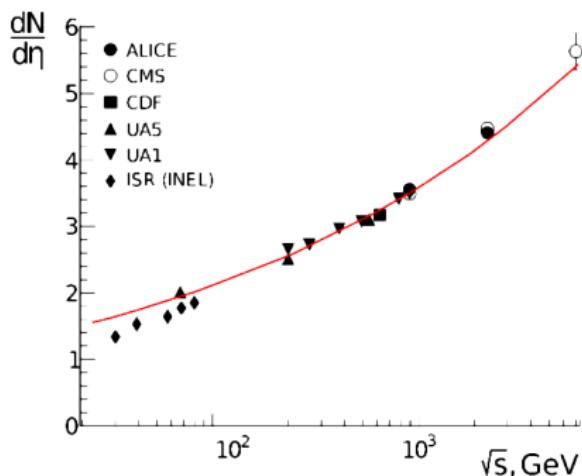


Fig. data points from: B. Abelev et al (ALICE Collaboration) Phys. Rev. Lett. 110, 032301, 2013

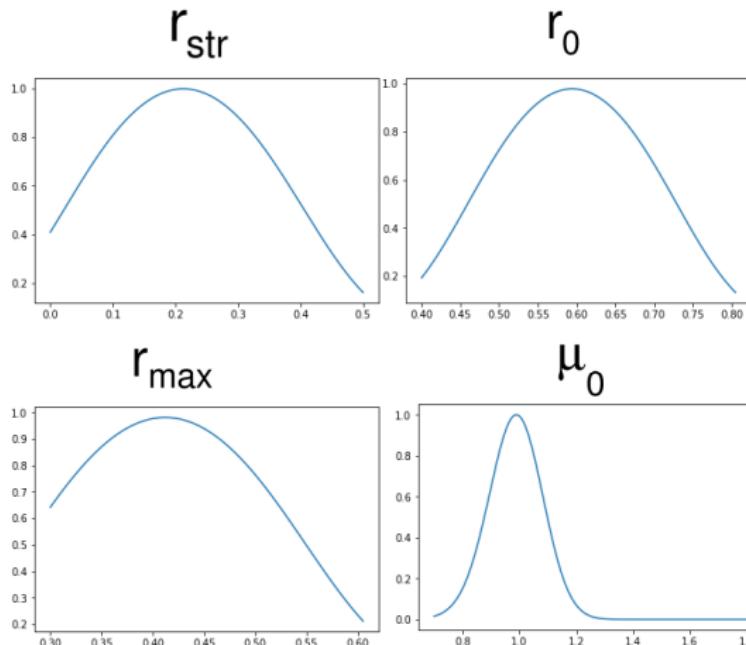
Initial distribution of parameters - uniform:

- r_0 : 0.4 – 0.7 fm
- r_{\max}/r_0 : 0.3 – 0.6
- α_s : 0.2 – 2.8
- r_{str} : 0 (no fusion) - 0.6 fm
- Energy range: 53 – 7000 GeV

Posterior distributions after accounting of pp multiplicity

pp multiplicity in the model

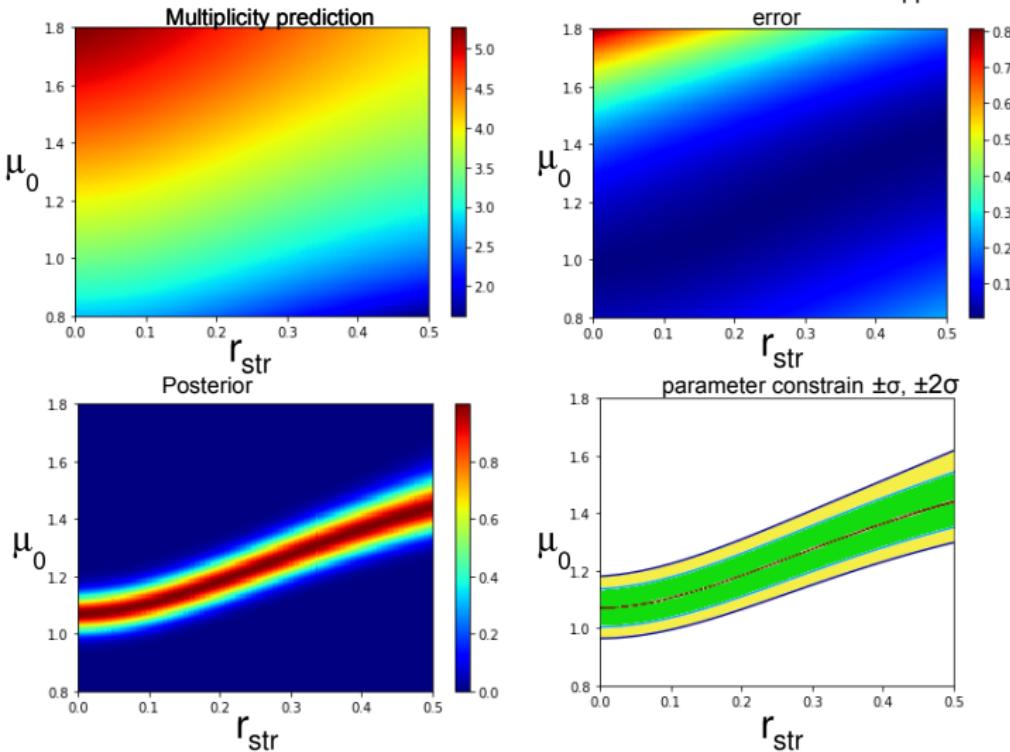
- Not much sensitive to string fusion r_{str}
- r_0 around 0.6 is favoured
- r_{max} and α_s : not well restricted
- μ_0 : peaks around 1.0



