EFT determination of heavy-hybrid spin potential

N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà, and A. Vairo
arXiv:1805.07713 [hep-ph]

Wai Kin Lai

Technical University of Munich

August 5, 2018

XIIIth Quark Confinement and the Hadron Spectrum
Maynooth University

Outline

- Quarkonium hybrids in pNRQCD

Outline

- Quarkonium hybrids in pNRQCD
- Nonperturbative matching for spin potential

Outline

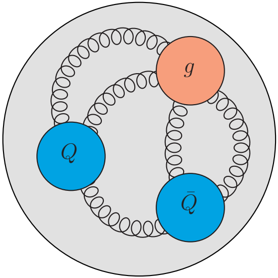
- Quarkonium hybrids in pNRQCD
- Nonperturbative matching for spin potential
- Result of spin-splitting

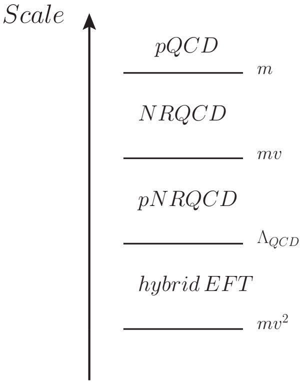
Outline

- Quarkonium hybrids in pNRQCD
- Nonperturbative matching for spin potential
- Result of spin-splitting
- Summary

Quarkonium hybrids in pNRQCD

- Quarkonium hybrid: $Q\bar{Q}$ in color octet with gluon excitation
- Separation of scales: $m \gg mv \gg \Lambda_{QCD} \gg m_Q v^2 \Longrightarrow$ suitable for EFT description ($m \equiv m_Q$)
- Integrate out d.o.f.: pQCD $\rightarrow$ NRQCD $\rightarrow$ pNRQCD $\rightarrow$ hybrid EFT





At the scale $\Lambda_{QCD} \ll \mu \ll mv$, we have the weakly-coupled pNRQCD:

Lagrangian of weakly-coupled pNRQCD

$$L_{\rm pNRQCD} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o \right) \mathrm{O} \right]$$
$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r} \cdot \boldsymbol{E} \mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r} \cdot \boldsymbol{E} \mathrm{S} + \frac{1}{2} \mathrm{O}^\dagger \boldsymbol{r} \cdot \{ \boldsymbol{E}, \mathrm{O} \} \right] + \frac{g}{4m} \mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}} \cdot [\boldsymbol{B}, \mathrm{O}] \right]$$
$$+ \frac{g c_F}{m} \mathrm{Tr}\left[\mathrm{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2) \cdot \boldsymbol{B} \mathrm{O} + \mathrm{O}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2) \cdot \boldsymbol{B} \mathrm{S} + \mathrm{O}^\dagger \boldsymbol{S}_1 \cdot \boldsymbol{B} \mathrm{O} - \mathrm{O}^\dagger \boldsymbol{S}_2 \mathrm{O} \cdot \boldsymbol{B} \right]$$
$$+ \frac{g c_s}{2m^2} \mathrm{Tr}\left[\mathrm{S}^\dagger (\boldsymbol{S}_1 + \boldsymbol{S}_2) \cdot (\boldsymbol{E} \times \boldsymbol{p}) \mathrm{O} + \mathrm{O}^\dagger (\boldsymbol{S}_1 + \boldsymbol{S}_2) \cdot (\boldsymbol{E} \times \boldsymbol{p}) \mathrm{S} \right.$$
$$\left. + \mathrm{O}^\dagger \boldsymbol{S}_1 \cdot (\boldsymbol{E} \times \boldsymbol{p}) \mathrm{O} + \mathrm{O}^\dagger \boldsymbol{S}_2 \mathrm{O} \cdot (\boldsymbol{E} \times \boldsymbol{p}) \right] \Big) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} + \dots \Big\}$$

$$h_s = -\frac{\boldsymbol{\nabla}_r^2}{m} + V_s(r)$$
$$h_o = -\frac{\boldsymbol{\nabla}_r^2}{m} + V_o(r)$$

$$V_o(r) = V_o^{(0)}(r) + \frac{V_o^{(1)}(r)}{m} + \frac{V_o^{(2)}(r)}{m^2} + \dots$$
$$V_o^{(2)}(r) = V_{o\,SD}^{(2)}(r) + V_{o\,SI}^{(2)}(r)$$
$$V_{o\,SD}^{(2)}(r) = V_{o\,SL}(r) \boldsymbol{L}_{Q\bar{Q}} \cdot \boldsymbol{S} + V_{o\,S^2}(r) \boldsymbol{S}^2 + V_{o\,S_{12}}(r) S_{12}$$

