Correct way to extract the dominant part of the Wilson loop in higher representations

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Problem of magnetic monopoles for sources in higher reps.

- In the fundamental representation, the string tension between a static quark and antiquark is almost fully reproduced by the contribution of magnetic monopoles which is extracted by Abelian projection procedure. This was confirmed in lattice studies.
- In higher representations, if we adapt the same procedure as fundamental representation naively, the monopole contribution doesn't reproduce the full string tension. For example in the adjoint representation of $SU(2)$, the monopole part of the string tension seems to be zero even in the intermediate region, c.f. Del Debbio et al. (1996).

In this talk, we claim that this is because the naive Abelian projection is not applicable to higher representations, and propose an appropriate operator to measure the monopole contribution. We support this claim by the lattice simulations.

Outline

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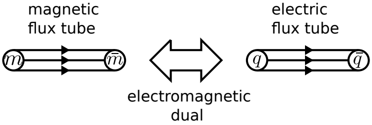
1. Dual superconductivity picture and Abelian projection
2. Wilson loops in higher representations
3. Suitable operator to extract the monopole contribution
4. Numerical results

Dual superconductivity picture

- The dual superconductor picture is a promising scenario for quark confinement proposed by Nambu, 't Hooft and Mandelstam, in which magnetic monopoles play an important role.
- In this scenario, the QCD vacuum is considered as a dual superconductor.
- QCD strings are considered as the electromagnetic dual of Abrikosov vortices.



Ordinary superconductivity is the result of condensation of Cooper pairs. Therefore we can suppose that, in order to be a dual superconductor, condensation of magnetic monopoles have to occur.

Abelian projection

The Abelian projection, which is proposed by 't Hooft (1981), consists of two steps.

1. We extract the Abelian part A^{Abel} of the gauge field
   which is defined as the Cartan component of the gauge field in some gauge, e.g., MA gauge where

   $$\int d^4x \sum_a (A_\mu^a A_\mu^a), \quad a \text{ denotes an off diagonal component}$$

   is minimized.
2. We define the magnetic current as usual by using the Abelian part of the gauge field as

   $$k^\nu = \partial_\mu {}^*F_d^{\mu\nu}, \quad {}^*F_d^{\mu\nu} := \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\partial_\rho A_\sigma^{\mathrm{Abel}} - \partial_\sigma A_\rho^{\mathrm{Abel}})$$

Abelian projection on lattices

On lattices, we can adapt the concept of the Abelian projection to extract the monopole contribution to the Wilson loop. We consider the $SU(N)$ pure Yang-Mills theory.

1. We define the Abelian part of the link variable $V_l \in U(1)^N$ by maximizing $\mathrm{Re}\,\mathrm{tr}(U_l V_l^\dagger)$ in the MA gauge. Then we define the Abelian projected Wilson loop as

$$\mathrm{tr} \prod_{l \in C} V_l .$$

   It was checked that the average of the Abelian projected Wilson loop reproduces the string tension for the full Wilson loop in SU(2) (Suzuki-Yotsuyanagi, 1990) and in SU(3) (Stack-Tucker-Wensley, 2002), which is called the Abelian dominance.

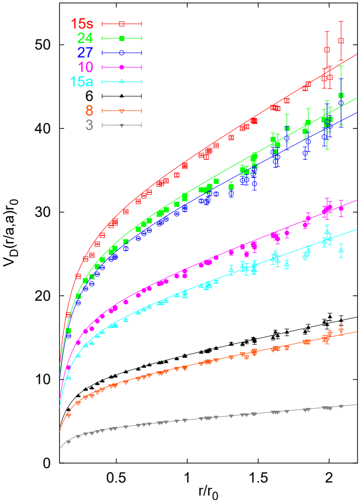
2. We extract the monopole contribution from the Abelian projected Wilson loop by adapting the Tousaint-DeGrand procedure which is originally adapted to the compact $U(1)$ gauge theory.
   It was checked that the monopole part of the Wilson loop reproduces the string tension in SU(2) (Suzuki-Yotsuyanagi, 1990) and in SU(3) (Stack-Tucker-Wensley, 2002), which is called the monopole dominance.

Outline

Wilson loops in higher representations

We can use Wilson loops in higher representations to test candidates of confinement mechanism by checking whether they reproduce the following behavior.

In $SU(3)$ Source: Bali (2000)



The potential between color sources in a higher representation has two characteristic features depending on the distance.

- At intermediate distance, the string tension is proportional to the quadratic Casimir.
- At asymptotic region, due to the screening by gluons, the string tension depends only on the N-ality of the representation.