Posterior parameter estimation from energy dependence of pp multiplicity

- Focus on string related parameters: r_{str} and μ_0 :

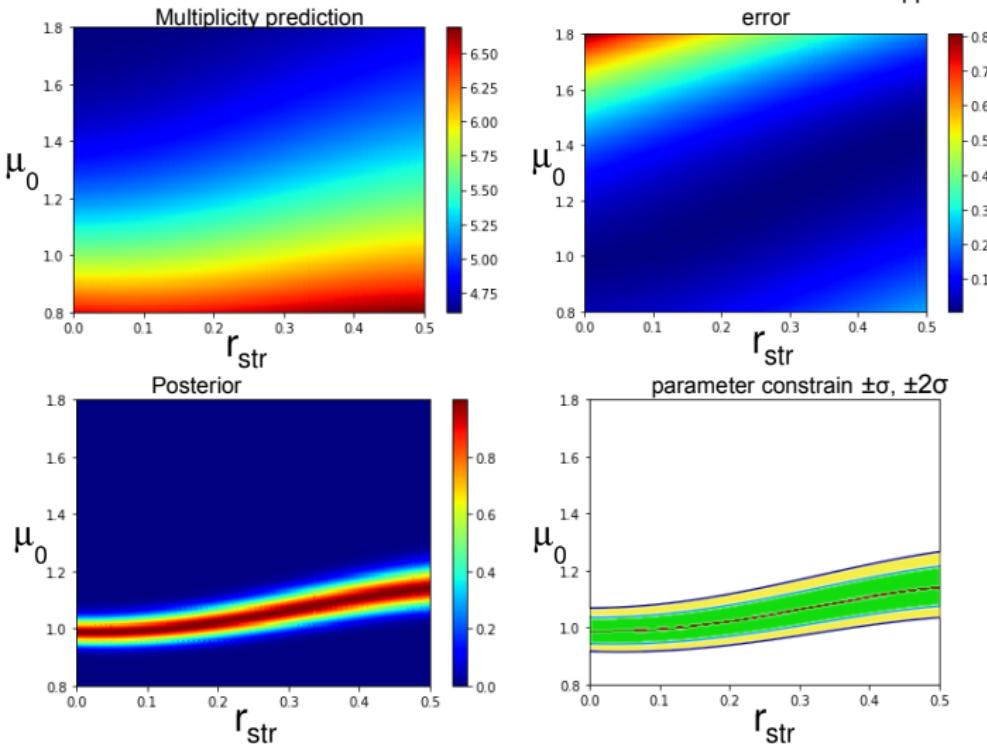
Experimental data:
 $\mu_{\text{pp}} = 3.61 \pm 0.17$ at 0.9 TeV