$\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$, $S_{12} = 12(\boldsymbol{S}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{S}_2 \cdot \hat{\boldsymbol{r}}) - 4\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$

The gluon excitatiom lives at the scale Λ_{QCD}. In the limit $r \to 0$, the gluon excitation can be characterized by the gluelump operator $G^{ia}_{\kappa}(\boldsymbol{R})$ ($\kappa = K^{PC}$):

$$h_0(\boldsymbol{R}) G^{ia}_{\kappa}(\boldsymbol{R})|0\rangle = \Lambda_{\kappa} G^{ia}_{\kappa}(\boldsymbol{R})|0\rangle$$

$$h_0(\boldsymbol{R}) = \frac{1}{2}\left(\boldsymbol{E}^a \cdot \boldsymbol{E}^a + \boldsymbol{B}^a \cdot \boldsymbol{B}^a\right)$$

Energy eigenstates in the octet sector of the weakly-coupled pNRQCD Hamiltonian:

$$|\kappa,\, \lambda\rangle = P^{i}_{\kappa\lambda} O^{a\,\dagger}(\boldsymbol{r}, \boldsymbol{R})\, G^{ia}_{\kappa}(\boldsymbol{R})|0\rangle$$

$P^{i}_{\kappa\lambda}$: projects the gluelump operator to an eigenstate of $\boldsymbol{K} \cdot \hat{\boldsymbol{r}}$ with eigenvalue λ
λ: characterize irreducible representation of $D_{\infty h}$

After integrating out Λ_{QCD}, we obtain the hybrid EFT:

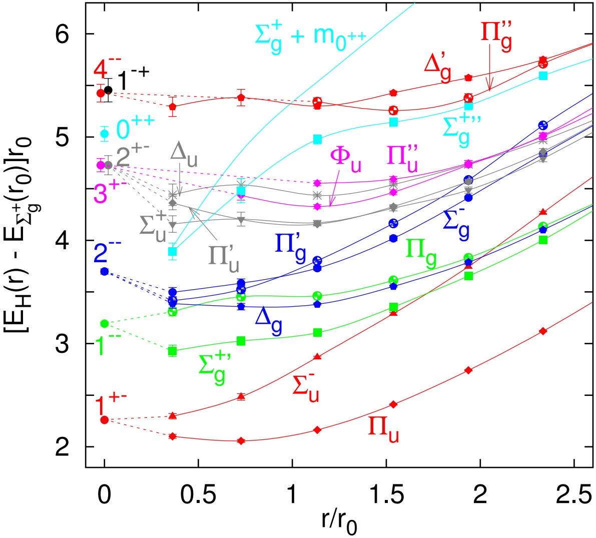
Lagrangian of hybrid EFT

$$L_{hybrid} = \int d^3R d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \Big\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m} P^{i}_{\kappa\lambda'} \Big\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) + \ldots$$

$\Psi_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R})$: wave function of $Q\bar{Q}$ in the quarkonium hybrid

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m^2} + \ldots$$
$$V^{(1)}_{\kappa\lambda\lambda'}(r) = V^{(1)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(1)}_{\kappa\lambda\lambda'\,SI}(r)$$
$$V^{(2)}_{\kappa\lambda\lambda'}(r) = V^{(2)}_{\kappa\lambda\lambda'\,SD}(r) + V^{(2)}_{\kappa\lambda\lambda'\,SI}(r)$$

$V^{(0)}_{\kappa\lambda}(r)$ is the static potential, which has been calculated on the lattice.



from G. S. Bali and A. Pineda, Phys. Rev. D **69**, 094001 (2004) [hep-ph/0310130]

The static energy has the form

$$V_{\kappa\lambda}^{(0)}(r) = \Lambda_\kappa + V_o^{(0)}(r) + \dots$$

- Λ_κ: gluelump mass, depends only on κ.
- $V_o^{(0)}(r)$: perturbative octet potential inherited from weakly-couple pNRQCD.
- "+..." nonperturbative r-dependent pieces, fitted to lattice data.