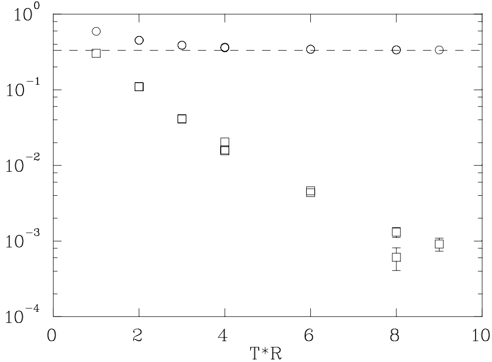
Naively extended Abelian projection in higher reps.

Naively extended Abelian projection does not reproduce the correct behavior of Wilson loops in higher representation.

For example, in the adjoint rep. in $SU(2)$ gauge theory, the average of the naive Abelian projected Wilson loop in the adjoint rep.,

$$W_{\text{adj}}^{\text{Abel}} = \frac{1}{3}\left(\exp\left(ig\oint A^3\right) + \exp\left(-ig\oint A^3\right) + 1\right),$$

approaches $1/3$ other than 0.



Source: Poulis (1996)

FIG. 7. The adjoint Wilson loop $W_{j=1}$ (2) versus the adjoint diagonal Wilson loop $W^{d}_{j=1}$ (o) in MA projection. The dashed line corresponds to the asymptotic value for the latter, $W^{d}_{j=1} = 1/3$.

Outline

The suitable operator to check "the Abelian dominance"

In an arbitrary representation of an arbitrary group, we claim that the suitable operator is

$$\tilde{W}_R = \exp\left(ig \oint \langle\Lambda| A_\mu |\Lambda\rangle\right), \quad |\Lambda\rangle : \text{ the highest weight state}$$

which is different from the naively defined Abelian projected Wilson loop in the representation R

$$\begin{aligned} W_R^{\text{Abel}} &= \mathrm{tr}_R \exp\left(ig \oint 2\,\mathrm{tr}\left(H_j A_\mu\right) H_j\right) \\ &= \sum_\mu \exp\left(ig \oint \langle\mu| A_\mu |\mu\rangle\right), \end{aligned}$$

where the sum is over the whole weights of R and

H_j: the Cartan generators

Example: adj. rep. in $SU(2)$ and adj. and $\mathbf{6}^*$ in $SU(3)$

In adj. rep. of $SU(2)$,

$$\tilde{W}_A = e^{ig\oint A^3},$$
$$W_A^{\mathrm{Abel}} = e^{ig\oint A^3} + e^{-ig\oint A^3} + 1$$

(c.f. Poulis(1996))
In adj. rep. of $SU(3)$,

$$\tilde{W}_A = e^{ig\oint A^3}, \quad (\Lambda = (1,0))$$
$$W_A^{\mathrm{Abel}} = e^{ig\oint A^3} + e^{-ig\oint A^3} + e^{ig\oint\left(\frac{1}{2}A^3+\frac{\sqrt{3}}{2}A^8\right)} + e^{-ig\oint\left(\frac{1}{2}A^3+\frac{\sqrt{3}}{2}A^8\right)}$$
$$+ e^{ig\oint\left(\frac{1}{2}A^3-\frac{\sqrt{3}}{2}A^8\right)} + e^{-ig\oint\left(\frac{1}{2}A^3-\frac{\sqrt{3}}{2}A^8\right)} + 2$$

In $\mathbf{6}^*$ of $SU(3)$

$$\tilde{W}_{\mathbf{6}} = e^{i\oint\frac{2}{\sqrt{3}}A^8}, \quad (\Lambda = (0, 1/2\sqrt{3}))$$
$$W_{\mathbf{6}}^{\mathrm{Abel}} = e^{ig\oint\frac{2}{\sqrt{3}}\oint A^8} + e^{ig\oint\left(A^3-\frac{1}{\sqrt{3}}A^8\right)} + e^{ig\oint\left(-A^3-\frac{1}{\sqrt{3}}A^8\right)}$$
$$+ e^{-ig\oint\frac{1}{\sqrt{3}}A^8} + e^{ig\oint\left(\frac{1}{2}\oint A^3+\frac{1}{2\sqrt{3}}A^8\right)} + e^{ig\oint\left(-\frac{1}{2}\oint A^3+\frac{1}{2\sqrt{3}}A^8\right)}$$

Where does this prescription come from?

