

# Neutron Stars Chirp about Vacuum Energy

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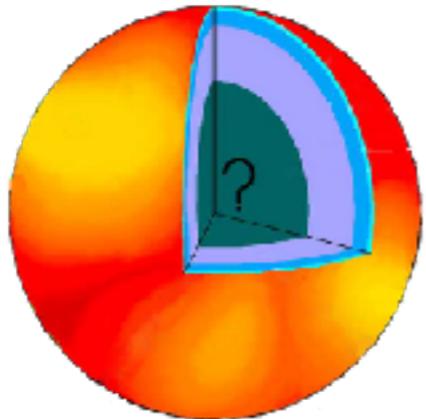
astro-ph.CO/1502.04702

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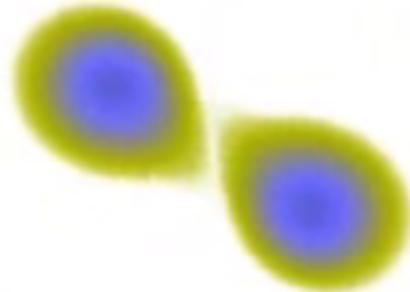
# Outline

$\Lambda$

Motivation: observe  
changing vacuum energy



Neutron Stars



Tidal Deformability

# The Evolution of Vacuum Energy

$\Lambda$

The cosmological constant is very small today

$$\Lambda \sim (10^{-3} \text{ eV})^4$$

from quantum field theory we expect

$$(TeV)^4, M_{Pl}^4$$

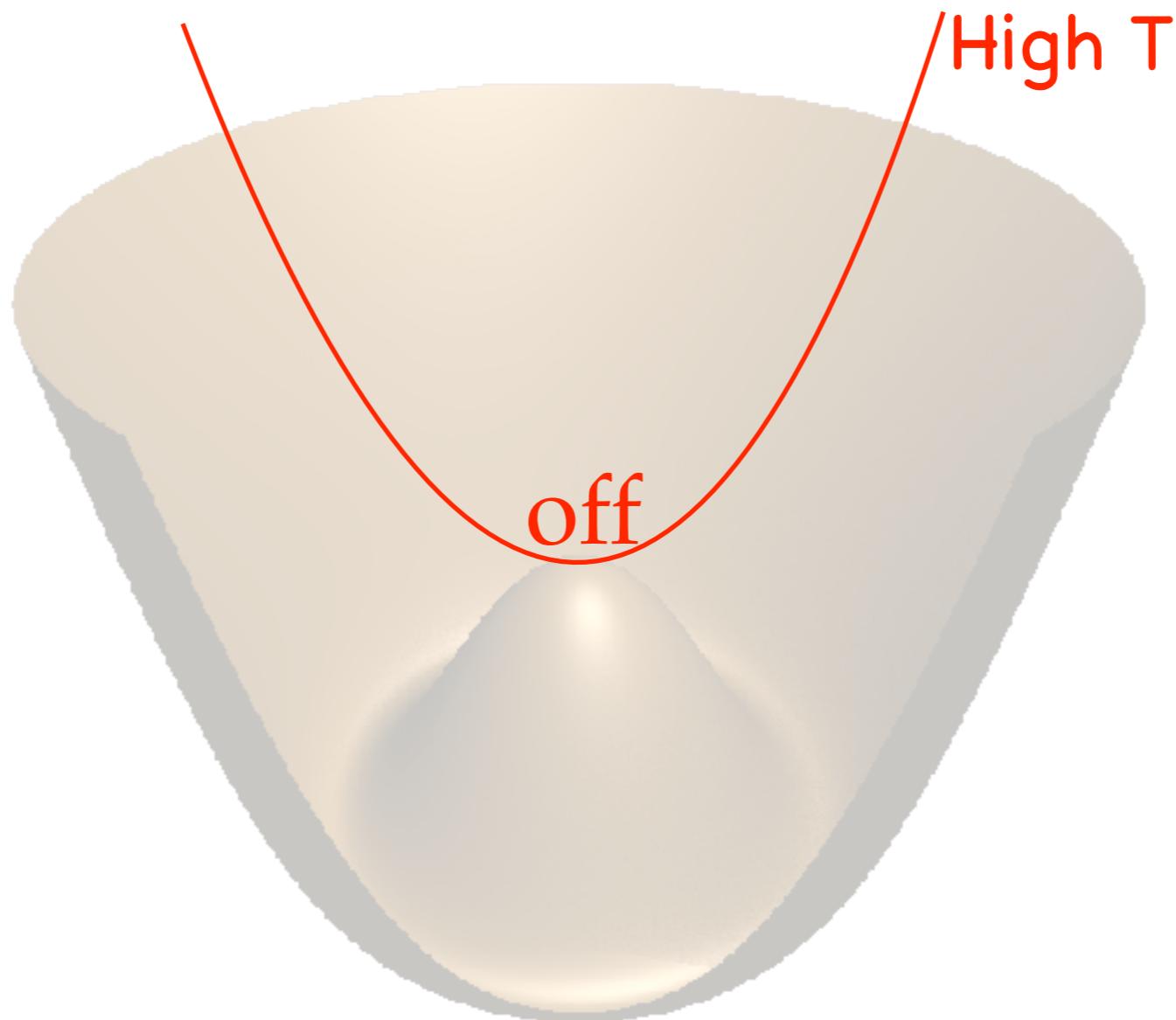
Why so small? Why not zero?

Is there an adjustment mechanism?  
Is it always very small?

# Vacuum Energy and Electroweak PT

$\Lambda$

$$\rho = \frac{\rho_R}{a^4} + \frac{\rho_m}{a^3} + \Lambda$$

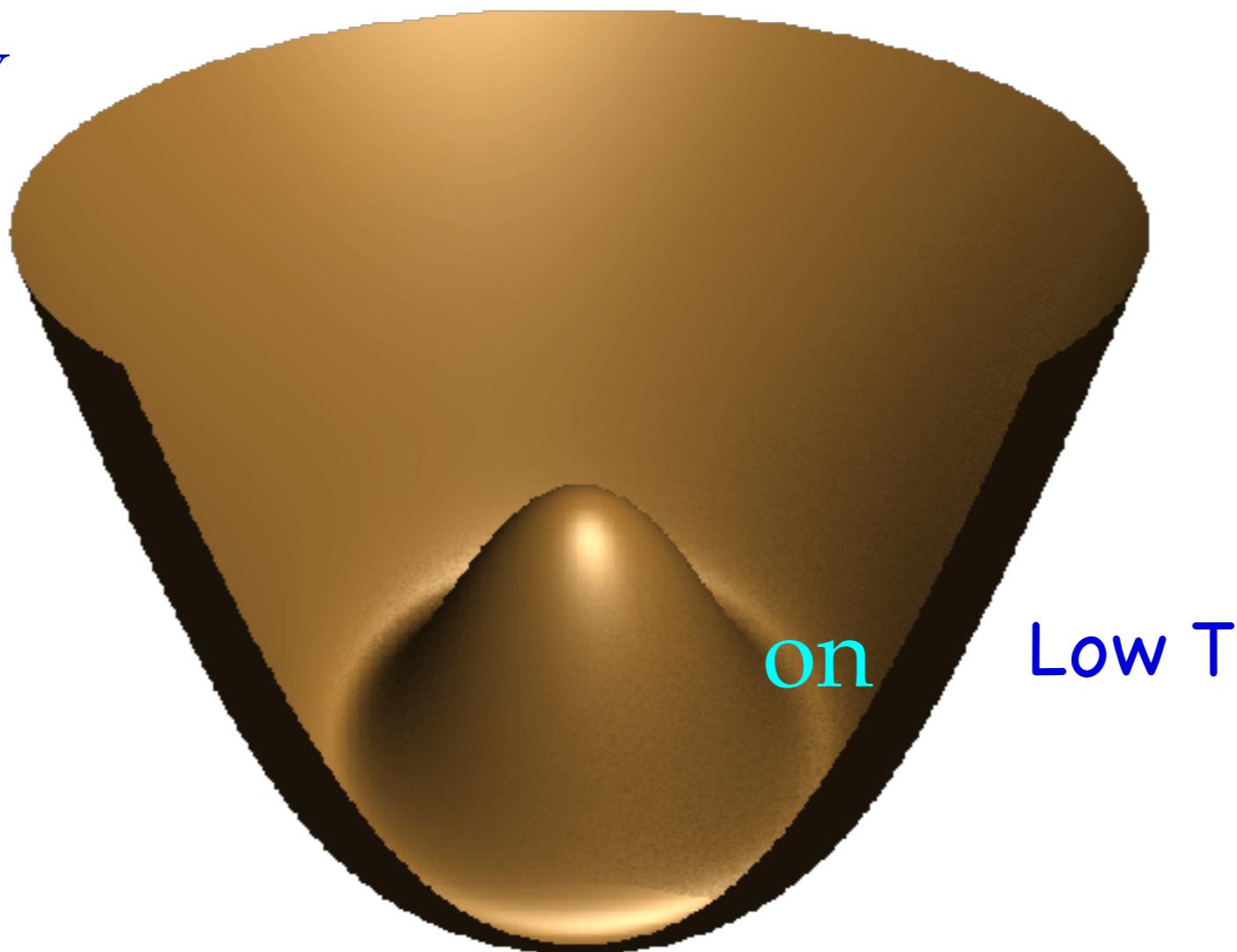


# Vacuum Energy and Electroweak PT

$\Lambda$

$$\rho = \frac{\rho_R}{a^4} + \frac{\rho_m}{a^3} + \Lambda$$

$$\Delta\Lambda \sim -M_W^4$$



# Vacuum Energy and Electroweak PT

$\Lambda$

$$\Delta\Lambda \sim -M_W^4$$

$$\Lambda + \Delta\Lambda \sim \mathcal{O}(\Lambda_{QCD}^4)$$

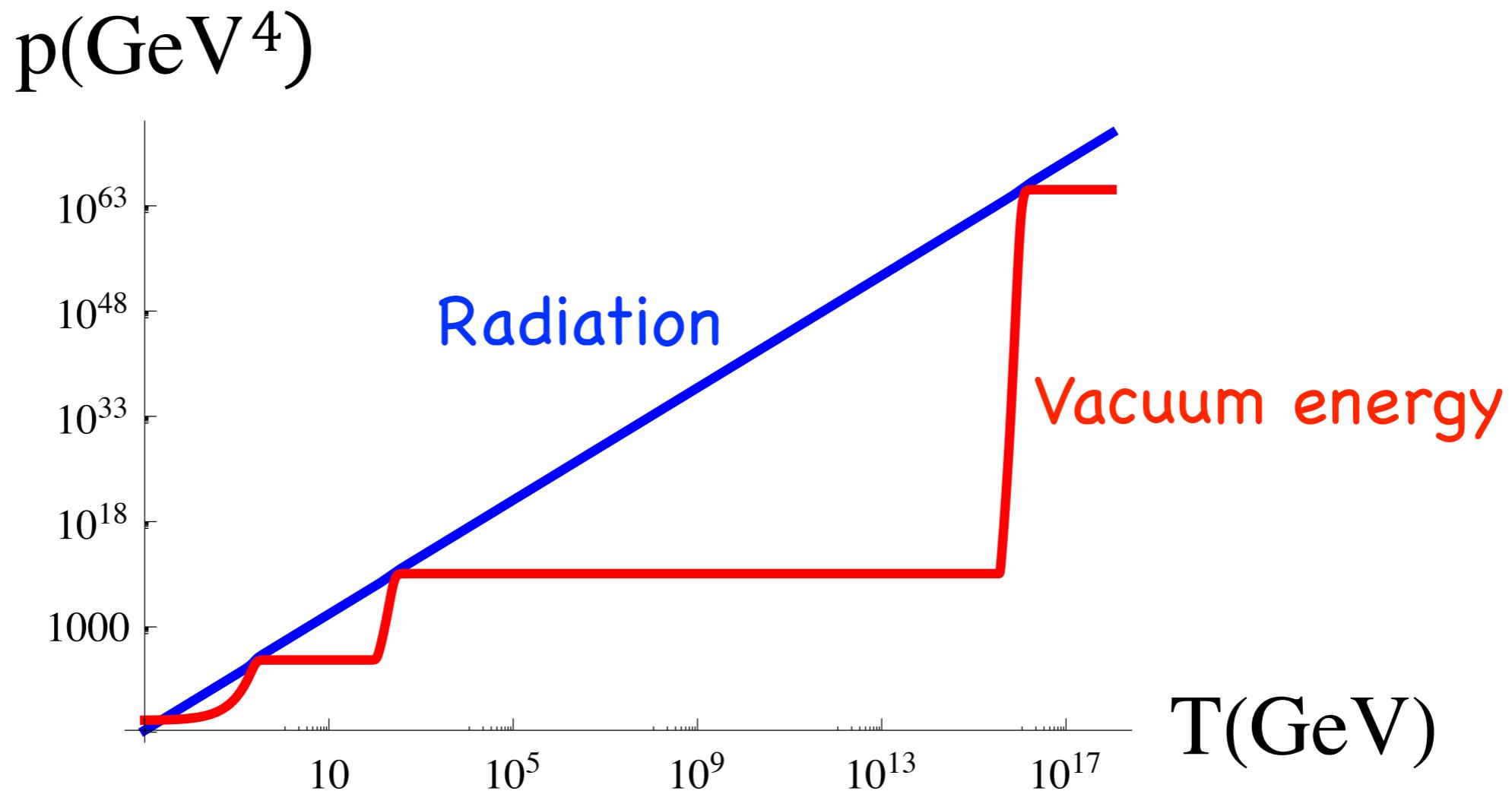
tuning

At one phase transition Universe already “knows”  
where the next phase transition will be

previously  $\Lambda$  was much larger than now,  
but never dominated previously!

# Sketch of vacuum energy evolution

$\Lambda$



Size of step of order  $T_{c,i}^4$

Amount of tuning:  $T_{c,i+1}^4$

$\Lambda$

# Steps or adjustment?

If verified:  
lends credence to anthropics

Difficulty:  $\Lambda$  always sub-dominant

Last transition at  $\Lambda_{\text{QCD}}$

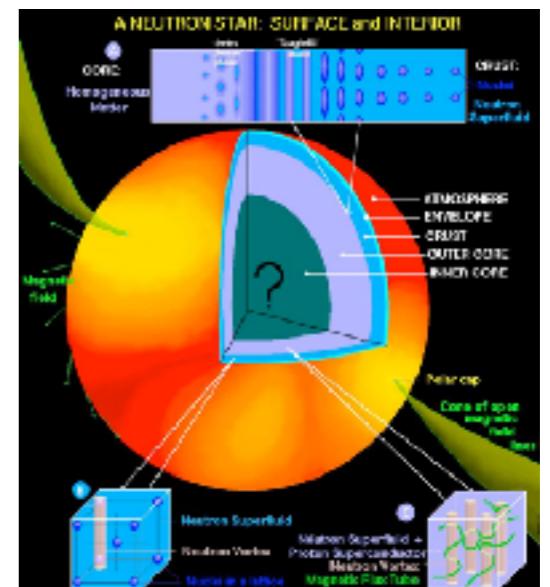
Above CMB, BBN, No direct tests...

