NTNU – Trondheim
Norwegian University of
Science and Technology

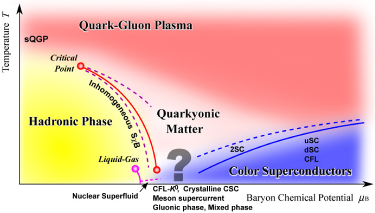
Pion condensation and QCD phase diagram at finite isospin density

XIIIth Quark Confinement and the Hadron Spectrum
Maynooth August 3 2018

Jens O. Andersen

1. Collaborators: Prabal Adhikari (St Olaf)
   Patrick Kneschke (UiS)

2. References: Phys. Rev. **D** 97 076005 (2018)
   Phys. Rev. **D** 95, 036017 (2017)
   e-Print: arXiv:1805.08599



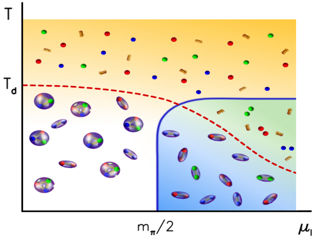
Fukushima et al. Rept. Prog. Phys. 74 (2011) 014001.

1. EoS relevant for early universe, heavy-ion collisions and compact stars.
2. Quark-gluon plasma at high temperature
3. CFL phase at large baryon chemical potential.
4. Only few exact results known (QGP, CFL) (talks Brauner and Kolesova).
5. Sign problem at finite μ_B. Monte Carlo simulations difficult.
6. No sign problem at finite μ_I and $\mu_B = 0$. Lattice simulations possible. [1]

[1] Kogut and Sinclair (2002), Brandt, Endrodi, and Schmalzbauer (2018).

Pion condensation and QCD phase diagram at finite isospin density

1. Isospin chemical potential μ_I introduces an imbalance between up and down-quarks $\mu_u = \mu + \mu_I$, $\mu_d = \mu - \mu_I$.
2. This talk [2]



B. B. Brandt, G. Endrodi, and S. Schmalzbauer, Phys.Rev. D **97**, 054514 (2018).

a) Phase diagram in $\mu - \mu_I$ plane. Inhomogeneous phases and competition with a pion condensate at $T = 0$? (Talk Buballa).

b) Phase diagram in the $\mu_I - T$ plane. BEC-BCS crossover. Chiral and deconfinement transitions? (Poster Brandt).

[2] Son and Stephanov (2001).

1. Quark-meson model

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[(\partial_\mu\sigma)(\partial^\mu\sigma)+(\partial_\mu\pi_3)(\partial^\mu\pi_3)\right]+(\partial_\mu+2i\mu_I\delta_\mu^0)\pi^+(\partial^\mu-2i\mu_I\delta_0^\mu)\pi^- \\ &\quad -\frac{1}{2}m^2(\sigma^2+\pi_3^2+2\pi^+\pi^-)-\frac{\lambda}{24}(\sigma^2+\pi_3^2+2\pi^+\pi^-)^2 \\ &\quad +h\sigma+\bar{\psi}\left[i\partial\!\!\!/+\mu_f\gamma^0-g(\sigma+i\gamma^5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\psi\end{aligned}$$

2. Chiral density wave and constant pion condensate

$$\sigma=\phi_0\cos(qz)\,,\qquad \pi_1=\pi_0\,,\qquad \pi_3=\phi_0\sin(qz)$$

3. Meson potential with $\Delta = g\phi_0$ and $\rho = g\pi_0$

$$V_0=\frac{1}{2}\frac{q^2}{g^2}\Delta^2+\frac{1}{2}\frac{m^2}{g^2}\Delta^2+\frac{1}{2}\frac{m^2-4\mu_I^2}{g^2}\rho^2+\frac{\lambda}{24g^4}\left(\Delta^2+\rho^2\right)^2-\frac{h}{g}\Delta\cos(qz)\delta_{q,0}$$

1. Quark energies

$$
\begin{aligned}
E_u^{\pm} &= E(\pm q, -\mu_I)\,, E_d^{\pm} = E(\pm q, \mu_I)\,, E_{\bar{u}}^{\pm} = E(\pm q, \mu_I)\,, E_{\bar{d}}^{\pm} = E(\pm q, -\mu_I)\,, \\
E(q, \mu_I) &= \left[\left(\sqrt{p_\perp^2 + \left(\sqrt{p_\parallel^2 + \Delta^2} + \frac{q}{2}\right)^2} + \mu_I\right)^2 + \rho^2\right]^{\frac{1}{2}}\,,
\end{aligned}
$$

