Confinement/deconfinement phase transition in SU(3) Yang-Mills theory and Non-Abelian dual Meissner effect

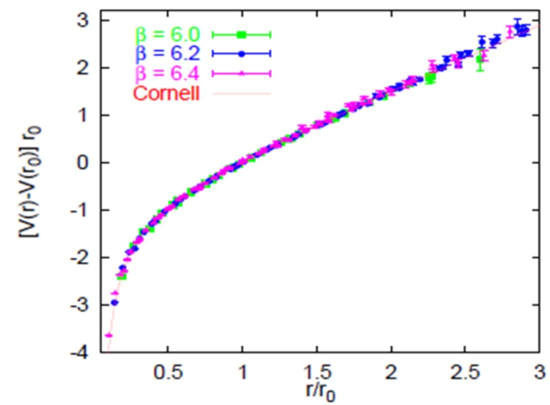
Akihiro Shibata (KEK)

In collaboration with:
Kei-Ichi. Konodo (Chiba U.), Seikou Kato (Oyama NTC),

Introduction(1)

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]
- **Dual superconductivity** is promising mechanism. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam(1976), A.M. Polyakov (1975)]

G.S. Bali Phys. Rept. **343, 1–136 (2001)**



- To establish this picture, we must show evidences of the dual version of the superconductivity in various situations.
- ➢ Dual super conductivity in the fundamental representation (Preceding works)
- ➢ Dual super conductivity in the higher representation (Matsudo's talk)
- ➢ Confinemet/deconfinement phase transition at finite temperature (this talk)

Dual superconductivity

Superconductor (condensed matter)

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

Dual superconductor (QCD)

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromoelectric flux tube) connecting quark and antiquark
- Linear potential between quark and anti-quark

Extracting dominant mode for confinement

Abelian projection method	Decomposition method
Extracting the dominant mode as the diagonal (Abelian) part in the maximal Abelian (MA) gauge. *U=XV* – SU(2) → U(1) – SU(3) → U(1)XU(1) Problems: ✓ The results of Abelian projection method depends on the gauge fixing of the Yang-Mills theory. ✓ The gauge fixing breaks (global) color symmetry.	**[a new formulation on a lattice]** Extracting the relevant mode *V* for quark confinement by solving the defining equation in the gauge independent way (gauge-invariant way). ➔ The Abelian projection method can be reformulated by using the decomposition method in the gauge invariant way.

A new formulation of Yang-Mills theory (on a lattice)

[Phys.Rept. 579 (2015) 1-226]

Decomposition of SU(N) gauge links For SU(N) YM gauge link, there are sever al possible options of decomposition *discriminated by its stability groups*:

- ☐ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$
- ☐ SU(3) Yang-Mills link variables: **Two options**

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

maximal option : $U(1) \times U(1) \subset SU(3)$

Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

Dual Superconductivity in SU(3) Yang-Mills

Abelian Dual superconductivity

☐Abelian projection in MA gauge ::
SU(3) ➔ U(1)xU(1) (Maximal torus)

•Perfect Abelian dominance in string tension[Sakumichi-Suganuma]

☐Decomposition method

•**Maximal option** of a new formulation [ours]

Cho-Faddev-Niemi-Shavanov decomposition [N Cundy, Y.M. Cho et.al]

Non-Abelian Dual superconductivity

☐Decomposition method

•**Minimal option:** (non-Abelian dual superconductivity) based on the U(2) stability sub-group.

we have showed in the series works

✓V-field dominance, non-Abalian magnetic monopole dominance in string tension

✓ chromo-flux tube and dual Meissner effect,

✓confinement/deconfinement phase transition in terms of dual Meissner effect at finite temperature

minimal option: The decomposition of SU(3) link variable

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

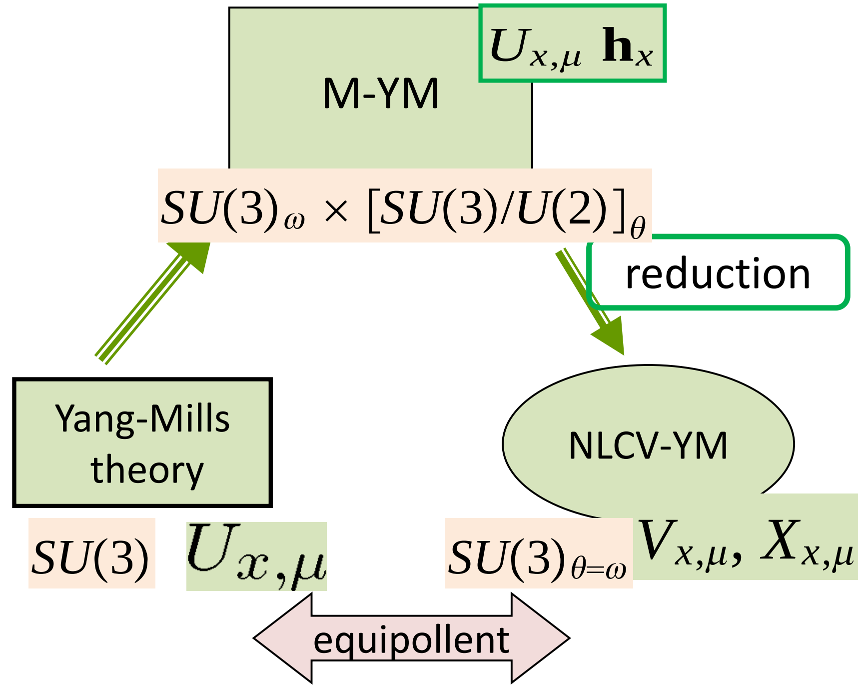
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Minimal option: Defining equation for the decomposition

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}^\dagger_{x,\mu}(\det \hat{L}_{x,\mu})^{1/N} g_x^{-1} \quad V_{x,\mu} = X^\dagger_{x,\mu}U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det \hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L^\dagger_{x,\mu}}\right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu})$$
$$+ 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}$$

continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

Minimal option: Non-Abelian magnetic monopole

For Wilson loop in the fundamental representation

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int [d\mu(\xi)]_\Sigma \exp\left(-ig \int_{S:C=\partial\Sigma} dS^{\mu\nu} \sqrt{\frac{N-1}{2N}} \operatorname{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x)) \right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left(ig\sqrt{\frac{N-1}{2N}}(k, \Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j, N_\Sigma) \right)$$

magnetic current $k := \delta^* F = {}^* dF, \quad \Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}$

electric current $j := \delta F, \quad N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$

$\Delta = d\delta + \delta d, \quad \Theta_\Sigma := \int_\Sigma d^2 S^{\mu\nu}(\sigma(x)) \delta^D(x - x(\sigma))$

k and j are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

K.-I. Kondo PRD77 085929(2008)

Note that field strength $F[V]$ is described by V-field in the minimal option.