# Where should we look?

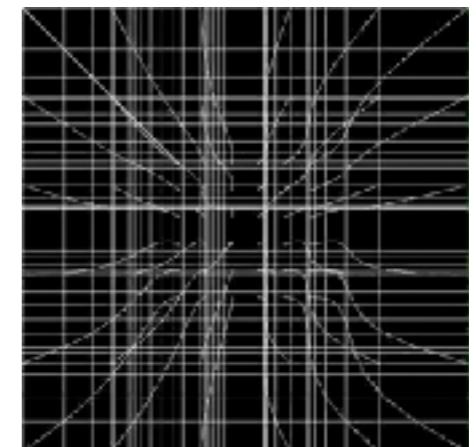
$\Lambda$

System where vacuum energy can be a significant fraction of total energy

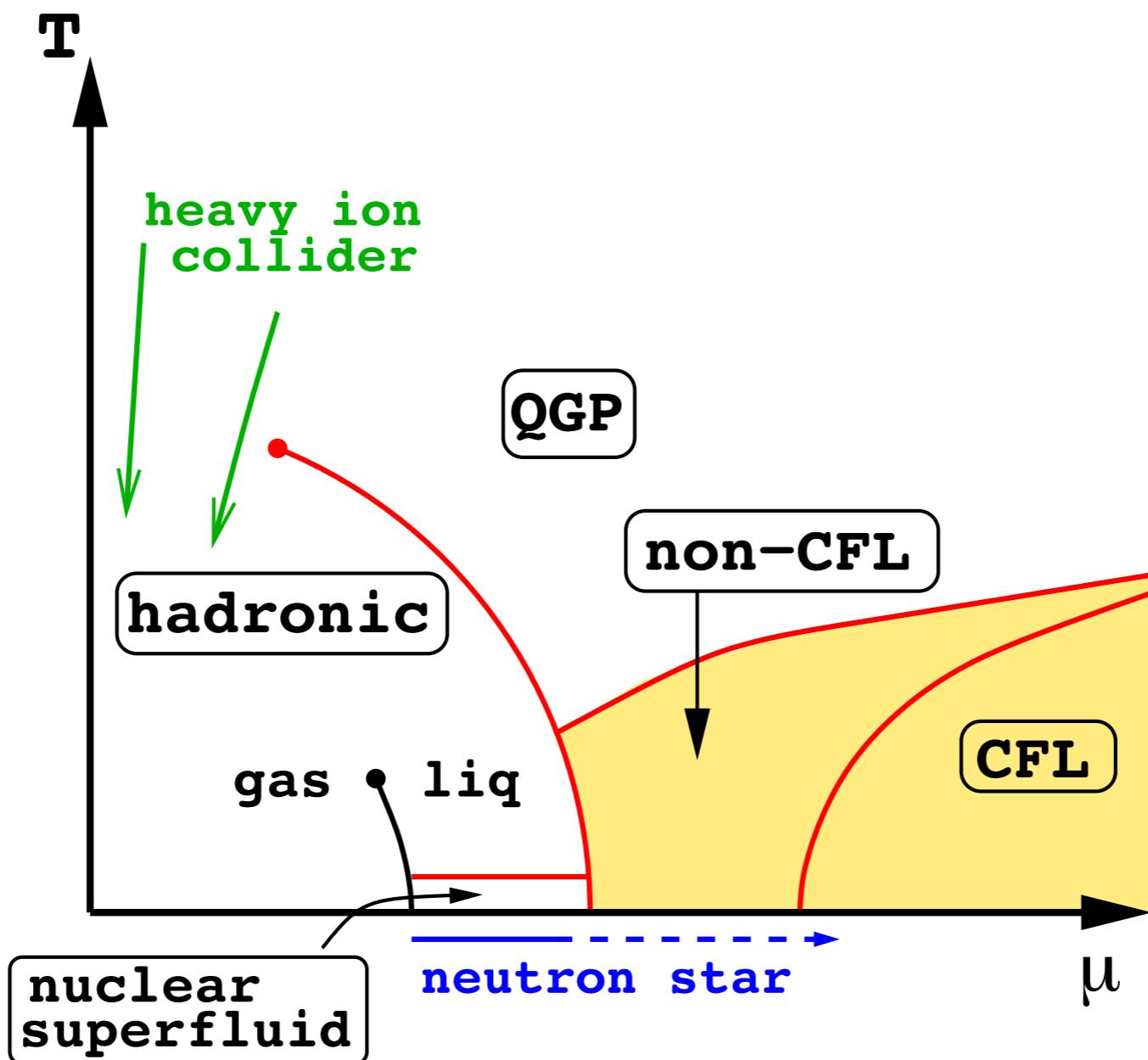
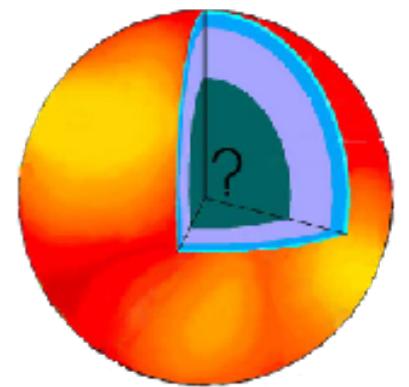
Neutron stars



primordial gravitational waves  
passing through  
Cosmic Phase Transitions

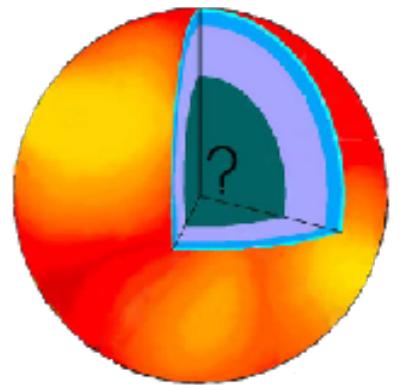


# High Density Phases of QCD



Alford, Schmitt, Rajagopal, Schaefer hep-ph/0709.4635

# Model for neutron stars



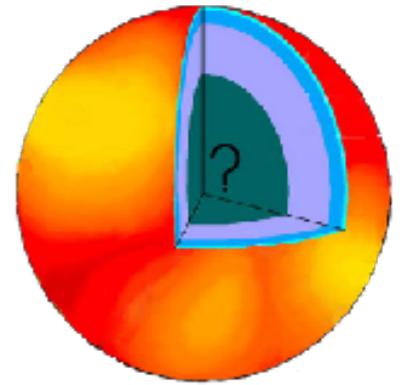
At zero temperature, gravitational pressure balanced by pressure of fluid

$$ds^2 = e^{\nu(r)} dt^2 - (1 - 2Gm(r)/r)^{-1} dr^2 - r^2 d\Omega^2$$

Einstein eqs (aka Tolman-Oppenheimer-Volkoff):



# Toy model for neutron stars



outer core

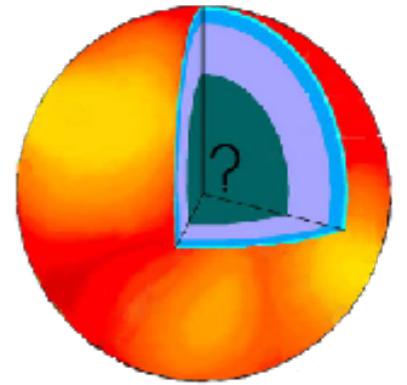
$$p_{\text{out}} = K_{\text{out}} \rho^{\gamma_{\text{out}}}$$

$$\epsilon_{\text{out}} = (1 + a_{\text{out}}) \rho + \frac{K_{\text{out}}}{\gamma_{\text{out}} - 1} \rho^{\gamma_{\text{out}}}$$

$$\rho = m_n n$$

$$d(\epsilon/\rho) = -p d(1/\rho)$$

# Toy model for neutron stars



outer core

$$p_{\text{out}} = K_{\text{out}} \rho^{\gamma_{\text{out}}}$$

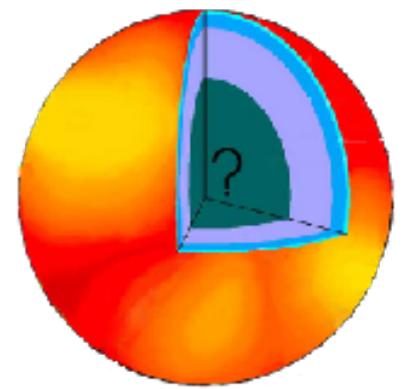
$$\epsilon_{\text{out}} = (1 + a_{\text{out}}) \rho + \frac{K_{\text{out}}}{\gamma_{\text{out}} - 1} \rho^{\gamma_{\text{out}}}$$

inner core

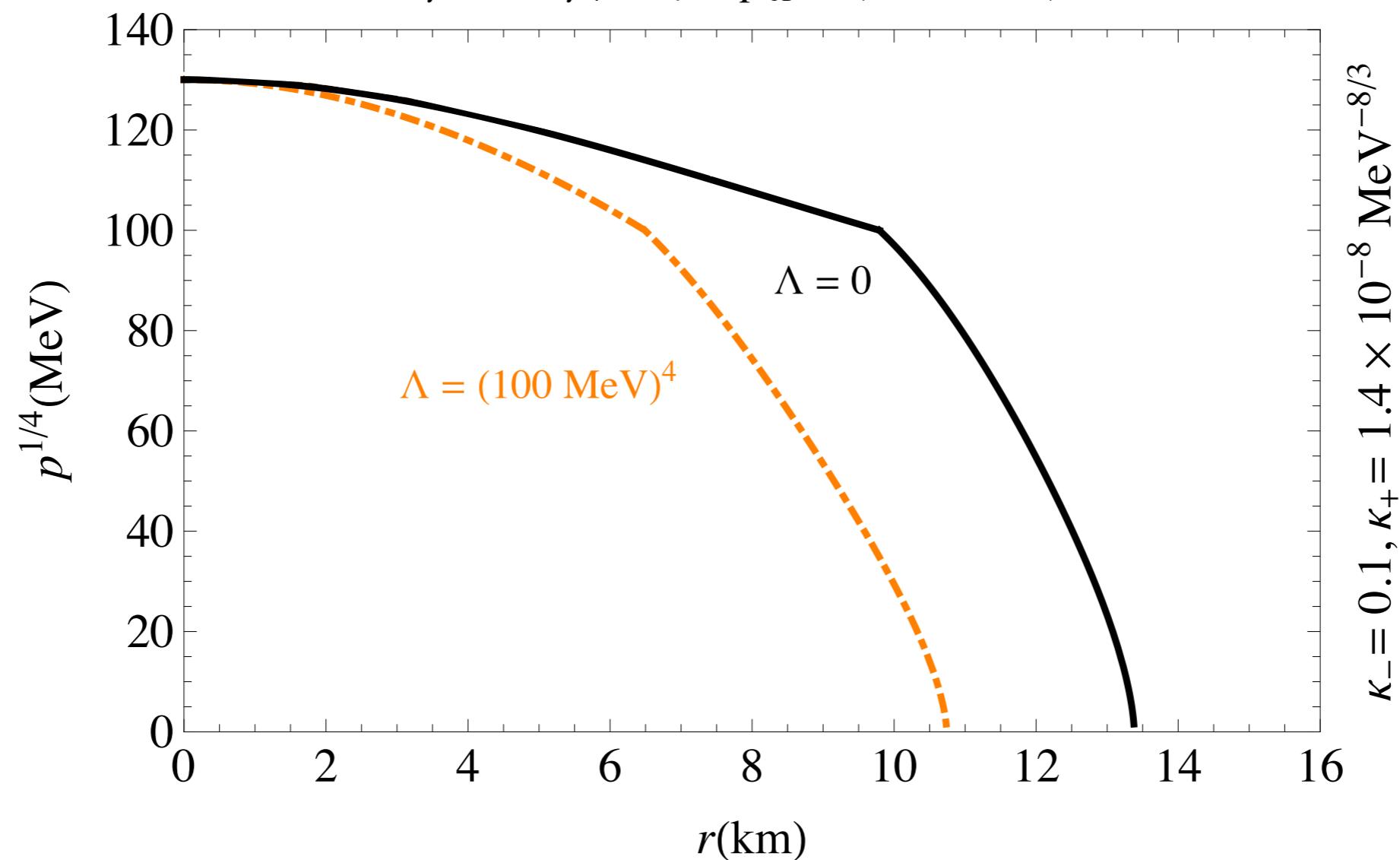
$$p_{\text{in}} = K_{\text{in}} \rho^{\gamma_{\text{in}}} - \Lambda$$

$$\epsilon_{\text{in}} = (1 + a_{\text{in}}) \rho + \frac{K_{\text{in}}}{\gamma_{\text{in}} - 1} \rho^{\gamma_{\text{in}}} + \Lambda$$

