



Thermodynamics and Non-Gaussian Measures in the Covariant Variational Approach to Yang-Mills Theory

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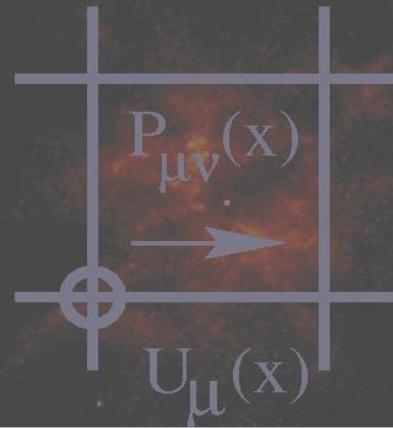


Overview

1. Covariant variational principle for the effective action
2. Propagators at $T=0$ and $T>0$
3. Effective potential of the Polyakov loop
4. Thermodynamics
5. Non-Gaussian measures
6. Summary



$$\rho\rangle + \langle \ln \mathcal{J} \rangle$$
$$+ \frac{1}{2} \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{ab}(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle - \frac{1}{2} \int_{\beta} d(x, y)$$
$$+ \frac{1}{2} \int_{\beta} d(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle \left\{ \bar{\omega}_{\mu\nu}^{ab}(x, y) - \chi_{\mu\nu}^{ab}(x, y) \right\}$$
$$\text{det} \left(\frac{2\pi}{\bar{\omega}} \right)^{-\frac{1}{2}} + \frac{1}{2} (N^2 - 1) \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{-1}(x, y) \left\{ \bar{\omega}_{\mu\nu}(x, y) - \chi_{\mu\nu}(x, y) \right\}$$
$$\ln \left(\frac{\bar{\omega}_{\mu\nu}^{ab}}{2\pi} \right) + \frac{1}{2} (N^2 - 1) \beta V \int_{\beta} dk \bar{\omega}_{\mu\nu}^{-1}(k) \left\{ \bar{\omega}_{\mu\nu}(k) - \chi_{\mu\nu}(k) \right\}$$



The Variation Principle



- Variation principle for path integral measure (theory space)

$$d\mu = dA \cdot \rho(A) = \mathcal{N} dA \exp[-R(A)]$$

$$\langle \hat{A}(x_1) \cdots \hat{A}(x_n) \rangle_\mu = \int d\mu(A) A(x_1) \cdots A(x_n) \quad \text{moments}$$

- Free action

$$F(\mu) = \langle S(A) \rangle_\mu - \hbar \mathcal{W}(\mu)$$

euclidean action

entropy $\mathcal{W} = -\langle \ln \rho \rangle_\mu = \langle R \rangle_\mu - \ln \mathcal{N}$



Target action: **S(A)**

trial action: **R(A)**



- Variation principle I

$$F(\mu) \stackrel{!}{=} \min \quad \implies \quad d\mu_0(A) = Z^{-1} \exp \left[\hbar^{-1} S(A) \right] dA \quad \text{Gibbs measure}$$

$$\langle A(x_1) \cdots A(x_n) \rangle_{\mu_0} \quad \text{Schwinger functions}$$

- Variation principle II (Quantum effective action)

$$F(\mu, \mathcal{A}) = \min_{\mu} \left\{ F(\mu) \mid \langle A \rangle_{\mu} = \mathcal{A} \right\} \quad \stackrel{\min}{\implies} \quad d\mu = d\mu_{\mathcal{A}}$$

$$\Gamma(\mathcal{A}) = F(\mu_{\mathcal{A}}, \mathcal{A}) \stackrel{!}{=} \min$$

Note: proper functions

$$\frac{\delta \Gamma(\mathcal{A})}{\delta \mathcal{A}(x_1) \cdots \delta \mathcal{A}(x_n)}$$

• Variation principle III (Yang-Mills theory)

$$d\mu_0(A) = dA \mathcal{J}(A) \exp \left[-\hbar^{-1} S_{\text{gf}}(A) \right]$$

FP determinant

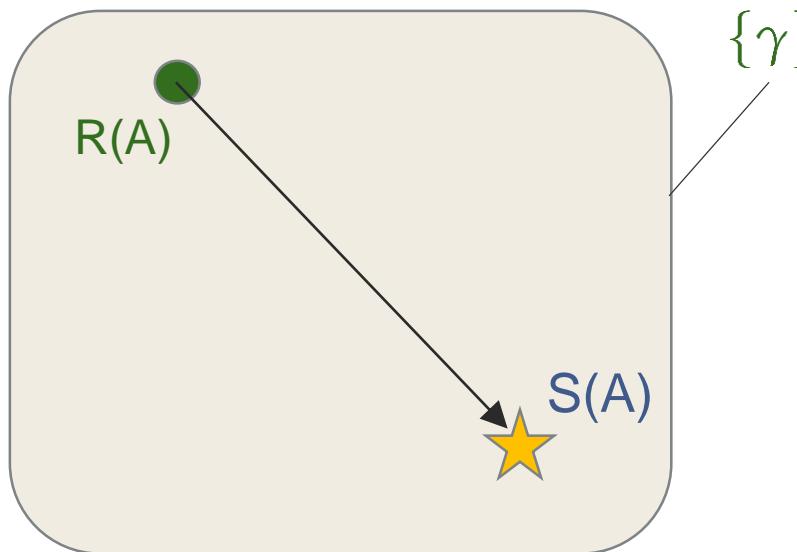
→ $\bar{\mathcal{W}}(\mu) = \mathcal{W}(\mu) + \langle \ln \mathcal{J} \rangle = -\langle \ln(\rho/\mathcal{J}) \rangle$ relative entropy

$$F(\mu, \mathcal{A}) \equiv \left\{ \langle S_{\text{gf}} \rangle_\mu - \left[\langle R \rangle_\mu + \langle \ln \mathcal{J} \rangle_\mu - \ln \mathcal{N} \right] \mid \langle A \rangle_\mu = \mathcal{A} \right\} \stackrel{!}{=} \min$$

→ Determines kernels in $R(A) = \frac{1}{2} A \gamma_2 A + \frac{1}{3!} \gamma_3 A^3 + \dots$

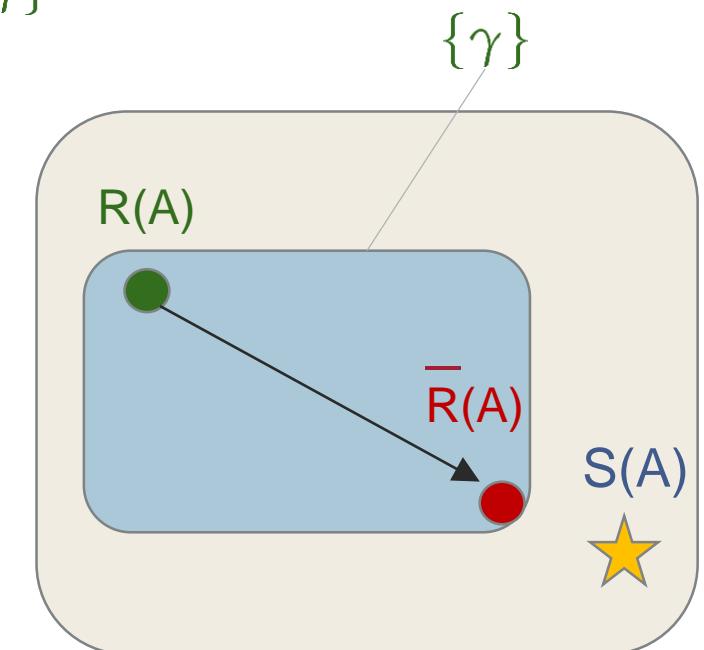
Target action: $S(A)$ has bare vertices $\{g\}$