The hybrid mass spectra for the lowest-lying gluelump ($\kappa = 1^{+-}$) have been calculated in M. Berwein, N. Brambilla, J. Tarrús Castellà and A. Vairo, Phys. Rev. D **92**, no. 11, 114019 (2015) [arXiv:1510.04299 [hep-ph]].
In this work, we will only consider the lowest-lying gluelump.

Nonperturbative matching for spin potential

The spin-dependent potentials have the form

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{K}^{ij}_{1}P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S} + V_{1\,SKb}(r)\left[\left(\boldsymbol{r}\cdot P^{\dagger}_{1\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}P^{j}_{1\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}P^{j\dagger}_{1\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot P_{1\lambda'}\right)\right] + \dots$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SLa}(r)\left(P^{i\dagger}_{1\lambda}\boldsymbol{L}_{Q\bar{Q}}P^{i}_{1\lambda'}\right)\cdot\boldsymbol{S} + V_{1\,SLb}(r)P^{i\dagger}_{1\lambda}\left(L^{i}_{Q\bar{Q}}S^{j} + S^{i}L^{j}_{Q\bar{Q}}\right)P^{j}_{1\lambda'} + V_{1\,S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r)S_{12}\delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r)P^{i\dagger}_{1\lambda}P^{j}_{1\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right) + \dots$$

$$\left(K^{ij}_{1}\right)^{k} = i\epsilon^{ijk}$$

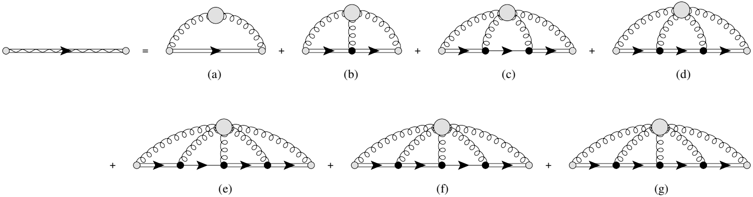
$$P^{i}_{10} = \hat{r}^{i}_{0} = \hat{r}^{i}$$

$$P^{i}_{1\pm1} = \hat{r}^{i}_{\pm} = \mp\left(\hat{\theta}^{i} \pm i\hat{\phi}^{i}\right)/\sqrt{2}$$

- $V_i(r) = V_i^{p}(r) + V_i^{np}(r)$, a sum of a perturbative part and a nonperturbative part.
- The nonperturbative matching is done in the small r regime, so that a multipole expansion is performed: $V_i^{np}(r) = V_i^{np\,(0)} + V_i^{np\,(1)}r^2 + \dots$.
- We work to the accuracy LO in the multipole expansion for the $1/m^2$-potentials, and NLO in the multipole expansion for the $1/m$-potentials.

$$
\begin{aligned}
V_{SK} &= V_{SK}^{np\,(0)} + V_{SK}^{np\,(1)} r^2 \\
V_{SKb} &= V_{SKb}^{np\,(0)} \\
V_{SLa} &= V_{o\,SL} + V_{SLa}^{np\,(0)} \\
V_{SLb} &= V_{SLb}^{np\,(0)} \\
V_{S^2} &= V_{o\,S^2} + V_{S^2}^{np\,(0)} \\
V_{S_{12}a} &= V_{o\,S_{12}} \\
V_{S_{12}b} &= V_{S_{12}b}^{np\,(0)}
\end{aligned}
$$

Nonperturbative matching at Λ_{QCD}:



$V_i^{np\,(j)}$ can be expressed as the following gluonic correlators (in the gauge $A_0 = 0$):

$$\left(U_B^K\right)^{ijk}_{abe} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \, \langle 0|G^{ia\dagger}(T/2) gB^{jb}(t) G^{ke}(-T/2)|0\rangle$$

$$\left(U_B^s\right)^{ijkl} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \, \langle 0|G^{ia\dagger}(T/2) gB^{ja}(t) gB^{kb}(t') G^{lb}(-T/2)|0\rangle$$

$$\left(U_B^o\right)^{ijkl}_{abde} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \, \langle 0|G^{ia\dagger}(T/2) gB^{jb}(t) gB^{kd}(t') G^{le}(-T/2)|0\rangle$$

$$\left(U_E^s\right)^{ijkl} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \, \langle 0|G^{ia\dagger}(T/2) gE^{ja}(t) gE^{kb}(t') G^{lb}(-T/2)|0\rangle$$

$$\left(U_E^o\right)^{ijkl}_{abde} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \, \langle 0|G^{ia\dagger}(T/2) gE^{jb}(t) gE^{kd}(t') G^{le}(-T/2)|0\rangle$$

$$\left(U_{EBE}^K\right)^{iljmk}_{abcde} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \int_{-T/2}^{t'} dt'' \times \langle 0|G^{ia\dagger}(T/2) gE^{lb}(t) gB^{jc}(t') gE^{md}(t'') G^{ke}(-T/2)|0\rangle$$

$$\left(U_{BEE}^K\right)^{iljmk}_{abcde} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \int_{-T/2}^{t'} dt'' \times \langle 0|G^{ia\dagger}(T/2) gB^{jb}(t) gE^{lc}(t') gE^{md}(t'') G^{ke}(-T/2)|0\rangle$$

$$\left(U_{EEB}^K\right)^{iljmk}_{abcde} = \lim_{T\to\infty} \frac{ie^{i\Lambda T}}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{t} dt' \int_{-T/2}^{t'} dt'' \times \langle 0|G^{ia\dagger}(T/2) gE^{lb}(t) gE^{mc}(t') gB^{jd}(t'') G^{ke}(-T/2)|0\rangle$$

Result of spin-splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.

Lowest-lying quarkonium hybrid multiplets

Multiplet	l	$J^{PC}(s=0)$	$J^{PC}(s=1)$
H_1	1	1^{--}	$(0,1,2)^{-+}$
H_2	1	1^{++}	$(0,1,2)^{+-}$
H_3	0	0^{++}	1^{+-}
H_4	2	2^{++}	$(1,2,3)^{+-}$

Result of spin-splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the ${V_{SK}^{np}}^{(0)}$-term, 1st order for the rest.

Lowest-lying quarkonium hybrid multiplets

Multiplet	l	$J^{PC}(s=0)$	$J^{PC}(s=1)$
H_1	1	1^{--}	$(0,1,2)^{-+}$
H_2	1	1^{++}	$(0,1,2)^{+-}$
H_3	0	0^{++}	1^{+-}
H_4	2	2^{++}	$(1,2,3)^{+-}$

Result of spin-splitting

Method:

- First, solve the Schrödinger equation with the static potential $V^{(0)}_{1\lambda}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V^{np\,(0)}_{SK}$-term, 1st order for the rest.
- Six nonperturbative parameters $V^{np\,(0)}_{SK}$, $V^{np\,(1)}_{SK}$, $V^{np\,(0)}_{SLa}$, $V^{np\,(0)}_{SLb}$, $V^{np\,(0)}_{S^2}$, $V^{np\,(0)}_{S_{12}b}$ are fitted to lattice data of charmonium hybrid spectrum.