The lattice version of magnetic monopole current is defined by using plaquette:

$$\Theta^8_{\mu\nu} := -\arg \operatorname{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x \right) V_{x,\mu} V_{x+\mu,\mu} V^\dagger_{x+\nu,\mu} V^\dagger_{x,\nu} \right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu \Theta^8_{\alpha\beta},$$

maximal option: Defining equation for the decomposition

By introducing color fields $\mathbf{n}_x^{(3)} = \Theta_x(\lambda^3/2)\Theta^\dagger$, $\mathbf{n}_x^{(8)} = \Theta_x(\lambda^8/2)\Theta^\dagger$ $\in SU(3)_\omega \times [SU(3)/(U(1)\times U(1))]_\theta$, a set of the defining equation for the decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\varepsilon[V]n_x^{(k)} = \frac{1}{\varepsilon}(V_{x,\mu}n_{x+\mu}^{(k)} - n_x^{(k)}V_{x,\mu}) = 0, \quad (k = 3,8)$$

$$g_x = \exp(2\pi i n/N)\exp(i\sum_{j=3,8} a^{(j)}n_x^{(j)}) = 1$$

Coressponding to the continuum version of the dexomposition $\mathcal{A}_\mu(x) = V_\mu(x) + \mathcal{X}_\mu(x)$

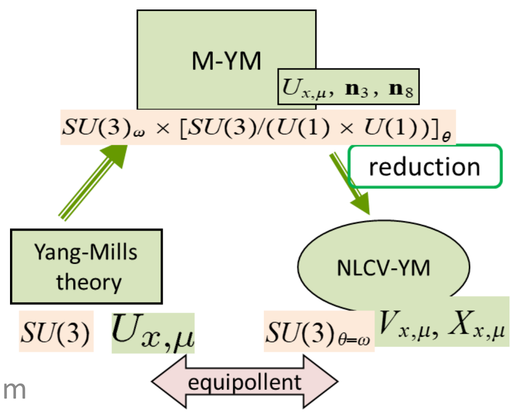
$$D_\mu[V_\mu]\mathbf{n}^{(k)}(x) = 0, \qquad tr(\mathbf{n}^{(k)}(x)\mathcal{X}_\mu(x)) = 0, \quad (k = 3,8)$$

$$X_{x,\mu} = \hat{K}_{x,\mu}^\dagger \det(K_{x,\mu})^{1/3}g_x^{-1}, \quad V_{x,\mu} = g_x\hat{K}_{x,\mu}\det(K_{x,\mu})^{-1/3}$$

where

$$\hat{K}_{x,\mu} := \left(\sqrt{K_{x,\mu}K_{x,\mu}^\dagger}\right)^{-1}K_{x,\mu}, \quad \hat{K}_{x,\mu}^\dagger = K_{x,\mu}^\dagger\left(\sqrt{K_{x,\mu}K_{x,\mu}^\dagger}\right)^{-1}$$

$$K_{x,\mu} = 1 + 6\mathbf{n}_x^{(3)}U_{x,\mu}\mathbf{n}_{x+\mu}^{(3)}U_{x,\mu}^\dagger + 6\mathbf{n}_x^{(8)}U_{x,\mu}\mathbf{n}_{x+\mu}^{(8)}U_{x,\mu}^\dagger$$



Reduction condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ and color fields which is given by minimizing the reduction condition.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory, i.e., defining an effective theory, i.e., gauge-Higgs model whose kinetic term is given by the reduction condition

for given $U_{x,\mu}$

$$F[\Theta;U] = \begin{cases} \sum_{x,\mu} \mathrm{tr}\left[\sum_{j=3,8} \left(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)}\right)^{\dagger} \left(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)}\right) \right] & \text{MAG (maximal)} \\ \sum_{x,\mu} \mathrm{tr}\left[\left(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(8)}\right)^{\dagger} \left(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(8)}\right) \right] & \text{n8 (minimal)} \end{cases}$$

where $\mathbf{n}_j := \Theta^{\dagger} H_j \Theta$, H_j Cartan generators, and $D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)} := U_{x,\mu}\mathbf{n}^{(j)}_{x+\mu} - \mathbf{n}^{(j)}_{x} U_{x,\mu}$

Maximal option with the MA reduction as gauge invariant version of Abelian projection in the Maximal Abelian gauge

MA reduction condition is rewritten into the gauge fixing of maximal Abelian gauge.

$$F_{MA}[\Theta;U] = \sum_{x,\mu} tr\left[\sum_{j=3,8}(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)})^{\dagger}(D^{\epsilon}_{\mu}[U]\mathbf{n}^{(j)})\right] = \sum_{x,\mu}\left[2-2\sum_{j=3,8}\mathrm{tr}(U^{\dagger}_{x,\mu}\mathbf{n}^{(j)}_{x}U_{x,\mu}\mathbf{n}^{(j)}_{x+\mu})\right]$$

$$= \sum_{x,\mu}\left[2-2\sum_{j=3,8}\mathrm{tr}\left([\Theta_x U^{\dagger}_{x,\mu}\Theta^{\dagger}_{x}]\frac{\lambda_j}{2}[\Theta_{x+\mu}U_{x,\mu}\Theta^{\dagger}_{x+\mu}]\frac{\lambda_j}{2}\right)\right] = \sum_{x,\mu}(2-F_{MAG}[\Theta;U])$$

Decomposition for maximal option id given by

$$K_{x,\mu} = U_{x,\mu} + 6\mathbf{n}^{(3)}_{x}U_{x,\mu}\mathbf{n}^{(3)}_{x+\mu} + 6\mathbf{n}^{(8)}_{x}U_{x,\mu}\mathbf{n}^{(8)}_{x+\mu} = \Theta_x\left[{}^{\Theta^{\dagger}}U_{x,\mu} + 6\frac{\lambda_3}{2}{}^{\Theta^{\dagger}}U_{x,\mu}\frac{\lambda_3}{2} + 6\frac{\lambda_8}{2}{}^{\Theta^{\dagger}}U_{x,\mu}\frac{\lambda_8}{2}\right]\Theta^{\dagger}_{x+\mu}$$

$$= \Theta_x[\mathrm{diag}({}^{\Theta^{\dagger}}u^{11}_{x,\mu}, {}^{\Theta^{\dagger}}u^{22}_{x,\mu}, {}^{\Theta^{\dagger}}u^{33}_{x,\mu},)]\Theta^{\dagger}_{x+\mu}$$

$$V_{x,\mu} = (K_{x,\mu}K^{\dagger}_{x,\mu})^{-1/2}K_{x,\mu}(\det K_{x,\mu})^{-1/3} = \mathrm{diag}\left(\frac{{}^{\Theta^{\dagger}}u^{11}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{11}_{x,\mu}\right|}, \frac{{}^{\Theta^{\dagger}}u^{22}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{22}_{x,\mu}\right|}, \frac{{}^{\Theta^{\dagger}}u^{33}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{33}_{x,\mu}\right|}\right)\left(\frac{{}^{\Theta^{\dagger}}u^{11}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{11}_{x,\mu}\right|}\times\frac{{}^{\Theta^{\dagger}}u^{22}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{22}_{x,\mu}\right|}\times\frac{{}^{\Theta^{\dagger}}u^{33}_{x,\mu}}{\left|{}^{\Theta^{\dagger}}u^{33}_{x,\mu}\right|}\right)^{-1/3}$$

LATTICE DATA

- Polyakov loops for Yang-Mills and restricted field
  - Distribution of Polyakov loop values
  - Polyakov loop average and center symmetry breaking/restoration
- Static potential of quark and antiquark
  - correlation function of Polyakov loops
  - Wilson loop average
- dual Meissner effect and confinement/deconfinement phase transition
  - Appearance/disappearance of chromoelectric flux tube
  - Induced magnetic (monopole) current