Trial action $R(A)$ has variation kernels $\{\gamma\}$



gap equation trivial:

$$\gamma = g$$

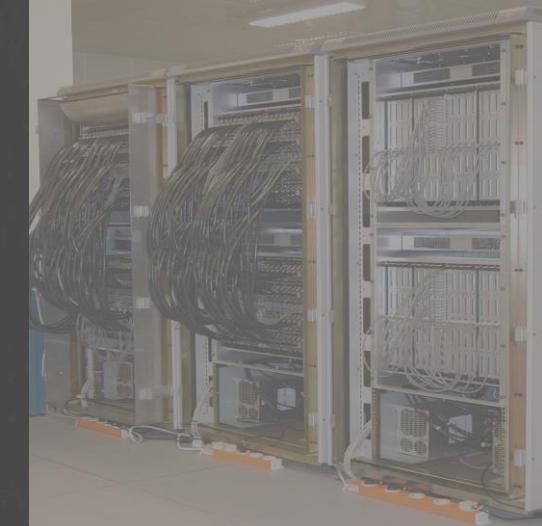
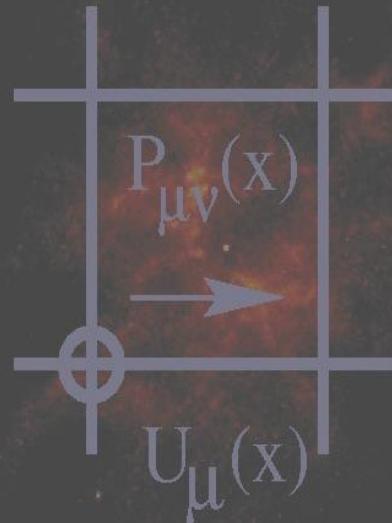


gap equation non-trivial:

$$\gamma = \bar{\gamma} \neq g$$



$$\begin{aligned} & \langle \rho \rangle + \langle \ln \mathcal{J} \rangle \\ & + \frac{1}{2} \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{ab}(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle - \frac{1}{2} \int_{\beta} d(x, y) \\ & + \frac{1}{2} \int_{\beta} d(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle \left\{ \bar{\omega}_{\mu\nu}^{ab}(x, y) - \chi_{\mu\nu}^{ab}(x, y) \right. \\ & \text{et } \left(\frac{2\pi}{\bar{\omega}} \right)^{-\frac{1}{2}} + \frac{1}{2} (N^2 - 1) \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{-1}(x, y) \left\{ \bar{\omega}_{\mu\nu}(x, y) - \right. \\ & \ln \left(\frac{\bar{\omega}_{\mu\nu}^{ab}}{2\pi} \right) + \frac{1}{2} (N^2 - 1) \beta V \int_{\beta} dk \bar{\omega}_{\mu\nu}^{-1}(k) \left\{ \bar{\omega}_{\mu\nu}(k) - \right. \end{aligned}$$



Gaussian Trial Measure In YM Theory



Gaussian ansatz

- UV : gluons weakly interacting
- IR : ghost dominance near Gribov horizon, self-interaction subdominant

$$d\mu(A) = \mathcal{N}(\omega) \cdot \mathcal{J}(A)^{-1} \cdot \exp \left[-\frac{1}{2} \int d(x,y) A_\mu^c(x) \delta^{ab} \omega_{\mu\nu}(x,y) A_\nu^b(y) \right]$$

Curvature Approximation

$$\ln \mathcal{J}(A) = -\frac{1}{2} \int d(x,y) A_\mu^a(x) \chi_{\mu\nu}^{ab}(x,y) A_\nu^b(y) + \dots$$

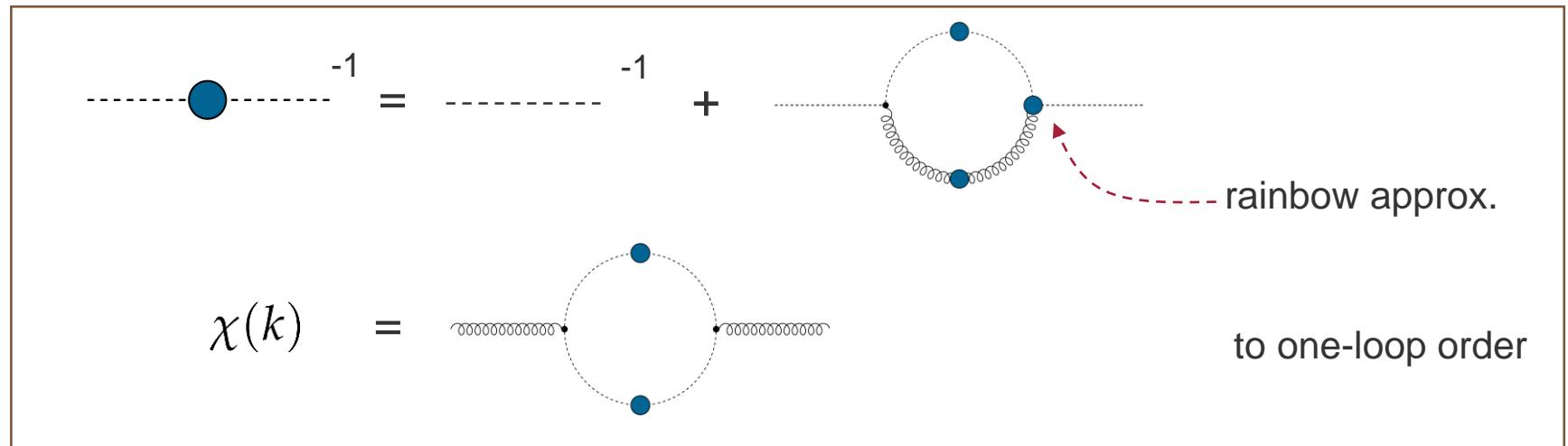
$$\chi_{\mu\nu}^{ab}(x,y) = -\left\langle \frac{\delta^2 \ln \mathcal{J}}{\delta A_\mu^a(x) \delta A_\nu^b(y)} \right\rangle \rightarrow \delta^{ab} t_{\mu\nu}(k) \chi(k)$$

curvature

Ghost sector (Landau gauge)

Use resolvent identity (DSE) on FP operator $G^{-1} = \partial_\mu \hat{D}^\mu = G_0^{-1} + h$

$$G_0 \langle G^{-1} \rangle = 1 + G_0 \langle hG \rangle \langle G^{-1} \rangle$$



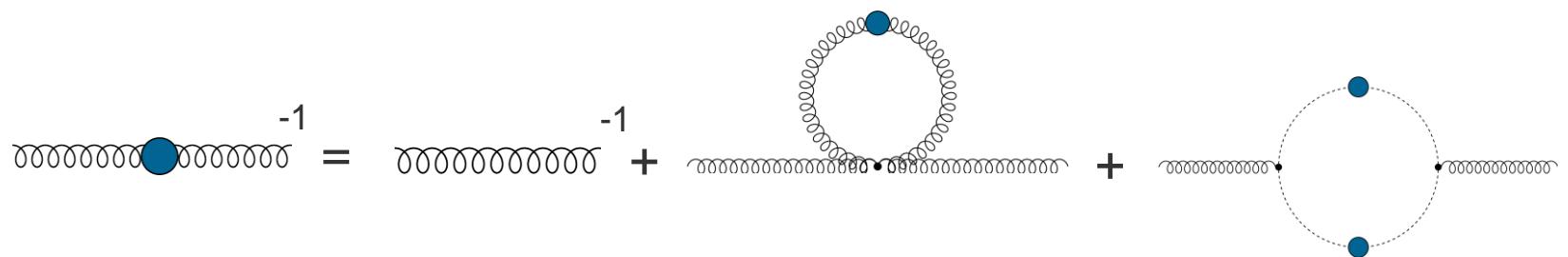
Free action

- evaluation of $F(\omega) = \langle S_{\text{gf}} \rangle_\omega - \bar{\mathcal{W}}(\omega)$: only requires Wick's theorem

Gap Equation

$$\frac{\delta}{\delta \omega(k)} F(\omega) = 0$$

$$\omega(k) = k^2 + M^2 + \chi(k)$$



Counterterms

Renormalization conditions (3 scales $0 \leq \mu_c \leq \mu_0 \ll \mu$)

- fix $\eta(\mu_c) \rightarrow Z_c$
 - fix $\omega(\mu) = Z \mu^2$
 - fix $\omega(\mu_0) = Z M_A^2$

scaling/decoupling ghost form factor $\langle G(k) \rangle = \frac{\eta(k)}{k^2}$