Lowest-lying quarkonium hybrid multiplets

Multiplet	l	$J^{PC}(s=0)$	$J^{PC}(s=1)$
H_1	1	1^{--}	$(0,1,2)^{-+}$
H_2	1	1^{++}	$(0,1,2)^{+-}$
H_3	0	0^{++}	1^{+-}
H_4	2	2^{++}	$(1,2,3)^{+-}$

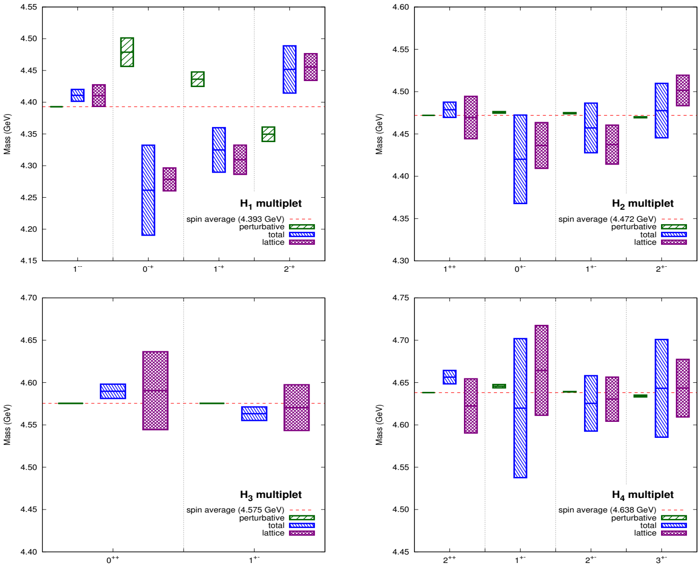
Result of spin-splitting

Method:

- First, solve the Schrödinger equation with the static potential $V_{1\lambda}^{(0)}(r)$.
- Apply perturbative theory to the spin-dependent terms, 2nd order for the $V_{SK}^{np\,(0)}$-term, 1st order for the rest.
- Six nonperturbative parameters $V_{SK}^{np\,(0)}$, $V_{SK}^{np\,(1)}$, $V_{SLa}^{np\,(0)}$, $V_{SLb}^{np\,(0)}$, $V_{S^2}^{np\,(0)}$, $V_{S_{12}b}^{np\,(0)}$ are fitted to lattice data of charmonium hybrid spectrum.
- Since the nonperturbative parameters are independent of heavy-quark flavor, we use them to predict spin splittings for bottomonium hybrids.

Lowest-lying quarkonium hybrid multiplets

Multiplet	l	$J^{PC}(s=0)$	$J^{PC}(s=1)$
H_1	1	1^{--}	$(0,1,2)^{-+}$
H_2	1	1^{++}	$(0,1,2)^{+-}$
H_3	0	0^{++}	1^{+-}
H_4	2	2^{++}	$(1,2,3)^{+-}$



Spectrum of the four lowest-lying charmonium hybrid multiplets. Lattice data are from G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP, 12, 089 (2016), arXiv:1610.01073 [hep-lat].

Nonperturbative parameters obtained from fitting to lattice data

$V_{SK}^{np\,(0)}/\Lambda_{QCD}^2$	$+1.03$
$V_{SK}^{np\,(1)}/\Lambda_{QCD}^4$	-0.51
$V_{SLa}^{np\,(0)}/\Lambda_{QCD}^3$	-1.32
$V_{SLb}^{np\,(0)}/\Lambda_{QCD}^3$	$+2.44$
$V_{S^2}^{np\,(0)}/\Lambda_{QCD}^3$	-0.33
$V_{S_{12}b}^{np\,(0)}/\Lambda_{QCD}^3$	-0.39

($\Lambda_{QCD} = 0.5$ GeV)

Features of the results

- Perturbative contributions in spin triplets have a pattern opposite to that in the lattice data, and also to that of ordinary quarkonia. This is due to the repulsive nature of the perturbative heavy-quark-anti-quark octet potential.

Nonperturbative parameters obtained from fitting to lattice data

$V_{SK}^{np\,(0)}/\Lambda_{QCD}^2$	$+1.03$
$V_{SK}^{np\,(1)}/\Lambda_{QCD}^4$	-0.51
$V_{SLa}^{np\,(0)}/\Lambda_{QCD}^3$	-1.32
$V_{SLb}^{np\,(0)}/\Lambda_{QCD}^3$	$+2.44$
$V_{S^2}^{np\,(0)}/\Lambda_{QCD}^3$	-0.33
$V_{S_{12}b}^{np\,(0)}/\Lambda_{QCD}^3$	-0.39

($\Lambda_{QCD} = 0.5$ GeV)

Features of the results

- Perturbative contributions in spin triplets have a pattern opposite to that in the lattice data, and also to that of ordinary quarkonia. This is due to the repulsive nature of the perturbative heavy-quark-anti-quark octet potential.
- The discrepancy with the lattice data can be reconciled by the nonperturbative contributions, in particular the contribution from the $V_{SK}^{np\,(0)}$ term $\sim \Lambda_{QCD}^2/m$, parametrically larger than the perturbative contributions $\sim mv^4$.