Polyakov loop

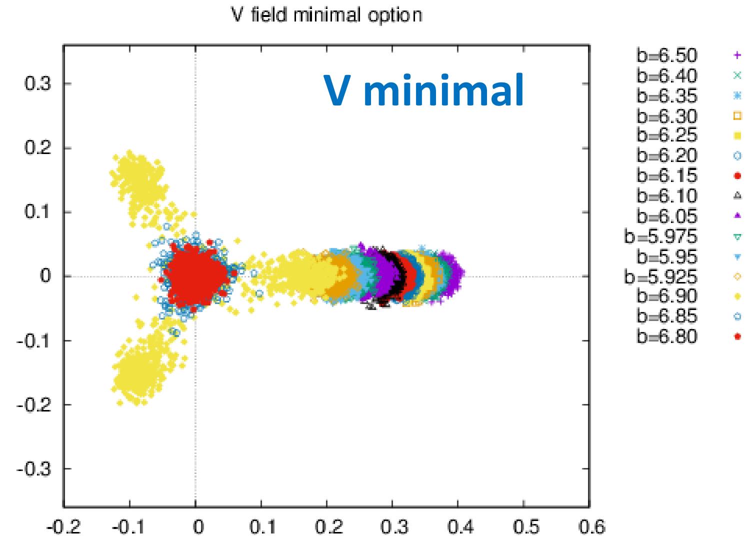
$$P_U(x) \ := \ \mathrm{tr}\left(\prod_{t=1}^{N_T} U_{(x,t),4} \right) \qquad \text{for YM field}$$

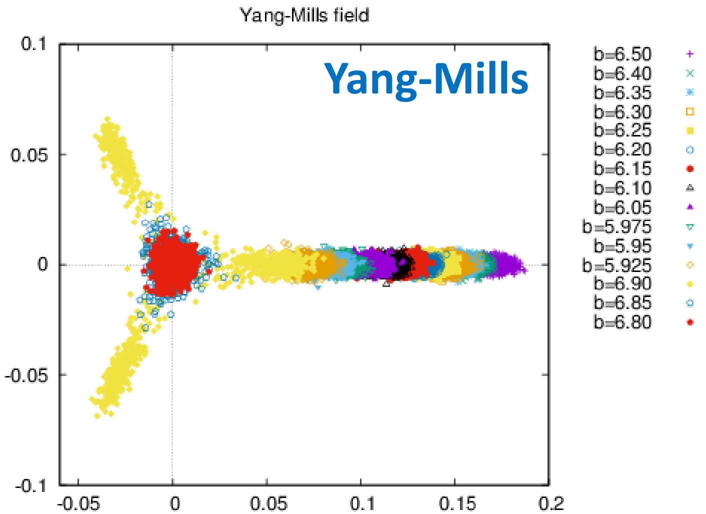
$$P_V(x) \ := \ \mathrm{tr}\left(\prod_{t=1}^{N_T} V_{(x,t),4} \right) \qquad \text{for restricted field}$$

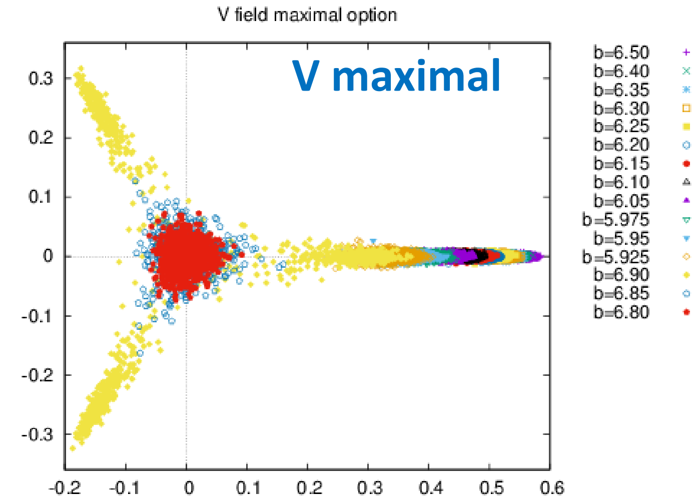
- Distribution of Polyakov loop values
- Polyakov loop average
  - ✓ center symmetry breaking/restoration
  - ✓ confinement/deconfinement phase transition

Distribution of Polyakov loop values

24^3x6 lattice
temperature for various β

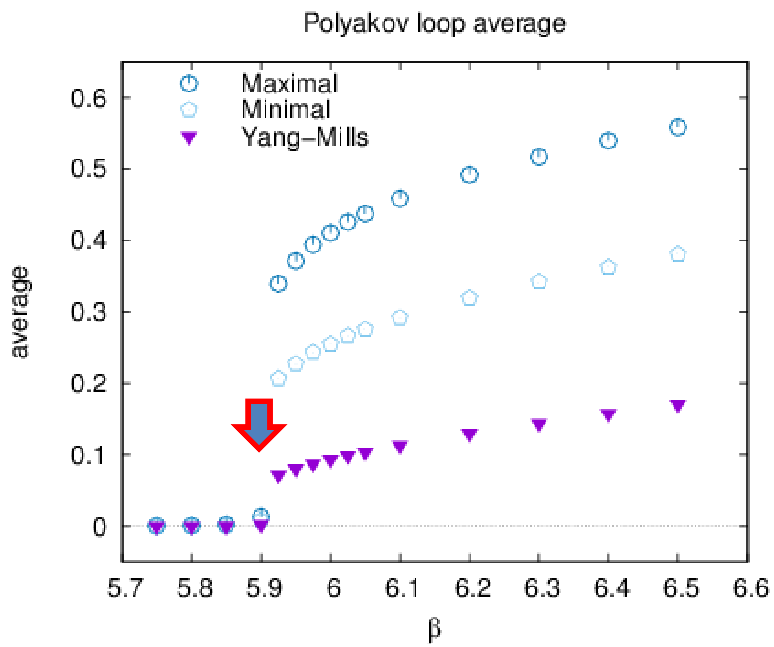




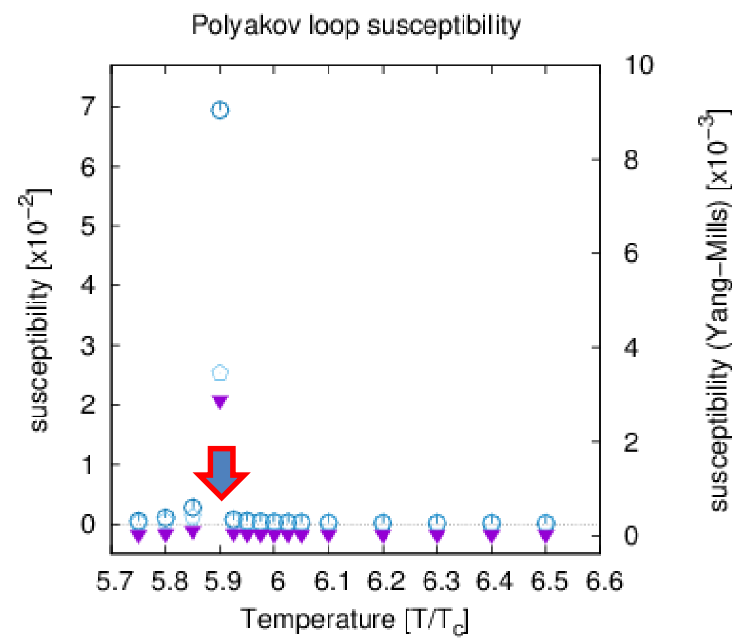


Polyakov loop average and center symmetry

Polyakov loop average



susceptibility



Magnitude of Polyakov-loop average is different, but gives the same phase transition temperature (β).

