# Direct CP violation in $K^0 \to \pi\pi$: SM status

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# What is $\varepsilon'/\varepsilon$?

- $\varepsilon'/\varepsilon$ constitutes a fundamental test for our understanding of CP violation within SM:

  $$\mathrm{Re}\,(\varepsilon'/\varepsilon) \propto \mathrm{Im}(V_{td}\,V_{ts}^*)$$

$$V_{\mathrm{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- $\varepsilon$ and $\varepsilon'$ parametrize different sources of CP violation in $K$ decays:

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon', \qquad \eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\,\varepsilon'.$$

- Dominant effect from CP violation in $K$ mixing is contained in $\varepsilon$:

$$|\varepsilon| = \tfrac{1}{3}|\eta_{00} + 2\eta_{+-}| = (2.228 \pm 0.011) \cdot 10^{-3}.$$

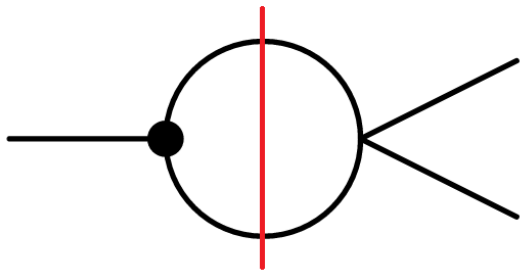
- $\varepsilon'$ is a tinier effect and accounts for direct CP violation in K decays:

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = \tfrac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3) \cdot 10^{-4}$$

  demonstrates the existence of direct CP violation in $K$ decays.

# Old history ...

- The theoretical prediction of $\varepsilon'/\varepsilon$ has a quite controversial history:
  1. First theoretical calculations claimed values of order $10^{-4}$, one order of magnitude smaller than the signal observed in 1993 by the CERN NA31 collaboration, giving support to the null result obtained by the E731 experiment.
  2. The final experimental confirmation that $\mathrm{Re}\,(\varepsilon'/\varepsilon) \approx 10^{-3}$, by NA48 and KTeV, triggering a large number of NP explanations.
  3. Old SM predictions had missed completely the important role of the final pion dynamics. When these contributions were taken into account, the theoretical prediction was found to be in good agreement with the experimental value.



$$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{SM}} = (17 \pm 9) \cdot 10^{-4}$$

(Pallante-Pich-Scimeni '01)

- Lattice QCD: Another tool to deal with non-perturbative effects.

1. For many years, Lattice QCD attempts to explain
   - Enhancement of the $\Delta I = 1/2$ but remained unsuccessful.
   - $\varepsilon'/\varepsilon$ were unreliable (some of them even negative).
2. The situation has changed due to the development of sophisticated techniques and the increasing power of modern computers. Explaining:
   - Successful $\Delta I = 3/2$ $K \to \pi\pi$: ✓

$$\sqrt{\tfrac{3}{2}}\,\mathrm{Re}A_2 = (1.50 \pm 0.17) \cdot 10^{-8}\ \mathrm{GeV} \quad 0.1\,\sigma \ \text{(RBC-UKQCD '15)}$$

   - First statistically-significant signal of $\Delta I = 1/2$ enhancement: ✓

$$\sqrt{\tfrac{3}{2}}\,\mathrm{Re}A_0 = (4.66 \pm 1.61) \cdot 10^{-7}\ \mathrm{GeV} \quad 1.0\,\sigma \ \text{(RBC-UKQCD '15)}$$

3. Lattice QCD estimation of $\varepsilon'/\varepsilon$: ✗

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.4 \pm 7.0) \cdot 10^{-4} \quad 2.1\,\sigma \ \text{(RBC-UKQCD '15)}$$

- Limitations of lattice result:

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.4 \pm 7.0)\cdot 10^{-4} \quad 2.1\,\sigma \ (\text{RBC-UKQCD '15})$$

  - This discrepancy has revived some of the old SM calculations predicting low values of $\varepsilon'/\varepsilon$ (missing AGAIN the crucial pion dynamics!) and has triggered several analysis of possible contributions from NP.
  - Before claiming any evidence for NP, one should realize the technical limitations of the current lattice result:

$$\delta_0 = +(23.8 \pm 5.0)^\circ \quad 2.9\,\sigma \ (\text{RBC-UKQCD '15}) \quad \text{✗}$$

$$\delta_0|_{\text{exp}} = +(39.2 \pm 1.5)^\circ$$

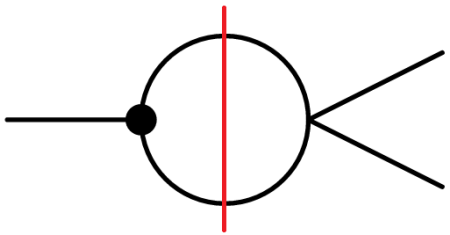
$$\delta_2 = -(11.6 \pm 2.8)^\circ \quad 1.0\,\sigma \ (\text{RBC-UKQCD '15}) \quad \text{✓}$$

$$\delta_2|_{\text{exp}} = -(8.5 \pm 1.5)^\circ$$

**Large uncertainty in $\delta_0$!**

1. **Unitarity:** $\delta_0(M_K) = (39.2 \pm 1.5)^\circ \longrightarrow \boxed{A_0 \approx 1.3 \times \mathrm{Dis}(A_I)}$