According to Diakonov-Petrov version of the non-Abelian Stokes theorem, the Wilson loop for the representation R can be written as

Non-Abelian Stokes theorem (Diakonov, Petrov(1989))

$$W_R[A] = \int DU \exp\left(\oint ig \langle\Lambda| A^U |\Lambda\rangle\right)$$
$$= \int DU \exp\left(\int_S ig\, d\left(\langle\Lambda| A^U |\Lambda\rangle\right)\right),$$

where

DU is the product of the Haar measure over the loop or a surface

$A^{U^\dagger} := UAU^\dagger + ig^{-1}UdU^\dagger$, and

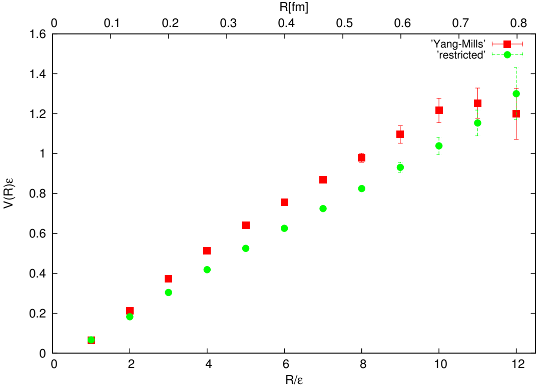
$|\Lambda\rangle$ is the highest weight state of the representation R.

Outline

$SU(2)$ adj. rep.
24^4lattice
$\beta = 2.5$
HYP smearing

Preliminary

$$V(R,T) = \log \frac{\langle W(R,T) \rangle}{\langle W(R,T+1) \rangle}$$



cf. Poulis (1996), Chernodub-Hashimoto-Suzuki (2004)

$SU(3)$ fund. rep. $(\mathbf{3},\ [1,0])$

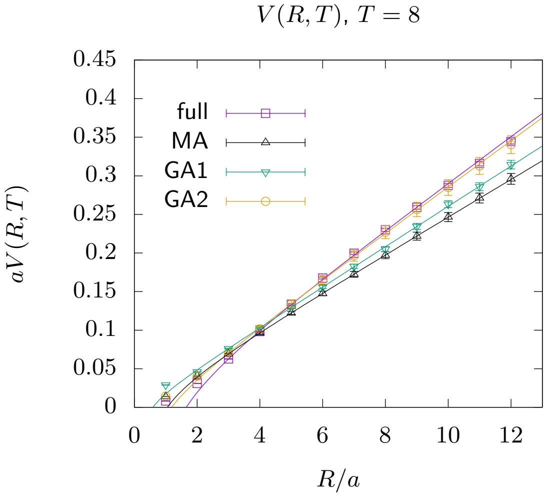
24^4lattice

$\beta = 6.2$

APE smearing

fit range: $3 \le R/a \le 11$

Preliminary

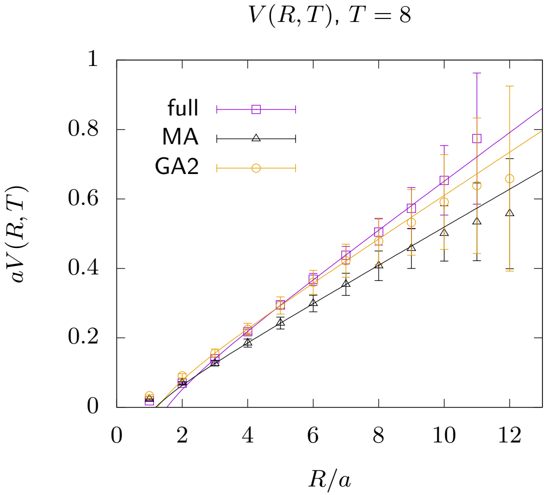


$a^2\sigma^F_{\rm full} \simeq 0.030$
$a^2\sigma^F_{\rm MA} \simeq 0.024 \simeq 0.81 a^2\sigma_{\rm full}$
$a^2\sigma^F_{\rm GA1} \simeq 0.026 \simeq 0.86 a^2\sigma_{\rm full}$
$a^2\sigma^F_{\rm GA2} \simeq 0.030 \simeq 1.0 a^2\sigma_{\rm full}$

c.f.
Sakumichi-Suganuma (2014)
Perfect Abelian dominance in MA gauge
32^4lattice $\beta = 6.4$

$SU(3)$ adj. rep. $(\mathbf{8}, [1,1])$
24^4lattice
$\beta = 6.2$
APE smearing
fit range: $3 \leq R/a \leq 8$