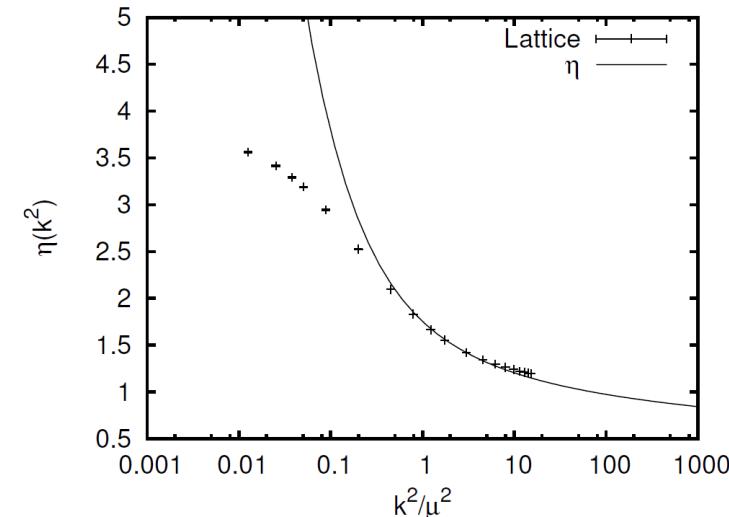
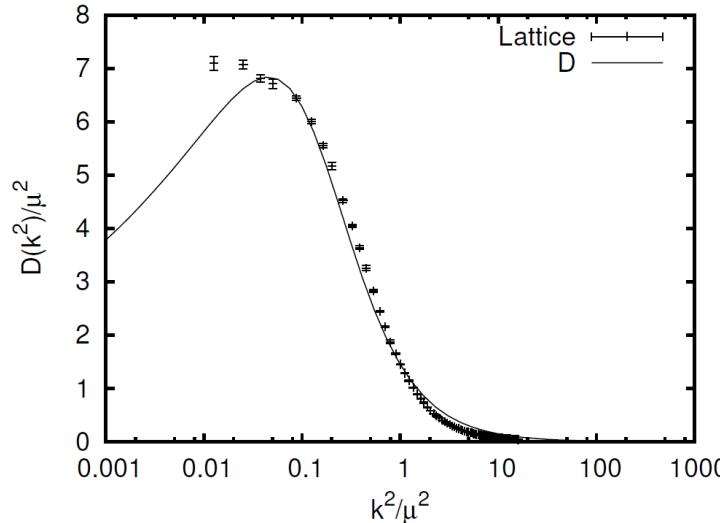
constituent *mass parameter*



Scaling Solution (G=SU(2))

MQ, H. Reinhardt, J. Heffner, Phys. Rev. **D89** 035037 (2014)

Lattice data from Bogolubsky et al., Phys. Lett. **B676** 69 (2009)



IR exponents: $\omega(k) \sim (k^2)^\alpha$

$$\alpha = \frac{1}{49} (44 - \sqrt{1201}) \approx 0.1907$$

$\eta(k) \sim (k^2)^{-\beta}$

$$\beta = \frac{1}{98} (93 - \sqrt{1201}) \approx 0.5953$$

numerical:

$$\alpha = 0.191(1) \quad \beta = 0.595(3)$$

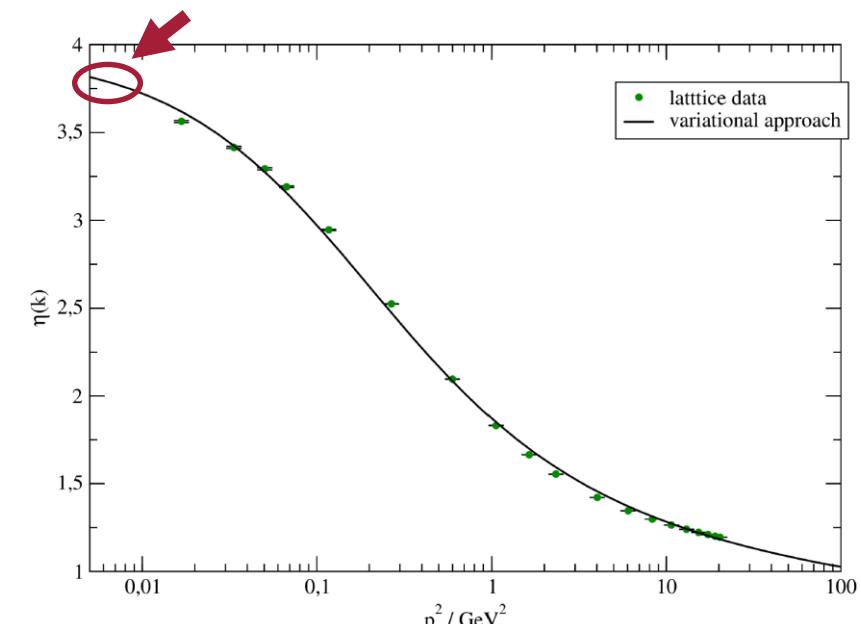
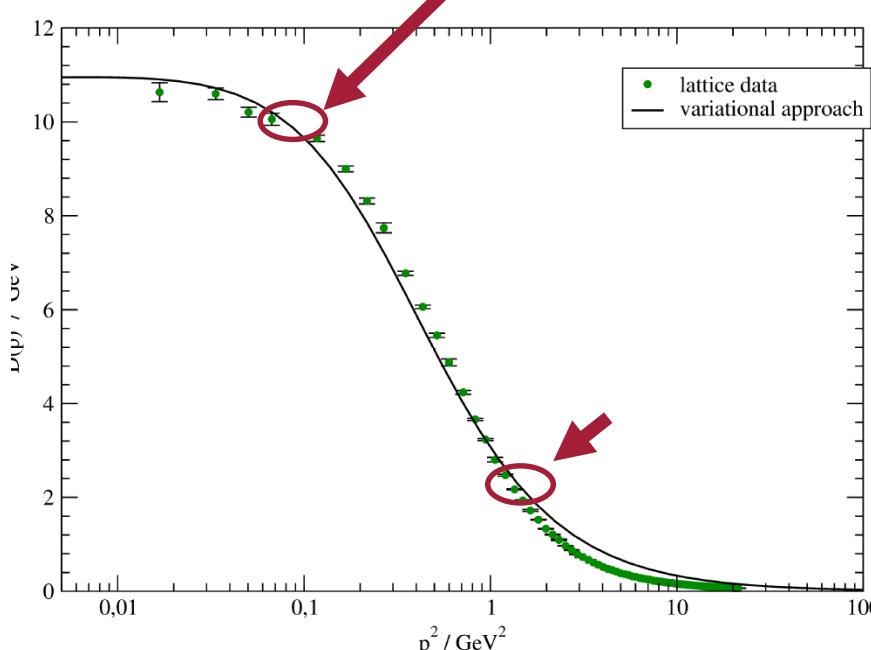
Lerche, v. Smekal PRD **65**

$$\alpha - 2\beta + \left(\frac{d}{2} - 1\right) < 10^{-3}$$

Decoupling Solution (G=SU(2))

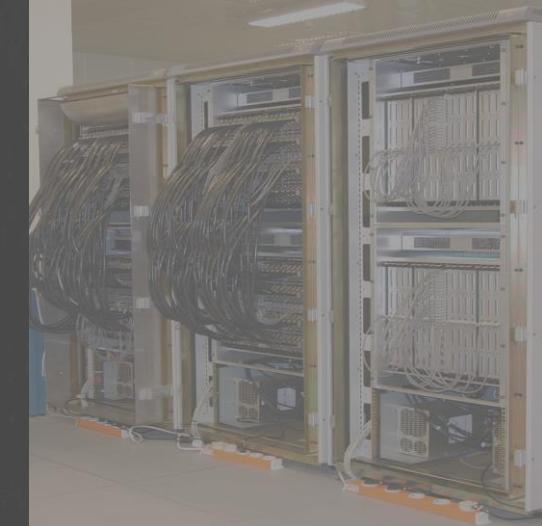
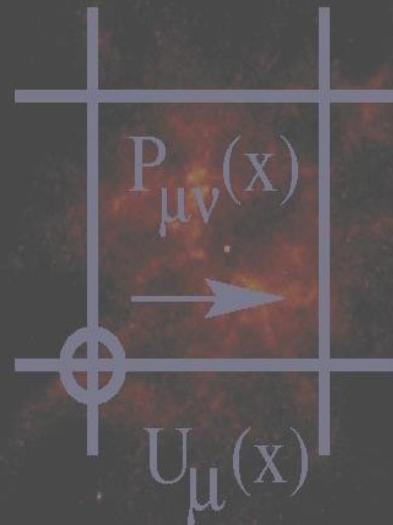
MQ, H. Reinhardt, J. Heffner, Phys. Rev. **D89** 035037 (2014)
Lattice data from Bogolubsky et al., Phys. Lett. **B676** 69 (2009)

Determines overall scale (not very precise !)





$$\begin{aligned}
 & \rho\rangle + \langle \ln \mathcal{J} \rangle \\
 & + \frac{1}{2} \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{ab}(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle - \frac{1}{2} \int_{\beta} d(x, y) \\
 & + \frac{1}{2} \int_{\beta} d(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle \left\{ \bar{\omega}_{\mu\nu}^{ab}(x, y) - \chi_{\mu\nu}^{ab}(x, y) \right. \\
 & \text{et } \left(\frac{2\pi}{\bar{\omega}} \right)^{-\frac{1}{2}} + \frac{1}{2} (N^2 - 1) \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{-1}(x, y) \left\{ \bar{\omega}_{\mu\nu}(x, y) - \right. \\
 & \ln \left(\frac{\bar{\omega}_{\mu\nu}^{ab}}{2\pi} \right) + \frac{1}{2} (N^2 - 1) \beta V \int_{\beta} dk \bar{\omega}_{\mu\nu}^{-1}(k) \left\{ \bar{\omega}_{\mu\nu}(k) - \right.
 \end{aligned}$$