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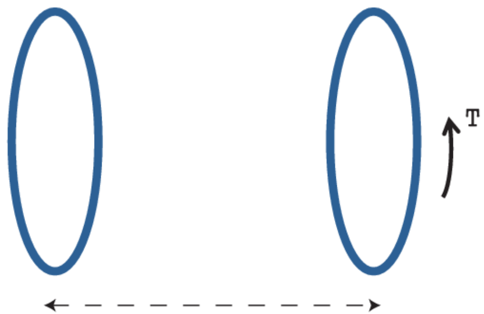
Static potential of quark and antiquark

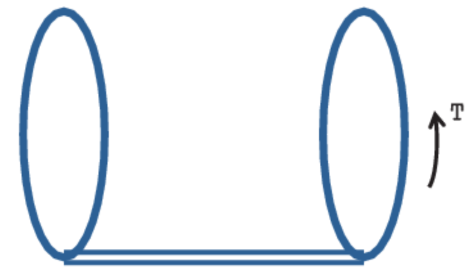
Correlation function of Polyakov loop



$$\tilde{V}(R;U) = -\log\langle P_U(x)P_U^\dagger(y)\rangle$$

$$\tilde{V}(R;V) = -\log\langle P_V(x)P_V^\dagger(y)\rangle$$

Wilson loop



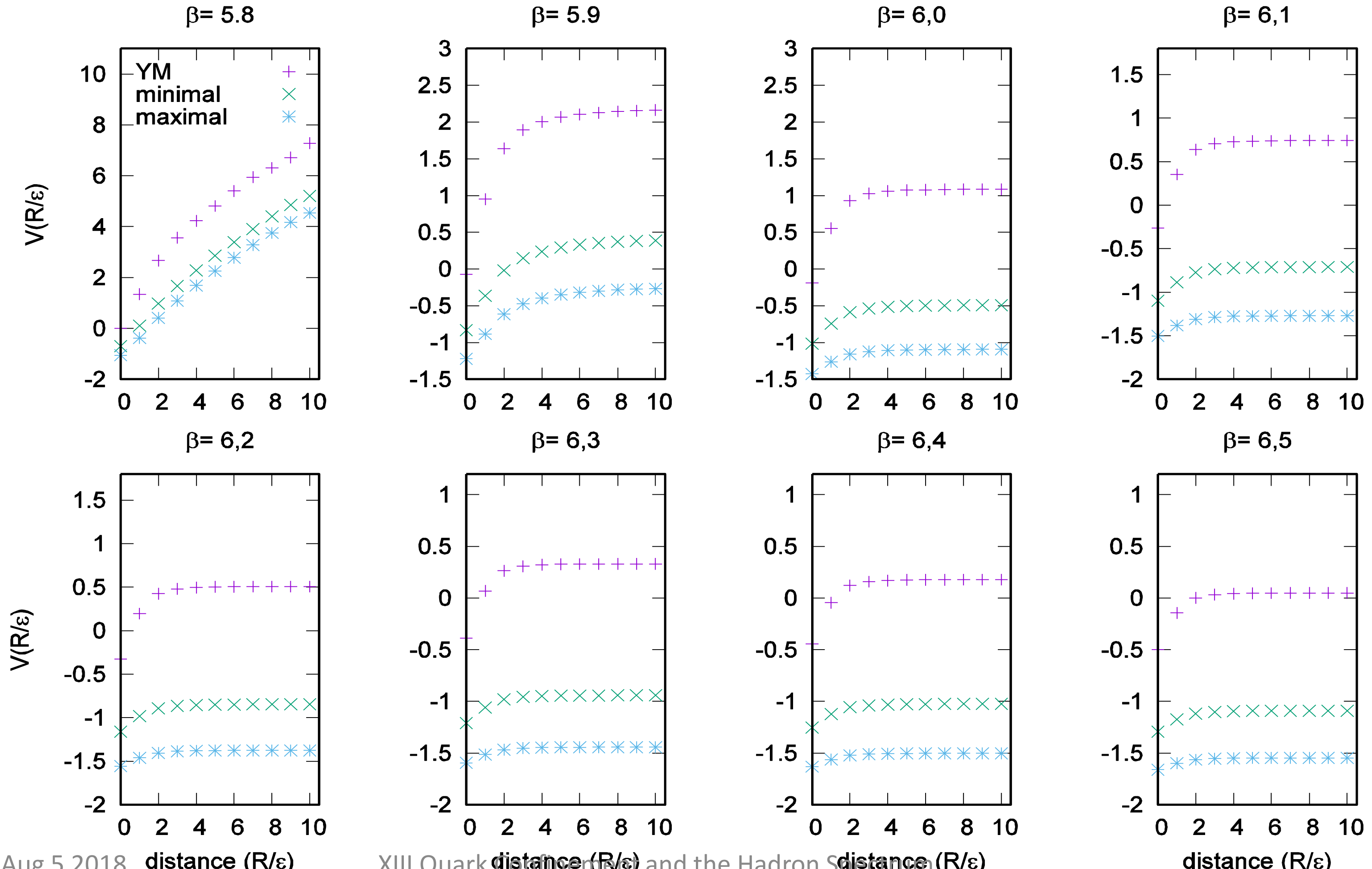
$$V(R;U) = -\log\langle W[U]\rangle$$

$$V(R;V) = -\log\langle W[V]\rangle$$

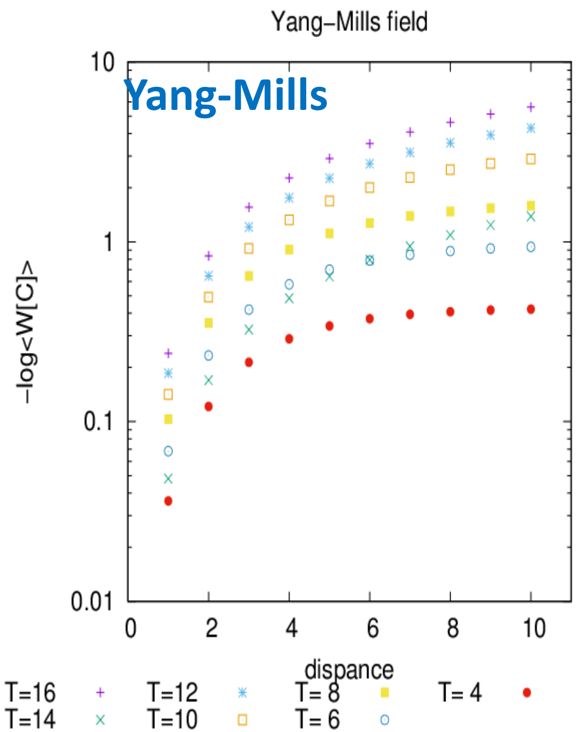
$$\langle P_U(\vec{x})P_U^*(\vec{y})\rangle$$