   (Colangelo-Gasser-Leutwyler '01)



$$A_I\, e^{i\,\delta_I} \;=\; \mathrm{Dis}(A_I) + i\mathrm{Abs}(A_I)$$

$$\tan\delta_I = \frac{\mathrm{Abs}(A_I)}{\mathrm{Dis}(A_I)}$$

$$A_I \;=\; \mathrm{Dis}(A_I)\,\sqrt{1+\tan^2\delta_I}$$

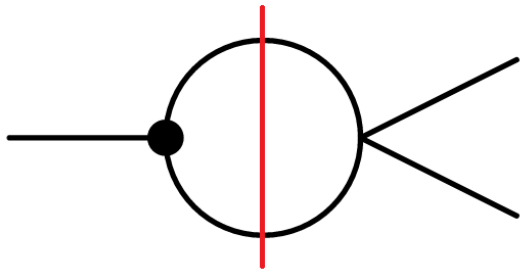
2. **Analicity:** $\boxed{\mathrm{Dis}(A_I) \propto \mathrm{Abs}(A_I)}$

3. **Message:** $\boxed{\text{Large } \delta_0 \longrightarrow \text{Large } \mathrm{Abs}(A_0) \longrightarrow \text{Large } \mathrm{Dis}(A_0)}$

   - Lattice still doesn't have a good control of the $I = 0$ amplitudes.
   - Still premature to derive strong implications and RBC-UKQCD collaboration is making efforts to improve the statistics.
   - Future lattice results will show if the discrepancy stays or not.

# Need of updated $\varepsilon'/\varepsilon$ within SM

- Although Pallante-Pich-Scimeni '01 within a theoretical framework which takes into account the important role of the pion dynamics obtained a theoretical value compatible with the experimental result.



$$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{SM}} = (17 \pm 9) \cdot 10^{-4}$$

- A lot of improvements since 2001:
  - Better knowledge of Low energy constants (LECs),
  - Much better precision of quark masses,
  - Strong coupling constant,
  - ...
- It is convenient to perform an updated of $\varepsilon'/\varepsilon$.

# Theoretical framework

| Energy Scale | Fields | Effective Theory |
|---|---|---|
| $M_W$ | $W, Z, \gamma, g$ $\tau, \mu, e, \nu_i$ $t, b, c, s, d, u$ | Standard Model |
| | ⬇ OPE | |
| $\lesssim m_c$ | $\gamma, g\ ;\ \mu, e, \nu_i$ $s, d, u$ | $\mathcal{L}_{\rm QCD}^{(n_f=3)}$, $\mathcal{L}_{\rm eff}^{\Delta S=1,2}$ |
| | ⬇ $N_C \to \infty$ | |
| $M_K$ | $\gamma\ ;\ \mu, e, \nu_i$ $\pi, K, \eta$ | $\chi$PT |

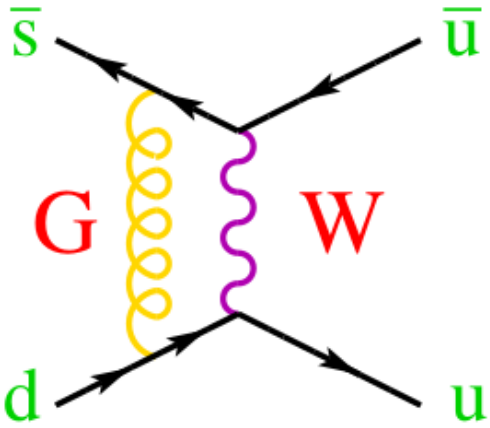
# Short-distance description

## $\Delta S = 1$ transitions for $K \to \pi\pi$

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu)\, Q_i(\mu)$$

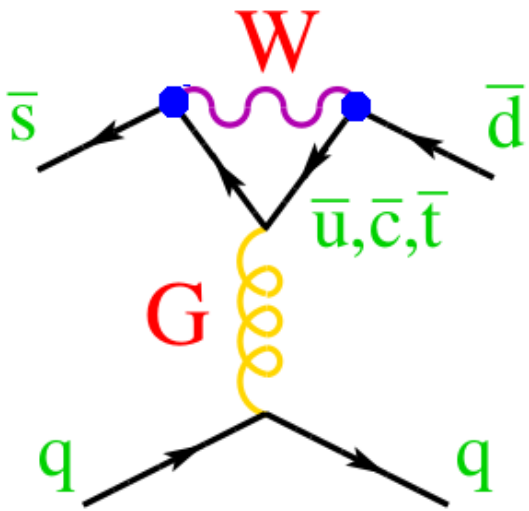
$$C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)$$