Finite Temperature

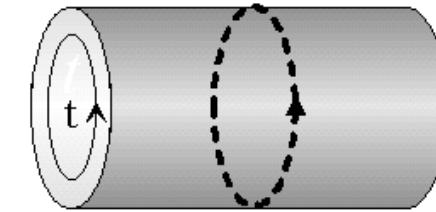
Extension to Finite Temperature

- imaginary time formalism

compactify euclidean time $t \in [0, \beta]$

periodic b.c. for gluons (up to center twists)

periodic b.c. for ghosts (even though fermions)



$$A(t, \mathbf{x}) = \beta^{-1} \sum_{n \in \mathbb{Z}} \int \frac{d^3 k}{(2\pi)^3} e^{i(\nu_n t + \mathbf{k} \cdot \mathbf{x})} A_n(\mathbf{k})$$

$$\nu_n = \frac{2\pi}{\beta} n \quad (n \in \mathbb{Z})$$



Extension to $T > 0$ straightforward

$$\int \frac{d^4 k}{(2\pi)^4} \longrightarrow \int_{\beta} d\mathbf{q} \equiv \beta^{-1} \sum_{n \in \mathbb{Z}} \int \frac{d^3 k}{(2\pi)^3}$$



- Lorentz structure of propagator

heat bath singles out restframe (1,0,0,0) breaks Lorentz invariance

two different 4-transversal projectors



$$\mathcal{P}_{\mu\nu}^T(k) = (1 - \delta_{\mu 0}) (1 - \delta_{\nu 0}) \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\mathbf{k}^2} \right) \quad \text{3-transversal}$$

$$\mathcal{P}_{\mu\nu}^L(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - \mathcal{P}_{\mu\nu}^T(k) \quad \text{3-longitudinal}$$



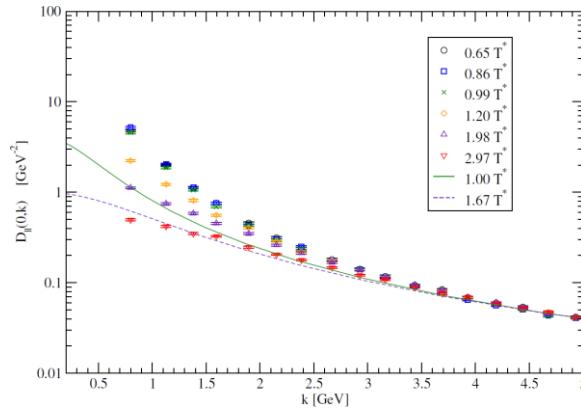
Two Lorentz structures for **kernel** and **curvature**

$$\omega_{\mu\nu}(k) = \omega_{\perp}(k) \cdot \mathcal{P}_{\mu\nu}^T(k) + \omega_{\parallel}(k) \cdot \mathcal{P}_{\mu\nu}^L(k) + \text{long.}$$

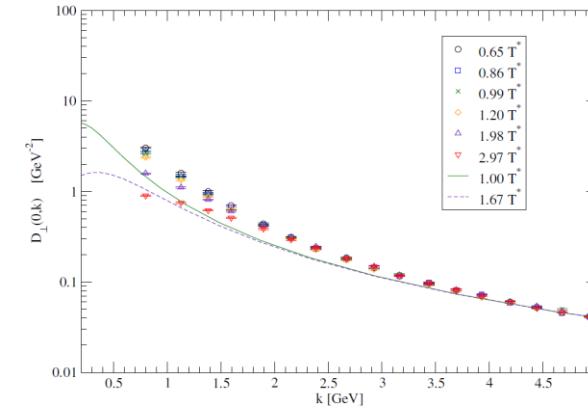
$$\chi_{\mu\nu}(k) = \chi_{\perp}(k) \cdot \mathcal{P}_{\mu\nu}^T(k) + \chi_{\parallel}(k) \cdot \mathcal{P}_{\mu\nu}^L(k) + \text{long.}$$



MQ, H. Reinhardt, Phys. Rev. D92 025051 (2015)



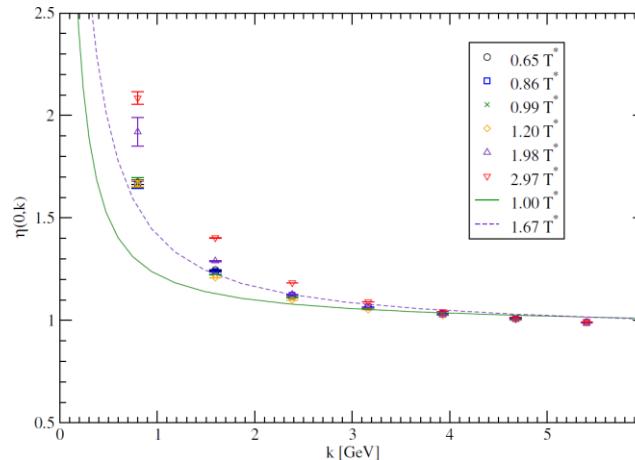
longitudinal gluon $D_{\parallel}(0, p)$



transversal gluon $D_{\perp}(0, p)$

ghost form factor

$$\eta(0, p)$$

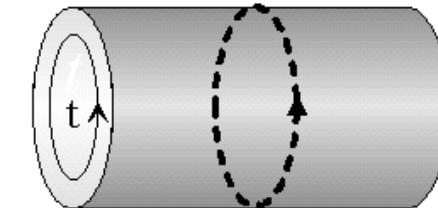


With increasing T

- Slight ghost enhancement
- Mild gluon suppression
- Temperature sensitivity larger in direction longitudinal to the heat bath

Polyakov loop

$$L(x) = P \exp \left[- \int_0^\beta dt A_0(t, x) \right]$$



Interpretation: free static quark energy

$$\langle \text{tr } L(x) \rangle = \exp [- \beta F_q(x)]$$

$$\langle \text{tr } L(x) L(y)^\dagger \rangle = \exp [- \beta F_{q\bar{q}}(x - y)]$$

Center symmetry

- maps $L \rightarrow z \cdot L$
- If unbroken $\langle L \rangle = 0$ [confinement]
- If broken $\langle L \rangle \neq 0$ [deconfinement]



Alternative order parameter

$$x \equiv \frac{\beta \langle A_0^3 \rangle}{2\pi} = \frac{\beta a}{2\pi} \in [0, 1]$$

G=SU(2)
Polyakov gauge [$\partial_0 A_0 = A_0^{\text{ch}} = 0$]
Background gauge [$\partial_0 a = 0$]