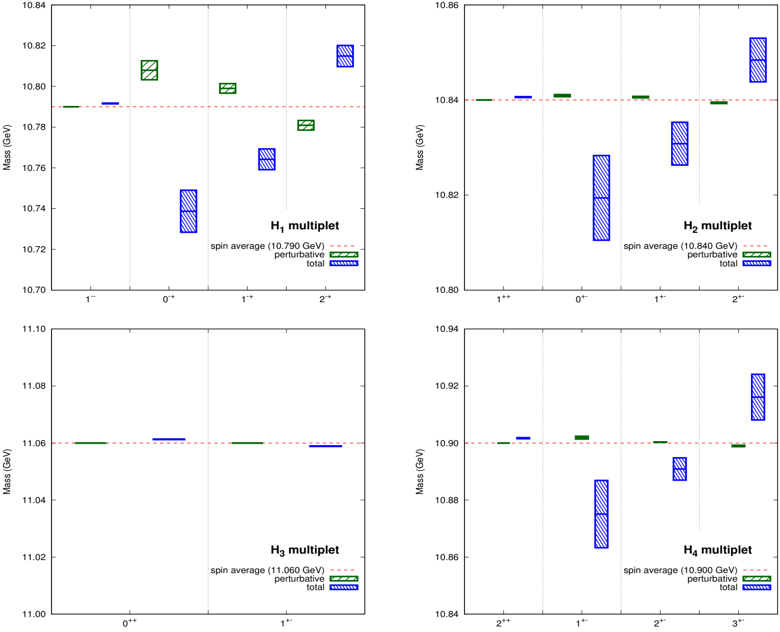
Nonperturbative parameters obtained from fitting to lattice data

$V_{SK}^{np\,(0)}/\Lambda_{QCD}^2$	$+1.03$
$V_{SK}^{np\,(1)}/\Lambda_{QCD}^4$	-0.51
$V_{SLa}^{np\,(0)}/\Lambda_{QCD}^3$	-1.32
$V_{SLb}^{np\,(0)}/\Lambda_{QCD}^3$	$+2.44$
$V_{S^2}^{np\,(0)}/\Lambda_{QCD}^3$	-0.33
$V_{S_{12}b}^{np\,(0)}/\Lambda_{QCD}^3$	-0.39

($\Lambda_{QCD} = 0.5$ GeV)

Features of the results

- Perturbative contributions in spin triplets have a pattern opposite to that in the lattice data, and also to that of ordinary quarkonia. This is due to the repulsive nature of the perturbative heavy-quark-anti-quark octet potential.
- The discrepancy with the lattice data can be reconciled by the nonperturbative contributions, in particular the contribution from the $V_{SK}^{np\,(0)}$ term $\sim \Lambda_{QCD}^2/m$, parametrically larger than the perturbative contributions $\sim mv^4$.
- The resultant values of the $V_i^{np\,(j)}$ from fitting to lattice data are consistent with the power counting.