$$\tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$



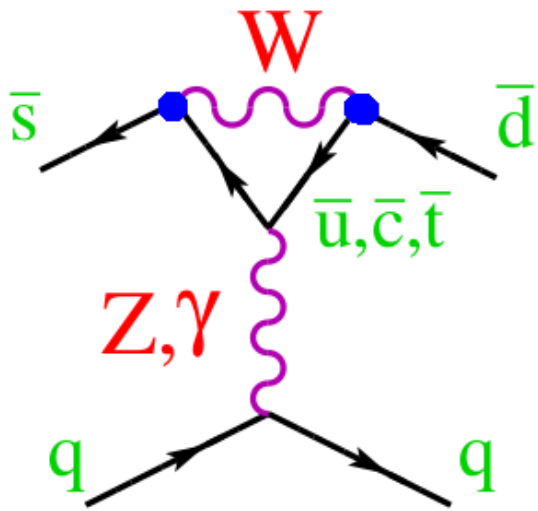

**Current - Current operators**

$Q_{1,2}$




**QCD - Penguins operators**

$Q_{3,4,5,6}$




**Electroweak Penguins operators**

$Q_{7,8,9,10}$

# Chiral Perturbation Theory ($\chi$PT) description

- QCD at low energies is $\chi$PT.
- $\chi$PT formulation is a consistent framework to describe the pseudoscalar-octet dynamics.
- Perturbative expansion in powers of $p^2/\Lambda_\chi^2$ where $\Lambda_\chi \sim 1\text{GeV}$.
- Chiral symmetry fix the allowed $\chi$PT operators, at given order in $p$.
- $\mathcal{O}(G_F\, p^2)$ : **Goldstone Interactions** $(\pi, K, \eta)$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda\, L_\mu\, L^\mu) + G_{27}\, F^4 \left(L_{\mu\,23}\, L_{11}^\mu + \frac{2}{3}\, L_{\mu\,21}\, L_{13}^\mu\right)$$

$$G_{8,27} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^*\, g_{8,27} \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda_{ij} \equiv \delta_{i3}\delta_{j2}; \quad U \equiv \exp\{i\sqrt{2}\phi/F\}$$

- Short-distance dynamics encoded in Low-Energy Couplings.
- LECs can be determined at $N_C \to \infty$ **(matching)**

# $K \to \pi\pi$ Isospin amplitudes

- Isospin decomposition:

$$A[K^0 \to \pi^+\pi^-] = A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \frac{1}{\sqrt{2}}(\mathcal{A}_{3/2} + \mathcal{A}_{5/2})$$

$$A[K^0 \to \pi^0\pi^0] = A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \sqrt{2}\,(\mathcal{A}_{3/2} + \mathcal{A}_{5/2})$$

$$A[K^+ \to \pi^+\pi^0] = \frac{3}{2} A_2^+\, e^{i\chi_2^+} = \frac{3}{2}\left(\mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2}\right).$$

  - $\chi_I$ can be identified with the S-wave $\pi\pi$ scattering phase shifts $\delta_I(M_K)$.
  - $\mathcal{A}_n = -G_8 F_\pi \left\{ (M_K^2 - M_\pi^2)\, \mathcal{A}_n^{(8)} - e^2 {F_\pi}^2 g_{\rm ewk}\, \mathcal{A}_n^{(g)} \right\} - G_{27} F_\pi (M_K^2 - M_\pi^2)\, \mathcal{A}_n^{(27)}$

    where $\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)}\right]$.

- $\varepsilon'$ in terms of the $K \to \pi\pi$ isospin amplitudes,

$$\varepsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2 - \chi_0)} \omega \left[ \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} \right]$$

1. $\Delta I = 1/2$ rule: $\varepsilon'$ is suppresed by the ratio $\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \approx 1/22$.
2. Strong Final States Interactions (FSI): $\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$.

# Estimation of the LECs

- From phenomenological data or with additional input from theory.
- Principle of calculation LECs: perform a matching between two EFTs.
- In the large $N_C$ limit, the T-product of two colour singlet currents factorizes:

$$\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle \left\{ 1 + \mathcal{O}\left(\frac{1}{N_C}\right) \right\}$$

Since quark currents have a well-known representation in $\chi$PT, the matching between the EFTs can be done at leading order in $1/N_C$.

- Weak couplings of $\mathcal{O}(G_F\, p^2)$ and $\mathcal{O}(e^2\, G_8\, p^0)$:

$$\begin{aligned}
g_8^{\infty} &= -\frac{2}{5}\, C_1(\mu_{\rm SD}) + \frac{3}{5}\, C_2(\mu_{\rm SD}) + C_4(\mu_{\rm SD}) - 16\, L_5\, B(\mu_{\rm SD})\, C_6(\mu_{\rm SD}), \\
g_{27}^{\infty} &= \frac{3}{5}\left[C_1(\mu_{\rm SD}) + C_2(\mu_{\rm SD})\right], \\
(e^2\, g_8\, g_{\rm ewk})^{\infty} &= -3\, B(\mu_{\rm SD})\, C_8(\mu_{\rm SD}) - \frac{16}{3}\, B(\mu_{\rm SD})\, C_6(\mu_{\rm SD})\, e^2\, (K_9 - 2\, K_{10}).
\end{aligned}$$

where $B(\mu_{\rm SD}) \equiv \frac{\langle \bar{q} q \rangle}{F_\pi^3} = \left[\frac{M_K^2}{(m_s+m_d)(\mu_{\rm SD}) F_\pi}\right]^2 \left[1 - \frac{16 M_K^2}{F_\pi^2}(2L_8 - L_5) + \frac{8 M_\pi^2}{F_\pi^2} L_5\right]$

# Simplified Estimate of $\varepsilon'/\varepsilon$

1. CP violation comes mainly from the QCD and EW penguin operators.
2. Chiral enhancement of the operators $(V-A)\otimes(V+A)$.
3. Good numerical approximation to consider only $Q_{6,8}$ operators.
4. **Simplified estimate:**