Background gauge

$$A_\mu = a \delta_{\mu 0} + Q_\mu$$

$$[D_\mu(a), Q_\mu] = 0$$

Transfer Landau -- Background

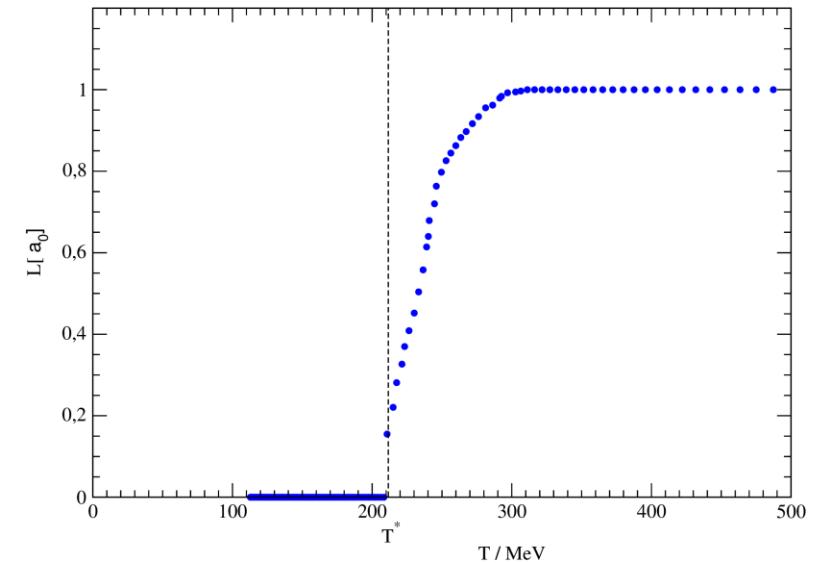
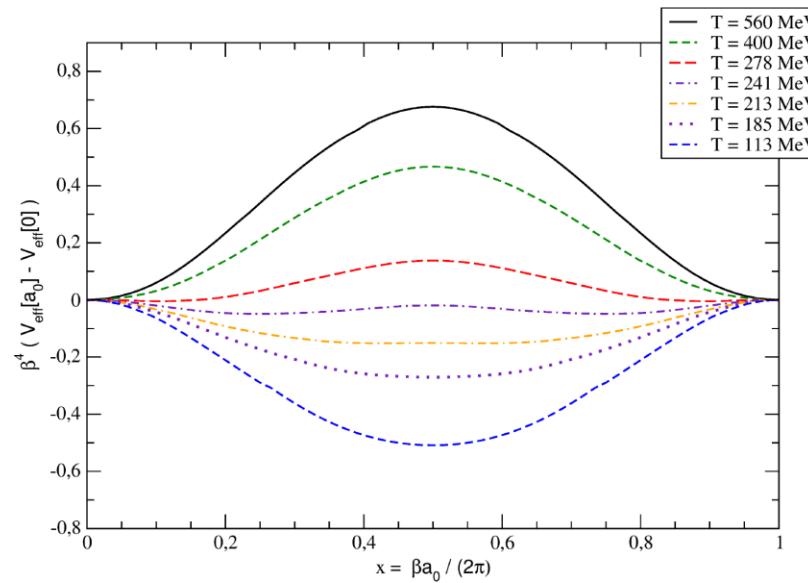
- replace $\partial_\mu \delta^{ab} \mapsto \hat{d}_\mu^{ab}$
- replace $p_\mu \mapsto p_\mu - \sigma a \delta_{\mu 0}$
- replace $N^2 - 1 = 3 \mapsto \sum_\sigma = 0, \pm 1$

in basis where rhs is diagonal
where σ are the simple roots
sum over simple roots



Phase transition for G=SU(2)

MQ, H. Reinhardt, Phys. Rev. D94, 065015 (2016)



Eff. Potential for Polyakov loop

- **2nd order** transition
- critical temperature $T^* = 216 \text{ MeV}$

Lattice

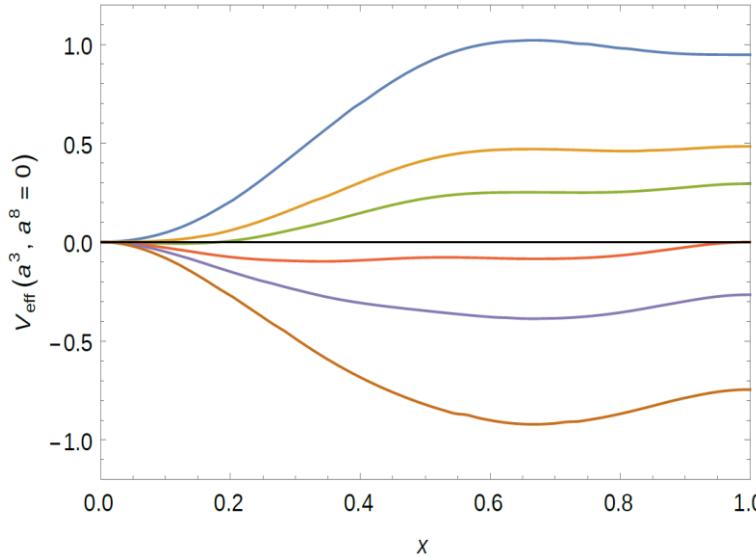
$T^* = 306 \text{ MeV}$

Lucini, Teper, Wenger, JHEP 01 (2004) 061

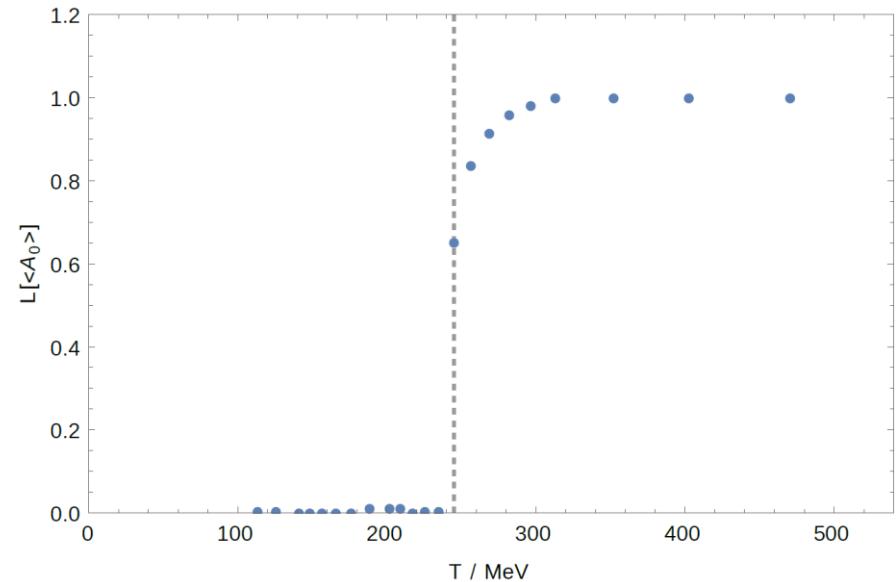


Phase transition for G=SU(3)

MQ, H. Reinhardt, Phys. Rev. **D94**, 065015 (2016)



slice of eff. Potential for
Polyakov loop



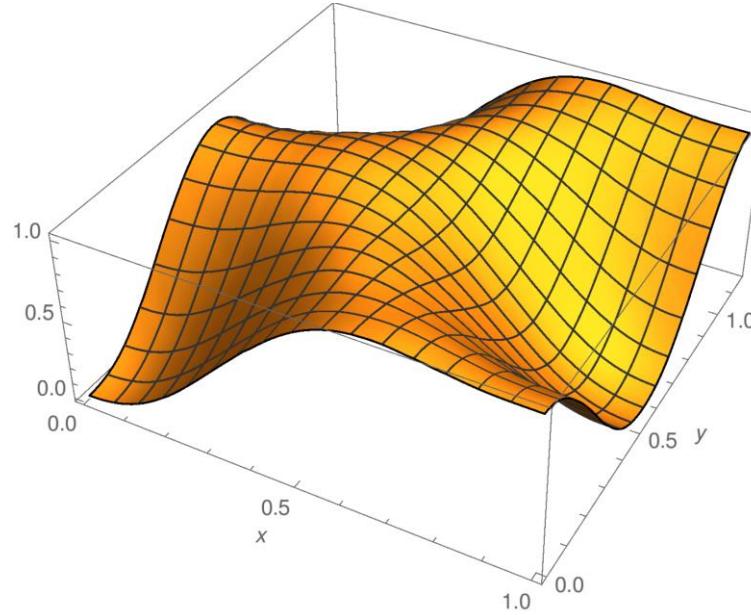
- 1st order transition
- critical temperature $T^* = 245 \text{ MeV}$
Lattice $T^* = 284 \text{ MeV}$

Lucini, Teper, Wenger, JHEP **01** (2004) 061



Effective potential for Polyakov loop in G=SU(3)

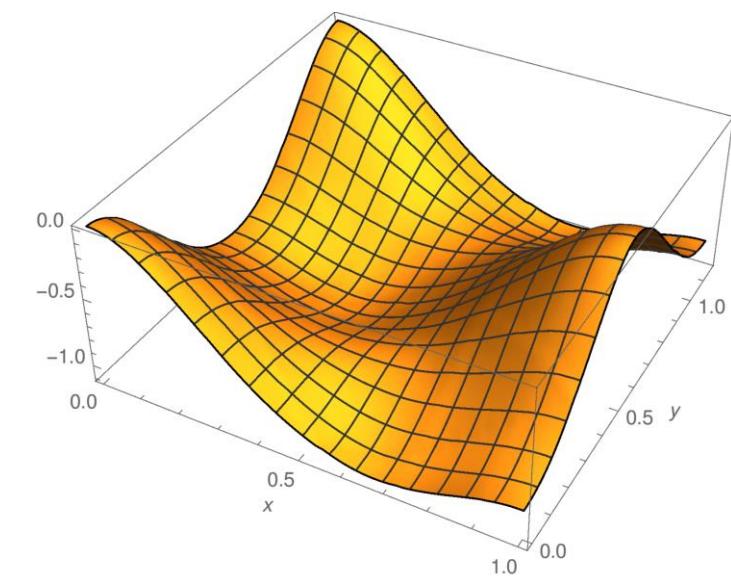
MQ, H. Reinhardt, Phys. Rev. **D94**, 065015 (2016)



