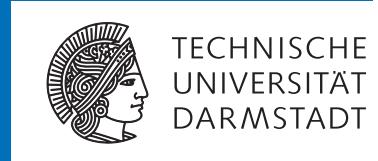


# Symmetry breaking patterns in QCD and the study of cold dense matter



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

Marc Leonhardt

Institut für Kernphysik, Technische Universität Darmstadt

with

Martin Pospiech and Jens Braun

XIII<sup>th</sup> Quark Confinement and the Hadron Spectrum  
Maynooth University, Dublin, Ireland



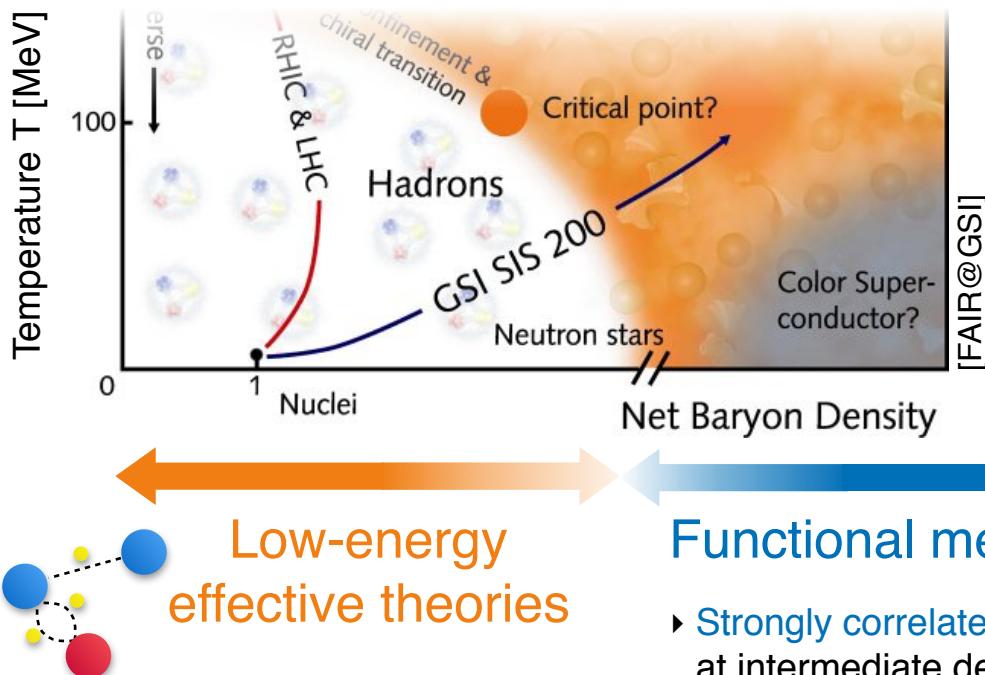
- [J. Braun, ML, M. Pospiech, PRD **96**, 076003 (2017)]
- [J. Braun, ML, M. Pospiech, PRD **97**, 076010 (2018)]
- [J. Braun, ML, J. M. Pawłowski, arXiv:1806.04432]
- [J. Braun, C. Drischler, K. Hebeler, ML, M. Pospiech, A. Schwenk, in preparation]

**HIC** | FAIR  
Helmholtz International Center

# The QCD phase diagram and the cold dense equation of state

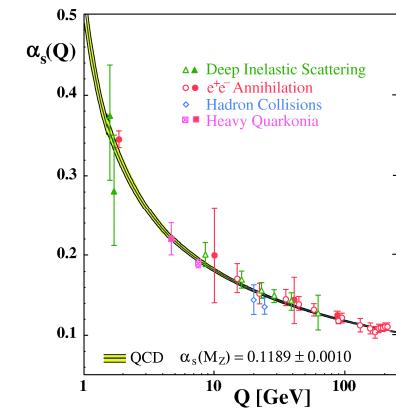


TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



## Functional methods

- ▶ Strongly correlated matter at intermediate densities
- ▶ variety of condensates as non-perturbative phenomena

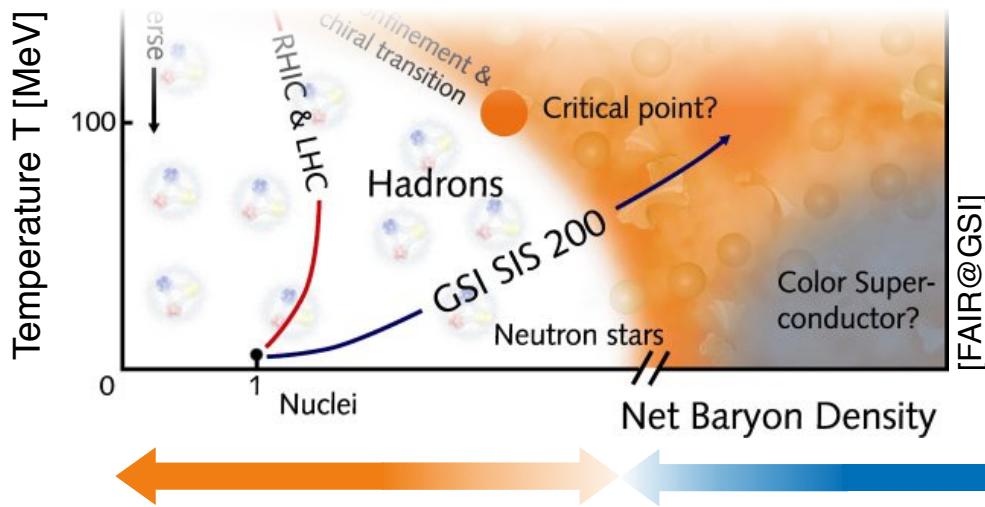


## Perturbative methods

- ▶ Quarks and gluons as only dofs
- ▶ Weak coupling expansion



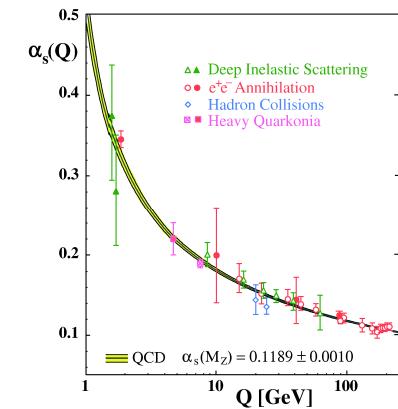
# The QCD phase diagram and the cold dense equation of state



Low-energy effective theories

## Functional methods

- ▶ Strongly correlated matter at intermediate densities
- ▶ variety of condensates as non-perturbative phenomena



$\lesssim 35 \rho_{\text{sat}}$  [Kurkela et al., 2014]

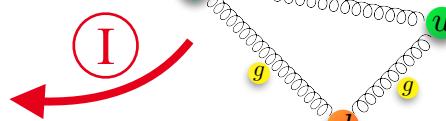
## Perturbative methods

- ▶ Quarks and gluons as only dofs
- ▶ Weak coupling expansion

EoS



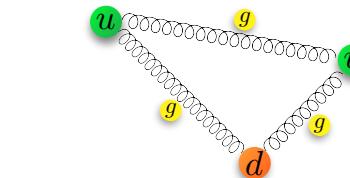
$$\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array} \xrightarrow{\lambda_{\alpha\beta\gamma\delta}}$$



# Outline

## ① Including dynamic gauge fields

- Framework and structure of  $\beta$  functions
- Symmetry breaking patterns
- Implications on the phase diagram



①

A diagram of a four-point interaction vertex. Four black arrows point towards a central cross. The top-left arrow is labeled  $\alpha$ , the top-right  $\beta$ , the bottom-left  $\gamma$ , and the bottom-right  $\delta$ . The central point is labeled  $\lambda_{\alpha\beta\gamma\delta}$ .

②

EoS

## ② Connecting to low-energy effective models (LEMs)

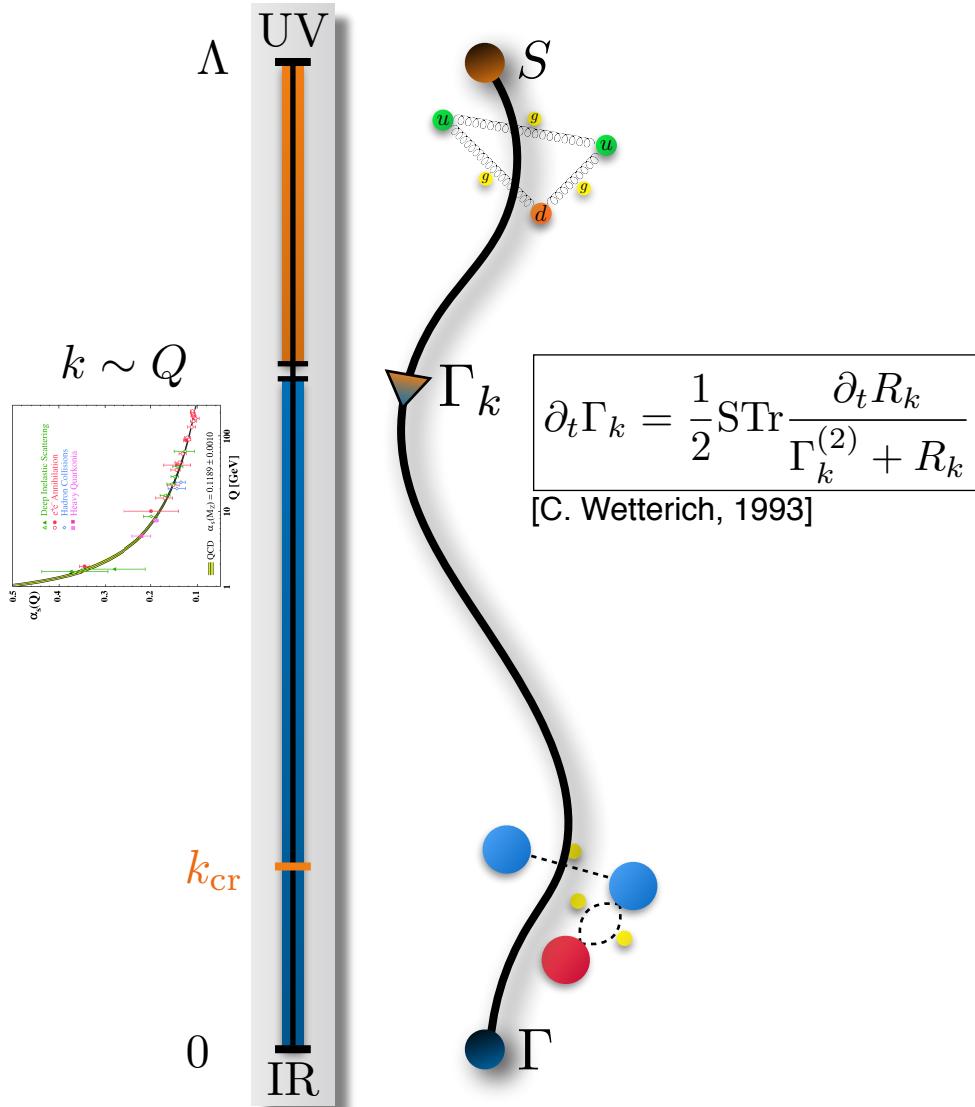
- Quark-meson-diquark model
- Findings on the equation of state (EoS)