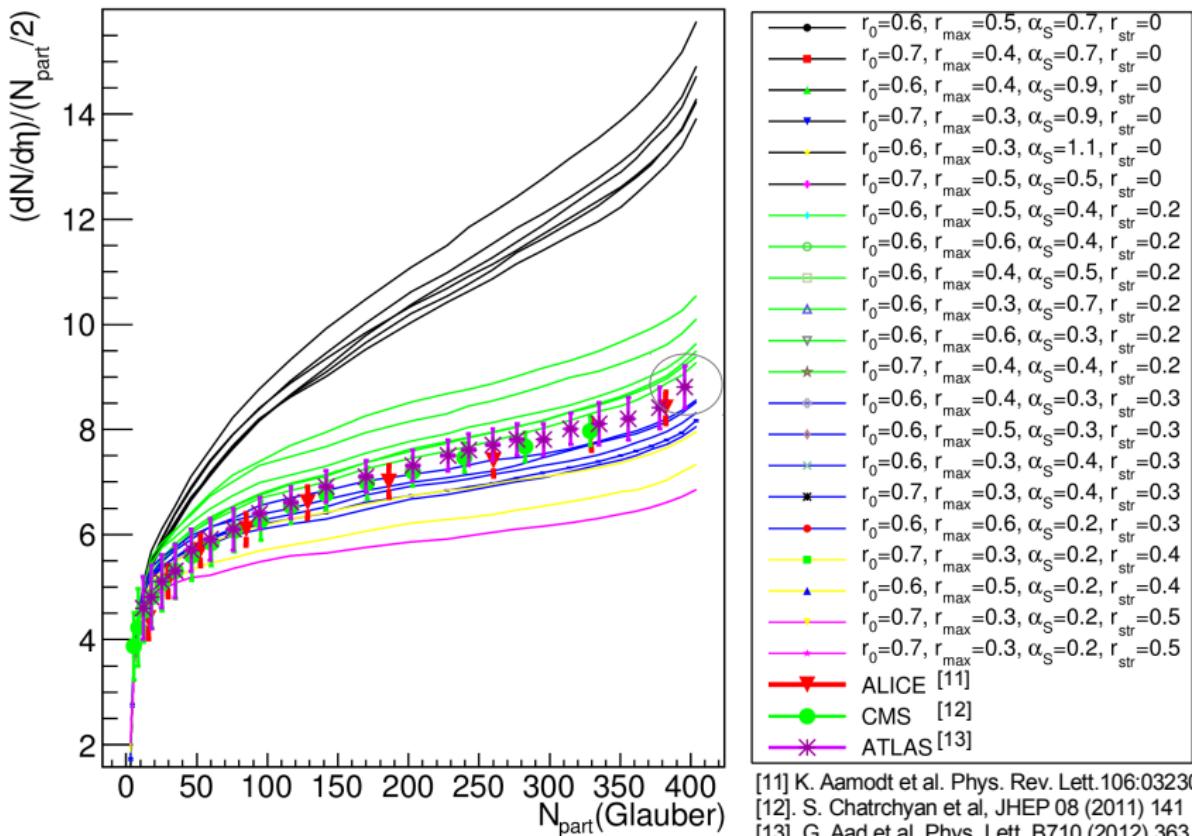
Posterior parameter estimation from energy dependence of pp multiplicity

- Focus on string related parameters: r_{str} and μ_0 :

Experimental data:
 $\mu_{\text{pp}} = 5.74 \pm 0.15$ at 7.0 TeV



Results: PbPb collisions at 2.76 TeV

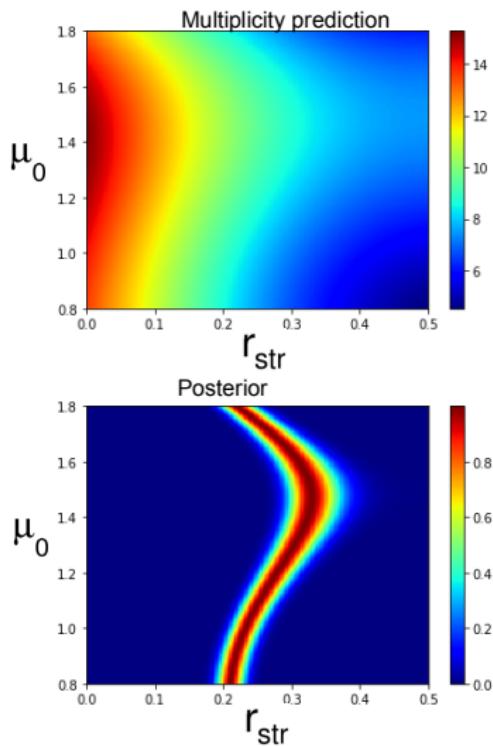


[11] K. Aamodt et al. Phys. Rev. Lett. 106:032301, 2011

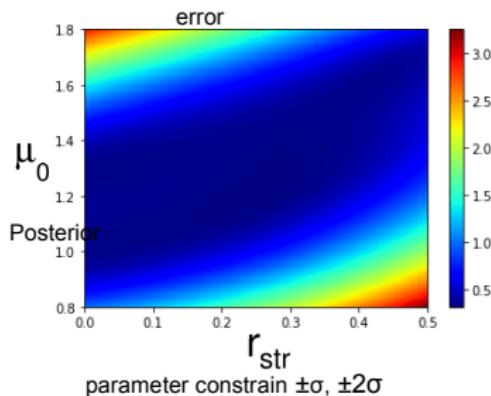
[12] S. Chatrchyan et al. JHEP 08 (2011) 141