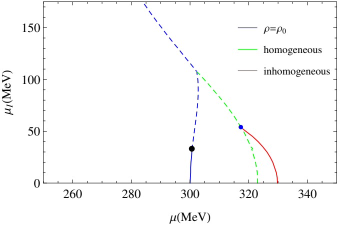
2. Integrate over fermions (regulator artefacts)

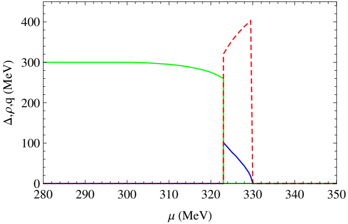
$$
V_1 = -\frac{1}{2}N_c \int_p \left(E_u^{\pm} + E_d^{\pm} + E_{\bar{u}}^{\pm} + E_{\bar{d}}^{\pm}\right) + \text{medium contribution}
$$

3. Model parameters are fixed using the on-shell renormalization scheme.

$$
\begin{aligned}
V_1 \quad = \quad & \frac{1}{2} f_\pi^2 q^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\log \frac{\Delta^2 + \rho^2}{m_q^2} + H(m_\pi^2) \right] \right\} \frac{\Delta^2}{m_q^2} \\
& + \frac{3}{4} m_\pi^2 f_\pi^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} m_\pi^2 F'(m_\pi^2) \right\} \frac{\Delta^2 + \rho^2}{m_q^2} \\
& - \frac{1}{4} m_\sigma^2 f_\pi^2 \left\{ 1 + \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\left(1 - \frac{4m_q^2}{m_\sigma^2} \right) F(m_\sigma^2) + \frac{4m_q^2}{m_\sigma^2} - H(m_\pi^2) \right] \right\} \frac{\Delta^2 + \rho^2}{m_q^2} \\
& - 2\mu_I^2 f_\pi^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\log \frac{\Delta^2 + \rho^2}{m_q^2} + H(m_\pi^2) \right] \right\} \frac{\rho^2}{m_q^2} \\
& + \frac{1}{8} m_\sigma^2 f_\pi^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\frac{4m_q^2}{m_\sigma^2} \left(\log \frac{\Delta^2 + \rho^2}{m_q^2} - \frac{3}{2} \right) - G(m_\sigma^2) + H(m_\pi^2) \right] \right\} \frac{(\Delta^2 + \rho^2)^2}{m_q^4} \\
& - \frac{1}{8} m_\pi^2 f_\pi^2 \left[1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} m_\pi^2 F'(m_\pi^2) \right] \frac{(\Delta^2 + \rho^2)^2}{m_q^4} - m_\pi^2 f_\pi^2 \left[1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} m_\pi^2 F'(m_\pi^2) \right] \frac{\Delta \delta_{q,0}}{m_q} \\
& + V_{\rm fin} + N_c \int_p \left[(E_u^\pm - \mu)\theta(\mu - E_u^\pm) + (E_d^\pm - \mu)\theta(\mu - E_d^\pm) \right] .
\end{aligned}
$$

1. Phase diagram and condensates in the chiral limit

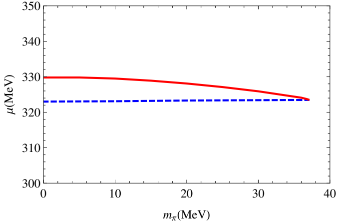


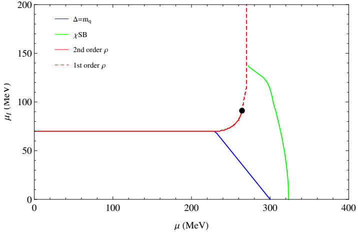


a) First - and second-order transitions with critical endpoints.

b) No coexistence of inhomogeneous chiral condensate and pion condensate.

2. m_π-dependence of inhomogeneous condensate and homogeneous phase diagram at the physical point





a) Critical m_π^c for existence of inhomogeneous condensate.

b) Critical $\mu_I^c = \frac{1}{2} m_\pi$ at $T = 0$ and Silver Blaze property. [3]

[3] T. D. Cohen, Phys. Rev. Lett. **91**, 222001 (2003).