Conclusions and outlook



# Functional renormalization group (FRG)

From high to low energies in QCD

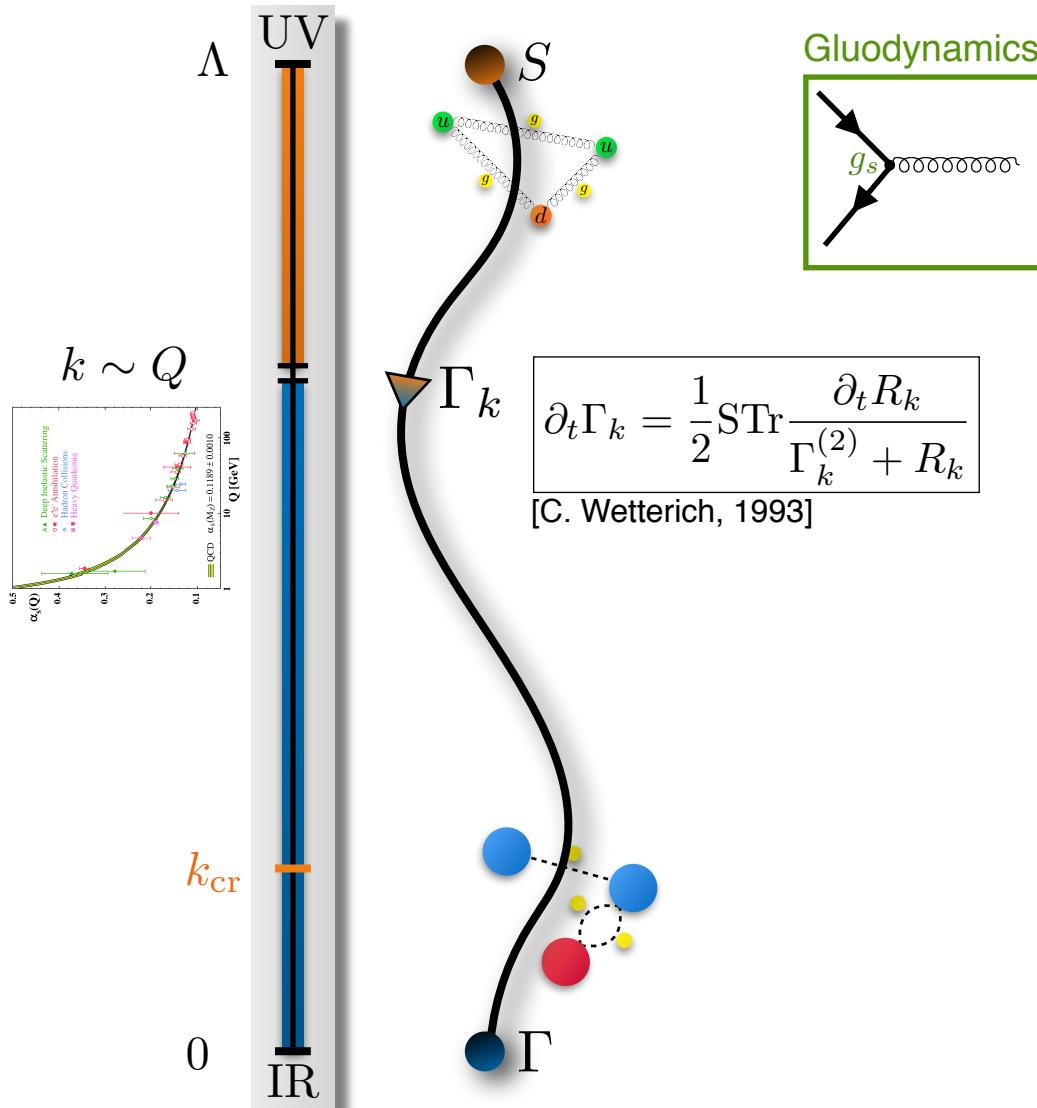


# Functional renormalization group (FRG)

From high to low energies in QCD



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



Strong coupling  $g_s(\Lambda)$  only parameter

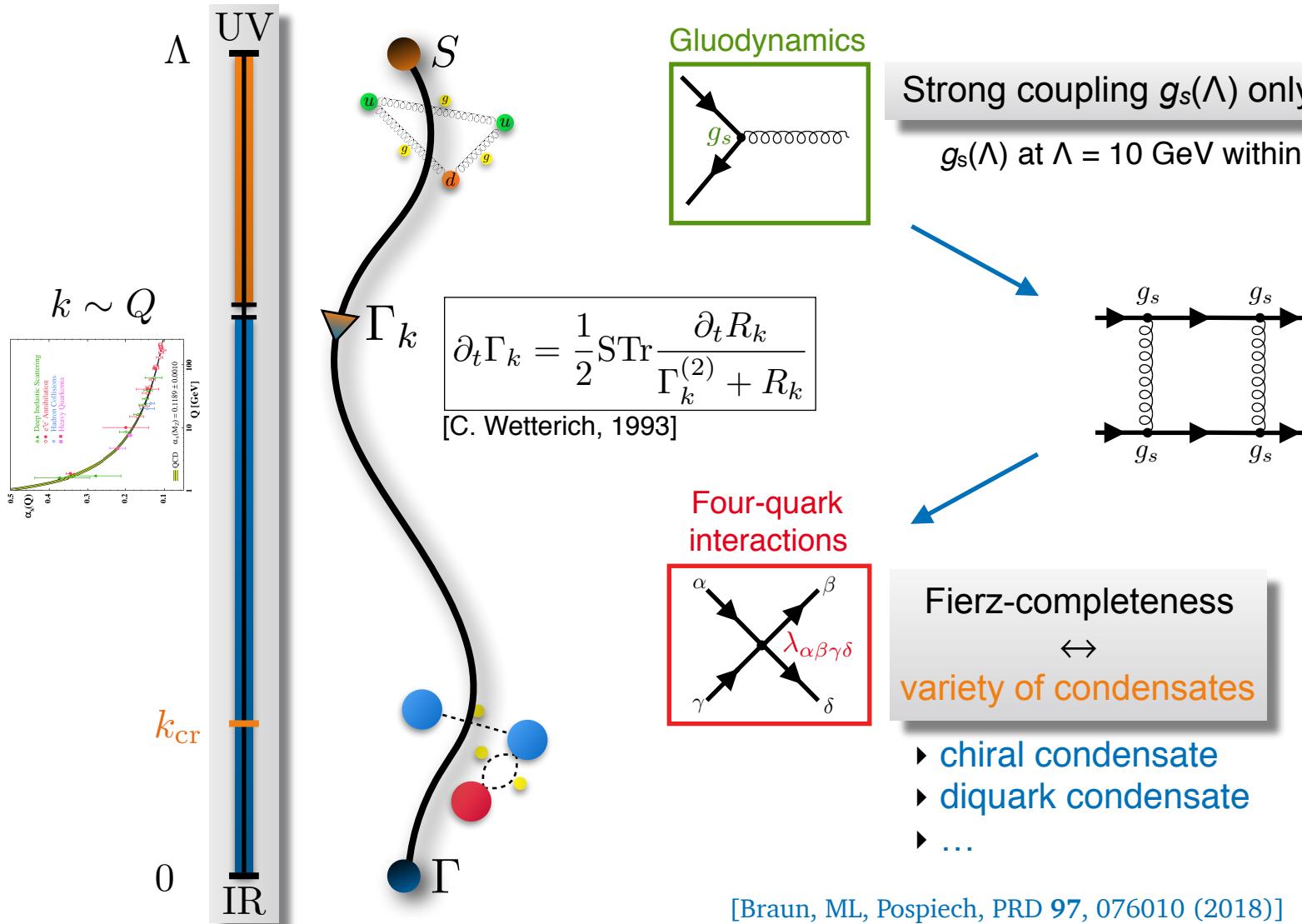
$g_s(\Lambda)$  at  $\Lambda = 10$  GeV within exp. error  
[Bethke, 2006]

# Functional renormalization group (FRG)

From high to low energies in QCD



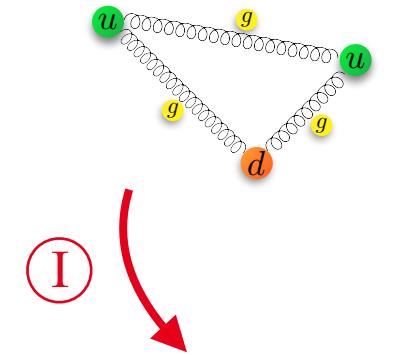
TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



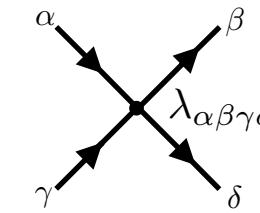
# Outline

## ① Including dynamic gauge fields

- Framework and structure of  $\beta$  functions
- Symmetry breaking patterns
- Implications on the phase diagram