# Larger $\Lambda$ leads to a smaller star

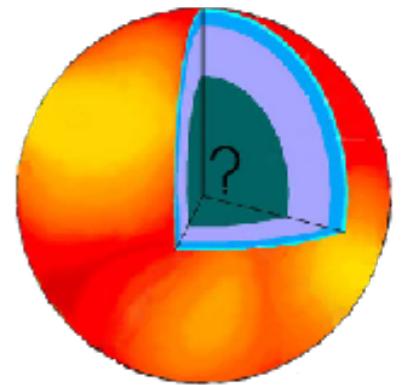


$$\gamma_- = 1, \gamma_+ = 5/3, p_{\text{cr}} = (100 \text{ MeV})^4$$



$$p'(r) = -\frac{p(r) + \epsilon(r)}{r(r - 2Gm(r))} G [m(r) + 4\pi r^3 p(r)]$$

# State of the Art models

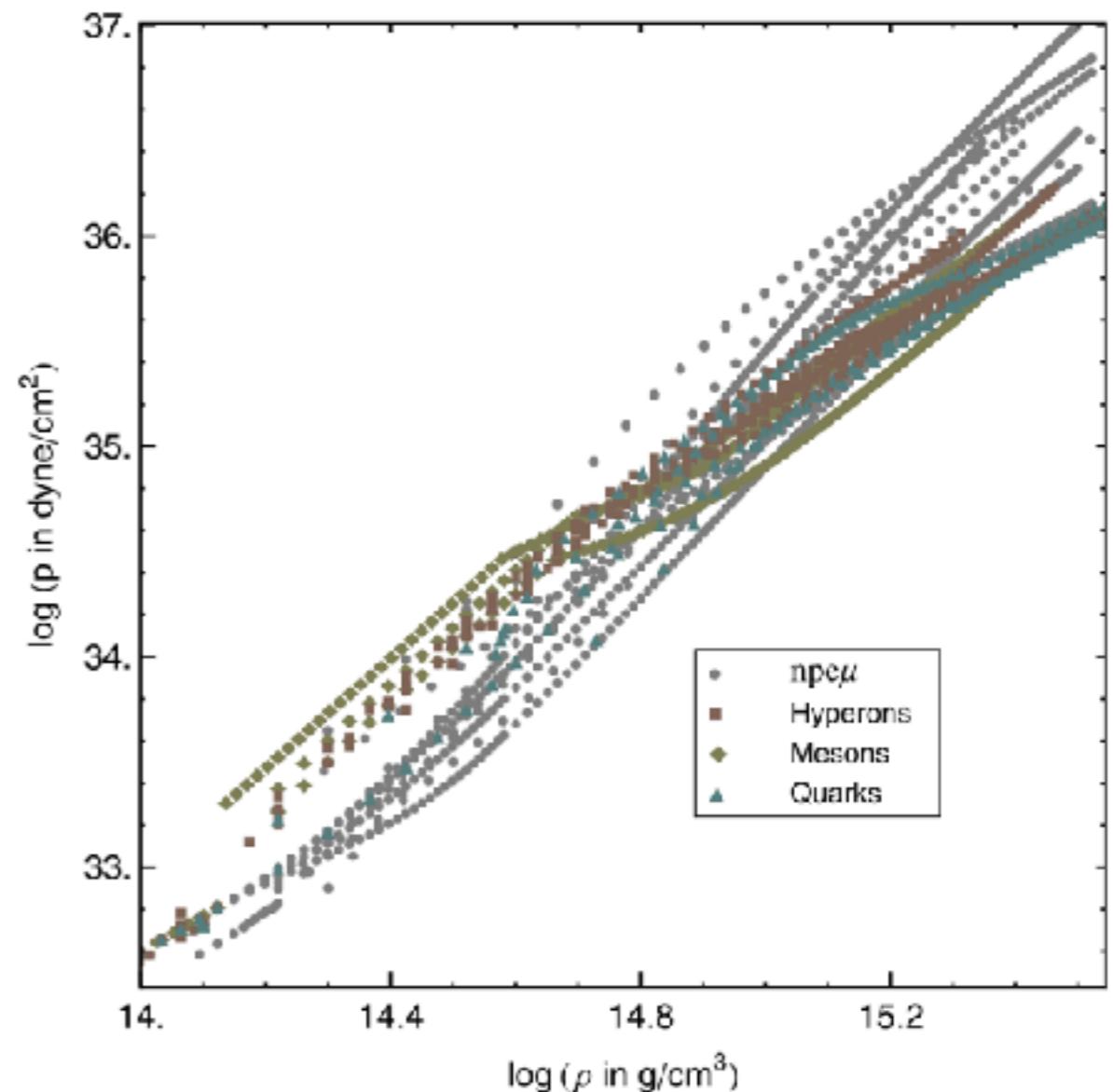


7 Layers

$$p = K_i \rho^{\gamma_i} , \quad p_{i-1} \leq p \leq p_i$$

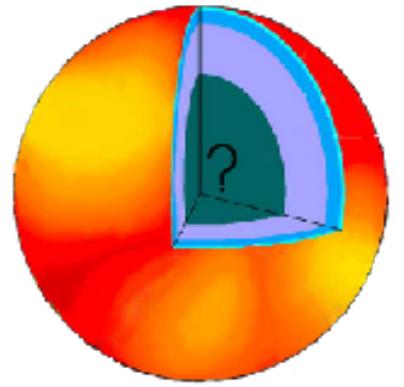
$$\epsilon = (1 + a_i) \rho + \frac{K_i}{\gamma_i - 1} \rho^{\gamma_i}$$

piece-wise polytopes



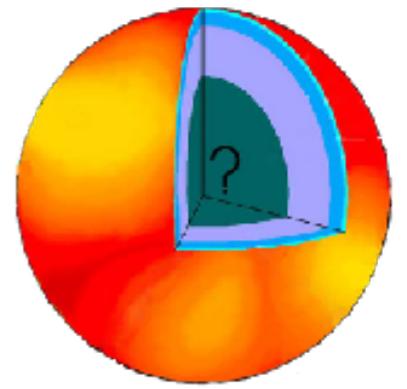
Read et. al. astro-ph/0812.2163

# Phase Transition



$$p_7 = K_7 \rho^{\gamma_7} - \Lambda$$

$$\epsilon_7 = (1 + a_7) \rho + \frac{K_7}{\gamma_7 - 1} \rho^{\gamma_7} + \Lambda$$



# Phase Transition

$$p_7 = K_7 \rho^{\gamma_7} - \Lambda$$

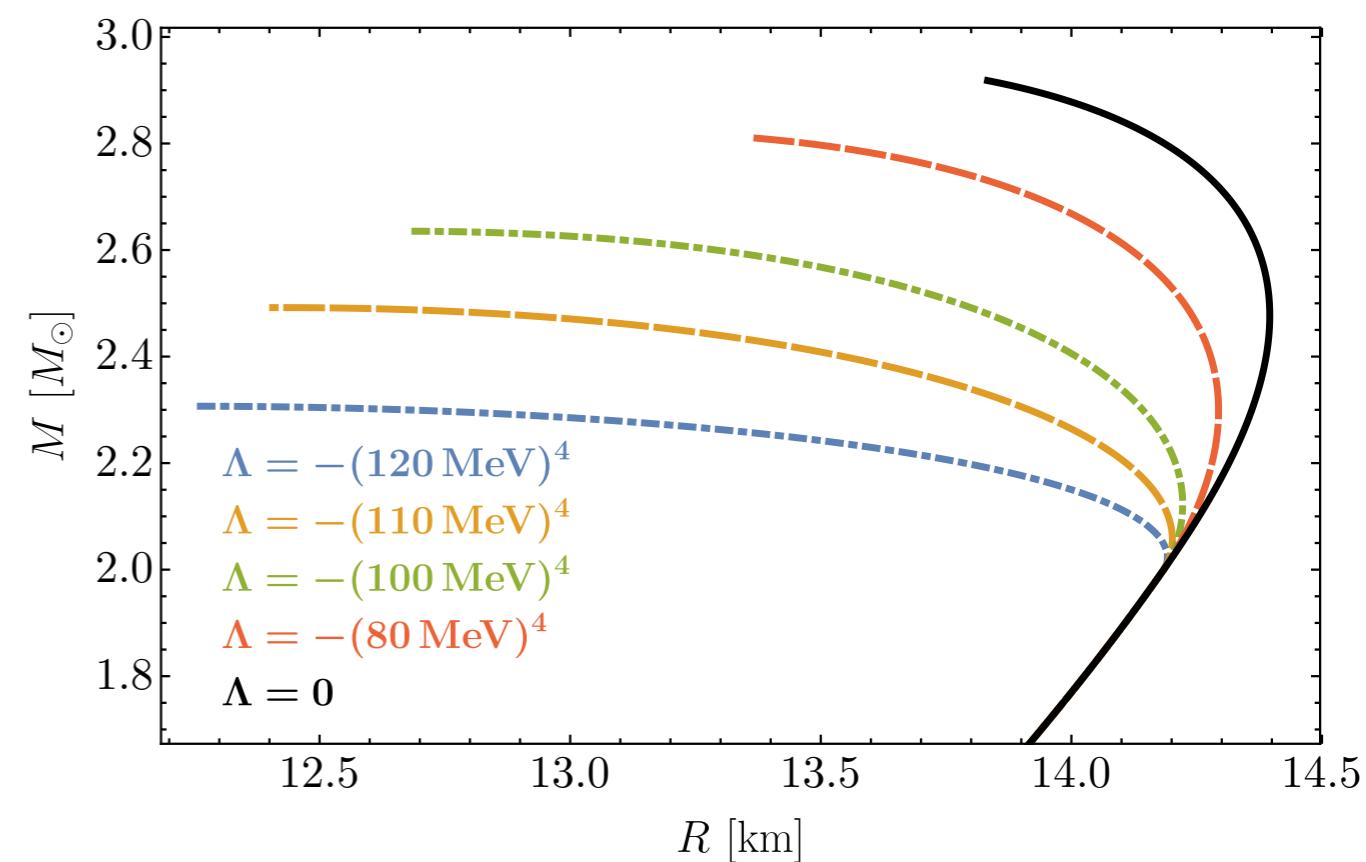
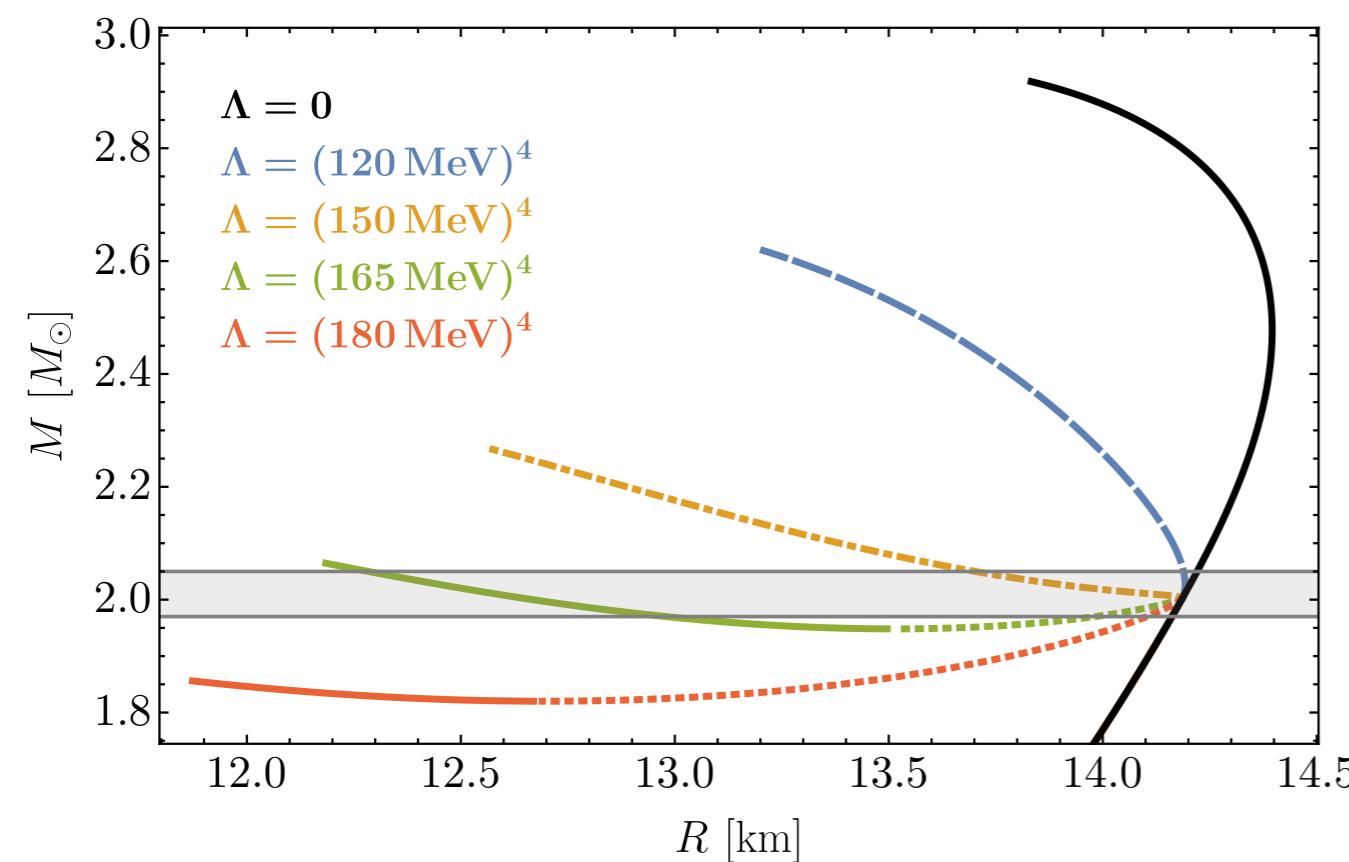
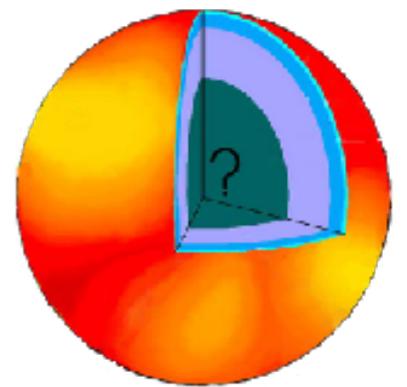
$$\epsilon_7 = (1 + a_7) \rho + \frac{K_7}{\gamma_7 - 1} \rho^{\gamma_7} + \Lambda$$

chemical potential:  $\mu = \frac{\epsilon + p}{n}$

$$\frac{\epsilon_+ + p_6}{\rho_+} = \frac{\epsilon_- + p_6}{\rho_-}$$

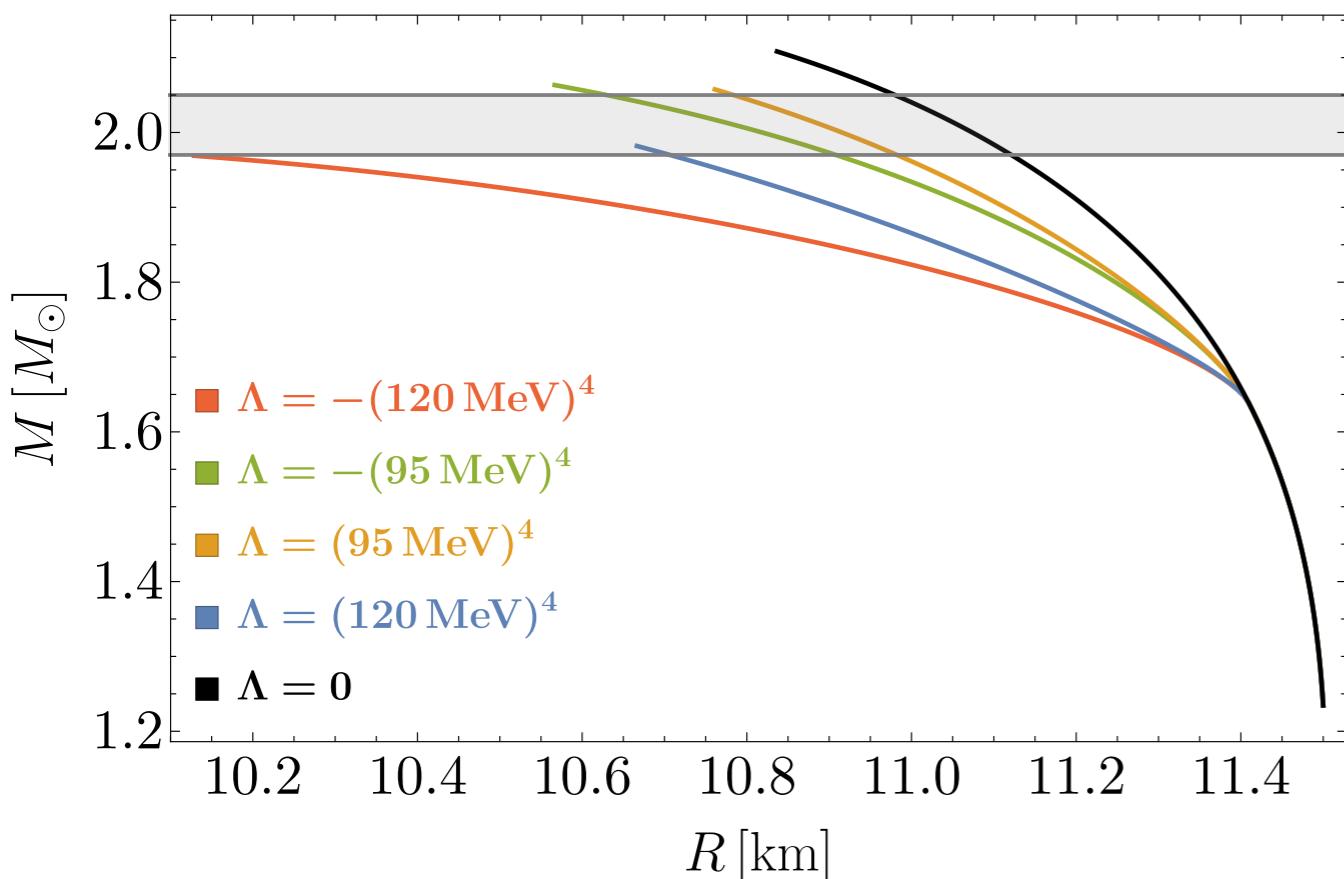
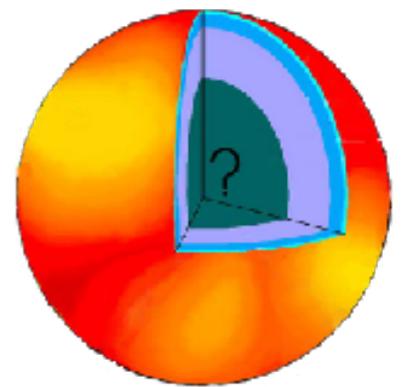
$$\epsilon_+ - \epsilon_- = \alpha |\Lambda|$$