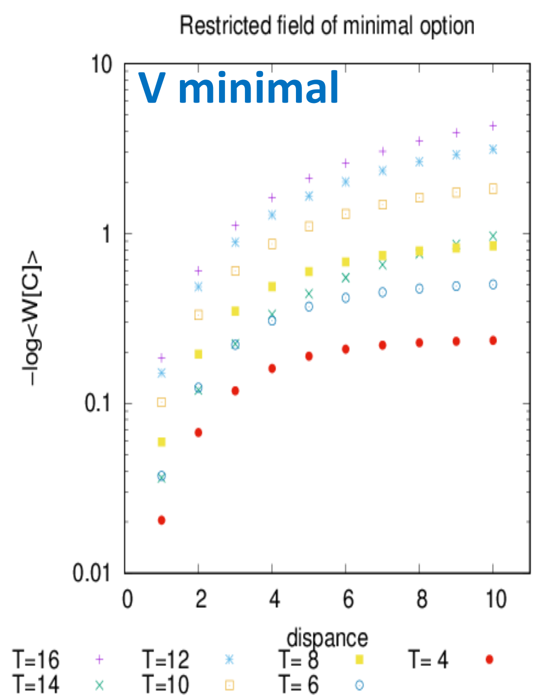
$$\simeq e^{-F_{q\bar{q}}/T} = \frac{1}{N_c^2}e^{-F^{(S)}/T} + \frac{N_c^2-1}{N_c^2}e^{-F^{(A)}/T}$$

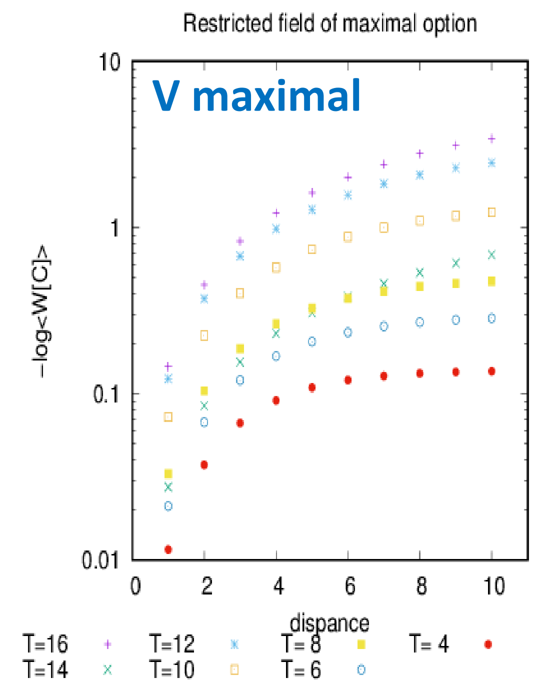
static potential (correlation function of Polyakov loops)



Static potential by Wilson loop







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Chromo flux

We exploit the operator proposed by Giacomo et.al. to measure "flux".

Adriano Di Giacomo et.al [PLB236:199,1990][NPB347:441-460,1990]

P. Cea, et al., PRD86 054501 (2012)

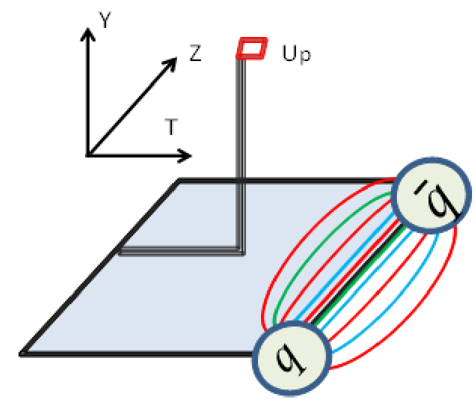
$$\boxed{\rho_W := \frac{\left\langle \mathrm{tr}\left(W[U]L_U U_p L_U^\dagger\right)\right\rangle}{\langle \mathrm{tr}(W[U])\rangle} - \frac{1}{\mathrm{tr}(\mathbf{1})}\frac{\langle \mathrm{tr}(W[U])\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W[U])\rangle}}$$

In the continuum limit $\epsilon \to 0$, ρ_W reduces to

$$\rho_W = ig\epsilon^2 \frac{\langle \mathrm{tr}(\mathcal{F}_{\mu\nu}[\mathcal{A}]L_U^\dagger W[U] L_U)\rangle}{\langle \mathrm{tr}(W[U])\rangle} + o(\epsilon^4)$$

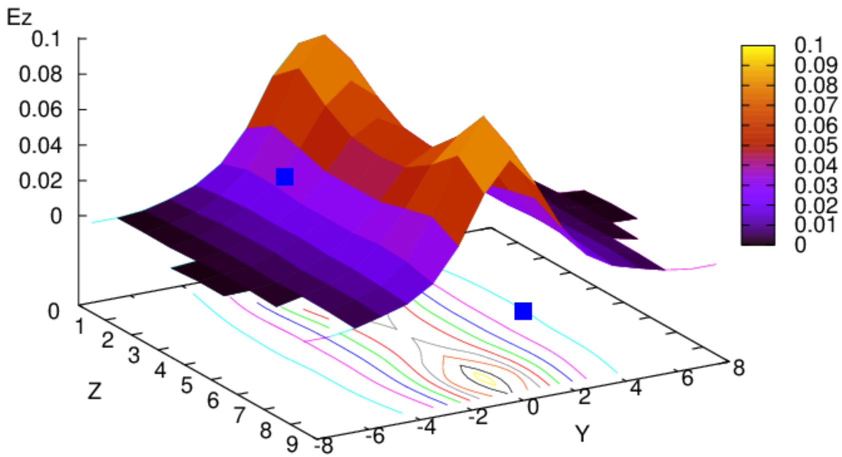
$$=: ig\epsilon^2 F_{\mu\nu}|_{q\bar{q}}$$