I



II

EoS

## ② Connecting to low-energy effective models (LEMs)

- Quark-meson-diquark model
- Findings on the equation of state (EoS)

Conclusions and outlook



# Ansatz for the effective average action

Quark self-interactions and gauge degrees of freedom



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

$$\Gamma_{\text{LO}}[\bar{\psi}, \psi, A] = \int_0^{\frac{1}{T}} d\tau \int d^3x \left\{ \bar{\psi} (\text{i}\gamma_0 \partial_0 + \text{i}\gamma_i \partial_i - \text{i}\mu\gamma_0) \psi + \frac{1}{2} \sum_{j \in \mathcal{B}} Z_j \bar{\lambda}_j \mathcal{L}_j \right\}$$

## Symmetries

Lorentz group:  $\text{SO}(1, 3) \longrightarrow \text{SO}(3)$

Flavor space:  $\text{SU}_L(2) \otimes \text{SU}_R(2) \otimes \text{U}_V(1)$

Color space:  $\text{SU}(N_c)$



Fierz-complete basis:  
**10 channels**

$$\mathcal{L}_{(\sigma-\pi)} = (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau_i\psi)^2$$

↔ formation of  
**chiral condensate**

$$\mathcal{L}_{\text{csc}} \sim (\text{i}\bar{\psi}\gamma_5\tau_A t_c^{A'} \mathcal{C} \bar{\psi}^T)(\text{i}\psi^T \mathcal{C} \gamma_5\tau_A t_c^{A'} \psi) \quad J^P = 0^+$$

e.g. [Rapp, Schäfer, Shuryak, Velkovsky, 1998]

↔ formation of  
**diquark condensate**



# Ansatz for the effective average action

Quark self-interactions and gauge degrees of freedom



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

$$\Gamma_{\text{LO}}[\bar{\psi}, \psi, A] = \int_0^{\frac{1}{T}} d\tau \int d^3x \left\{ \bar{\psi} (\text{i}\gamma_0 \partial_0 + \text{i}\gamma_i \partial_i - \text{i}\mu\gamma_0) \psi + \frac{1}{2} \sum_{j \in \mathcal{B}} Z_j \bar{\lambda}_j \mathcal{L}_j \right.$$
$$+ \frac{Z_A}{4} (F_{\mu\nu}^a)^2 + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 \quad \text{kinetic term for gluons}$$
$$\left. + \bar{g}_s \bar{\psi} \gamma_\mu A_\mu \psi \right\} \quad \text{quark-gluon interaction}$$

RG flow of the gauge coupling:  $k \frac{\partial}{\partial k} g_s^2(k) = \eta_{A,k} g_s^2(k)$  [L. F. Abbott, 1980]

$$\eta_{A,k} = \eta_{A,k}^{\text{YM}}(T) + \Delta\eta_{A,k}(T, \mu)$$

[J. Braun and H. Gies, 2007; J. Braun, 2009;  
J. Braun, L. M. Haas, F. Marhauser, J. M. Pawłowski, 2011;  
J. M. Pawłowski, 2011;  
J. Braun, L. Fister, J. M. Pawłowski and F. Rennecke, 2014]

$$\Delta\eta_{A,k}(T, \mu) = \frac{Z_A^{-1}}{3(N_c^2 - 1)} \left( \frac{\partial^2}{\partial p^2} \Pi^\perp \cdot \text{---o---o---o---o---} \right) \Big|_{p=0}$$



# RG flow of four-quark interactions

Structure of the  $\beta$  functions and formation of condensates



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

RG flow equation:

$$k \frac{\partial}{\partial k} \lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j + \sum_j g_s + g_s$$

# RG flow of four-quark interactions

Structure of the  $\beta$  functions and formation of condensates



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

RG flow equation:

$$k \frac{\partial}{\partial k} \lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j + \sum_j g_s + g_s$$

At  $k = \Lambda$ :  
 $\lambda_i^{(\text{UV})} = 0$   
 $g_s(\Lambda)$  only parameter

# RG flow of four-quark interactions

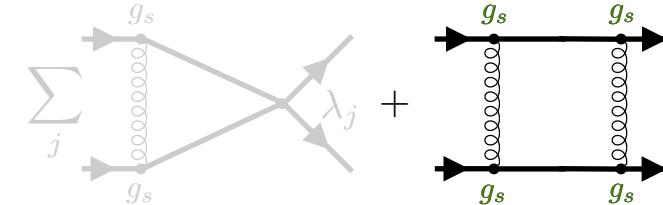
Structure of the  $\beta$  functions and formation of condensates



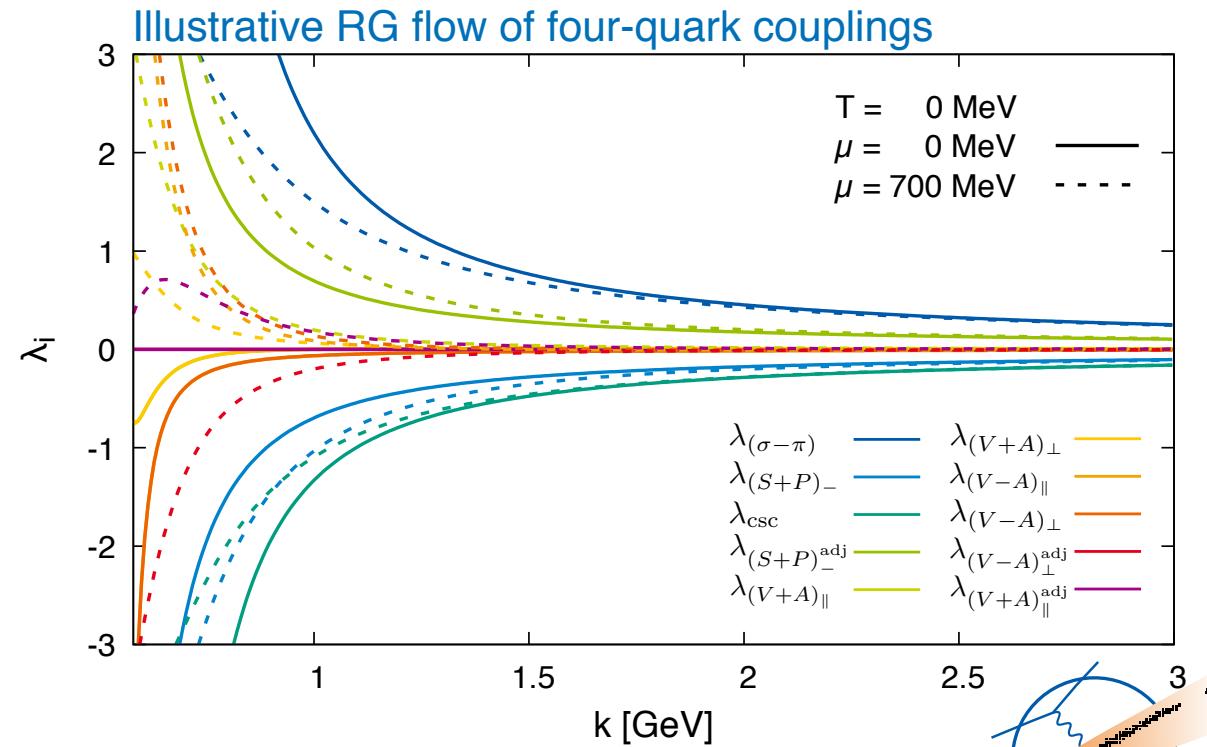
TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

RG flow equation:

$$k \frac{\partial}{\partial k} \lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j$$



At  $k = \Lambda$ :  
 $\lambda_i^{(\text{UV})} = 0$   
 $g_s(\Lambda)$  only parameter



[see also Braun, 2006; Mitter, Pawłowski, Strodthoff 2014]

# RG flow of four-quark interactions

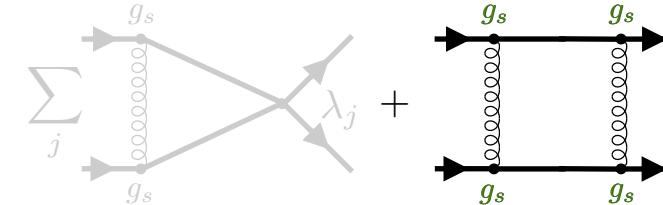
Structure of the  $\beta$  functions and formation of condensates



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

RG flow equation:

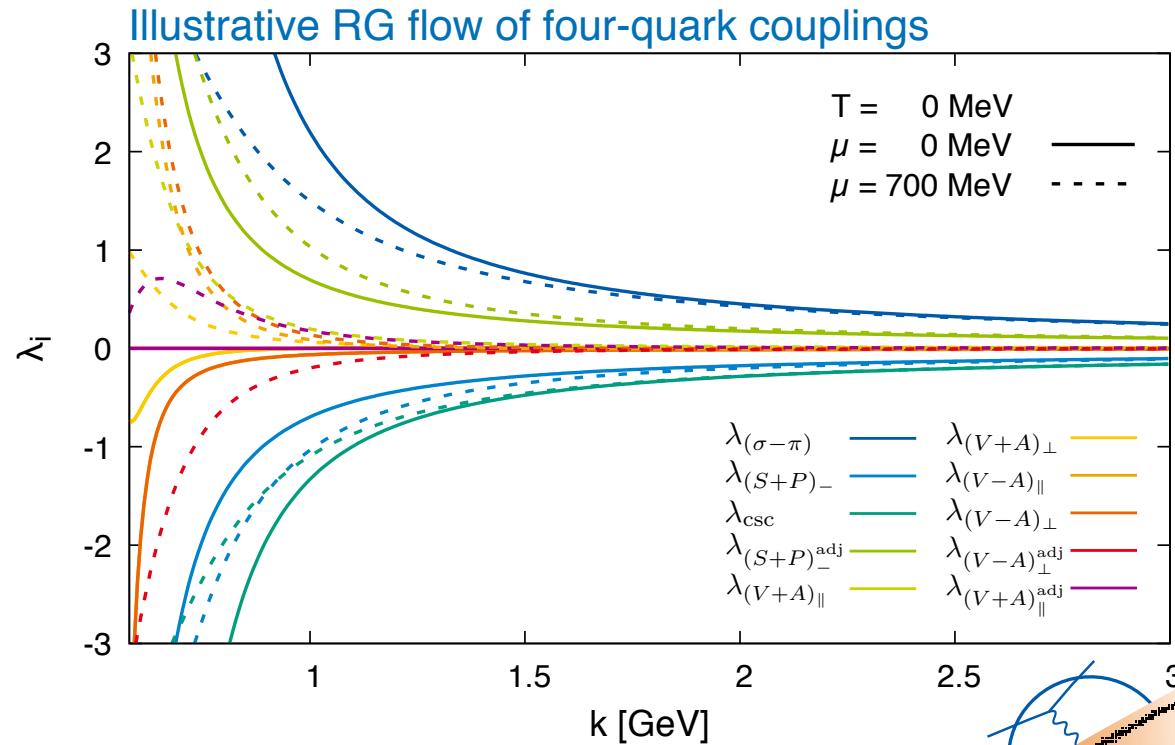
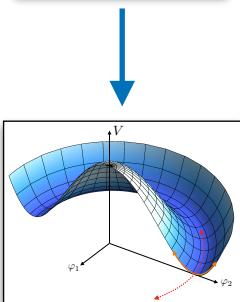
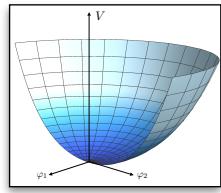
$$k \frac{\partial}{\partial k} \lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j$$



At  $k = \Lambda$ :  
 $\lambda_i^{(\text{UV})} = 0$   
 $g_s(\Lambda)$  only parameter

Onset of  
condensate  
formation:

$$\left. \frac{1}{\bar{\lambda}_i} \right|_{k_{\text{cr}}} \rightarrow 0$$



[see also Braun, 2006; Mitter, Pawłowski, Strodthoff 2014]

# RG flow of four-quark interactions

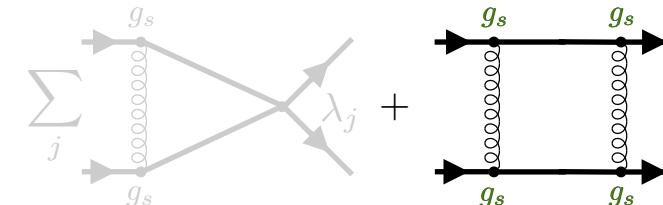
Structure of the  $\beta$  functions and formation of condensates



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

RG flow equation:

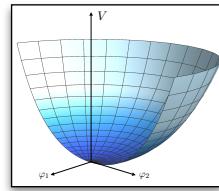
$$k \frac{\partial}{\partial k} \lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j$$



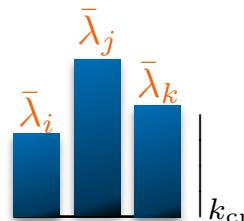
At  $k = \Lambda$ :  
 $\lambda_i^{(\text{UV})} = 0$   
 $g_s(\Lambda)$  only parameter

Onset of condensate formation:

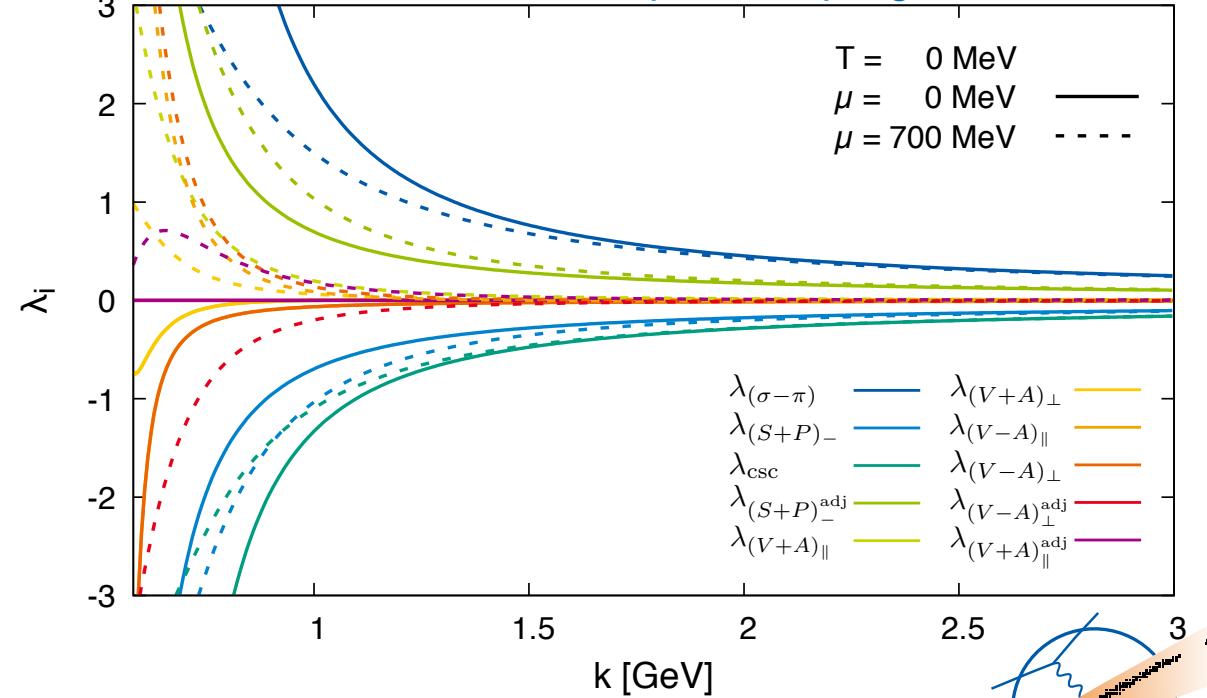
$$\frac{1}{\bar{\lambda}_i} \Big|_{k_{\text{cr}}} \rightarrow 0$$



Relative interaction strengths:



Illustrative RG flow of four-quark couplings



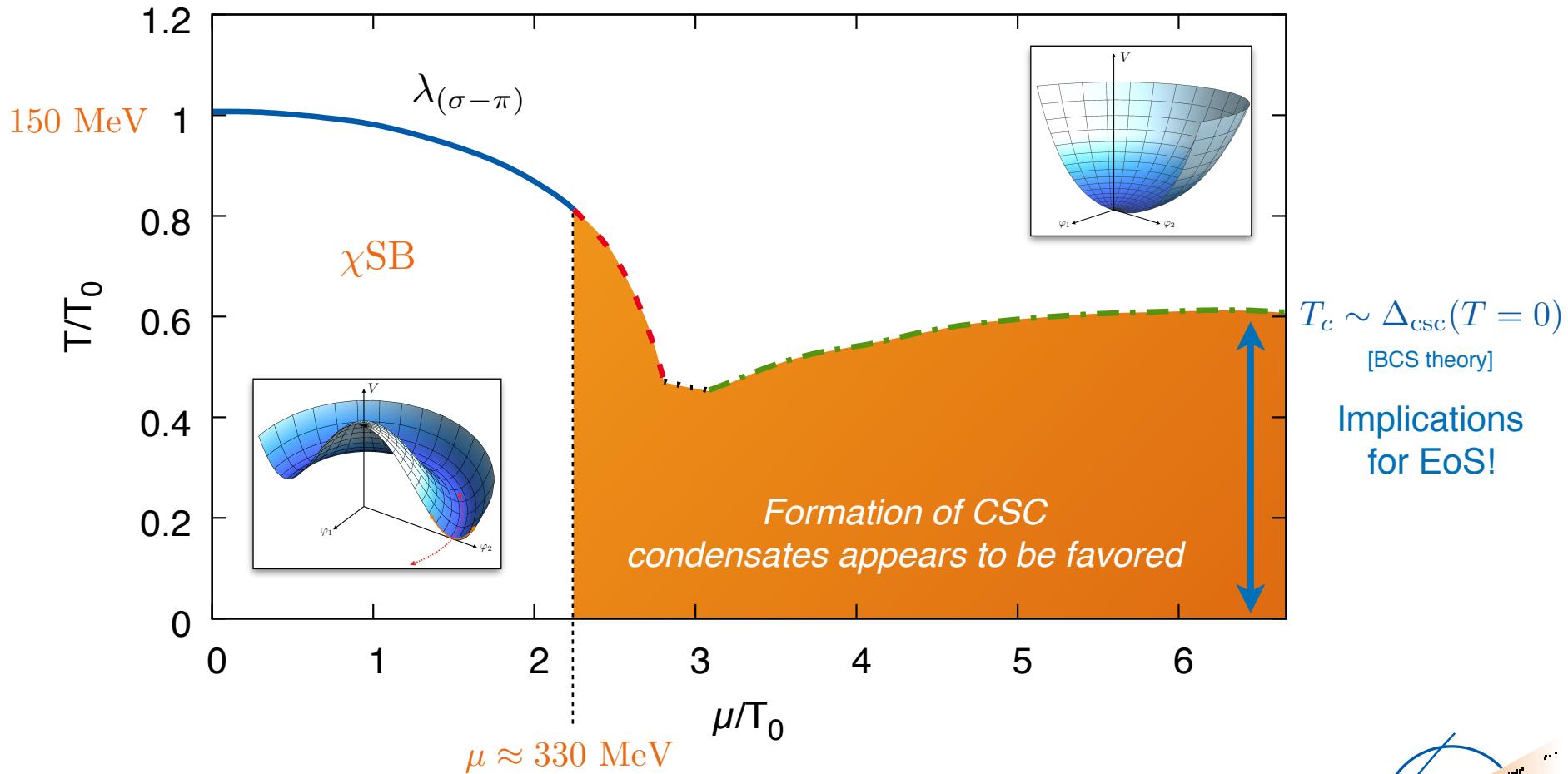
[see also Braun, 2006; Mitter, Pawłowski, Strodthoff 2014]