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.2\cdot 10^{-3}\left\{ B_6^{(1/2)}\left(1-\underbrace{\Omega_{\rm eff}}_{\rm IB}\right) - 0.48\, B_8^{(3/2)}\right\}$$

$$\mathrm{IB} \equiv \mathbf{O}[(m_u-m_d)p^2, e^2p^2]$$

- Large $N_C$:

$$B_6^{(1/2)} = B_8^{(3/2)} = 1 \longrightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 10.0\cdot 10^{-4} \sim \mathcal{O}(10^{-3})$$

- Buras-Gorbahn-Jäger-Jamin '15:

$$B_6^{(1/2)} = 0.57,\ B_8^{(3/2)} = 0.76 \rightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.6\cdot 10^{-4} \sim \mathcal{O}(10^{-4})$$

**Strong cancellation between $Q_6$ and $Q_8$!**

# Anatomy of $\varepsilon'/\varepsilon$ calculation

$$\mathrm{Re}(\varepsilon'/\varepsilon) = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\rm eff}) - \frac{\mathrm{Im}A_2^{\rm emp}}{\mathrm{Re}A_2^{(0)}}\right]$$

Strong cancelation: $\quad Q_6 \quad - \quad Q_8 \quad = 0$

1. **$O(p^4)$ $\chi$PT Loops:** **Large correction** $\Delta_L\mathcal{A}_n^{(X)} \to \frac{1}{N_C}\log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3}\times 2$

$$\mathcal{A}_n^{(X)} = a_n^{(X)}\left[1 + \Delta_L\,\mathcal{A}_n^{(X)} + \Delta_C\,\mathcal{A}_n^{(X)}\right] \qquad \text{Pallante-Pich-Scimemi Gisbert-Pich}$$

$$\Delta_L\mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + 0.47\,i \quad ;$$

$$\Delta_L\mathcal{A}_{1/2}^{(27)} = 1.03 \pm 0.63 + 0.47\,i \quad ; \quad \Delta_L\mathcal{A}_{3/2}^{(27)} = -0.04 \pm 0.05 - 0.21\,i$$

$$\Delta_L\mathcal{A}_{1/2}^{(g)} = 0.27 \pm 0.01 + 0.47\,i \quad ; \quad \Delta_L\mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.19 - 0.21\,i$$

2. **$O(p^4)$ LECs fixed at $N_C \to \infty$:** **Small correction** $\Delta_C\mathcal{A}_n^{(X)}$

   $\frac{1}{N_C}\log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3}\times 4$ but no large logarithm in the matching!

3. **Isospin Breaking $O[(m_u - m_d)p^2, e^2p^2]$:** **Sizeable correction**

$$\Omega_{\rm eff} = 0.06 \pm 0.08 \quad \text{(Cirigliano-Ecker-Neufeld-Pich '04)}$$

# Anatomy of $\varepsilon'/\varepsilon$ calculation

$$\mathrm{Re}(\varepsilon'/\varepsilon) = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\rm eff}) - \frac{\mathrm{Im}A_2^{\rm emp}}{\mathrm{Re}A_2^{(0)}}\right]$$

~~Strong cancelation:~~ $\quad Q_6 \uparrow \quad - \quad Q_8 \downarrow \quad \neq 0$

1. **$O(p^4)$ $\chi$PT Loops:** **Large correction** $\Delta_L \mathcal{A}_n^{(X)} \to \frac{1}{N_C}\log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3}\times 2$