# $M(R)$ for neutron stars

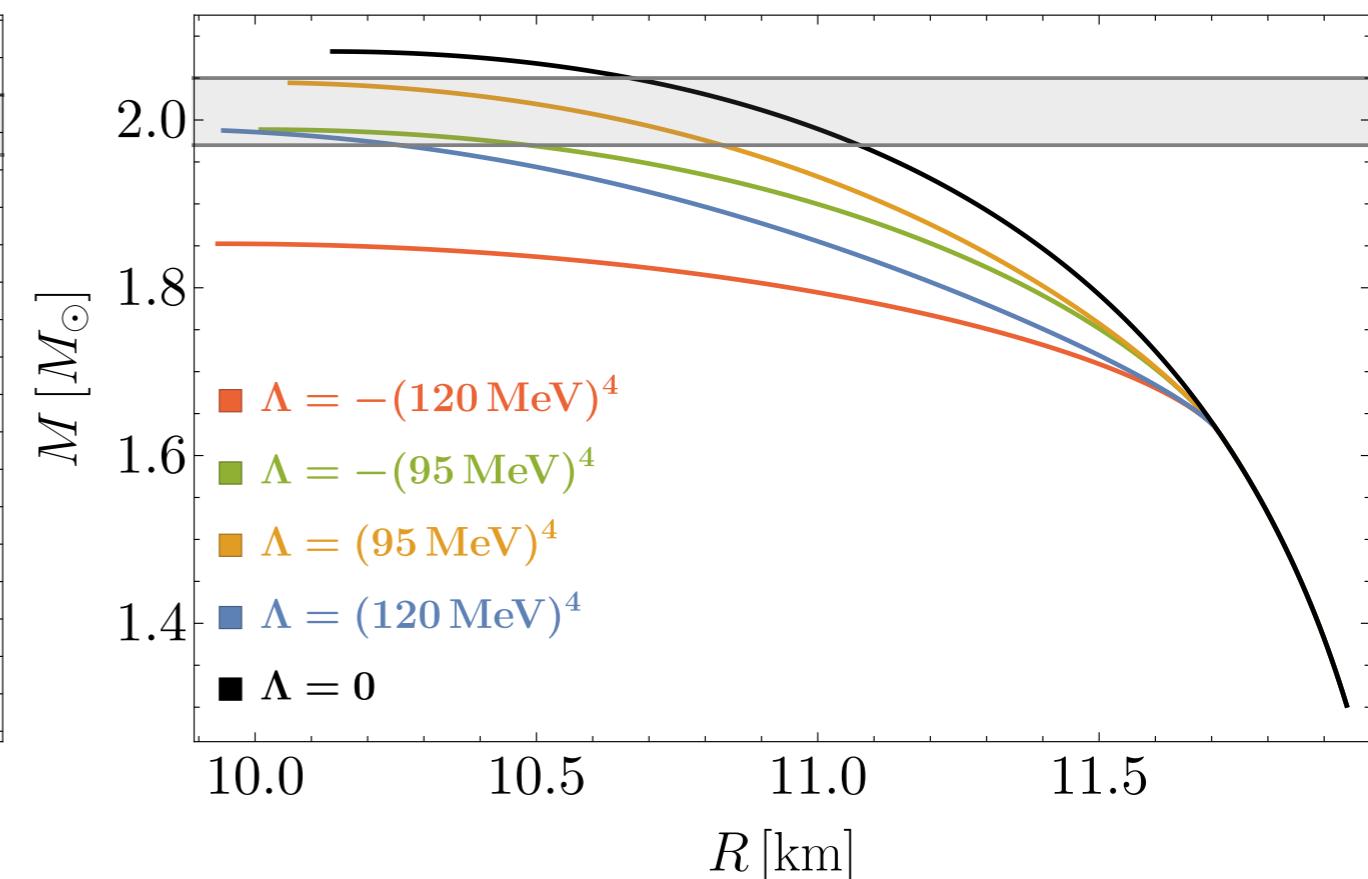


Hebeler parameterization [astro-ph/1303.4662](https://arxiv.org/abs/1303.4662)

# $M(R)$ for neutron stars

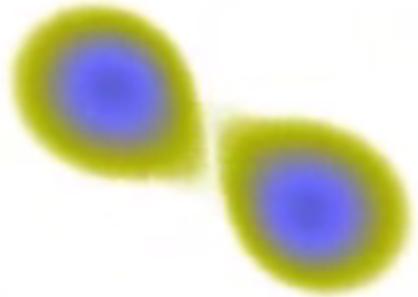


AP4



SLy

# Tidal Love Number



external tidal field

$$\frac{1 + g_{tt}}{2} \approx \frac{GM}{r} + \frac{3GQ_{ij}}{2r^5}x^i x^j - \frac{1}{2}\mathcal{E}_{ij}x^i x^j \dots$$

induced quadrupole  
moment

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\ell = 2 \text{ tidal Love number} \quad k_2 = \frac{3}{2} \frac{G\lambda}{R^5}$$



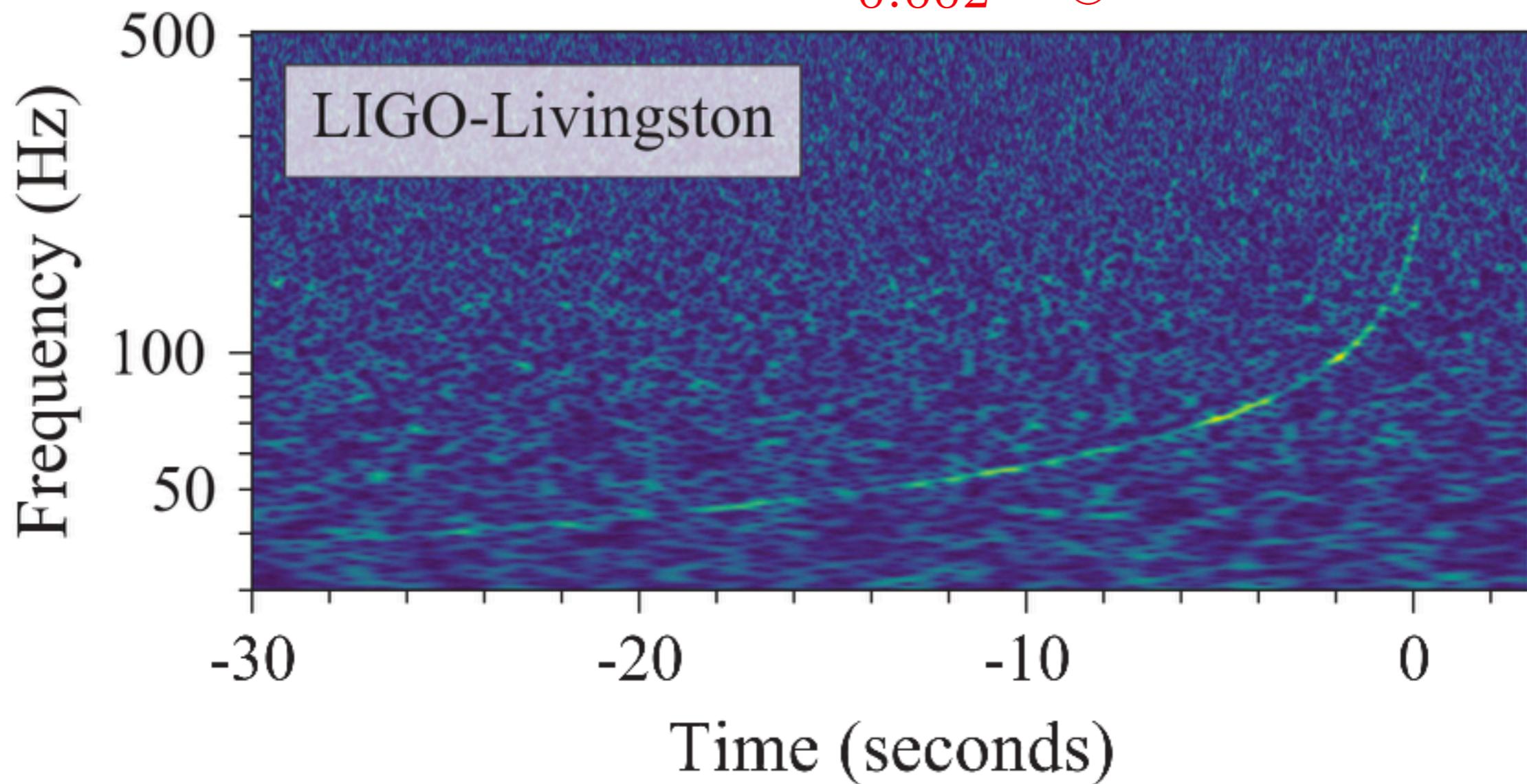
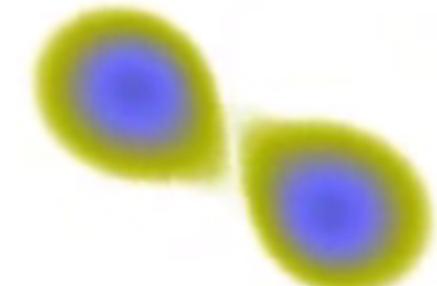
dimensionless  
tidal Love number