$\mathrm{tr}(U_p L W L^\dagger)$

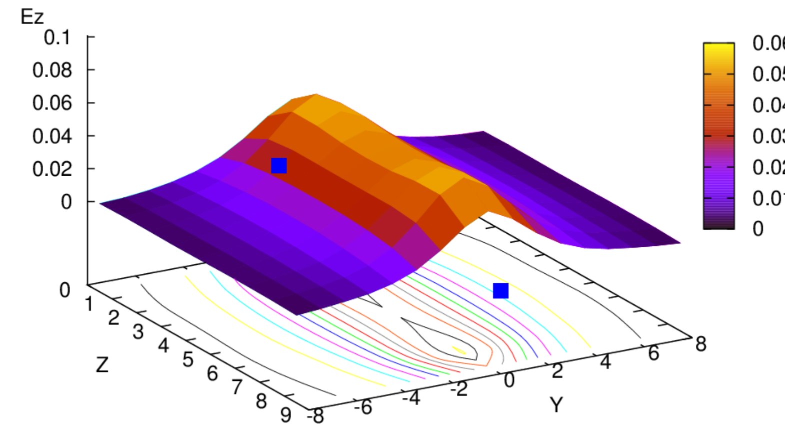


Chromoelectric flux tubes at zero temperature

Full Yang-Mills



Restricted field V in minimal option

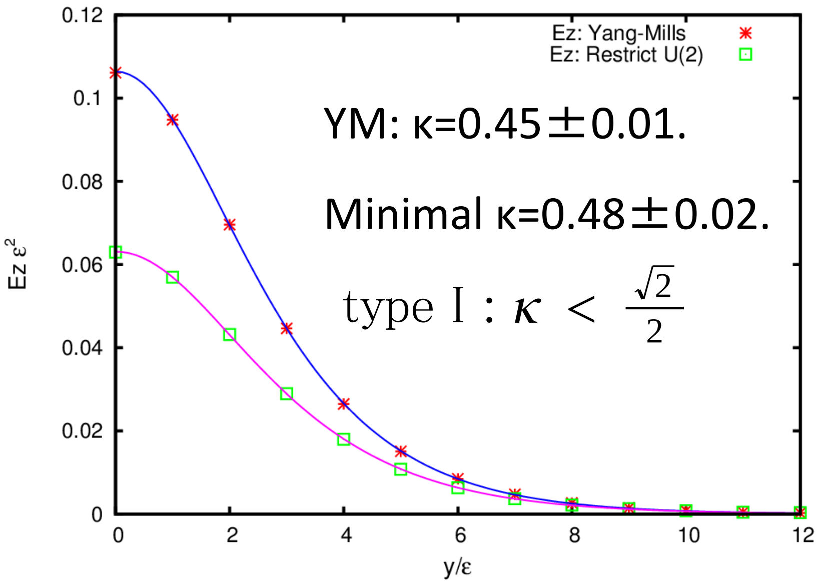


Phys.Rev. D87 (2013) no.5, 054011

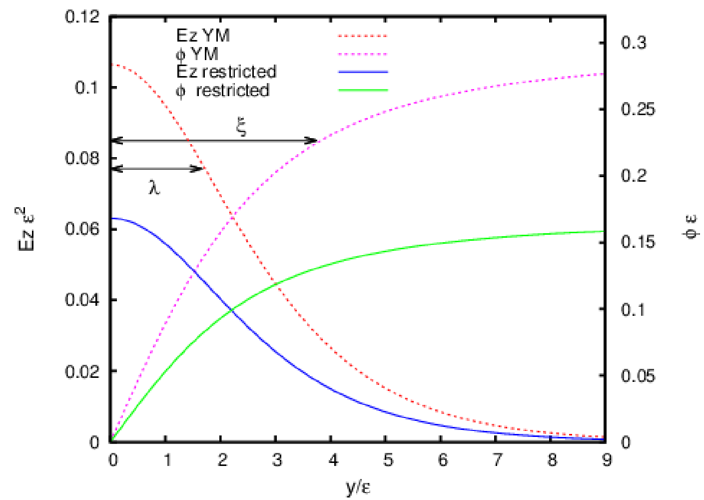
Dual Meissner effect and type of vacuum

Clem's method

$$\kappa = \sqrt{2}\,\frac{\lambda}{\zeta}\sqrt{1 - K_0^2(\zeta/\lambda)/K_1^2(\zeta/\lambda)}$$

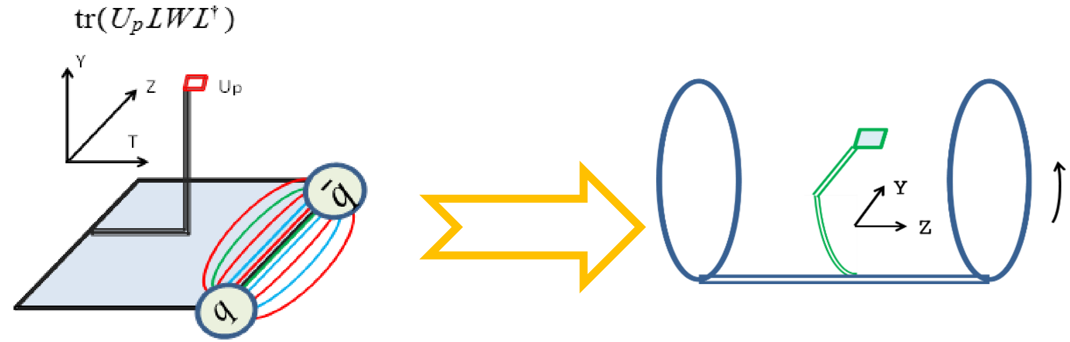


GL parameter

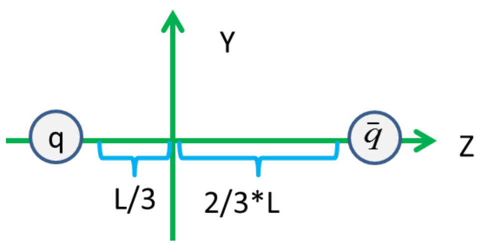


Using U(1) model and Ansatz for scalar field. For improved method [Poster]

Measurement of chromo flux at finite temperature



$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger) \rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p) \rangle}{\langle \mathrm{tr}(W) \rangle}$$



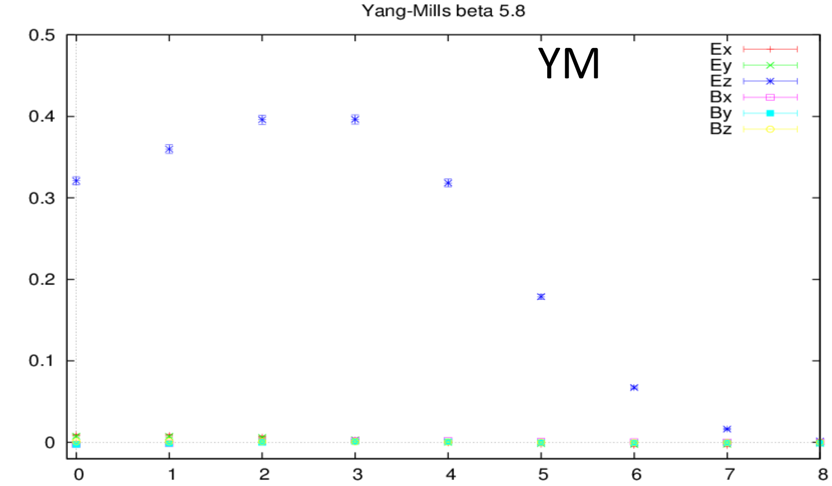
- Using the same operator with that of zero temperature.
- Size of Wilson loop T-direction = Nt

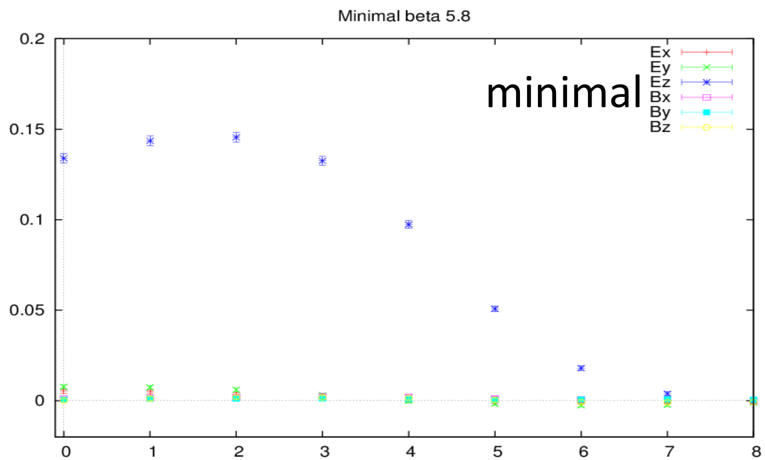
➔The source of quark and antiquark are given by **Plyakov loops** connecting by Wilson line.