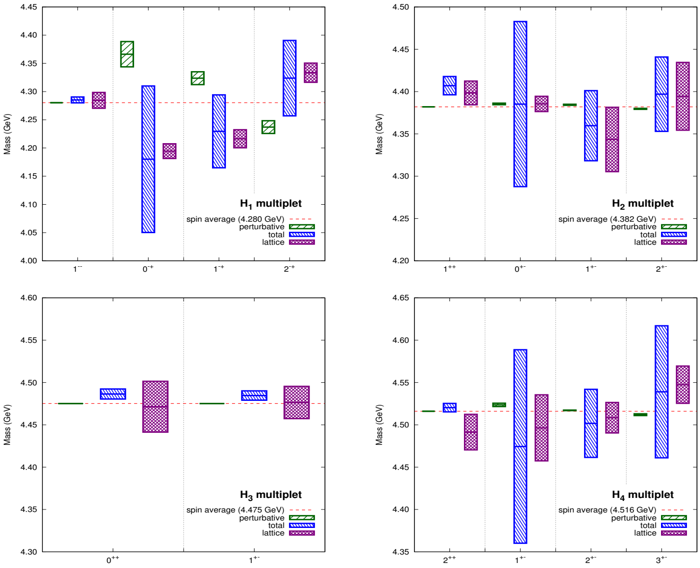
Spectrum of the four lowest-lying bottomonium hybrids predicted from hybrid EFT.

Summary

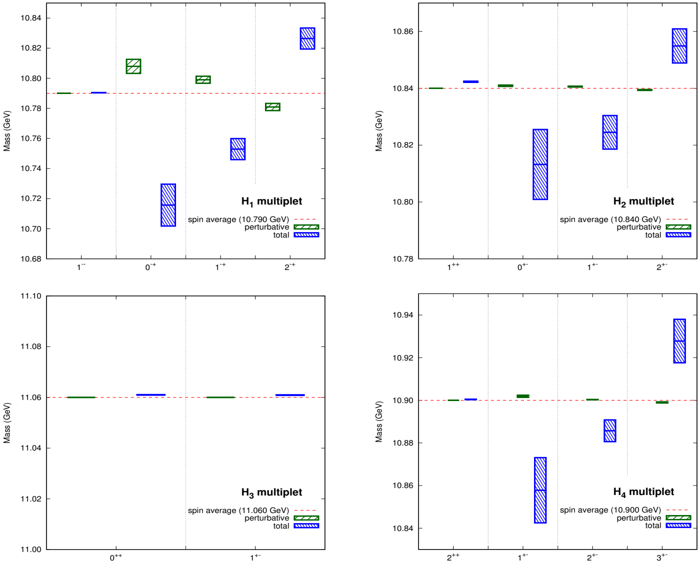
- We studied spin splittings in the quarkonium hybrid spectrum using a non-relativistic EFT.
- We obtained the spin-dependent potentials in the hybrid EFT by integrating out the scale Λ_{QCD}. The spin-dependent potentials have nonperturbative contributions given by gluonic correlators.
- Six nonperturbative parameters were fitted to the latest lattice data of the charmonium hybrid spectrum. The obtained values of these nonperturbative parameters are consistent with the power counting.
- With the obtained nonperturbative parameters, we predicted the bottomonium hybrid spectrum.

Thank you.

Back up slides



Spectrum of the four lowest-lying charmonium hybrid multiplets. The lattice results from Liu et al. with $m_\pi \approx 400$ MeV are plotted in purple.



Spectrum of the four lowest-lying bottomonium hybrids. Parameters from fitting to Liu et al.

Nonperturbative matching coefficients determined by fitting charmonium hybrid spectrum obtained from the hybrid EFT to the lattice spectrum from the Hadron Spectrum Collaboration data of Liu et al. and Cheung et al. with pion masses of $m_\pi \approx 400$ MeV and $m_\pi \approx 240$ MeV respectively. The matching coefficients are normalized to their parametric natural size. We take the value $\Lambda_{QCD} = 0.5$ GeV.

	Liu et al.	Cheung et al.
$V_{SK}^{np\,(0)}/\Lambda^2_{QCD}$	$+1.50$	$+1.03$
$V_{SK}^{np\,(1)}/\Lambda^4_{QCD}$	-0.65	-0.51
$V_{SLa}^{np\,(0)}/\Lambda^3_{QCD}$	$+0.81$	-1.32
$V_{SLb}^{np\,(0)}/\Lambda^3_{QCD}$	$+1.18$	$+2.44$
$V_{S^2}^{np\,(0)}/\Lambda^3_{QCD}$	-0.26	-0.33
$V_{S_{12}b}^{np\,(0)}/\Lambda^3_{QCD}$	$+0.69$	-0.39