$$\bar{\lambda} = \frac{2k_2}{3C^5} = \frac{\lambda}{G^4 M^5}$$



# LIGO GW170817

$$\mathcal{M} = 1.188_{-0.002}^{+0.004} M_{\odot}$$



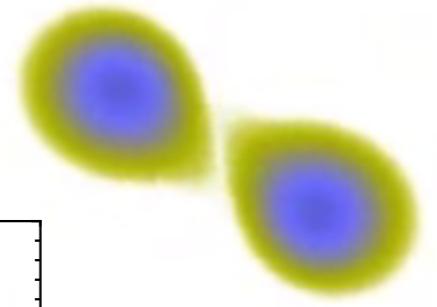
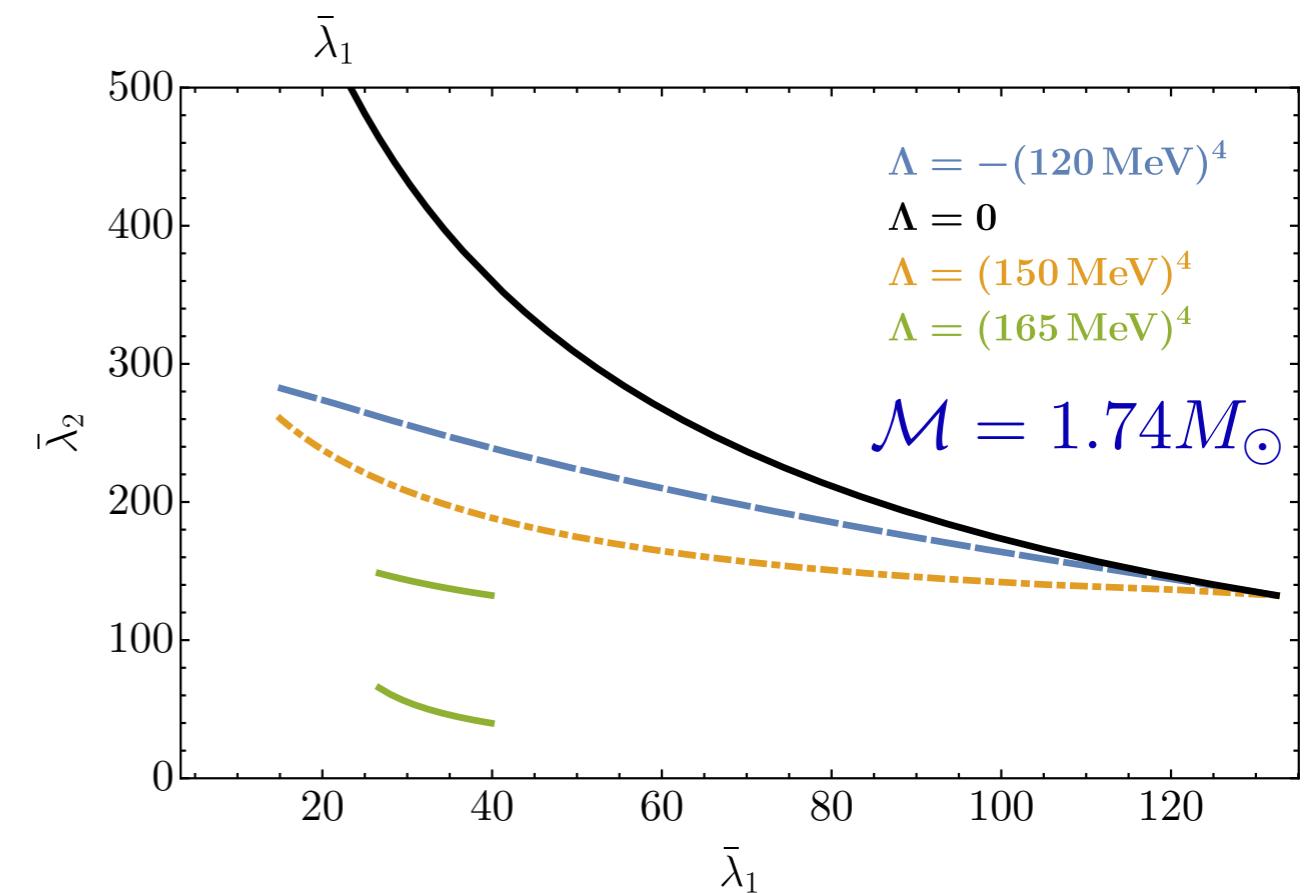
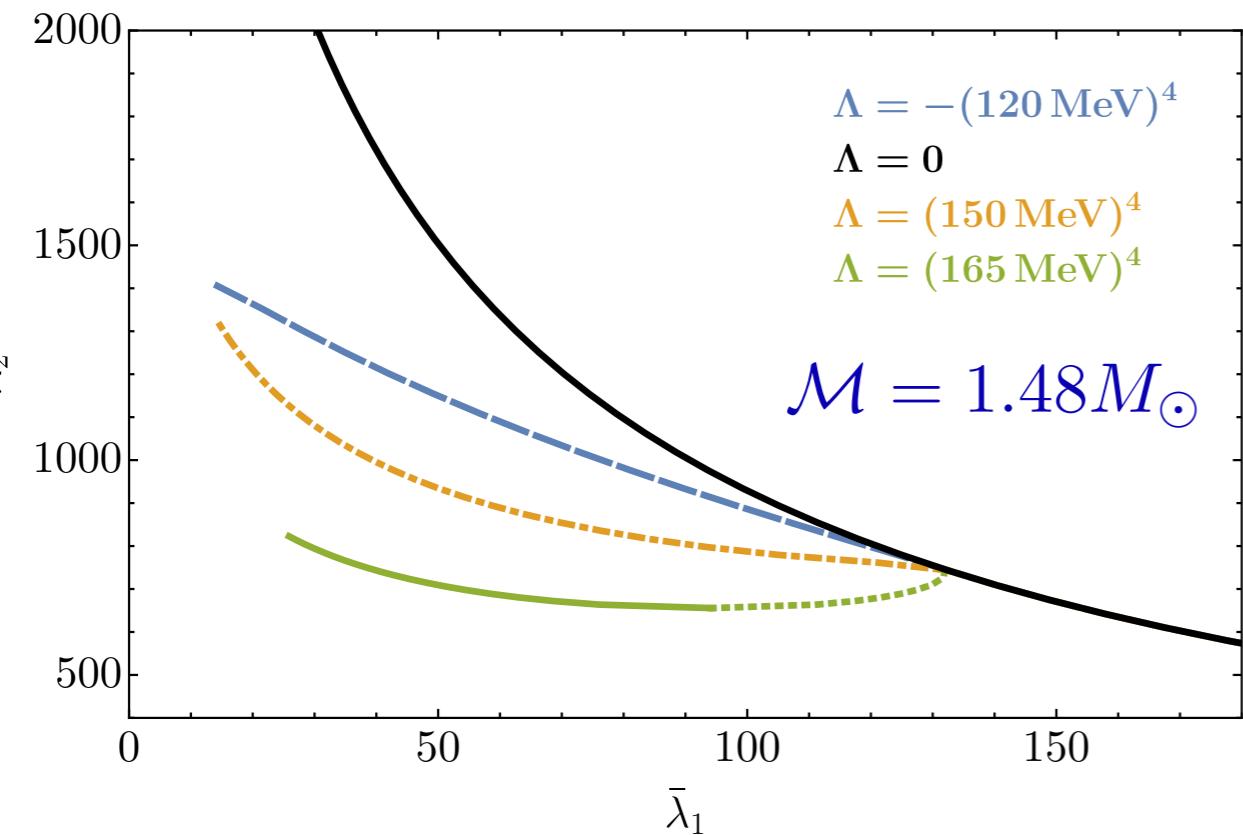
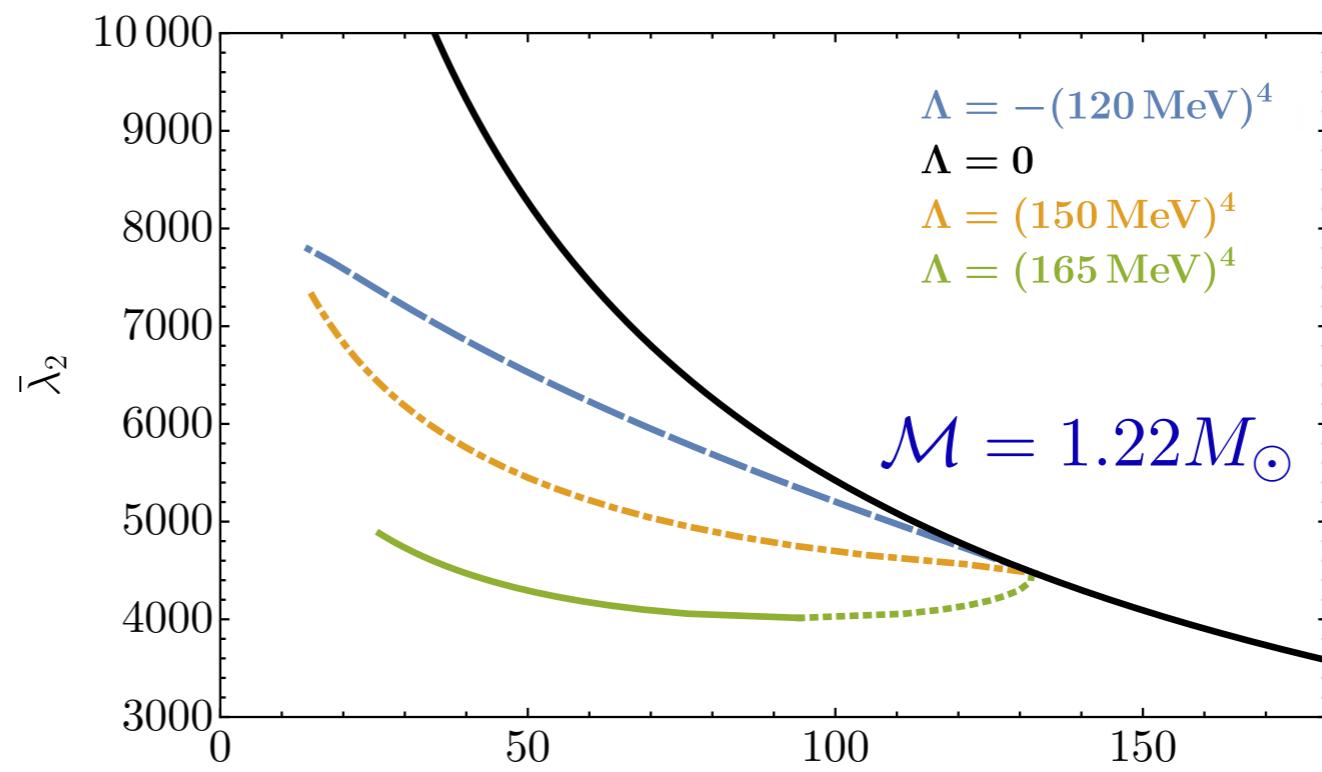
$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2) M_1^4 \bar{\lambda}_1 + (M_2 + 12M_1) M_2^4 \bar{\lambda}_2}{(M_1 + M_2)^5}$$

LIGO/Virgo gr-qc/1710.05832

# Tidal Deformabilities

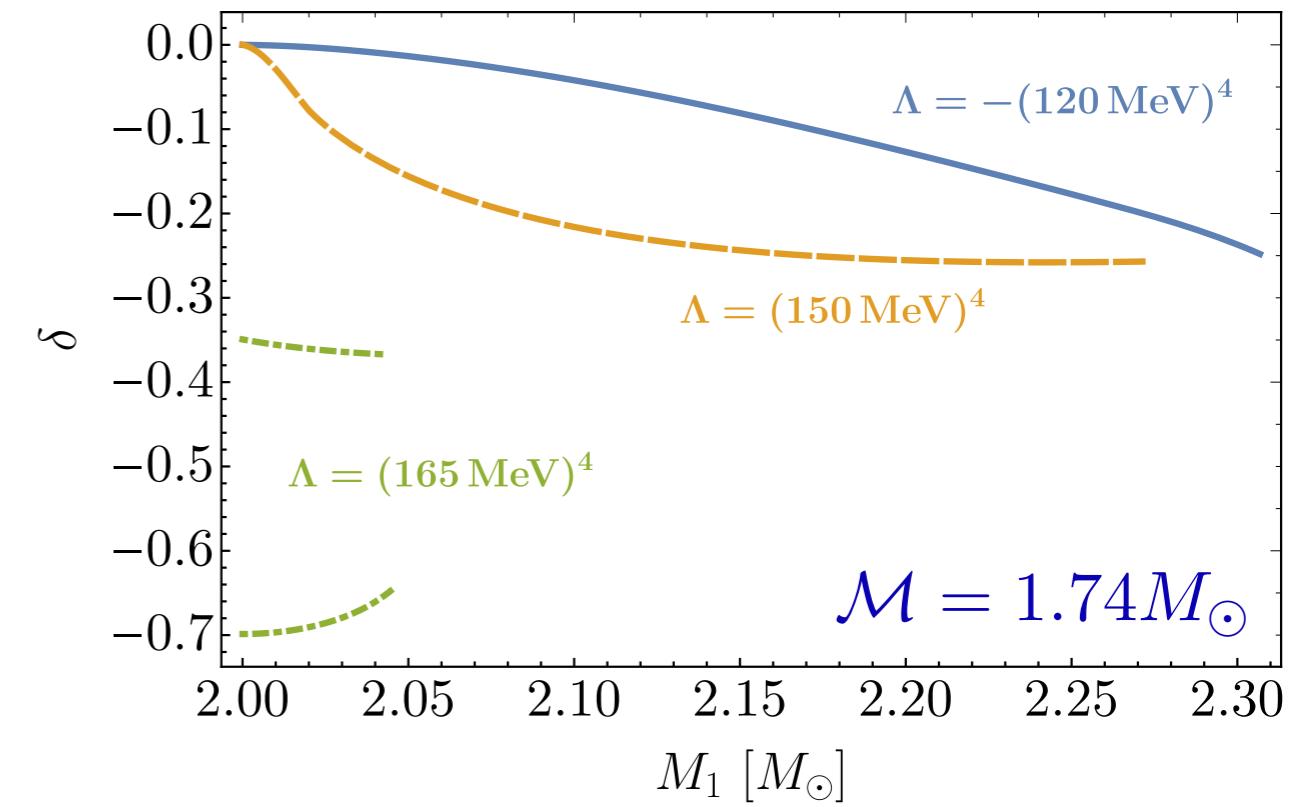
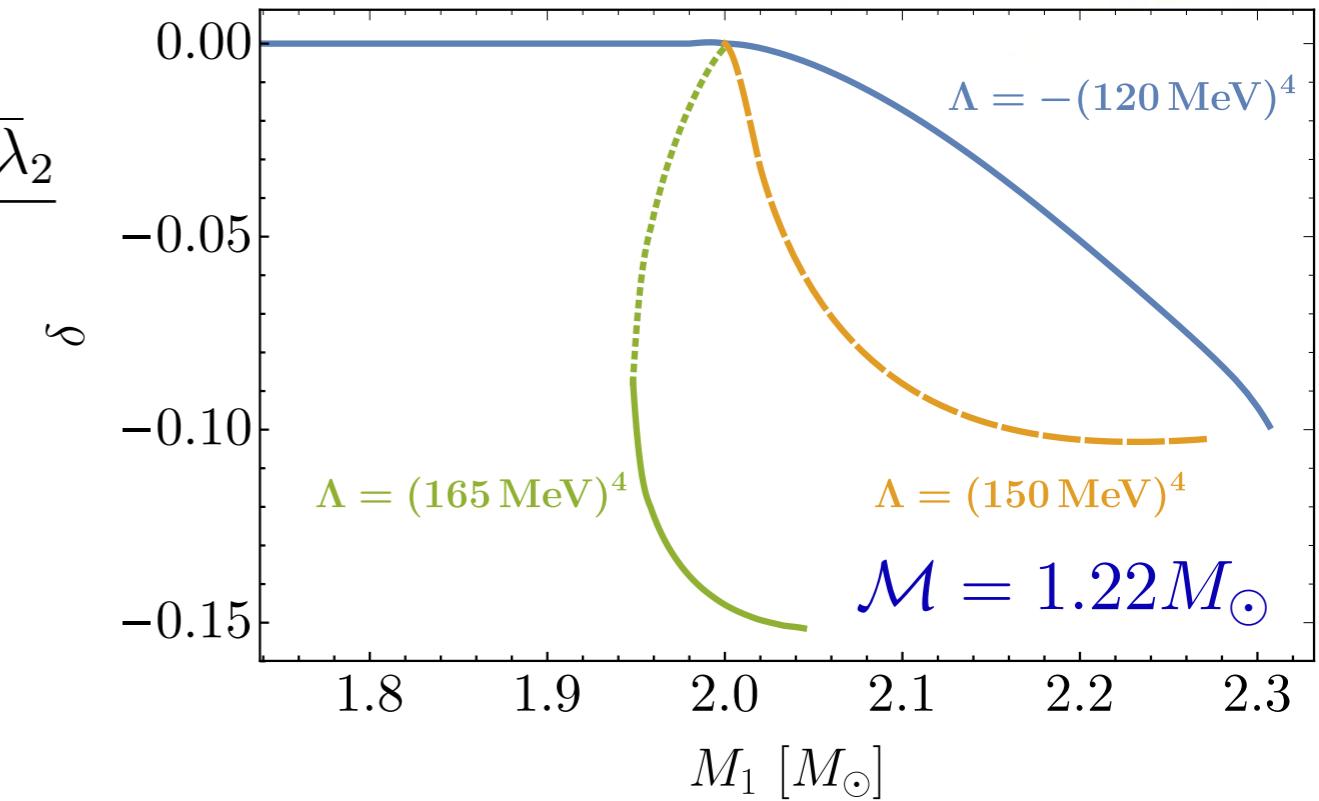
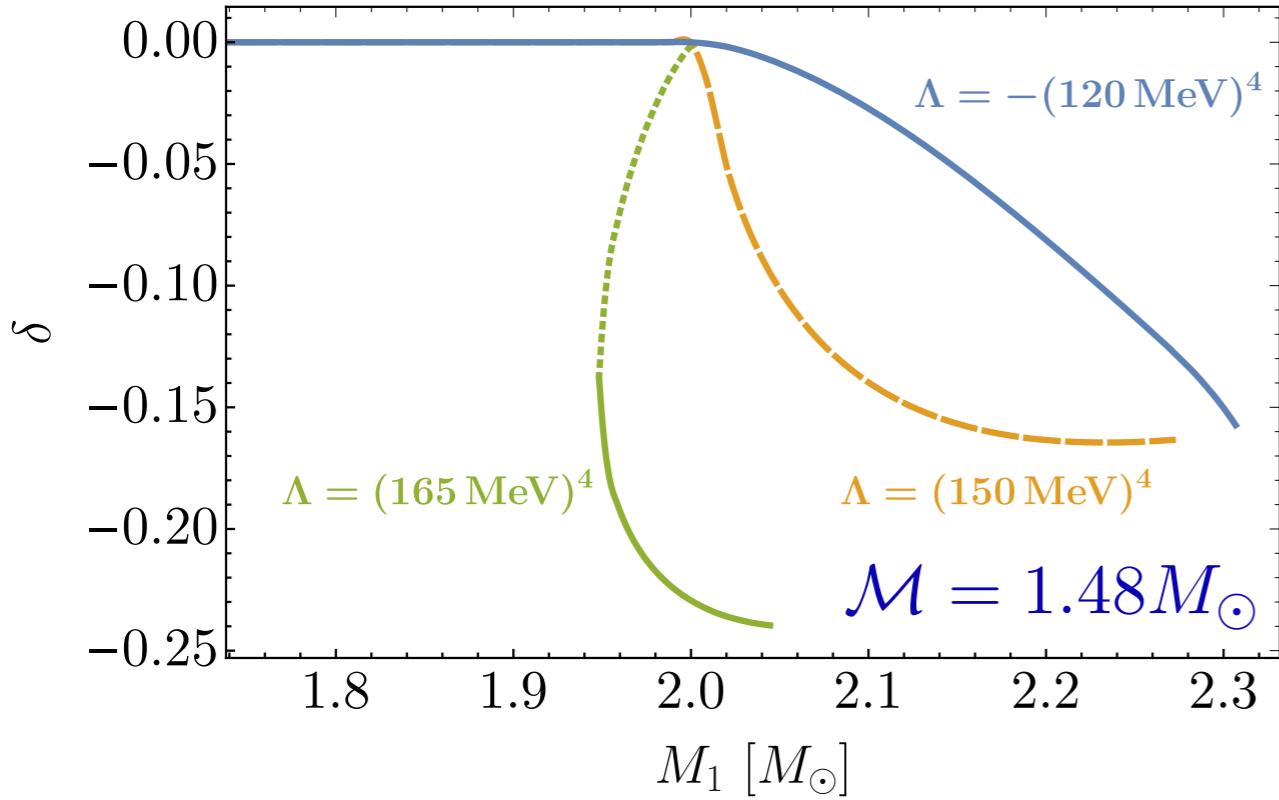
$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$