1. Wilson line

$$L(\mathbf{x}) \quad = \quad \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\mathbf{x},\tau)\right] \ .$$

2. Polyakov loop order parameter for deconfinement

$$\Phi = \frac{1}{N_c}\langle \mathrm{Tr}L\rangle \ , \quad \bar{\Phi} = \frac{1}{N_c}\langle \mathrm{Tr}L^\dagger\rangle \ .$$

3. Polyakov gauge and coupling to quarks [4]

$$A_4 \quad = \quad t_3 A_4^3 + t_8 A_4^8 \ .$$

[4] K. Fukushima Phys.Lett. B **591**, 277 (2004)

4. Fermionic contribution to effective potential

$$
\begin{aligned}
V_T \quad = \quad & -2T\int\frac{d^3p}{(2\pi)^3}\Big\{\mathrm{Tr}\log\left[1+3(\Phi+\bar{\Phi}e^{-\beta E_u})e^{-\beta E_u}+e^{-3\beta E_u}\right] \\
& +\mathrm{Tr}\log\left[1+3(\bar{\Phi}+\Phi e^{-\beta E_{\bar{u}}})e^{-\beta E_{\bar{u}}}+e^{-3\beta E_{\bar{u}}}\right] \\
& +\mathrm{Tr}\log\left[1+3(\Phi+\bar{\Phi}e^{-\beta E_d})e^{-\beta E_d}+e^{-3\beta E_d}\right] \\
& +\mathrm{Tr}\log\left[1+3(\bar{\Phi}+\Phi e^{-\beta E_{\bar{d}}})e^{-\beta E_{\bar{d}}}+e^{-3\beta E_{\bar{d}}}\right]\Big\}\,.
\end{aligned}
$$

5. $\Phi=\bar{\Phi}=1$ one recovers the usual fermion contribution.

6. Glue potential [5]

$$
\begin{aligned}
\frac{\mathcal{U}}{T^4} &= -\frac{1}{2}b_2\Phi\bar{\Phi} - \frac{1}{6}b_3\left(\Phi^3+\bar{\Phi}^3\right) + \frac{1}{4}b_4\left(\Phi\bar{\Phi}\right)^2 \ , \\
b_2 &= a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 \ , \\
b_3 &= \frac{3}{4} \ , \\
b_4 &= \frac{30}{4} \ .
\end{aligned}
$$

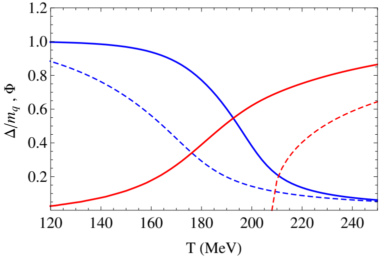
7. μ_I-dependent parameter [6]

$$
T_0(N_f,\mu_I) = T_\tau e^{-1/(\alpha_0 b(\mu_I))} \ .
$$

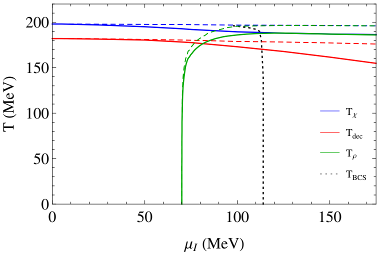
[5] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D **73** , 014019 (2006)

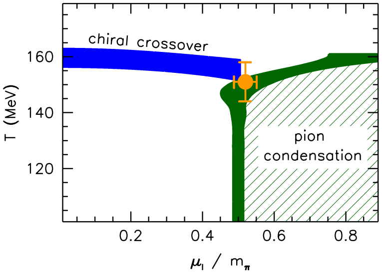
[6] B.-J. Schaefer, J. M. Pawlowski, and J. Wambach, Phys. Rev. D **76**, 074023 (2007).

8. Order parameters at vanishing μ and μ_I in the QM and PQM models.



1. Phase diagram[7]





1. BEC line always second order.
2. BEC line and chiral line merge.

[7] Endrodi B. Brandt, G. Endrodi, and S. Schmalzbauer Phys. Rev. D **97**, 054514 (2018)

Conclusions:

1. Rich phase diagrams. Onset of pion condensation at exactly $\mu_I^c = \frac{1}{2} m_\pi$.
2. No inhomogeneous chiral condensate for physical quark masses.
3. Good agreement between lattice simulations and model calculations.
   a) Second-order transition to a BEC state.
   b) BEC and chiral transition lines merge at large μ_I.

Outlook:

1. Mesonic fluctuations. [8]
2. Pion stars. [9]

[8] Kamikado, Strodthoff, von Smekal, and Wambach, PLB 718 (2013).

[9] Brandt et al, arXiv:1802.06685, JOA and P. Kneschke, arXiv:1807.08951.

THANK YOU FOR YOUR ATTENTION