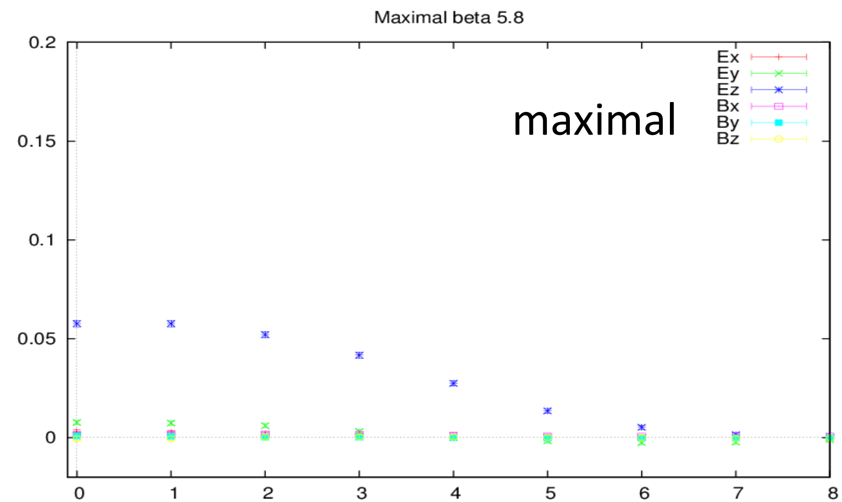
- The three types of probes and compare them.

$$O^{[YM]} = L[U]U_p L[U]^{-1} \quad :: \text{ original YM}$$
$$O^{[\mathrm{nin}]} = L[V^{[\min]}]V_p^{[\min]} L[V^{[\min]}]^{-1} \quad :: \text{ V field in minimal option}$$
$$O^{[\max]} = L[V^{[\max]}]V_p^{[\max]} L[V^{[\max]}]^{-1} \quad :: \text{ V field in maximal option}$$

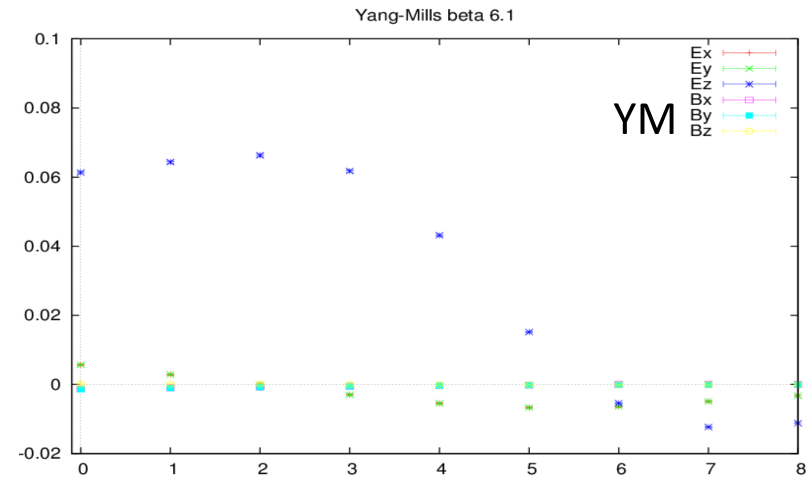
Chromo flux in confining phase

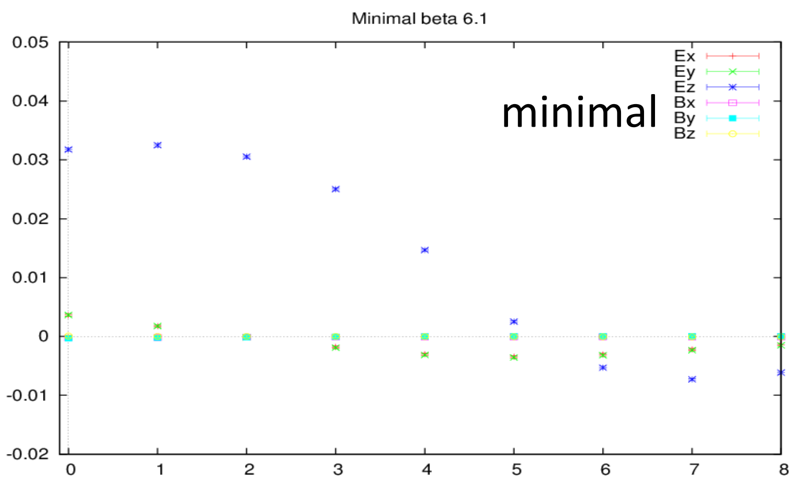


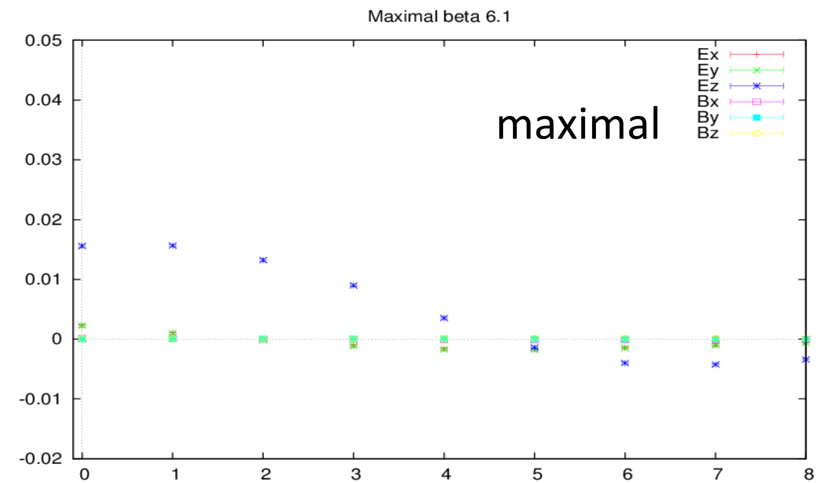




Chrono flux in deconfining phase





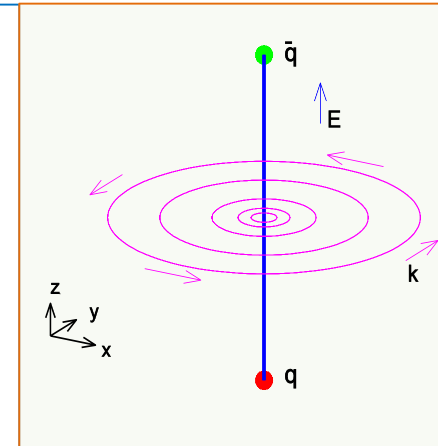


Induced magnetic (monopole) current and confinememt/deconfinement phase transition

Yang–Mills equation (Maxell equation) fo rrestricted field V_μ, the magnetic current (monopole) can be calculated as

$$k = \delta^* F[V] = {}^* dF[V],$$

where $F[V]$ is the field strength of V, d exterior derivative, $*$ the Hodge dual and δ the coderivative $\delta := {}^* d^*$, respectively.