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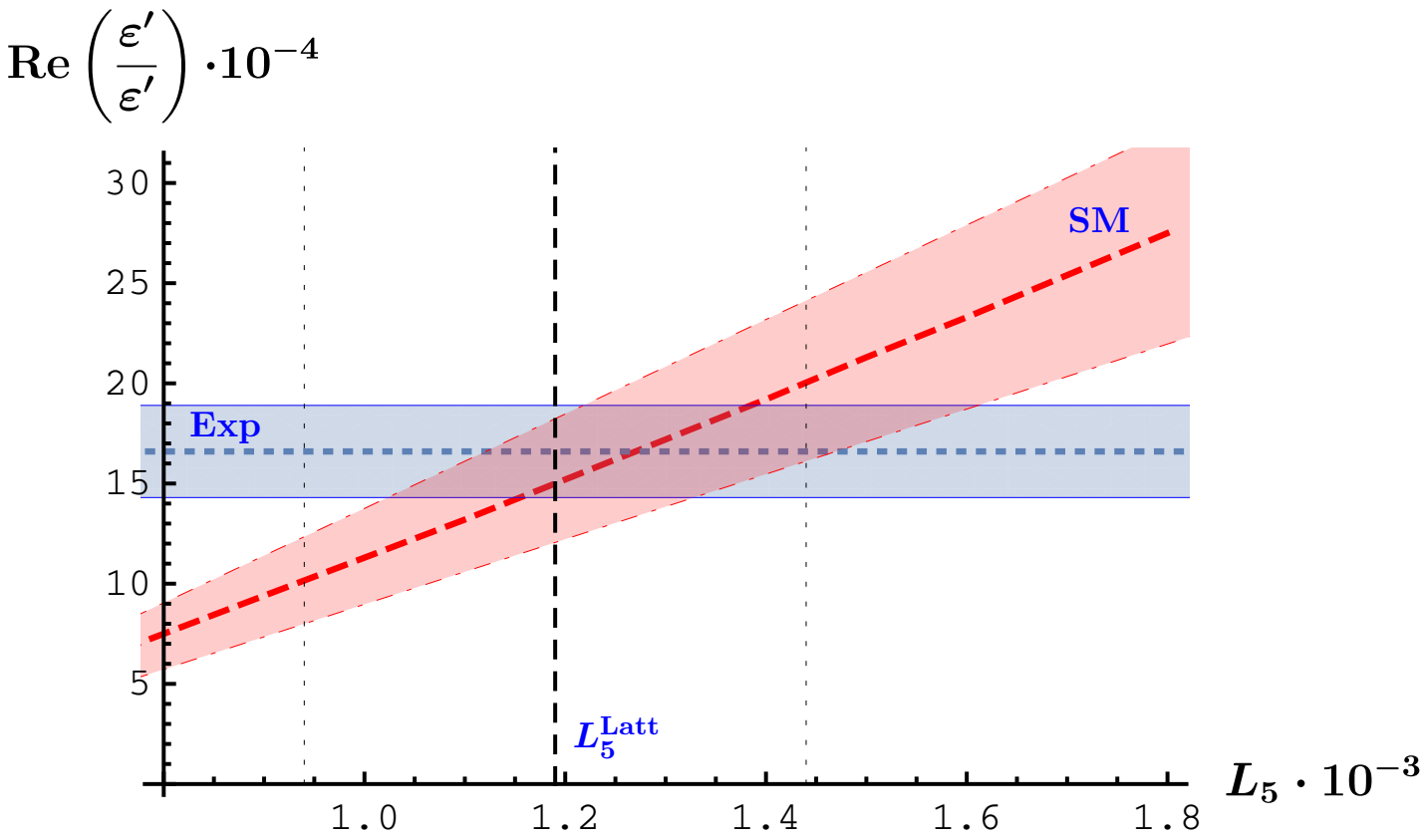
# SM prediction of $\mathrm{Re}\,(\varepsilon'/\varepsilon)$




$$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{SM}} = (15 \pm 2_{\mu} \pm 2_{m_s} \pm 2_{\Omega_{\mathrm{eff}}} \pm 6_{\frac{1}{N_C}}) \cdot 10^{-4}$$

(HG - A.Pich, arXiv:1712.06147)

# Needed Improvements

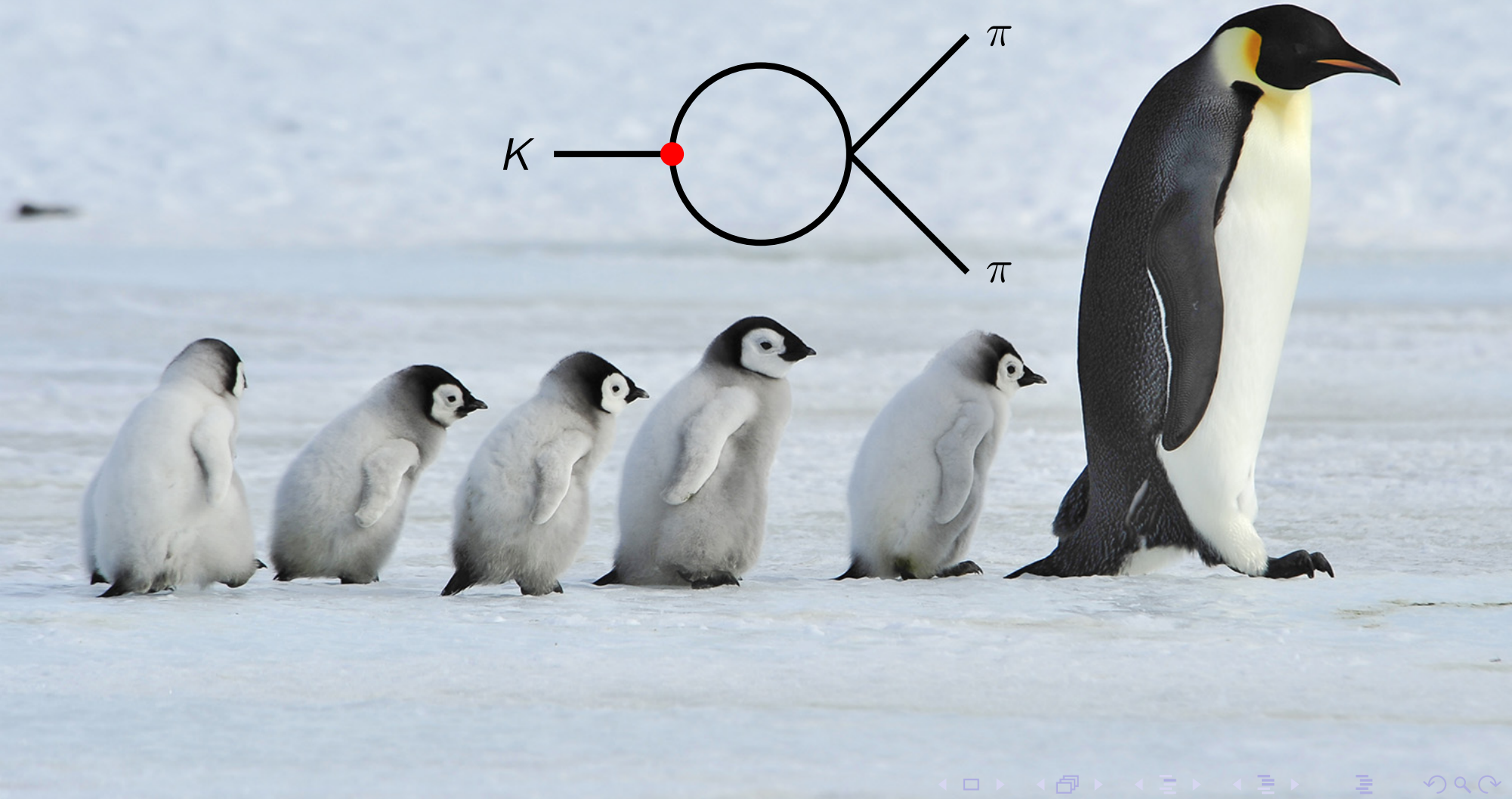
- **Wilson coefficients at NNLO** (Cerdà et al)
- **Updated value of $\Omega_{\text{eff}}$** (Cirigliano et al)
- **$g_8$ $g_{\text{ew}}$ at NLO in $1/N_C$** (A. Pich and Rodriguez-Sanchez)
- **$g_8$ and higher-order LECs at NLO** New ideas needed
- **$\chi$PT logarithms at NNLO** Feasible
- **Improved lattice input** Expected