[13] G. Aad et al. Phys. Lett. B710 (2012) 363

Posterior parameter estimation from central PbPb collisions at 2.76 TeV

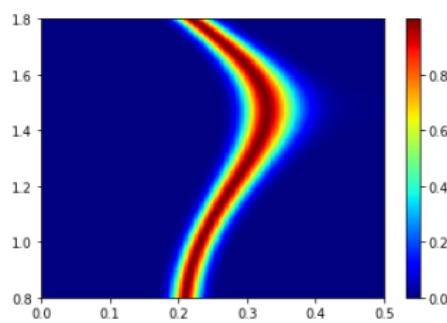


Experimental data:
 $\mu_{\text{PbPbcentral}}/\text{Npart}^2 = 8.4 \pm 0.4$

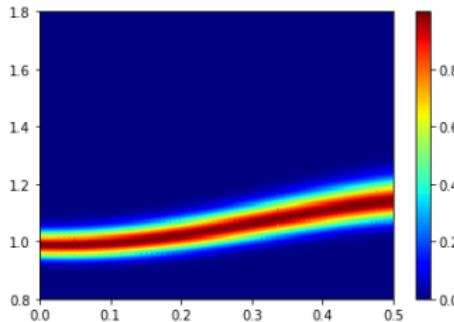


Combined results on Posterior and parameter estimation

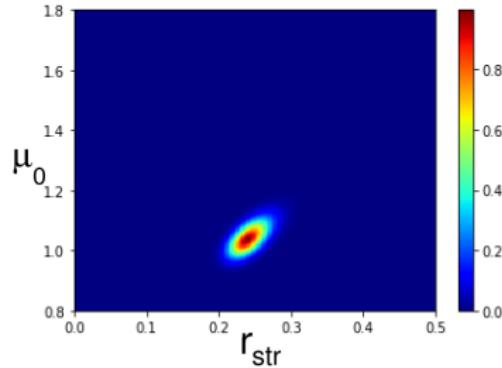
PbPb collisions at 2.76 TeV plus energy dependence of multiplicity in pp collisions



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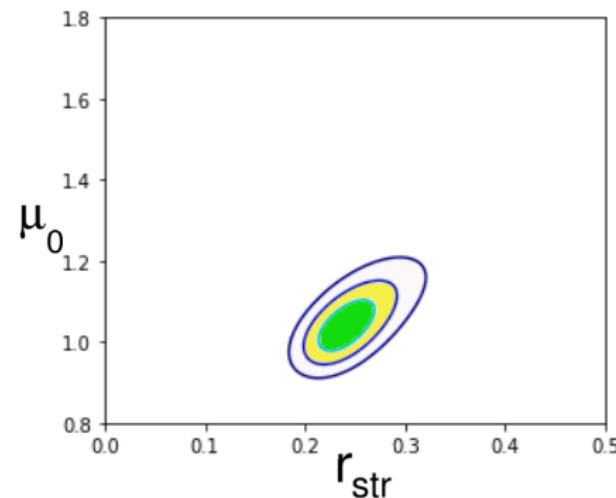
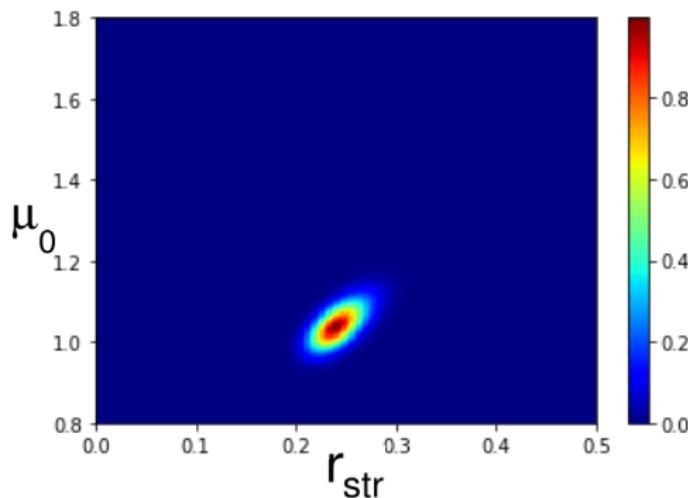


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Combined results on Posterior and parameter estimation

PbPb collisions at 2.76 TeV plus energy dependence of multiplicity in pp collisions



Estimation

$$r_{\text{str}} = 0.25 \pm 0.03 \text{ fm}$$

$$\mu_0 = 1.1 \pm 0.03$$

Conclusions

- Bayesian Gaussian process optimization has been applied for the parameter tuning of the non-Glauber Monte-Carlo with string fusion
- In the model the inelastic cross section and multiplicity are described wide energy range and for different colliding systems.
- Multiplicity per rapidity from one single string μ_0 is constrained by the energy dependence of the multiplicity in pp collisions
- The transverse radius of string r_{str} (string fusion parameter) is constrained by multiplicity in central PbPb collisions.

Outlook

- Improvement of the parameter estimation by considering more data
- Extension of the energy range
- Application of the Principal Component Analysis to observables

Backup

Model predictions for PbPb collisions at 2.76 TeV