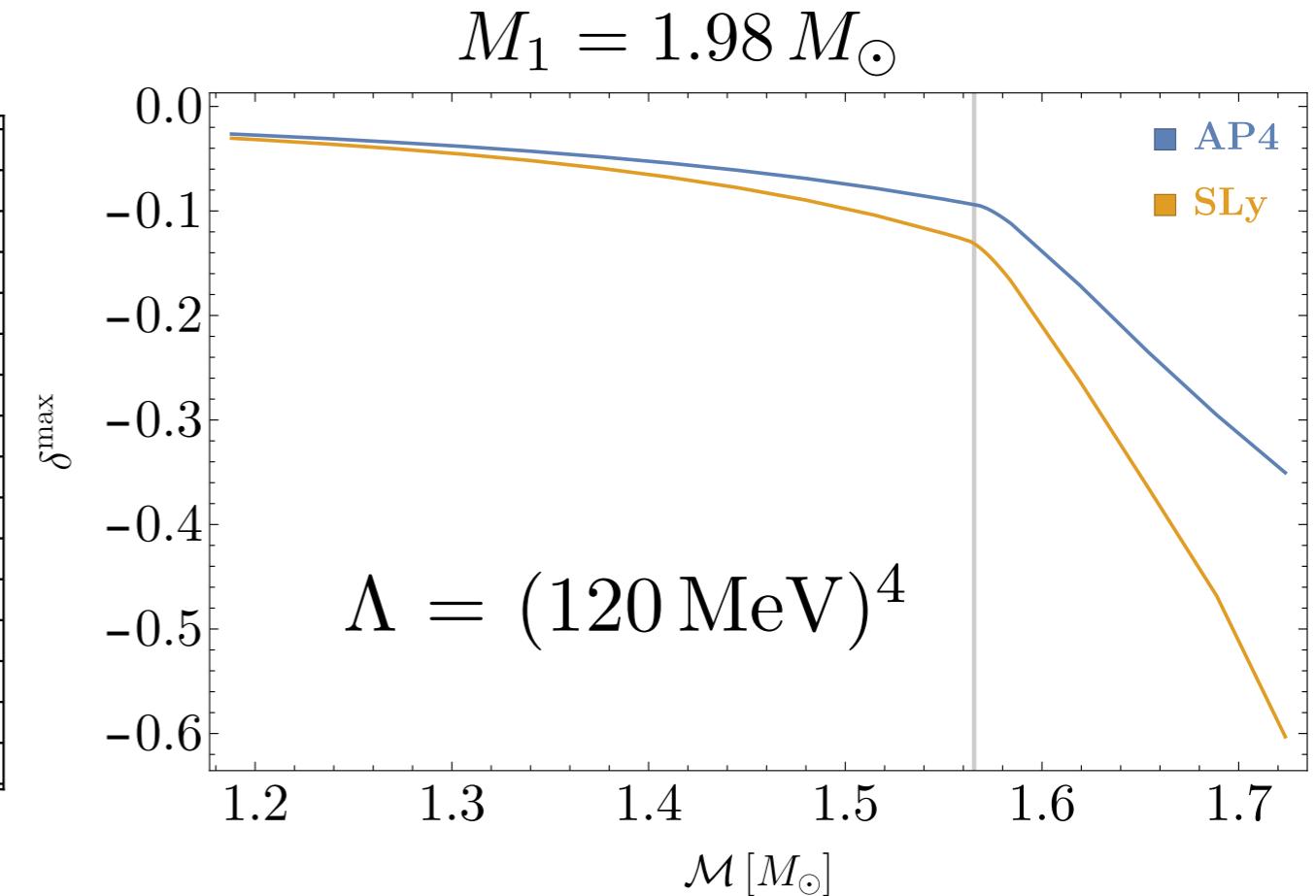
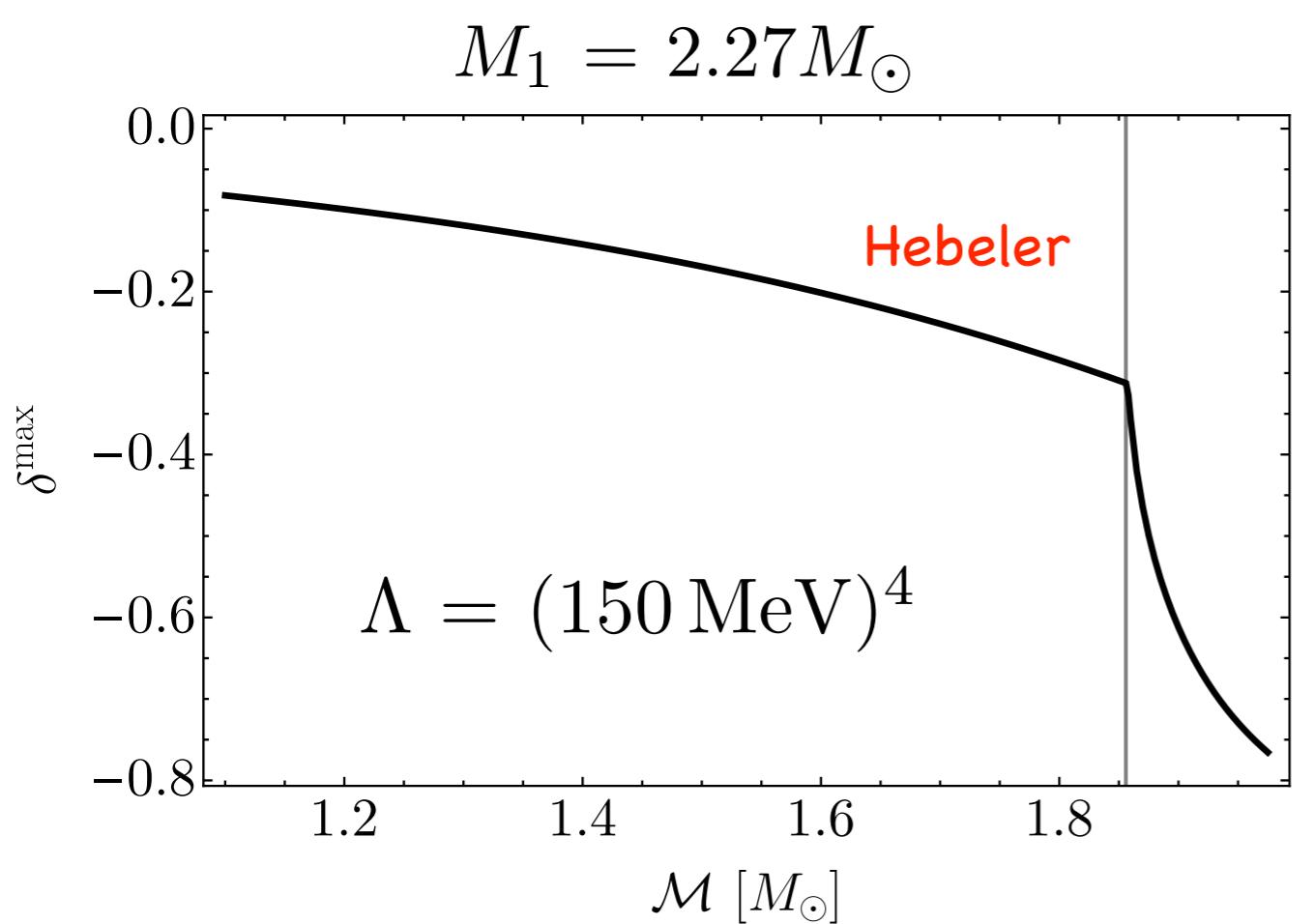
# Deviations

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4\bar{\lambda}_1 + (M_2 + 12M_1)M_2^4\bar{\lambda}_2}{(M_1 + M_2)^5}$$

$$\delta \equiv \frac{\tilde{\Lambda} - \tilde{\Lambda}_0}{\tilde{\Lambda}_0}$$

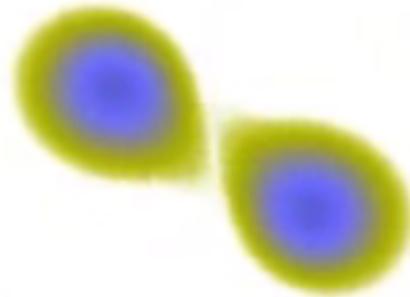


# Sensitivity



# Summary

$\Lambda$



vacuum energy should change  
during phase transitions

VE can cause measurable deviation  
in maximal mass,  $M(R)$ , and  
tidal deformability

# Backup Slides

# Neutron Stars

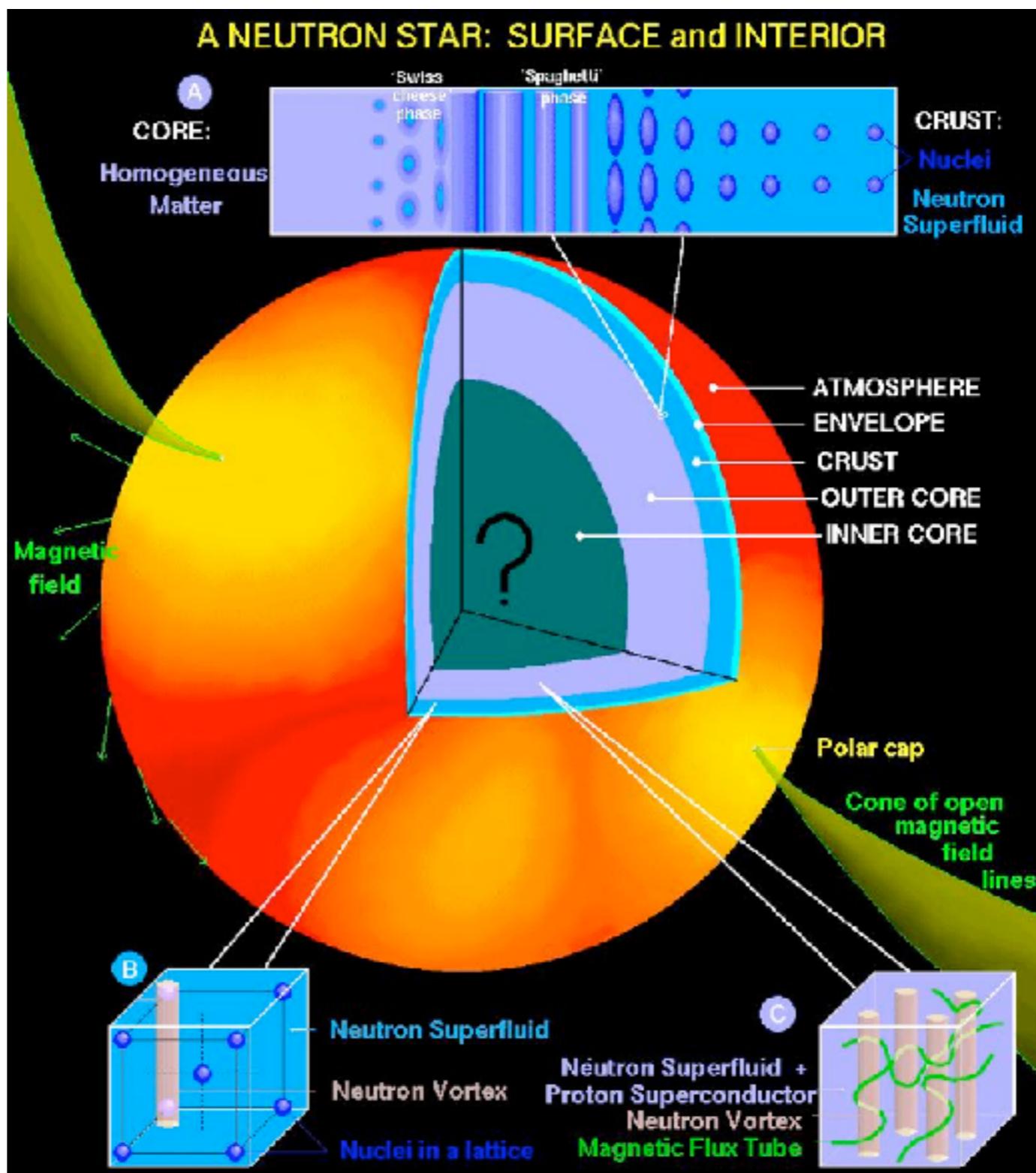
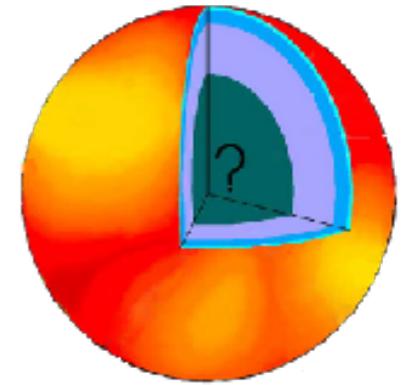


image by Dany P. Page <http://bit.ly/nscross>

# Model for neutron stars



At zero temperature, gravitational pressure balanced by pressure of fluid

$$ds^2 = e^{\nu(r)} dt^2 - (1 - 2Gm(r)/r)^{-1} dr^2 - r^2 d\Omega^2$$

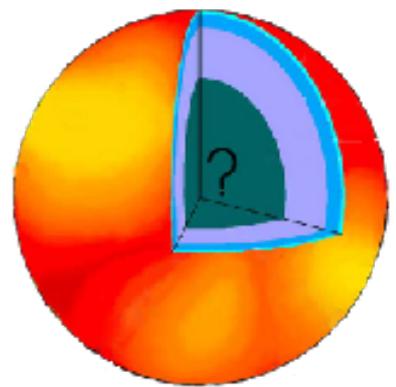
Einstein eqs (aka Tolman-Oppenheimer-Volkoff):

$$m'(r) = 4\pi r^2 \epsilon(r)$$

$$p'(r) = -\frac{p(r) + \epsilon(r)}{r(r - 2Gm(r))} G [m(r) + 4\pi r^3 p(r)]$$

$$\nu'(r) = -\frac{2p'(r)}{p(r) + \epsilon(r)}$$

# Toy model for neutron stars



outer core

$$p_{\text{out}} = K_{\text{out}} \rho^{\gamma_{\text{out}}}$$

$$\epsilon_{\text{out}} = (1 + a_{\text{out}}) \rho + \frac{K_{\text{out}}}{\gamma_{\text{out}} - 1} \rho^{\gamma_{\text{out}}}$$