**Best strategy: $\chi$PT (amplitudes) + Lattice (LECs)**

## SM prediction of $\varepsilon'/\varepsilon$ agrees well with the mesured value!

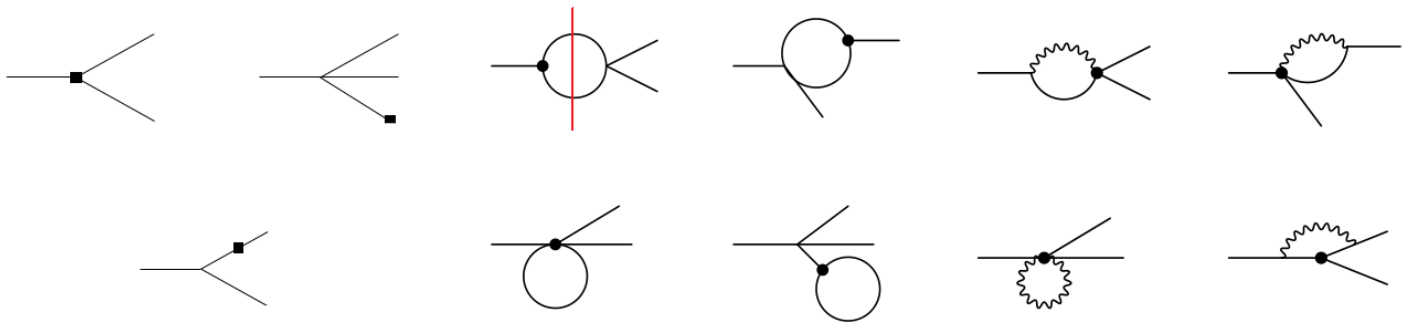
$$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{SM}} = (15 \pm 2_{\mu} \pm 2_{m_s} \pm 2_{\Omega_{\mathrm{eff}}} \pm 6_{\frac{1}{N_C}}) \cdot 10^{-4}$$

$$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$



# Amplitudes at NLO of $K \to \pi\pi$ $\mathcal{O}(p^4,\ (m_u - m_d)p^2,\ e^2p^0,\ e^2p^2)$

Including strong isospin violation and electromagnetic corrections (Cirigliano et al '04)



$$\mathcal{A}_n = G_{27}\, F_\pi\, (M_K^2 - M_\pi^2)\, \mathcal{A}_n^{(27)} + G_8\, F_\pi \left\{ (M_K^2 - M_\pi^2) \left[ \mathcal{A}_n^{(8)} + \varepsilon^{(2)}\, \mathcal{A}_n^{(\varepsilon)} \right] - e^2\, {F_\pi}^2 \left[ \mathcal{A}_n^{(\gamma)} + Z\, \mathcal{A}_n^{(Z)} + g_{\rm ewk}\, \mathcal{A}_n^{(g)} \right] \right\}$$

where $\mathcal{A}_n^{(X)} = a_n^{(X)} \left[ 1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)} \right] \quad ; \quad X = 27,\ 8,\ \varepsilon,\ \gamma,\ Z,\ g.$

$G_{8,27} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^*\, g_{8,27}, \quad \varepsilon^{(2)} = (\sqrt{3}/4)(m_d - m_u)/(m_s - \hat{m}) \approx 0.011; \quad Z \approx (M_{\pi^\pm}^2 - M_{\pi^0}^2)/(2e^2F_\pi^2) \approx 0.8$

# Simplified Estimate of $\varepsilon'/\varepsilon$

1. CP violation comes mainly from the QCD and EW penguin operators.

   QCD: $Q_{3,4}$ $(V-A)\otimes(V-A)$ and $Q_{5,6}$ $(V-A)\otimes(V+A)$.