Deconfined phase

$V(x,y)$ maximal at center symmetric points

$$T = 400 \text{ MeV}$$



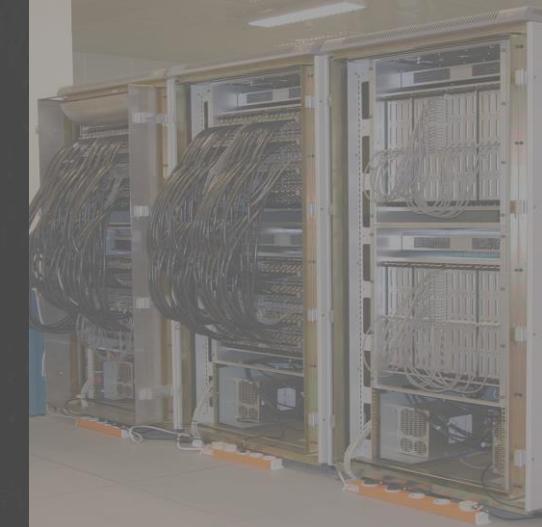
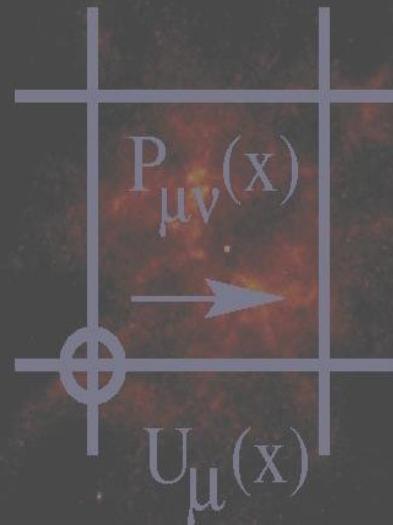
Confined phase

$V(x,y)$ minimal at center symmetric points

$$T = 141 \text{ MeV}$$



$$\begin{aligned} & \rho\rangle + \langle \ln \mathcal{J} \rangle \\ & + \frac{1}{2} \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{ab}(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle - \frac{1}{2} \int_{\beta} d(x, y) \\ & + \frac{1}{2} \int_{\beta} d(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle \left\{ \bar{\omega}_{\mu\nu}^{ab}(x, y) - \chi_{\mu\nu}^{ab}(x, y) \right. \\ & \text{et } \left(\frac{2\pi}{\bar{\omega}} \right)^{-\frac{1}{2}} + \frac{1}{2} (N^2 - 1) \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{-1}(x, y) \left\{ \bar{\omega}_{\mu\nu}(x, y) - \right. \\ & \ln \left(\frac{\bar{\omega}_{\mu\nu}^{ab}}{2\pi} \right) + \frac{1}{2} (N^2 - 1) \beta V \int_{\beta} dk \bar{\omega}_{\mu\nu}^{-1}(k) \left\{ \bar{\omega}_{\mu\nu}(k) - \right. \end{aligned}$$



Thermodynamics

Thermodynamics of the YM plasma

MQ, H. Reinhardt, Phys. Rev. D96, 054029 (2017)

- Free energy density:

$$F(\beta) = \min_{\mu} F_{\beta}(\mu) = \min_a \Gamma_{\beta}[a] = -\ln Z(\beta) = V_3 \beta \cdot f(\beta)$$

- pressure: $p(\beta) = -f(\beta)$
- energy density: $\epsilon(\beta) = f(\beta) + \beta \partial f / \partial \beta$
- Interaction strength: $\Delta(\beta) = -\beta \partial(p\beta^4) / \partial \beta = \beta^4(\epsilon - 3p)$
 Free relativistic gas: $p \sim T^4 \implies \Delta = 0$

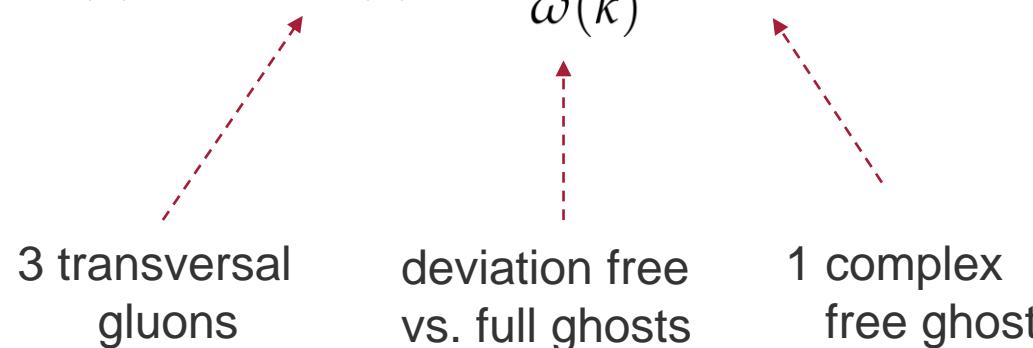


From variational solution of gluon and curvature

$$u(x, \beta) \equiv \beta^4 f_\beta(x) = 4\pi \int_0^\infty dq q^2 \sum_{n \in \mathbb{Z}} \left[\Phi(k_n(x)) + \frac{1}{2} \Phi(k_n(0)) \right]$$

$$k_n(x) \equiv \frac{2\pi}{\beta} \sqrt{(n+x)^2 + q^2} \quad x \equiv \frac{\beta a_0}{2\pi} \in [0, 1]$$

$$\Phi(k) = 3 \ln \omega(k) - 3 \frac{\chi_R(k)}{\omega(k)} - \ln k^2$$



Poisson resummation

T=0 subtracted (vacuum energy)

$$\beta^4 f_\beta(x) = u(x, \beta) - \lim_{\beta \rightarrow \infty} u(x, \beta) = -\frac{2}{\pi^2} \sum_{\nu=1}^{\infty} \frac{\cos(2\pi\nu x) + \frac{1}{2}}{\nu^4} h(\beta\nu)$$

$$h(\lambda) = -\frac{1}{4} \int_0^\infty d\tau \tau^2 J_1(\tau) \Phi(\tau/\lambda)$$

- Free (massive) boson $\Phi_M(k) = \ln(k^2 + M^2)$ $\Rightarrow h_M(\lambda) = \frac{1}{2} (\lambda M)^2 K_2(\lambda M)$

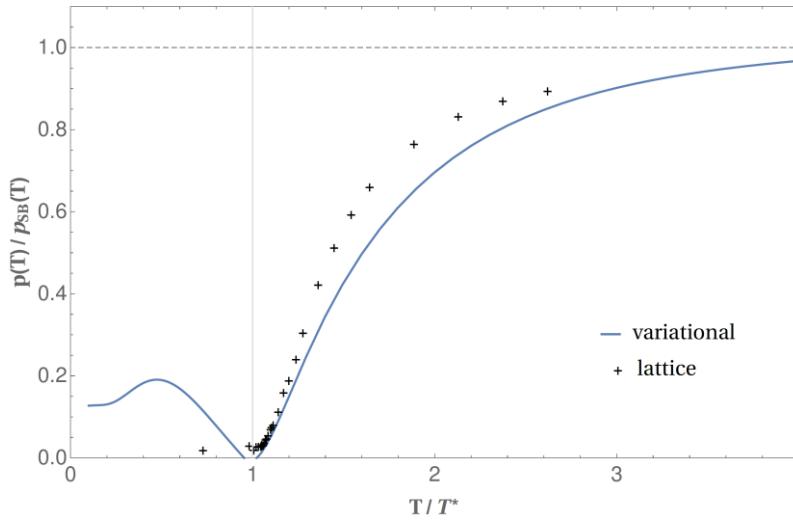
$$\lim_{\beta \rightarrow \infty} h_M(\beta\nu) = \begin{cases} 0 & : M > 0 \\ 1 & : M = 0 \end{cases}$$