# Exploring the phase diagram

Patterns of symmetry breaking and formation of condensates



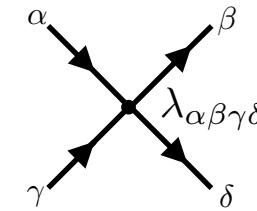
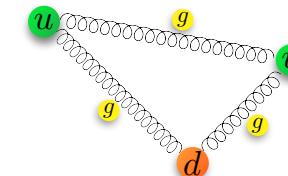
TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



# Outline

## ① Including dynamic gauge fields

- Framework and structure of  $\beta$  functions
- Symmetry breaking patterns
- Implications on the phase diagram



EoS

## ② Connecting to low-energy effective models (LEMs)

- Quark-meson-diquark model
- Findings on the equation of state (EoS)

Conclusions and outlook



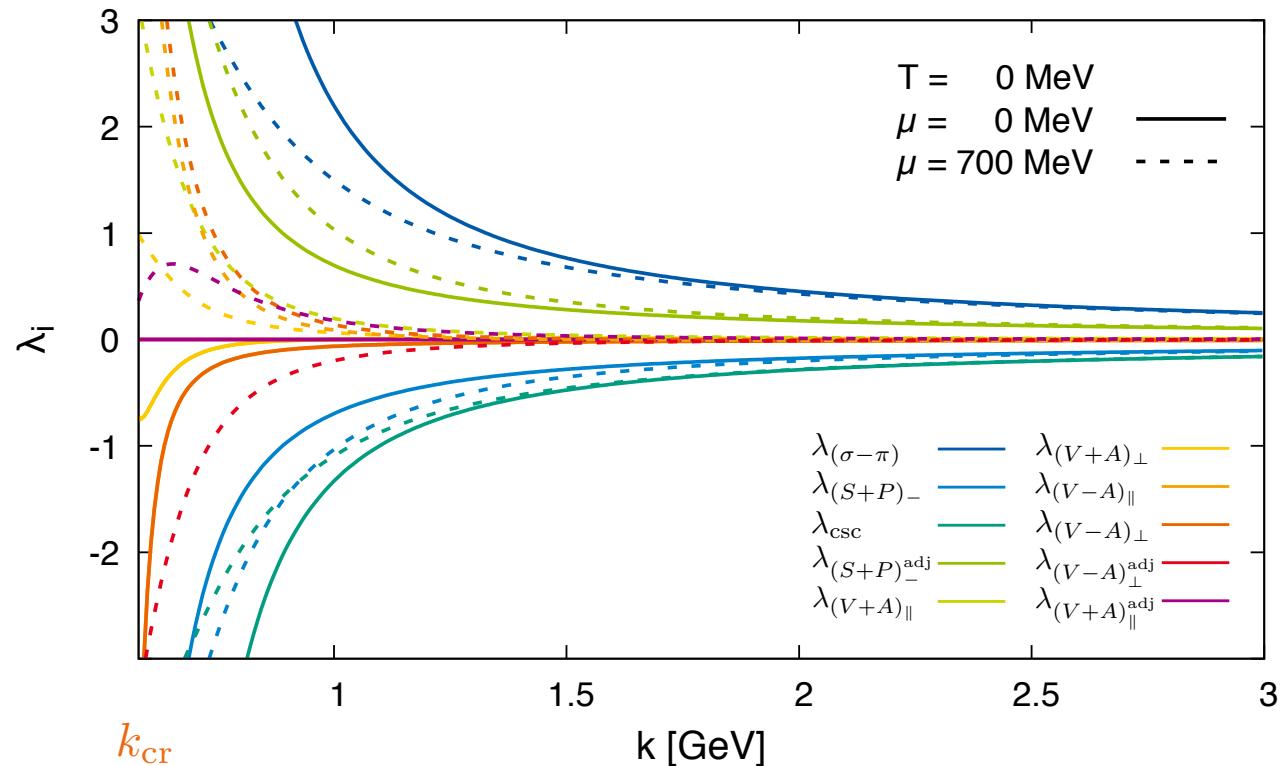
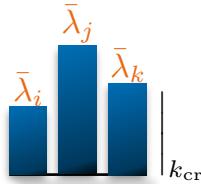
# RG flow of four-quark interactions

Illustration of the RG flow at zero temperature



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

- Diverging coupling signals onset of condensate formation
- Assessing relative interactions strengths  
→ Formation of specific condensates



[Braun, ML, Pospiech, (2018) in preparation]

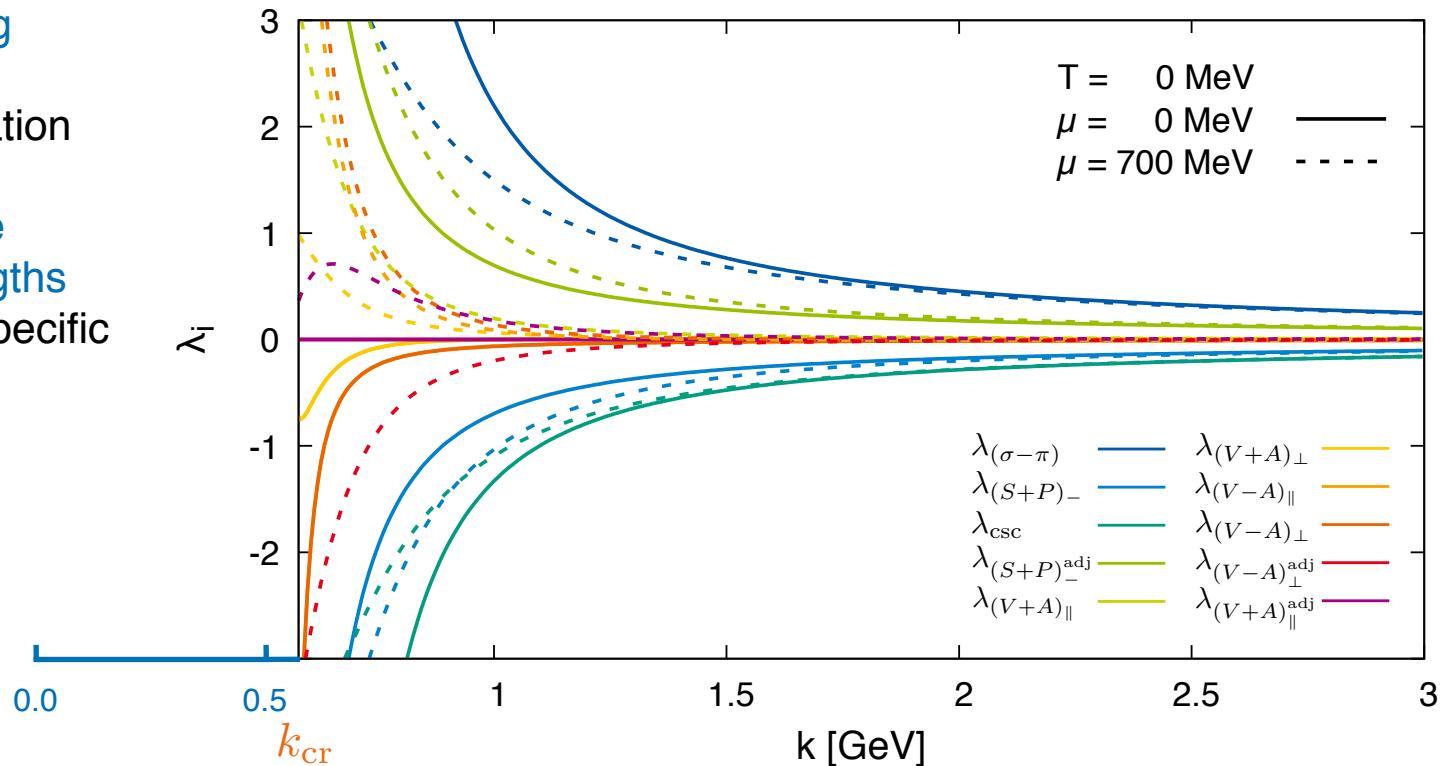
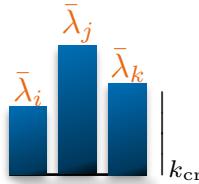
# RG flow of four-quark interactions

Illustration of the RG flow at zero temperature



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

- ▶ Diverging coupling signals onset of condensate formation
- ▶ Assessing relative interactions strengths  
→ Formation of specific condensates