   EW: $Q_{7,8}$ $(V-A)\otimes(V+A)$ and $Q_{9,10}$ $(V-A)\otimes(V-A)$.

2. Chiral enhancement of the operators $(V-A)\otimes(V+A)$.

   - Fierz rearrangement of numerically relevant operators:

     $$Q_4 = \sum_q (\bar{s}q)_{V-A}\,(\bar{q}d)_{V-A}, \quad Q_{10} = \tfrac{3}{2}\sum_q e_q\,(\bar{s}q)_{V-A}\,(\bar{q}d)_{V-A},$$
     $$Q_6 = \sum_q (\bar{s}_L q_R)\,(\bar{q}_R d_L), \quad Q_8 = -12\sum_q e_q\,(\bar{s}_L q_R)\,(\bar{q}_R d_L).$$

   - Large $N_C$: $\langle J\cdot J\rangle = \langle J\rangle\langle J\rangle\,\{1+\mathcal{O}(1/N_C)\}$

     $$\mathcal{A}_{LL} \equiv \langle\pi^+\pi^-|(\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)|K^0\rangle = \langle\pi^+|\bar{u}_L\gamma_\mu d_L|0\rangle\,\langle\pi^-|\bar{s}_L\gamma^\mu u_L|K^0\rangle = \tfrac{i\sqrt{2}}{4}F_\pi\,(M_K^2-M_\pi^2),$$

     $$\mathcal{A}_{LR}(\mu) \equiv \langle\pi^+\pi^-|(\bar{s}_L u_R)(\bar{u}_R d_L)|K^0\rangle = \langle\pi^+|\bar{u}_R d_L|0\rangle\,\langle\pi^-|\bar{s}_L u_R|K^0\rangle = \tfrac{i\sqrt{2}}{4}F_\pi\left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2.$$

     At $\mu = 1$ GeV: $\mathcal{A}_{LR}(\mu)/\mathcal{A}_{LL} \sim M_K^2/[m_s(\mu)+m_d(\mu)]^2 \sim 14$.

3. Good numerical approximation to consider only $Q_{6,8}$ operators.

4. Ignoring all other contributions to the CP-violating decay amplitudes:

$$\mathrm{Im}A_0|_{Q_6} = \frac{G_F}{\sqrt{2}} A^2\lambda^5\eta\, V_{td}V_{ts}^*\, y_6(\mu)\, 4\sqrt{2}\,(F_K - F_\pi)\left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2 B_6^{(1/2)},$$

$$\mathrm{Im}A_2|_{Q_8} = -\frac{G_F}{\sqrt{2}} A^2\lambda^5\eta\, V_{td}V_{ts}^*\, y_8(\mu)\, 2\,F_\pi \left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2 B_8^{(3/2)}.$$

5. Including the isospin breaking effects (Cirigliano-Ecker-Neufeld-Pich '03):

$$\mathrm{Re}(\varepsilon'/\varepsilon) \;=\; -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\rm eff}) - \frac{\mathrm{Im}A_2^{\rm emp}}{\mathrm{Re}A_2^{(0)}}\right]$$

with $\omega \equiv \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = \omega_+(1+f_{5/2})$, $\omega_+ \equiv \frac{\mathrm{Re}A_2^+}{\mathrm{Re}A_0}$ and $\Omega_{\rm eff} = 0.06 \pm 0.08$.

6. Simplified estimate: **Strong cancellation between $Q_6$ and $Q_8$!**

$$\mathrm{Re}(\varepsilon'/\varepsilon) \;\approx\; 2.2\cdot 10^{-3}\left\{B_6^{(1/2)}\,(1-\Omega_{\rm eff}) - 0.48\, B_8^{(3/2)}\right\}$$

- Large $N_C$:

  $B_6^{(1/2)} = B_8^{(3/2)} = 1$ and $\Omega_{\rm eff} = 0.06 \longrightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 10.0\cdot 10^{-4} \sim \mathcal{O}(10^{-3})$

- Buras-Gorbahn-Jäger-Jamin '15:

  $B_6^{(1/2)} = 0.57$, $B_8^{(3/2)} = 0.76$ and $\Omega_{\rm eff} = 0.15 \longrightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.6\cdot 10^{-4} \sim \mathcal{O}(10^{-4})$

# BBG Model (Bardeen-Buras-Gerard)

$$\mathcal{L}_{\rm BBG} = \frac{f_\pi}{4}\left\{\langle D_\mu U^\dagger\, D^\mu U\rangle + r\,\langle m\,(U + U^\dagger)\rangle - \frac{r}{\Lambda_\chi^2}\,\langle m\,(D^2 U + D^2 U^\dagger)\rangle\right\}$$

1. QCD at low energies is $\chi$PT, therefore any QCD inspired model has to obtain compatible predictions with $\chi$PT.
2. Inconsistencies of BBG Lagrangian:
   - $\mathcal{O}(p^2)$ equivalent to $\chi$PT. ✓
   - $\mathcal{O}(p^4)$ just $L_5$ term $(L_i = 0,\ i \neq 5)$ ✗
     - $(2L_8 - L_5)^\infty = -\frac{1}{4}L_5^\infty \longrightarrow \varepsilon'/\varepsilon$ depends strongly on $L_5$!
   - $\mathcal{O}(p^6)$ put to zero! ✗