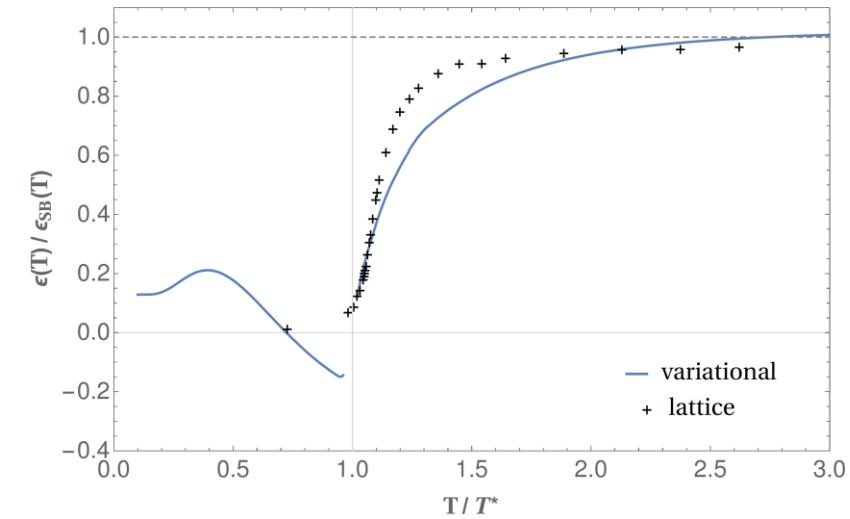
Pressure counts
massless modes
at T=0



pressure



energy density



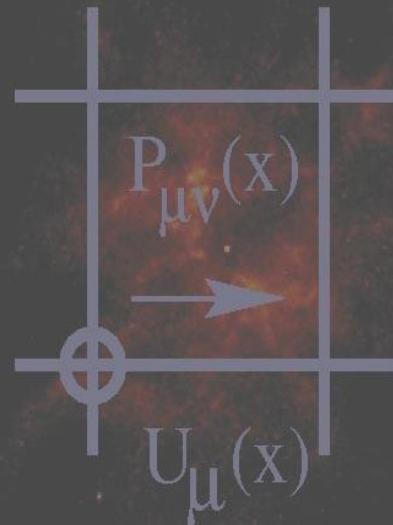
MQ, H. Reinhardt, Phys. Rev. D96, 054029 (2017)

- Correct SB limit at $T \rightarrow \infty$
- Infrared limit wrong $\lim_{T \rightarrow 0} \frac{p(T)}{p_{SB}(T)} = \frac{1}{8} \neq 0$
- Energy density jumps at T^* and becomes negative in region below T^*

cf:
 Canfora et al. (2015)
 Kondo (2015)
 Reinoso et al. (2015)
 Fischer, Lucker (2013)



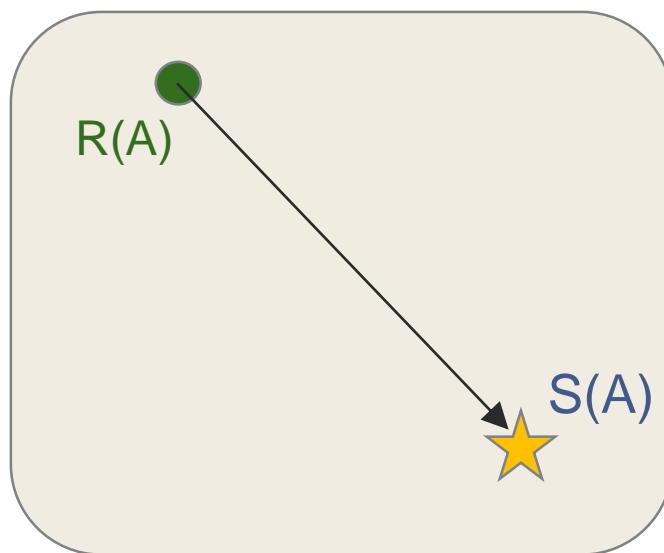
$$\begin{aligned}
 & \rho\rangle + \langle \ln \mathcal{J} \rangle \\
 & + \frac{1}{2} \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{ab}(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle - \frac{1}{2} \int_{\beta} d(x, y) \\
 & + \frac{1}{2} \int_{\beta} d(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle \left\{ \bar{\omega}_{\mu\nu}^{ab}(x, y) - \chi_{\mu\nu}^{ab}(x, y) \right. \\
 & \text{et } \left(\frac{2\pi}{\bar{\omega}} \right)^{-\frac{1}{2}} + \frac{1}{2} (N^2 - 1) \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{-1}(x, y) \left\{ \bar{\omega}_{\mu\nu}(x, y) - \right. \\
 & \ln \left(\frac{\bar{\omega}_{\mu\nu}^{ab}}{2\pi} \right) + \frac{1}{2} (N^2 - 1) \beta V \int_{\beta} dk \bar{\omega}_{\mu\nu}^{-1}(k) \left\{ \bar{\omega}_{\mu\nu}(k) - \right.
 \end{aligned}$$



Non-Gaussian Trial Measures

Target action: $S(A)$ has bare vertices $\{g\}$

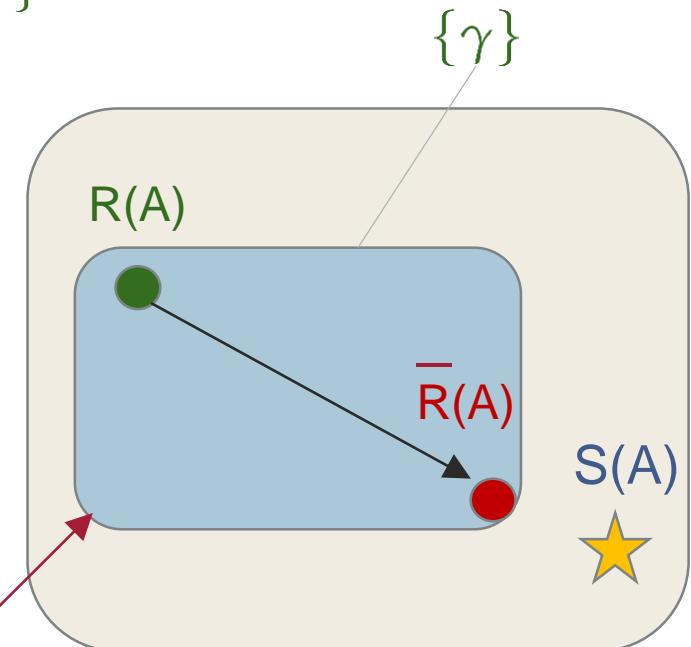
Trial action $R(A)$ has variation kernels $\{\gamma\}$



gap equation trivial:

$$\gamma = g$$

applies also to
truncated DSE



gap equation non-trivial:

$$\gamma = \bar{\gamma} \neq g$$

Example: Ghost dominance model

(D. Campagnari)

$$R = \frac{1}{2} \omega(3,4) A(3) A(4) + \bar{\sigma}(3) \left[-\Delta(3,4) + \Gamma_0(3,4;5) A(5) \right] c(4)$$

DSE

$$\text{~~~~~}^{-1} = \text{~~~~~} \square \text{~~~~~} + \text{~~~~~} \circlearrowleft \bullet \text{~~~~~}$$
$$\bullet \text{~~~~~} = -2 \quad \begin{array}{c} \text{~~~~~} \\ \diagdown \quad \diagup \\ \bullet \text{-----} \bullet \\ \diagup \quad \diagdown \end{array} \quad + \quad \text{~~~~~} \circlearrowleft \bullet \text{~~~~~}$$

Auto-Tune these DSE: determine optimal kernel ω from gap-equation

$$\left\langle \frac{\delta R}{\delta \omega} (S - R) \right\rangle_R = \left\langle \frac{\delta R}{\delta \omega} \right\rangle_R \left\langle S - R \right\rangle_R$$

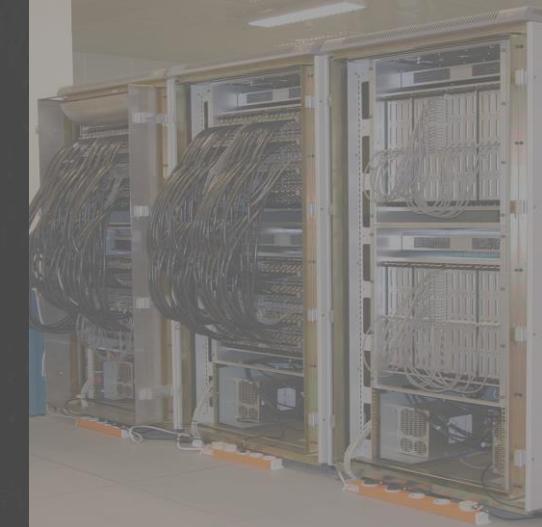
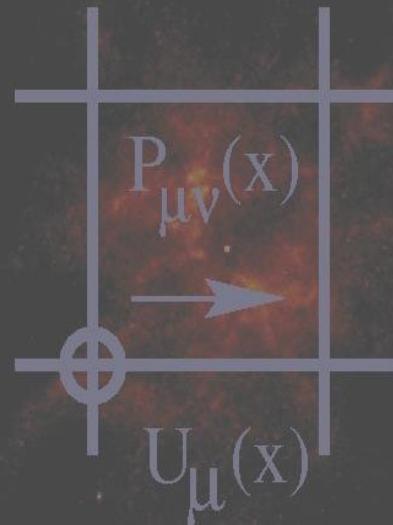
Gap equation:

$$\begin{aligned} \text{---} \square \text{---} &= \text{---} \square \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \text{---} - \frac{1}{2} \text{---} \bullet \text{---} - \frac{1}{2} \text{---} \bullet \text{---} \\ &\quad - \frac{1}{2} \text{---} \bullet \text{---} + \frac{1}{2} \text{---} \bullet \text{---} + \frac{1}{2} \text{---} \bullet \text{---} \\ &\quad + \frac{1}{3!} \text{---} \bullet \text{---} + \frac{1}{3!} \text{---} \bullet \text{---} + \frac{1}{3!} \text{---} \bullet \text{---} \\ &\quad + \frac{1}{4!} \text{---} \bullet \text{---} + \frac{1}{4} \text{---} \bullet \text{---} + \frac{1}{4!} \text{---} \bullet \text{---} \end{aligned}$$