customized ansatz  
for low energy part

now: two most dominant channels

chiral fields  $\varphi$   
diquark fields  $\Delta$  }  $S_{\text{QMD}}$

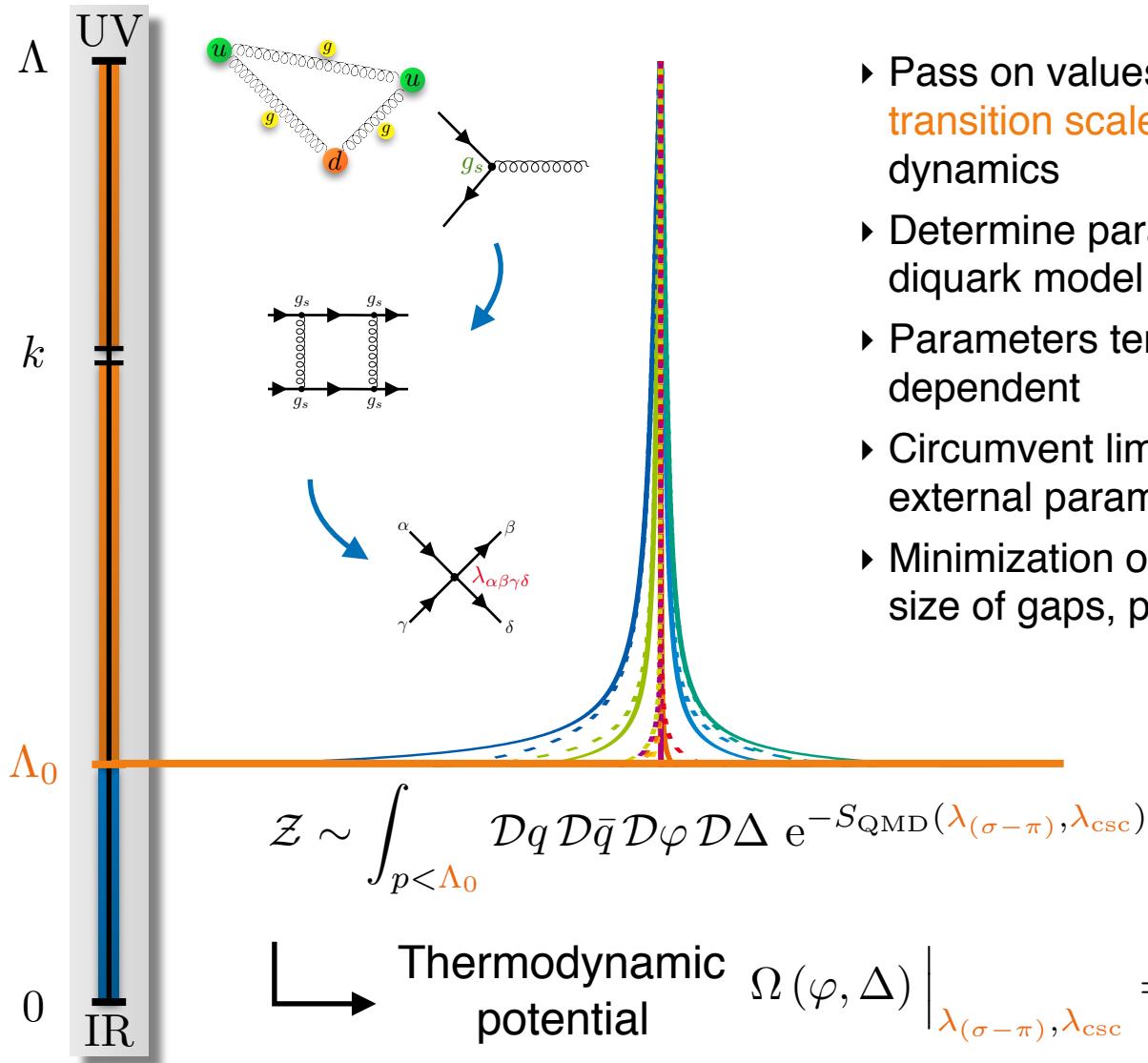
[Braun, ML, Pospiech, (2018) in preparation]

# Connecting to low-energy dynamics

Determining parameters at high densities



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



- ▶ Pass on values of four-quark couplings at **transition scale  $\Lambda_0$**  to access low-energy dynamics
- ▶ Determine parameters of a quark-meson-diquark model
- ▶ Parameters temperature and density dependent
- ▶ Circumvent limit of **accessible range** of external parameters
- ▶ Minimization of thermodynamic potential: size of gaps, pressure, density, ...

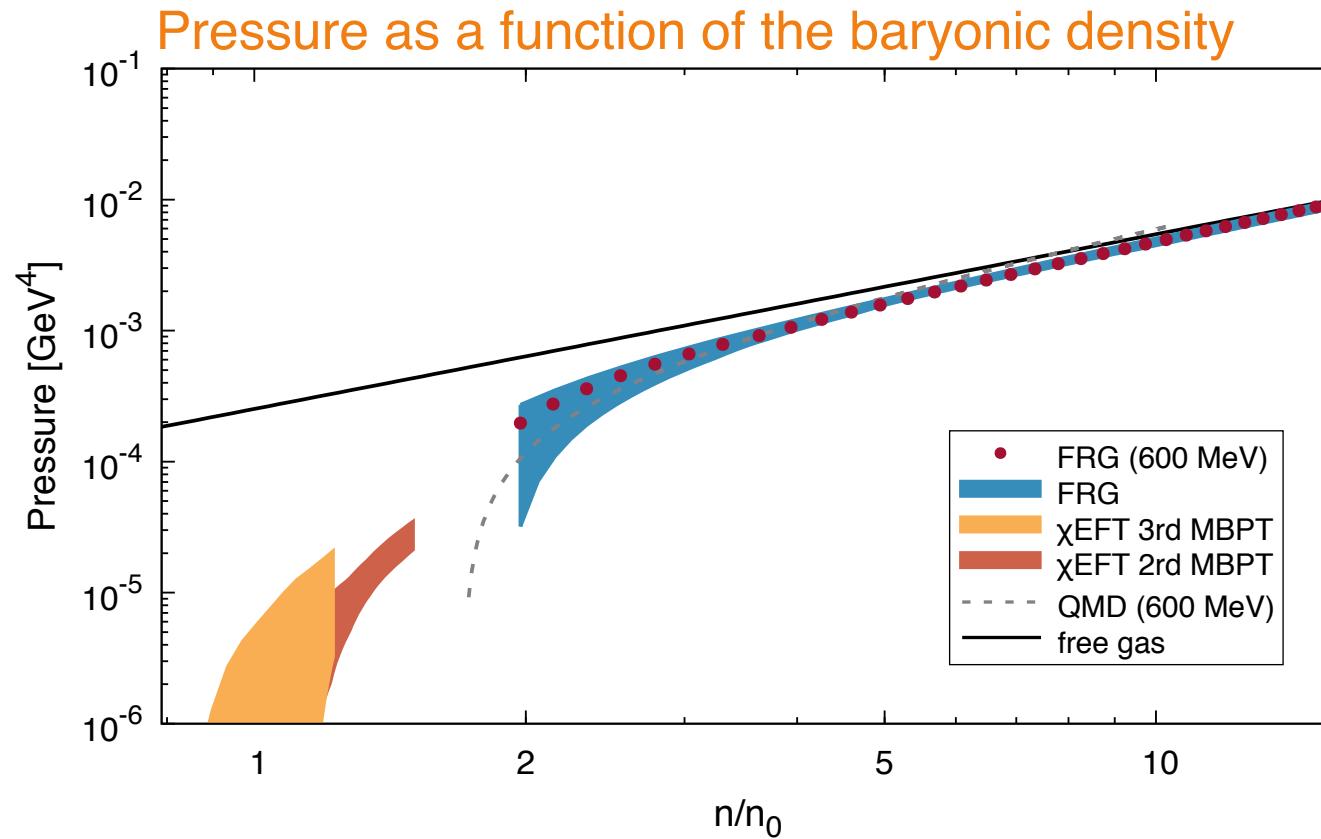


# Implications on the equation of state

Connecting to the quark-meson-diquark model



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



[ $\chi$ EFT data by C. Drischler, K. Hebeler and A. Schwenk]

- Compatible to pQCD [E. S. Fraga, A. Kurkela, and A. Vuorinen, 2015]
- Degrees of freedom at smaller densities [e.g. M. Drews and W. Weise, 2016]

# Conclusions and outlook



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

## Dynamic gauge fields

- Dynamical generation of four-quark interactions by gluodynamics
- Importance of Fierz-completeness at high quark chemical potential and low temperature
- Formation of CSC condensates appears to be favored at high densities

## Connecting to low-energy dynamics

- Utilizing RG flow of four-quark couplings to determine parameters of low-energy effective model at high densities
- Equation of state:
  - Connects to free gas pressure at asymptotically large densities
  - Pressure towards smaller densities lifted to higher values; compatibility with  $\chi$ EFT results appears to be improved

## Outlook

- Improve connection between high- and low-energy regime by employing dynamical hadronization techniques



# Backup



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

# Four-quark interactions and symmetry breaking



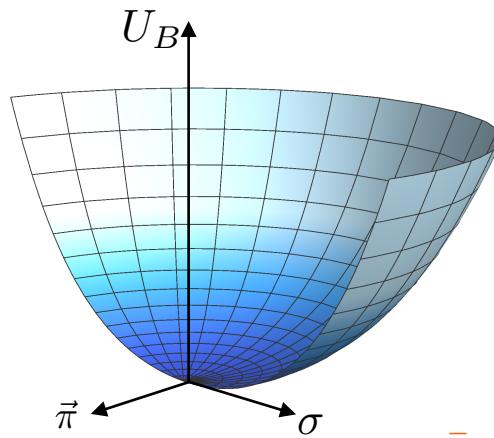
$$S[\bar{\psi}, \psi] = \int_x \left\{ \bar{\psi} i \not{\partial} \psi + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} [(\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \vec{\tau} \psi)^2] \right\}$$

**Partial bosonization**

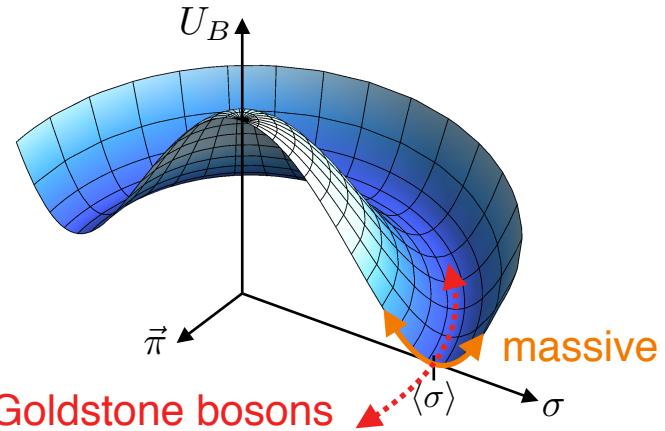
$$\begin{aligned}\sigma &\sim \bar{\psi} \psi \\ \vec{\pi} &\sim \bar{\psi} \gamma_5 \vec{\tau} \psi\end{aligned}$$