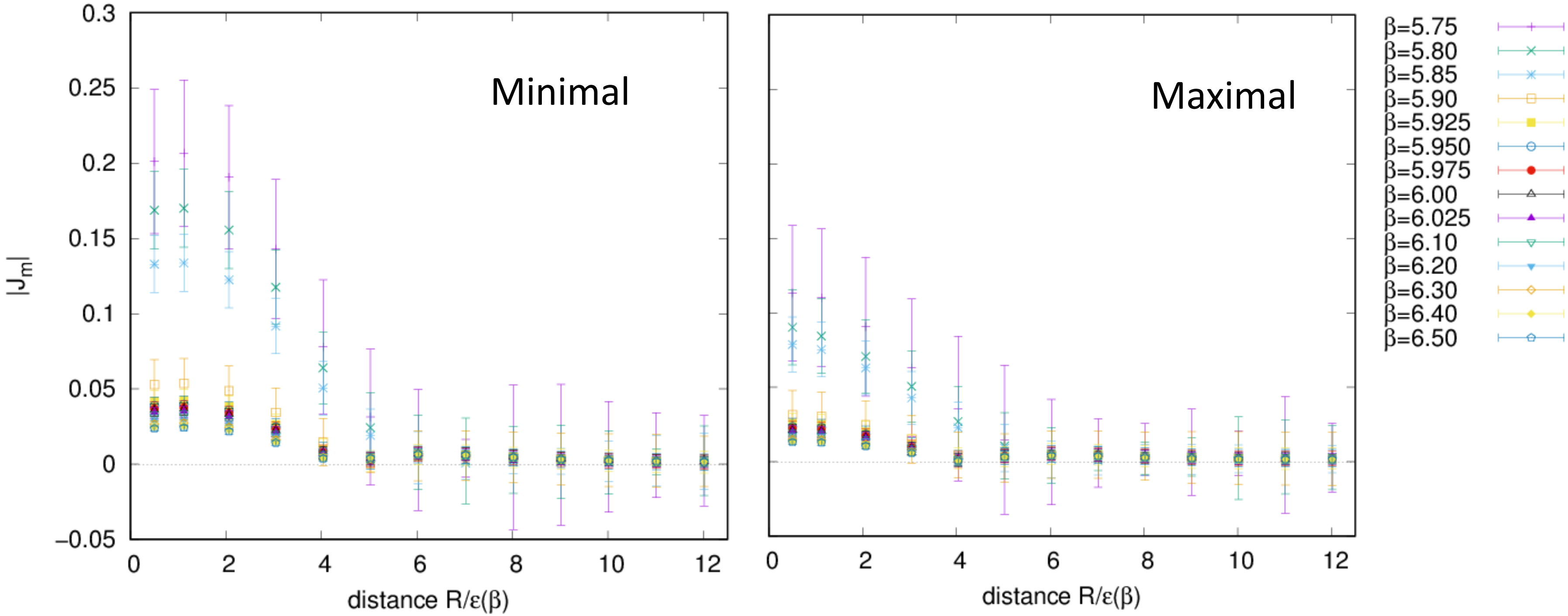


The magnetic current k *must* be ***zero*** for regular function due to Bianchi Identity.

Non zero k suggests the monopole condensation

Induced The magnetic current (monopole) k can be a **order parameter** of the dual Meissner effect.

Phase transition of the dual Meissner effect



The amplitude of induced magnetic current sharply decreases toward the critical point($\beta=5.9$)

Summary

- We investigate the confinement/deconfinement phase transition in view of the dual superconductivity in SU(3) YM theory by using the new formulation.
- The restricted fields in both options reproduce the profile of Yang-Mills field in both confinement/deconfinement phases
- The critical temperature of the center symmetry breaking is the exactly same for the restricted fields and Yang-Mills theory.
- The static potential (string tension) of YM field is reproduced by V-fields
- Confinement/deconfinement phase transition is observed due to the phase transition of the dual Meissner effect.
- The appearance/disappearance of the magnetic (monopole) currents according to the phase transition.
- The critical point of the center symmetry breaking and the critical point of the the dual Meissner effect is almost same.

BACKUP

Chromo flux probe by the restricted field (1)

Replacing the Yang–Mills field U by the restricved filed V to define the operator for the restricted field

$$\rho_W := \frac{\langle \mathrm{tr}(W[V] L_V V_p L_V^\dagger) \rangle}{\langle \mathrm{tr}(W[V]) \rangle} - \frac{1}{\mathrm{tr}(\mathbf{1})} \frac{\langle \mathrm{tr}(W[V]) \mathrm{tr}(V_p) \rangle}{\langle \mathrm{tr}(W[V]) \rangle}$$

For the **maximal option**, V obay the identity

$$V_p = \frac{\mathrm{tr}(V_p)}{\mathrm{tr}(\mathbf{1})} \mathbf{1} + 2 \sum_{J=3,8} tr(V_p \mathbf{n}^{(j)}) \mathbf{n}^{(j)}$$

by using color fields, $\mathbf{n}^{(j)}$ $(j = 3, 8)$. The fact that color fields and the restricted field commute covariantly by constraction (defineing equation):

$$\mathbf{n}^{(j)} V_p = V_p \mathbf{n}^{(j)} \qquad (j = 3, 8)$$

yields the exxpression free from the Shwinger line L_V and $L_V^\dagger$:

$$\rho_W := 2 \sum_{J=3,8} \frac{\langle \mathrm{tr}(W[V] \mathbf{n}^{(j)}) \ (\mathrm{tr}(V_p \mathbf{n}^{(j)}) \rangle}{\langle \mathrm{tr}(W[V]) \rangle}$$

Chromo flux probe by the restricted field(2)

$$\rho_W := 2 \sum_{J=3,8} \frac{\langle \mathrm{tr}(W[V]\mathbf{n}^{(j)})\ (\mathrm{tr}(V_p \mathbf{n}^{(j)}))\rangle}{\langle \mathrm{tr}(W[V])\rangle}$$

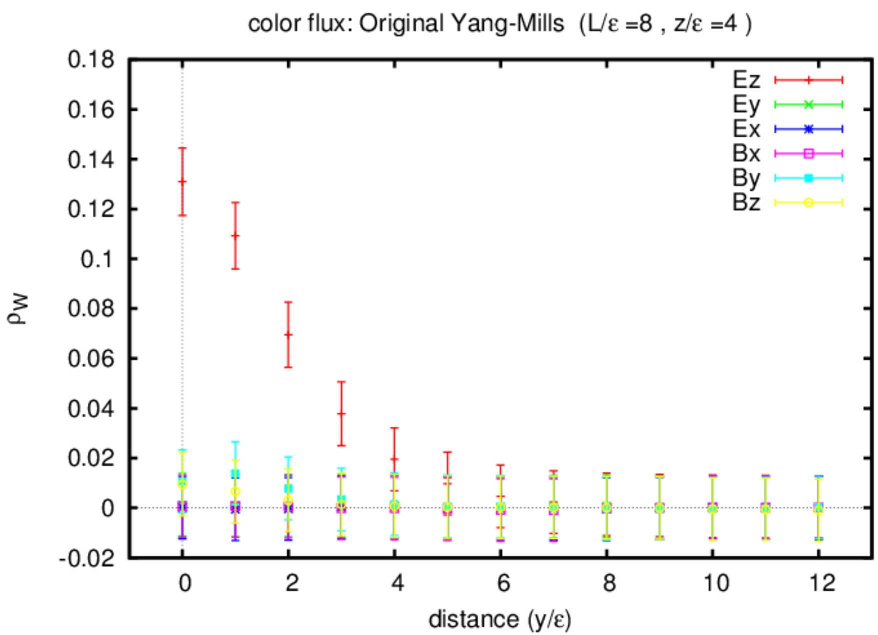
In the continuum limit, $(\mathrm{tr}(V_p \mathbf{n}^{(j)})$ is reduces to the gauge invariant field strength $\mathcal{F}_{\mu\nu}[\mathcal{V}](y) \cdot \mathbf{n}^{(j)}(y)$ at the probe. Therefore, ρ_W measures the chromofield strength for the restricted "Abelian" part of the gauge field.

There could exist a colorsinglet contribution in $(\mathrm{tr}(V_p \mathbf{n}^{(j)})$ as a functional of U, i.e., $n_z = n_z[U]$, because the color field is determined by minimizing the reduction functional.

ρ_W is at least meaningful operator to measure the restricted field strength in the gauge-invariant way.

chromo flux at zero temperature

Full Yang-Mills field



Restriced field in minimal option