## 3 Hadronic Matrix Elements (HME) within BBG model: Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln\left(1+\frac{\Lambda^2}{\tilde{m}_6^2}\right) = 1 - 0.66\ln\left(1+\frac{\Lambda^2}{\tilde{m}_6^2}\right)$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln\left(1+\frac{\Lambda^2}{\tilde{m}_8^2}\right) = 1 + 0.08\ln\left(1+\frac{\Lambda^2}{\tilde{m}_8^2}\right)$$

$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln\left(1+\frac{\Lambda^2}{\tilde{m}_8^2}\right) = 1 - 0.17\ln\left(1+\frac{\Lambda^2}{\tilde{m}_8^2}\right)$$

$$\boxed{B_6^{(1/2)} \leq B_6^{(1/2)} < 1}$$

- HME = Large $N_C$ estimation + some dispersive corrections.
- Missing $L_i$, $i \neq 5$ corrections!
- Missing large absorptive corrections of order $\mathcal{O}(1/N_C)$
- $\text{Abs}(A_I) = 0 \longrightarrow \delta_I = 0$, $\forall I$ (Not in QCD!)

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$$\cancel{B_6^{(1/2)} \leq B_6^{(1/2)} < 1} \longrightarrow \text{Not true in QCD!}$$

- HME = Large $N_C$ estimation + some dispersive corrections.
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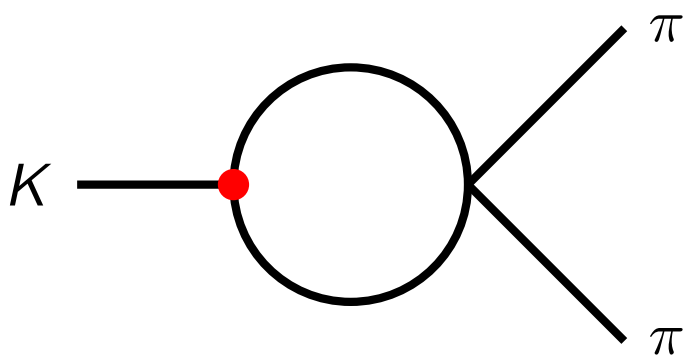
# Absortive contributions in $\chi$PT




Absorptive contribution fully originated by this diagram, since it is the only one where the two intermediate pions can be put on their mass-shell.

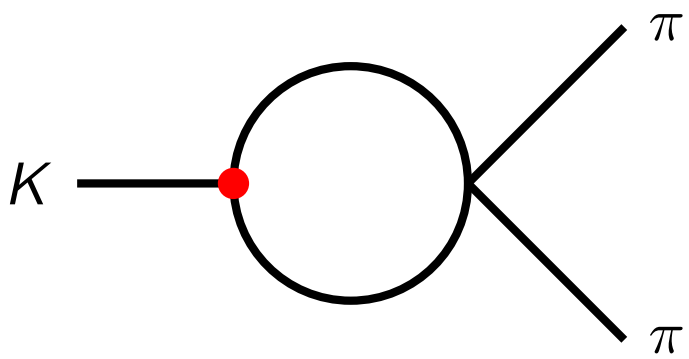
$$\Delta_L \mathcal{A}_{1/2}^{(X)} = \frac{M_K^2}{(4\pi F_\pi)^2}\left(1-\frac{M_\pi^2}{2M_K^2}\right)\widetilde{B}(M_\pi^2,M_\pi^2,M_K^2)+\cdots,$$

$$\Delta_L \mathcal{A}_{3/2}^{(X)} = -\frac{1}{2}\frac{M_K^2}{(4\pi F_\pi)^2}\left(1-\frac{2M_\pi^2}{M_K^2}\right)\widetilde{B}(M_\pi^2,M_\pi^2,M_K^2)+\cdots,$$

where

$$\widetilde{B}(M_\pi^2,M_\pi^2,M_K^2) = \sigma_\pi\left[\log\left(\frac{1-\sigma_\pi}{1+\sigma_\pi}\right)+i\pi\right]+\log\left(\frac{\nu_\chi^2}{M_\pi^2}\right)+1, \quad \sigma_\pi \equiv \sqrt{1-4M_\pi^2/M_K^2}.$$

# LO $\chi$PT values for the strong $\pi\pi$ scattering phase shift




For $J = 0$ and $I = 0, 2$, at $s = M_K^2$:

$$\boxed{\tan\delta_0(M_K^2) = \frac{\sigma_\pi}{32\pi F_\pi^2}\left(2M_K^2 - M_\pi^2\right)}, \qquad \boxed{\tan\delta_2(M_K^2) = \frac{\sigma_\pi}{32\pi F_\pi^2}\left(2M_\pi^2 - M_K^2\right)}.$$

The predicted phase-shift difference, $\delta_0(M_K^2) - \delta_2(M_K^2) = 37^\circ$, is somewhat lower than its experimental value showing that higher-order rescattering contributions are numerically relevant.The one-loop integral $\widetilde{B}(M_\pi^2, M_\pi^2, M_K^2)$ contains a large chiral logarithm of the ultraviolet scale $\nu_\chi$ over the infrared scale $M_\pi$, which enhances the dispersive component of the $I = 0$ amplitude and suppresses the $I = 2$ one.