Match critical pressure at boundary

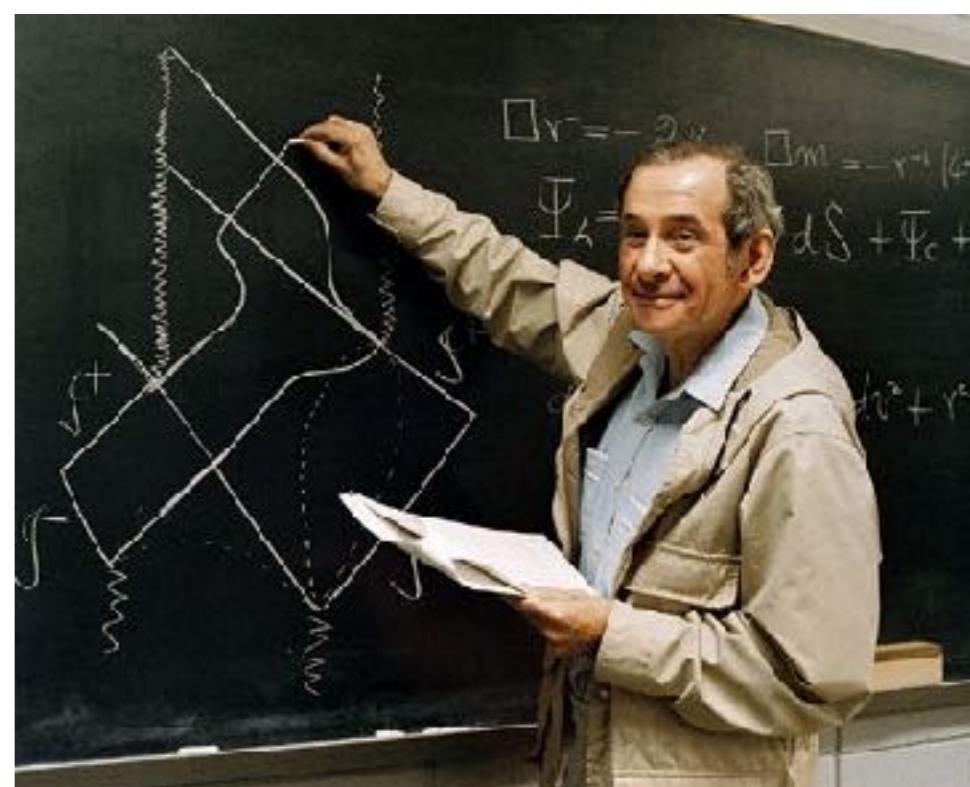
inner core

$$p_{\text{in}} = K_{\text{in}} \rho^{\gamma_{\text{in}}} - \Lambda$$

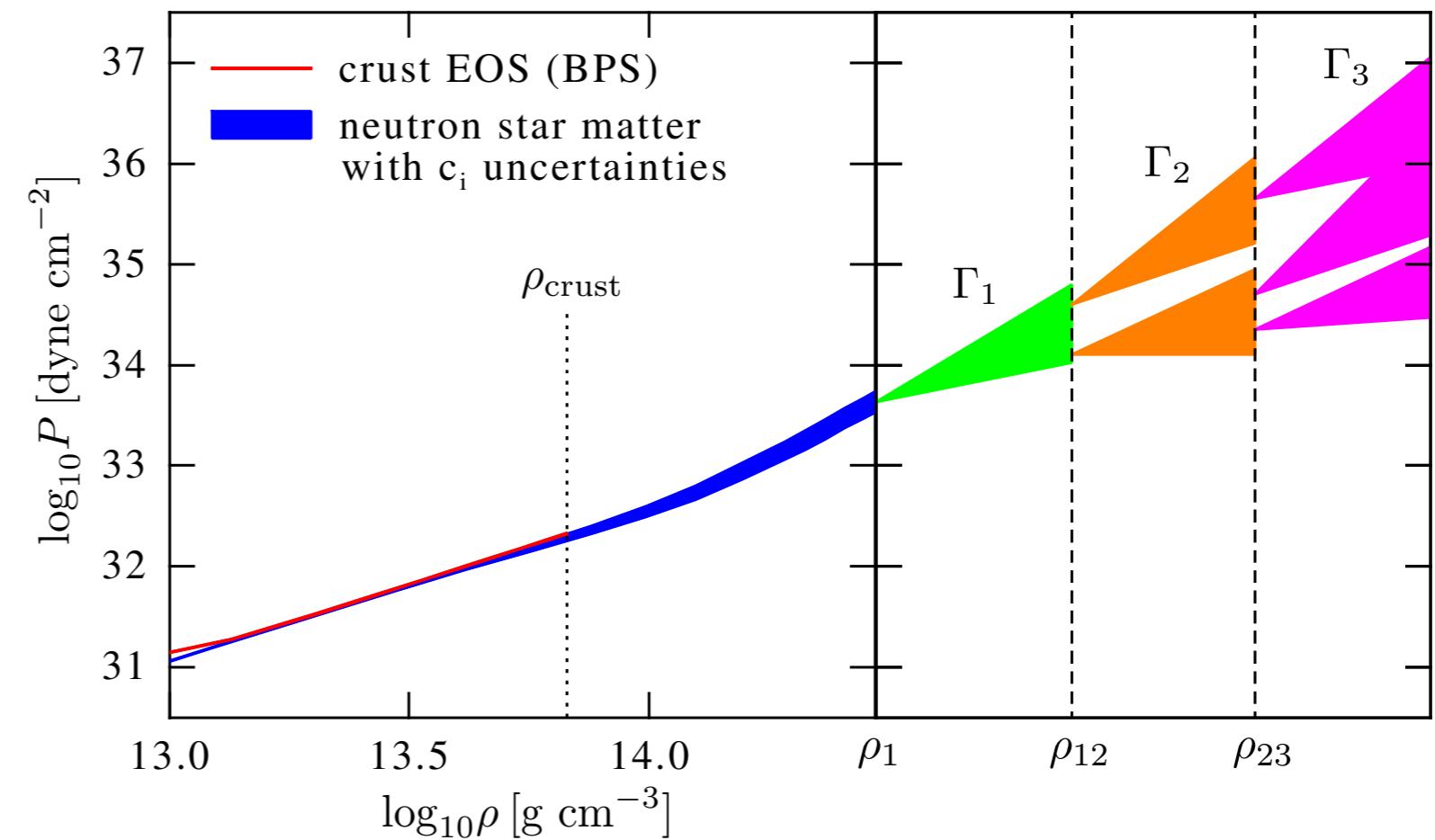
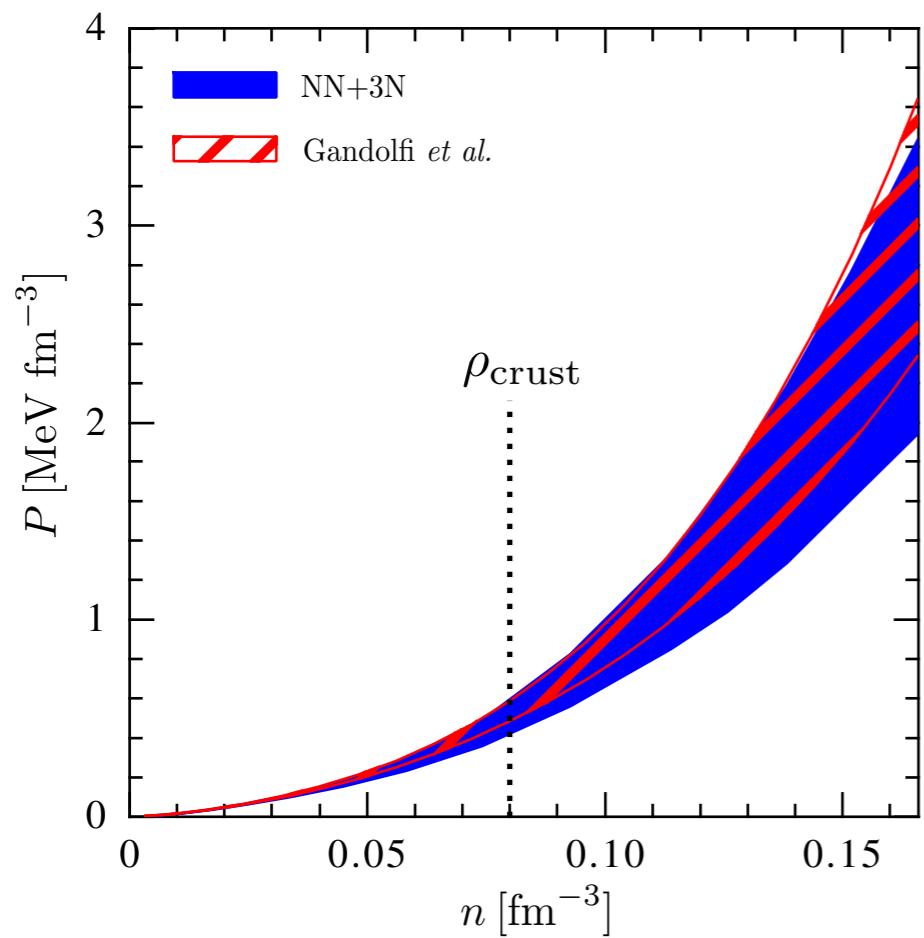
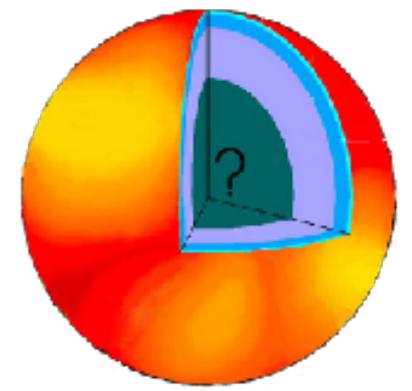
$$\epsilon_{\text{in}} = (1 + a_{\text{in}}) \rho + \frac{K_{\text{in}}}{\gamma_{\text{in}} - 1} \rho^{\gamma_{\text{in}}} + \Lambda$$

Israel Junction condition:

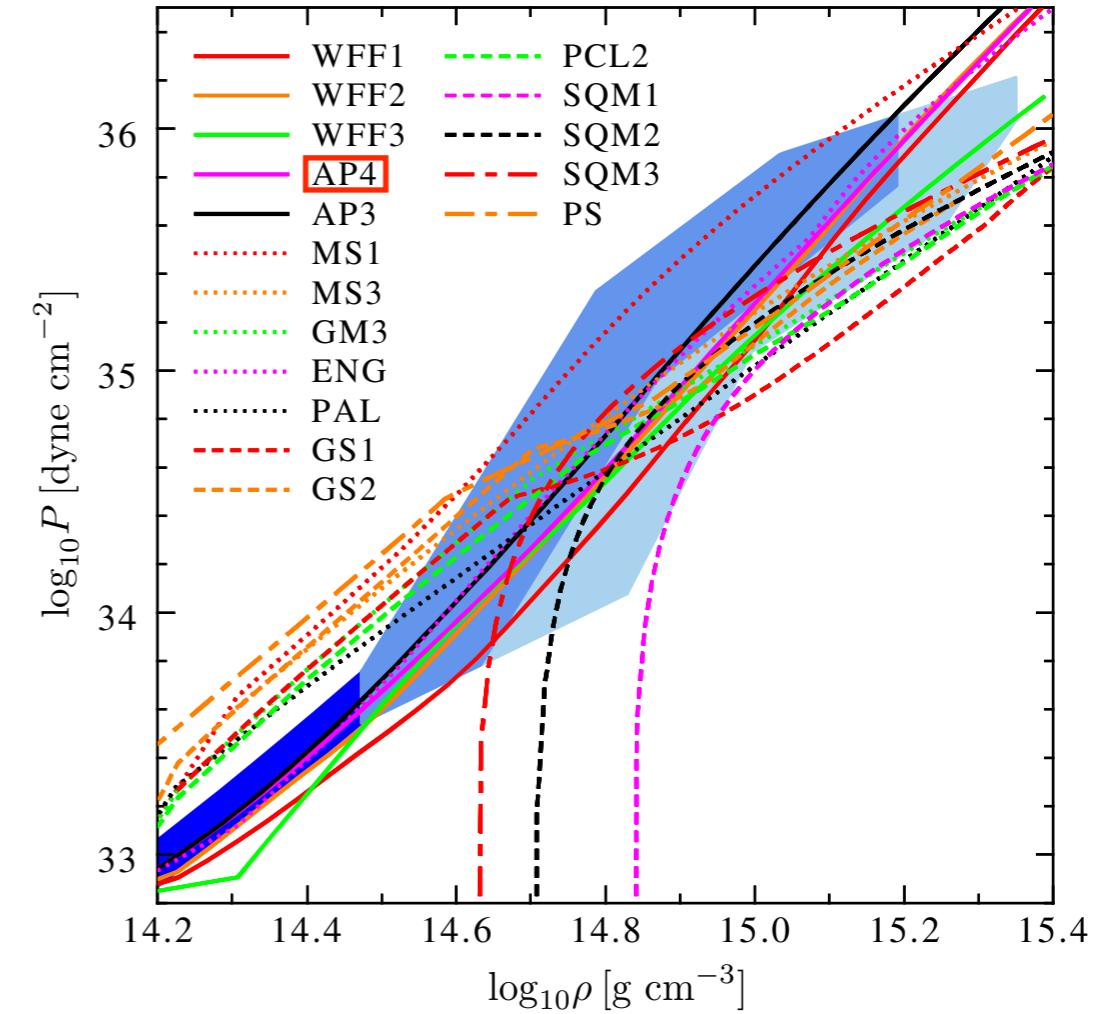
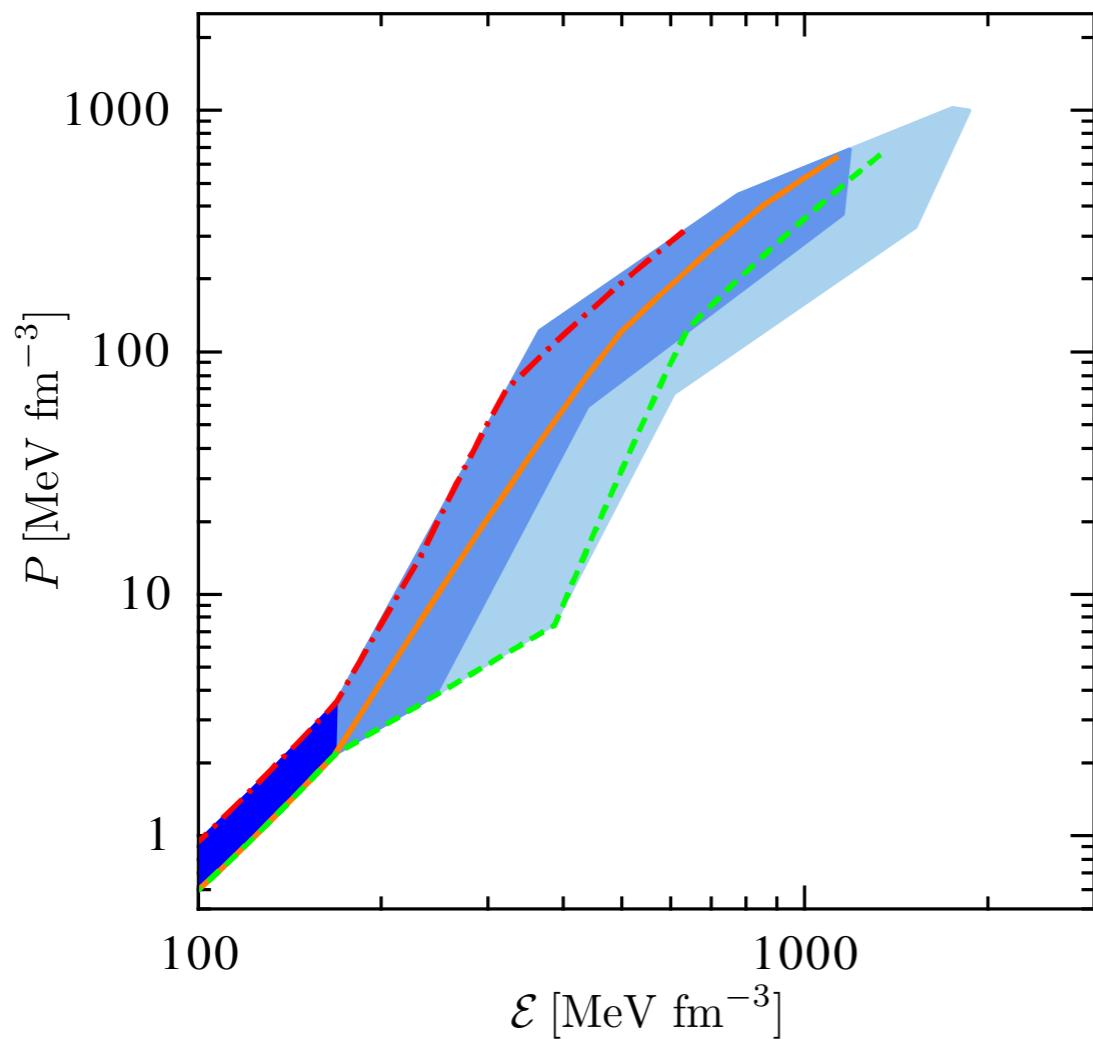
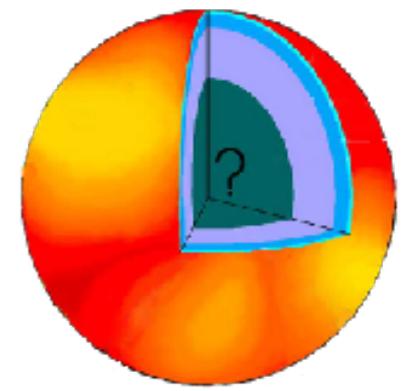
$\nu'(r), M(r)$  continuous,  
thus  $p(r)$  also continuous



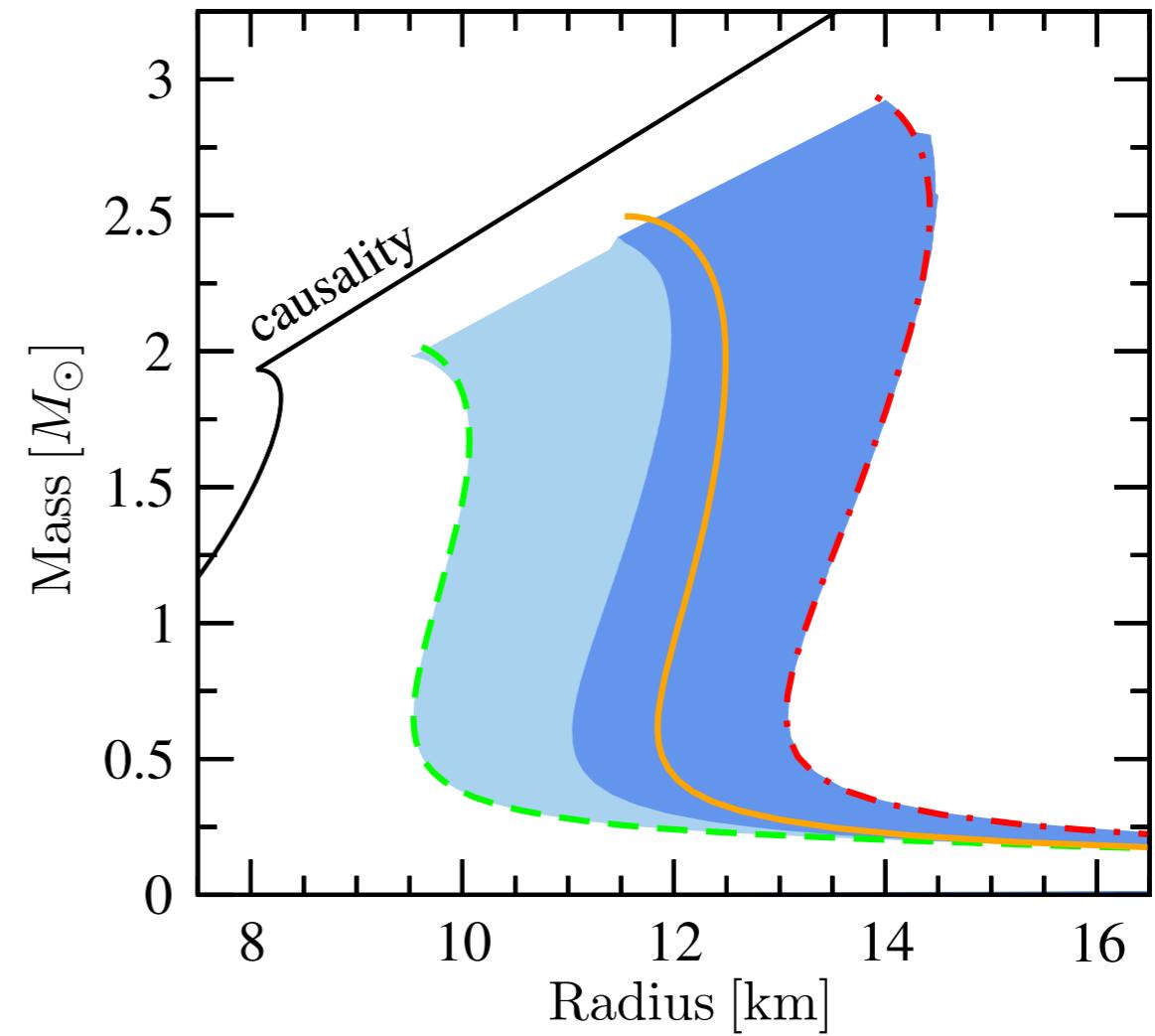
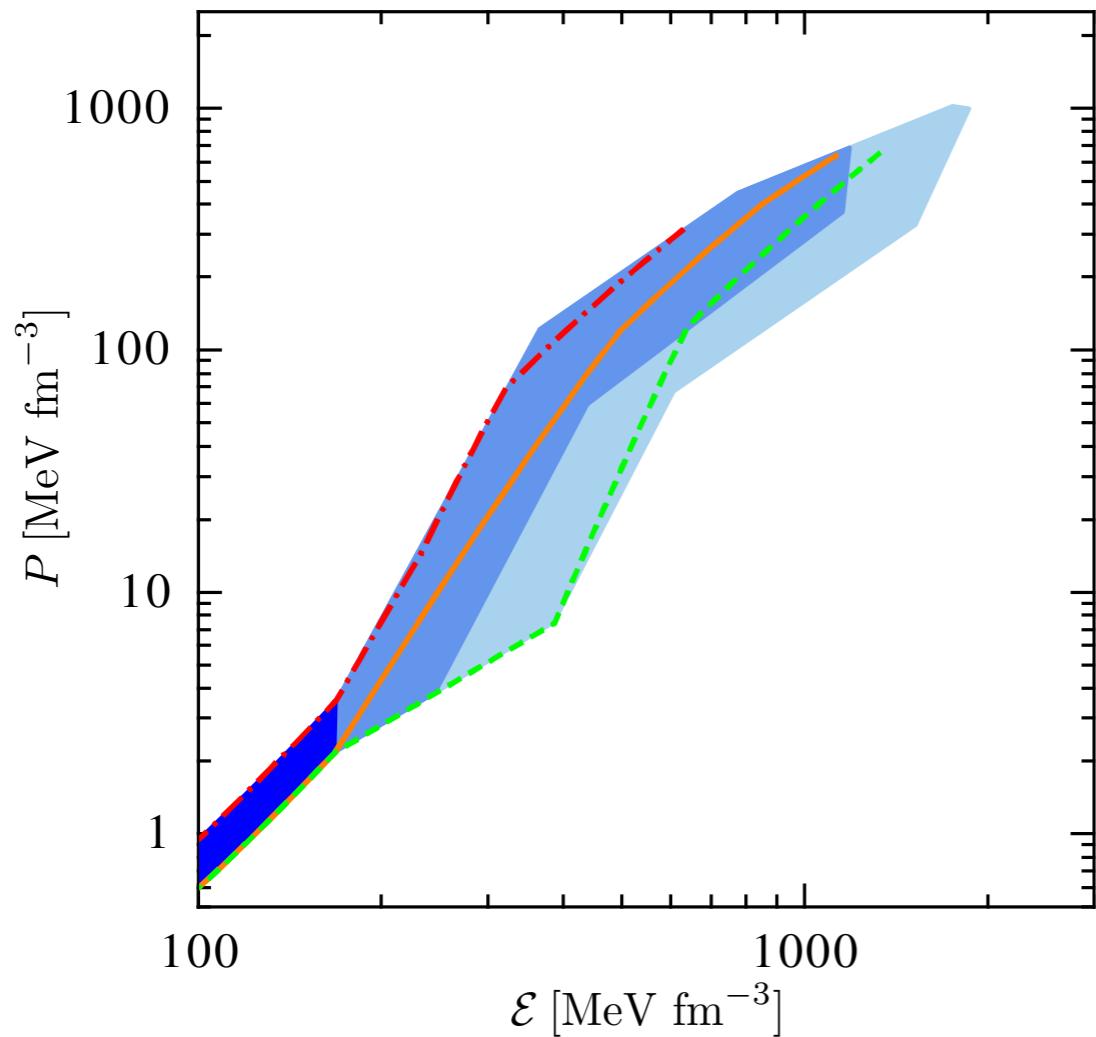
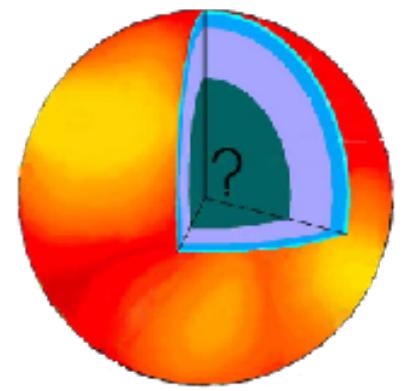
# EOS for neutron stars



# EOS for neutron stars



# EOS for neutron stars



|            | <b>SLy</b>               | <b>AP4</b>    | <b>Hebeler</b> |
|------------|--------------------------|---------------|----------------|
| $K_1$      | $9.27637 \times 10^{-6}$ | See [22]      |                |
| $p_1$      | $(0.348867)^4$           |               |                |
| $p_2$      | $(7.78544)^4$            | See [22]      |                |
| $p_3$      | $(10.5248)^4$            |               |                |
| $p_4$      | $(40.6446)^4$            | $(41.0810)^4$ | $(72.2274)^4$  |
| $p_5$      | $(103.804)^4$            | $(97.1544)^4$ | $(102.430)^4$  |
| $p_6$      | $(176.497)^4$            | $(179.161)^4$ | $(149.531)^4$  |
| $\gamma_1$ | 1.58425                  |               |                |
| $\gamma_2$ | 1.28733                  |               |                |
| $\gamma_3$ | 0.62223                  |               | See [22]       |
| $\gamma_4$ | 1.35692                  |               |                |
| $\gamma_5$ | 3.005                    | 2.830         | 4.5            |
| $\gamma_6$ | 2.988                    | 3.445         | 5.5            |
| $\gamma_7$ | 2.851                    | 3.348         | 3              |

Table 1: The parameters used for each EoS. The exponents  $\gamma_i$  are dimensionless, the various pressures have units of  $\text{MeV}^4$ , and  $K_1$  is in units of  $\text{MeV}^{4-4\gamma_1}$ . The Hebeler et al. parametrization [22] uses a semi-analytic expression which is not piecewise polytropic in the outer region of the star, and thus cannot be displayed in the table.

# Mass Bound