Symmetry of the  
ground state

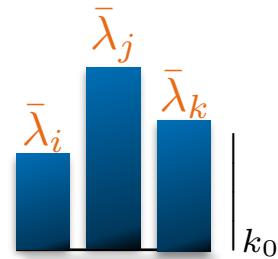
$$U_B \sim \frac{1}{\bar{\lambda}_{(\sigma-\pi)}} (\sigma^2 + \pi^2) + \dots$$



$$\frac{1}{\bar{\lambda}_{(\sigma-\pi)}} \Big|_{k_0} \rightarrow 0$$



Assessing relative  
interaction strengths



Formation of  
specific condensates



# Four-quark interactions and symmetries

Fierz-complete basis of interactions



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

In total **20 channels** meet  
symmetry constraints

**Fierz identities**

Fierz-complete basis:  
**10 channels**

$U_A(1)$  breaking channels:

$$\mathcal{L}_{(\bar{\psi}\psi)^2}^{(\sigma-\pi)} = (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2$$

$\leftrightarrow$  formation of  
chiral condensate

$$\mathcal{L}_{(\bar{\psi}\psi)^2}^{\text{csc}} \sim (i\bar{\psi}\gamma_5\tau_A t_c^{A'} \mathcal{C} \bar{\psi}^T)(i\psi^T \mathcal{C} \gamma_5\tau_A t_c^{A'} \psi) \quad J^P = 0^+$$

[Rapp, Schäfer, Shuryak, Velkovsky, 1998]

$\leftrightarrow$  formation of  
diquark condensate

$$\mathcal{L}_{(\bar{\psi}\psi)^2}^{\det} = (\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 - (\bar{\psi}\vec{\tau}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2$$

$$\mathcal{L}_{(\bar{\psi}\psi)^2}^{(S+P)_-^{\text{adj}}} = (\bar{\psi}T^a\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}T^a\psi)^2 + (\bar{\psi}\gamma_5T^a)^2 - (\bar{\psi}\vec{\tau}T^a\psi)^2$$

$U_A(1)$  symmetric channels:

$$\mathcal{L}_{(\bar{\psi}\psi)^2}^{(V+A)_\parallel} \quad \mathcal{L}_{(\bar{\psi}\psi)^2}^{(V+A)_\perp} \quad \mathcal{L}_{(\bar{\psi}\psi)^2}^{(V-A)_\parallel} \quad \mathcal{L}_{(\bar{\psi}\psi)^2}^{(V-A)_\perp} \quad \mathcal{L}_{(\bar{\psi}\psi)^2}^{(V+A)_\parallel^{\text{adj}}} \quad \mathcal{L}_{(\bar{\psi}\psi)^2}^{(V-A)_\perp^{\text{adj}}}$$

[Braun, ML, Pospiech '17]

# Exploring the phase diagram

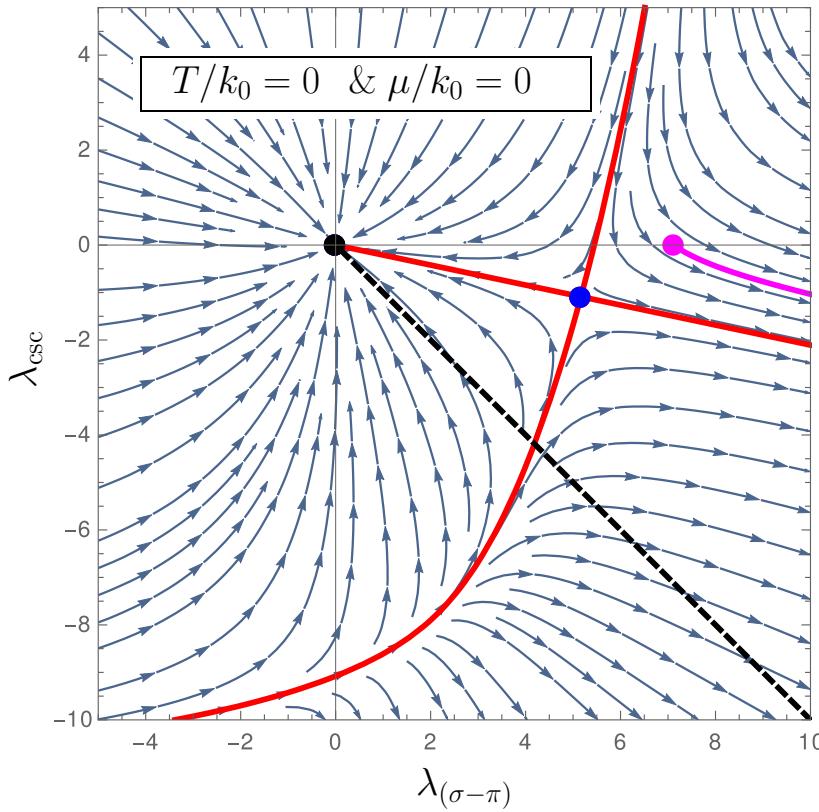
Fixed-point structure and patterns of symmetry breaking



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

## Ansatz two-channel approximation

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} (\sigma - \pi) + \frac{1}{2} \bar{\lambda}_{\csc} (\csc) \right\}$$



# Exploring the phase diagram

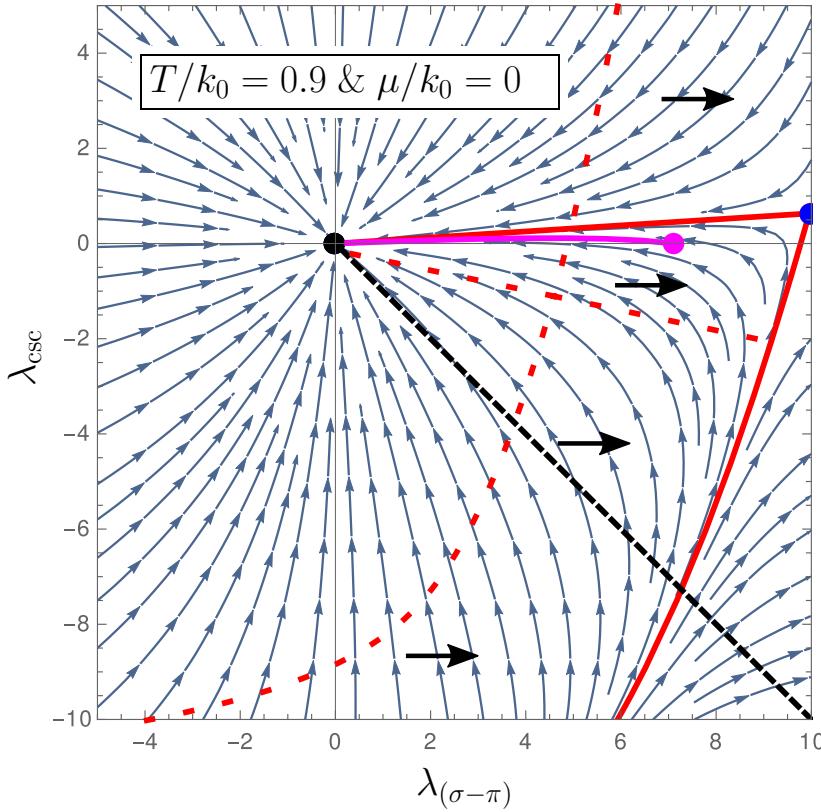
Fixed-point structure and patterns of symmetry breaking



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

## Ansatz two-channel approximation

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} (\sigma - \pi) + \frac{1}{2} \bar{\lambda}_{\text{csc}} (\text{csc}) \right\}$$



# Exploring the phase diagram

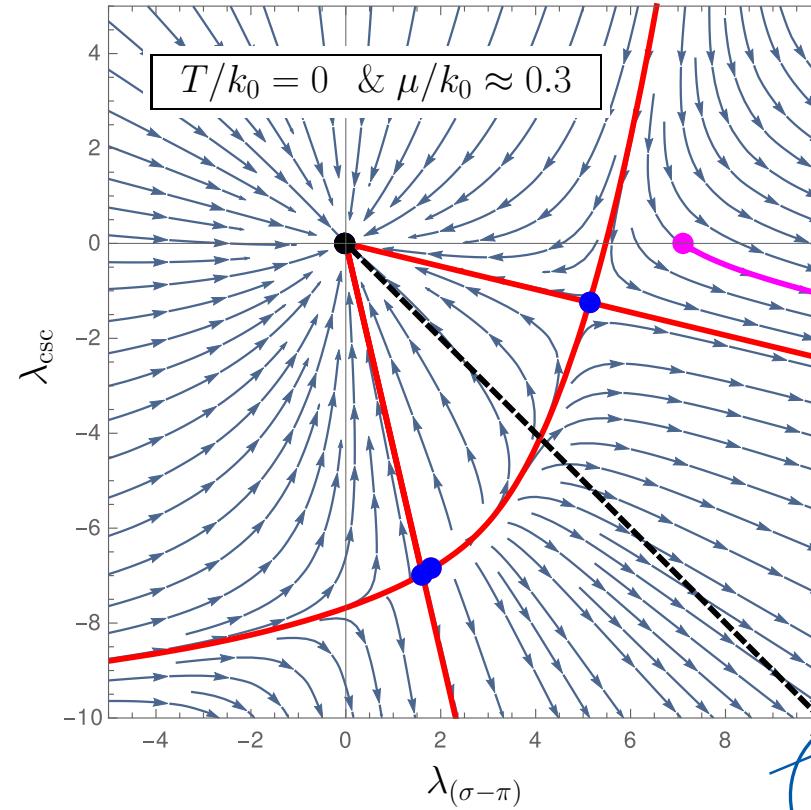
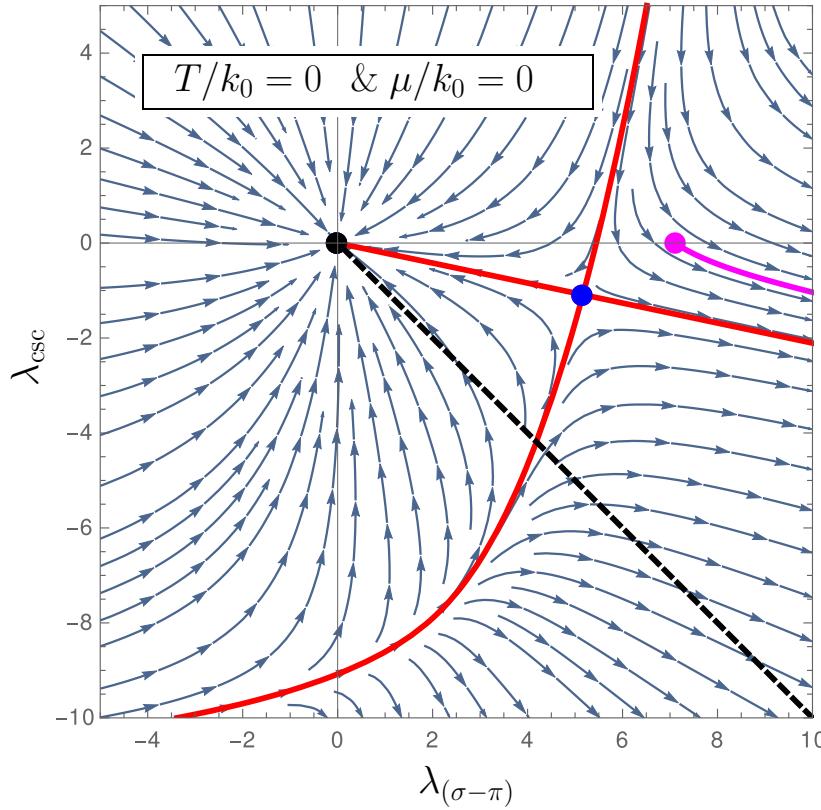
Fixed-point structure and patterns of symmetry breaking