A red diagonal line starts from the top right and points downwards towards the bottom left, passing through the terms involving shaded circles.



$$\begin{aligned}
 & \rho\rangle + \langle \ln \mathcal{J} \rangle \\
 & + \frac{1}{2} \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{ab}(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle - \frac{1}{2} \int_{\beta} d(x, y) \\
 & + \frac{1}{2} \int_{\beta} d(x, y) \langle A_{\mu}^a(x) A_{\nu}^b(y) \rangle \left\{ \bar{\omega}_{\mu\nu}^{ab}(x, y) - \chi_{\mu\nu}^{ab}(x, y) \right. \\
 & \text{et } \left(\frac{2\pi}{\bar{\omega}} \right)^{-\frac{1}{2}} + \frac{1}{2} (N^2 - 1) \int_{\beta} d(x, y) \bar{\omega}_{\mu\nu}^{-1}(x, y) \left\{ \bar{\omega}_{\mu\nu}(x, y) - \right. \\
 & \ln \left(\frac{\bar{\omega}_{\mu\nu}^{ab}}{2\pi} \right) + \frac{1}{2} (N^2 - 1) \beta V \int_{\beta} dk \bar{\omega}_{\mu\nu}^{-1}(k) \left\{ \bar{\omega}_{\mu\nu}(k) - \right.
 \end{aligned}$$



Summary



Summary

1. Covariant variational principle for the effective action
2. Propagators at $T=0$ and renormalization
3. Finite temperatures: Deconfinement + Thermodynamics
4. Non-Gaussian measures, Auto-Tuned DSE

Outlook

1. Fermions
2. Chemical Potential



Example: Gaussian trial measure

ϕ^4 Theory in d dim: $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4$

Trial measure : $d\mu = \det \omega^{\frac{1}{2}} \cdot \exp \left[-\frac{1}{2} \int (\phi - \varphi) \omega(\phi - \varphi) \right]$

minimal free action $\omega = \omega_\varphi$

$$\begin{aligned} \rightarrow \Gamma(\varphi) &= \int dx \frac{1}{2} \varphi \omega_\varphi \varphi = \int dx \frac{1}{2} \varphi \omega_0 \varphi + \mathcal{O}(\varphi^4) \\ &= \int dx \left[\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{M^2}{2} \varphi^2 + \frac{\lambda}{4!} [1 + \dots] \varphi^4 + \mathcal{O}(\varphi^6) \right] \end{aligned}$$

dynamical mass $M^2 = m^2 + \frac{\hbar \lambda}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + M^2}$

Ghost sector

Use resolvent identity on FP operator $G^{-1} = \partial_\mu \hat{D}^\mu = G_0^{-1} + h$

$$G_0 \langle G^{-1} \rangle = 1 + G_0 \langle hG \rangle \langle G^{-1} \rangle$$

in terms of ghost form factor $G(k) = \frac{\eta(k)}{k^2}$

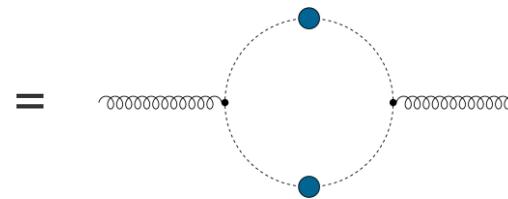
$$\eta(k)^{-1} = 1 - Ng^2 \int \frac{d^4 q}{(2\pi)^4} \frac{\eta(k-q)}{(k-q)^2} \frac{1 - (\hat{k} \cdot \hat{q})}{\bar{\omega}(q)}$$



Curvature Equation

To given loop order

$$\chi(k) = \text{Tr} \left\langle G \frac{\delta(-\partial D)}{\delta A(2)} G \frac{\delta(-\partial D)}{\delta A(1)} \right\rangle \approx \text{Tr} \langle G \rangle \Gamma_0 \langle G \rangle \Gamma_0$$

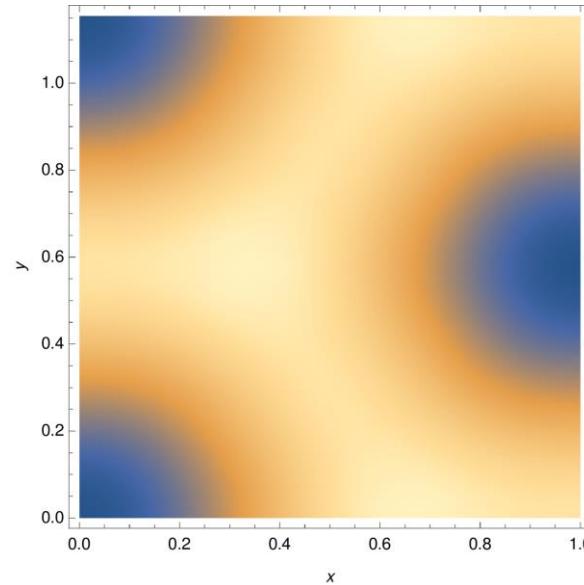


$$\chi(k) = \frac{1}{3} N g^2 \int \frac{d^4 q}{(2\pi)^4} \frac{\eta(k-q) \eta(q)}{(k-q)^2} \left[1 - (\hat{k} \cdot \hat{q}) \right]$$



Effective potential for Polyakov loop in G=SU(3)

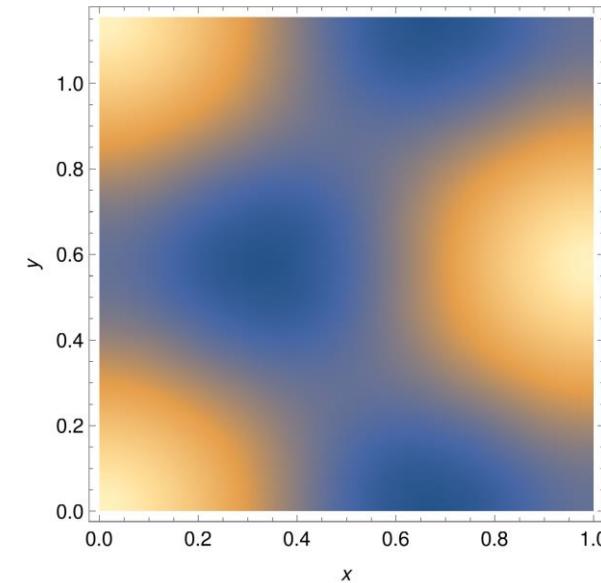
MQ, H. Reinhardt, Phys. Rev. D94, 065015 (2016)



Deconfined phase

$V(x,y)$ maximal at center symmetric points

$$T = 400 \text{ MeV}$$



Confined phase

$V(x,y)$ minimal at center symmetric points

$$T = 141 \text{ MeV}$$



Effective potential of background field (Polyakov Loop)

$$\beta^4 V_{\text{eff}}(x) = \beta^4 W(x) + \frac{6}{\pi^2} \sum_{m=1}^{\infty} \frac{1 - \cos(2\pi mx)}{m^4} h(\beta m)$$

$$\beta^4 W(x) \equiv 8\pi \int_0^\infty dq q^2 \sum_{n \in \mathbf{Z}} \ln \frac{(n+x)^2 + q^2}{n^2 + q^2} = \frac{4}{3} \pi^2 x^2 (1-x)^2$$

Weiß potential

$$h(\lambda) = -\frac{1}{4} \int_0^\infty d\xi \xi^2 J_1(\xi) \left[\ln \left[\frac{\bar{\omega}(\xi/\lambda)}{(\xi/\lambda)^2} \right] - \frac{\chi_R(\xi/\lambda)}{\bar{\omega}(\xi/\lambda)} \right]$$

where $x = \frac{\beta a_0}{2\pi} \in [0, 1]$

Similarly for G=SU(3)



Interaction strength

MQ, H. Reinhardt, Phys. Rev. D96, 054029 (2017)