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

## Ansatz two-channel approximation

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} (\sigma - \pi) + \frac{1}{2} \bar{\lambda}_{\text{csc}} (\text{csc}) \right\}$$



# Exploring the phase diagram

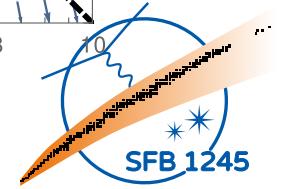
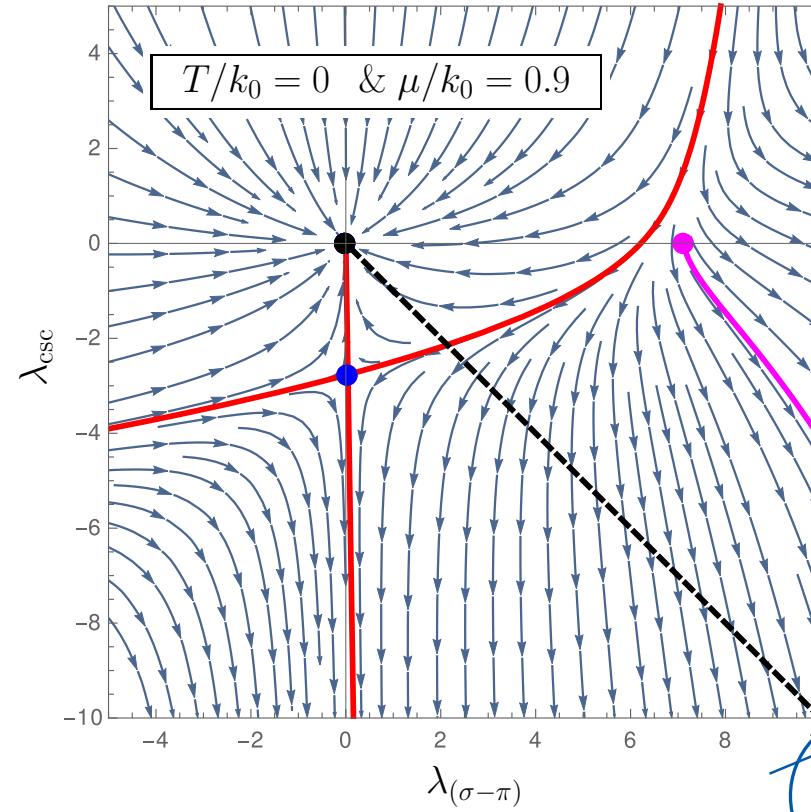
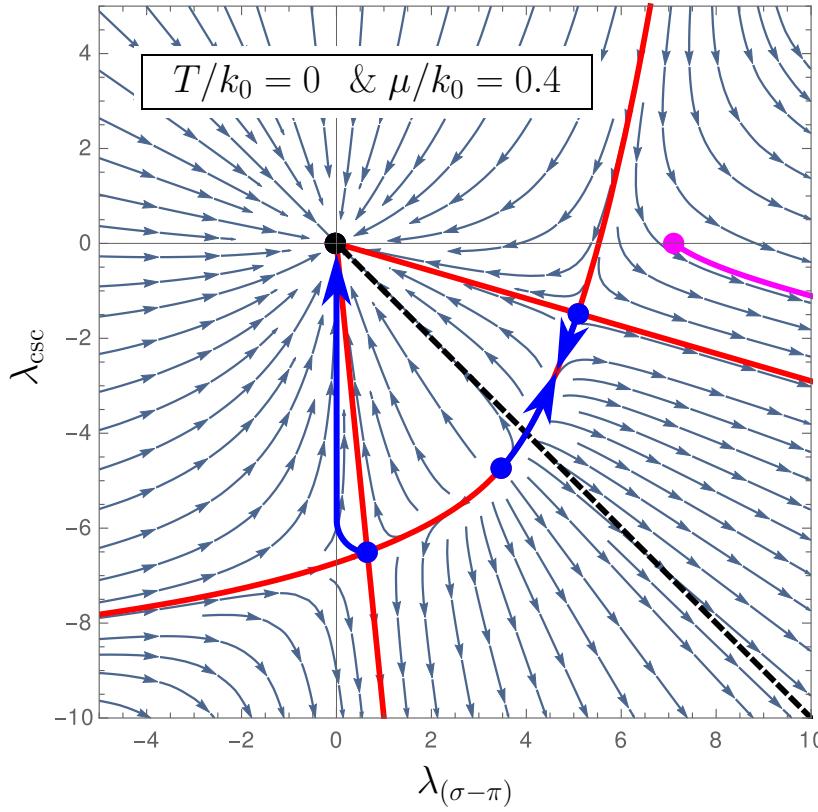
Fixed-point structure and patterns of symmetry breaking



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

## Ansatz two-channel approximation

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} (\sigma - \pi) + \frac{1}{2} \bar{\lambda}_{\text{csc}} (\text{csc}) \right\}$$

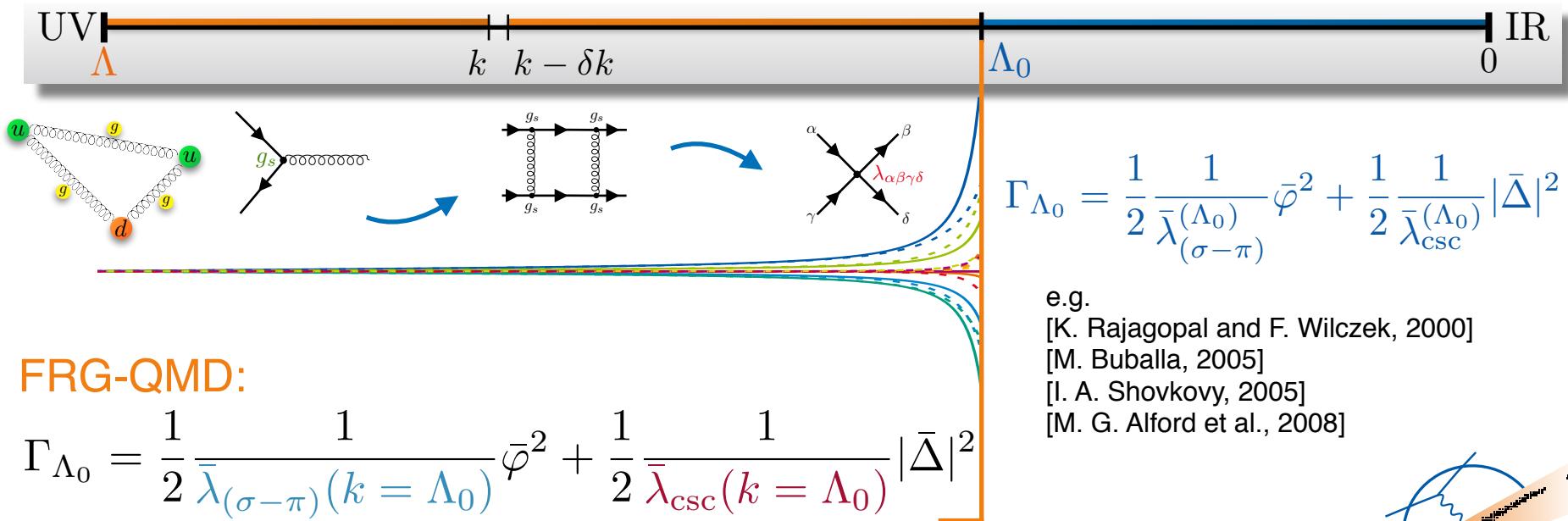


# Connecting to low energy effective models

Determining parameters at high densities

## Quark-meson-diquark model (QMD)

$$S = \int_x \left\{ \bar{q} \left( i\cancel{\partial} - i\mu\gamma_0 + i(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5) \right) q + \frac{1}{2} \frac{1}{\bar{\lambda}_{(\sigma-\pi)}} \varphi^2 \right. \quad \text{QM part} \\ \left. + \bar{q}\gamma_5\tau_2\Delta_A^* T^A \mathcal{C} \bar{q}^T - q^T \mathcal{C} \gamma_5\tau_2\Delta_A T^A \mathcal{C} q + \frac{1}{2} \frac{1}{\bar{\lambda}_{\text{csc}}} |\Delta|^2 \right\} \text{diquarks}$$



## FRG-QMD:

$$\Gamma_{\Lambda_0} = \frac{1}{2} \frac{1}{\bar{\lambda}_{(\sigma-\pi)}(k = \Lambda_0)} \bar{\varphi}^2 + \frac{1}{2} \frac{1}{\bar{\lambda}_{\text{csc}}(k = \Lambda_0)} |\bar{\Delta}|^2$$