Preliminary

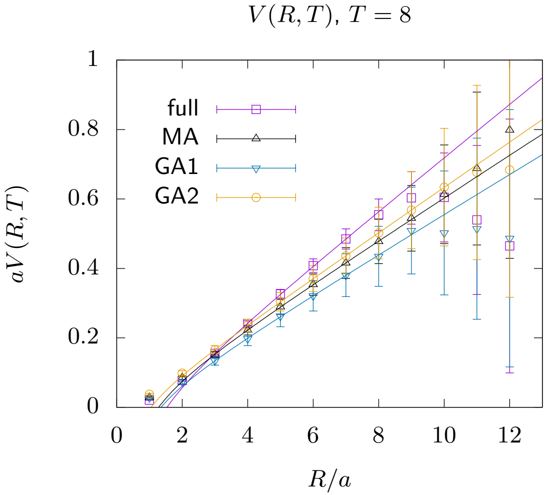


$C_2(A)/C_2(F) = 2.25$

$a^2\sigma^A_{\rm full} \simeq 0.069 \simeq 2.3a^2\sigma^F_{\rm full}$
$a^2\sigma^A_{\rm MA} \simeq 0.054 \simeq 0.78a^2\sigma^A_{\rm full}$
$a^2\sigma^A_{\rm GA2} \simeq 0.062 \simeq 0.89a^2\sigma^A_{\rm full}$

$SU(3)$ $\mathbf{6}^*$ ([0, 2])
24^4 lattice
$\beta = 6.2$
APE smearing
fit range: $3 \leq R/a \leq 8$

Preliminary



$C_2(\mathbf{6})/C_2(F) = 2.5$

$a^2\sigma^{02}_{\text{full}} \simeq 0.076 \simeq 2.6a^2\sigma^F_{\text{full}}$
$a^2\sigma^{02}_{\text{MA}} \simeq 0.061 \simeq 0.80a^2\sigma_{\text{full}}$
$a^2\sigma^{02}_{\text{GA1}} \simeq 0.057 \simeq 0.75a^2\sigma_{\text{full}}$
$a^2\sigma^{02}_{\text{GA2}} \simeq 0.065 \simeq 0.85a^2\sigma_{\text{full}}$

Summary

- The (naive) Abelian projected Wilson loop for a higher representation does not reproduce the correct behavior of the original Wilson loop.
- Through the NAST, we obtain another projected Wilson loop, which is essentially same as the Abelian projected Wilson loop in the fundamental representation, and is different from that in in higher representations.
- According to the lattice simulation, the proposed operator reproduce the correct behavior in the adjoint representation in the $SU(2)$ gauge theory and in the adjoint representation and $\mathbf{6}^*$ in $SU(3)$ gauge theory.

Buckups

The meaning of our prescription

In $SU(2)$, the Wilson loop in the representation j can be written using the untraced Wilson loop W in the fundamental representation, $\mathrm{tr}_j W_C = F_j(W)$. Thus the average is determined if the probability distribution $P(W;C)$ of the untraced Wilson loop is determined as

$$\langle \mathrm{tr}_j W_C \rangle = \int dW \, P(W;C) F_j(W),$$

$$P(W;C) := \int DU \delta(W, \prod_{l \in C} U_l) e^{-S[U]} / \int DU e^{-S[U]}.$$

In the same way, the (naive) Abelian projected Wilson loop in the representation j can be expressed as

$$\langle \mathrm{tr}_j V_C \rangle = \int dV \, \tilde{P}(V;C) F_j(V),$$

$$\tilde{P}(V;C) := \int DU \delta(V, \prod_{l \in C} V_l) e^{-S[U]} / \int DU e^{-S[U]},$$

where V_l is an Abelian projected link variable.
Note that dW is the Haar measure on $SU(2)$ while dV is the Haar measure on $U(1)$.
We expect when the size of C goes to infinity,

$$P(W;C) \to 1, \quad \tilde{P}(V;C) \to 1.$$

Because of the gauge invariance of $P(W;C)$, we can express $P(W;C)$ by using the eigenvalue $e^{i\theta}$ of W only as

$$P(W;C) = P(\theta;C),$$

while because we can write $V = \mathrm{diag}(\exp(i\theta), \exp(-i\theta))$, we can express $\tilde{P}(V;C)$ as

$$\tilde{P}(V;C) = \tilde{P}(\theta;C)$$

Then we can write

$$\langle \mathrm{tr}_j W_C \rangle = \frac{2}{\pi} \int_0^{\pi} d\theta \sin^2\theta P(\theta;C) F_j(\theta),$$
$$\langle \mathrm{tr}_j V_C \rangle = \frac{1}{\pi} \int_0^{\pi} d\theta \tilde{P}(\theta;C) F_j(\theta),$$
$$F_j(\theta) = \sum_{m=-j}^{j} e^{im\theta},$$

where $\sin^2\theta$ comes from the Haar measure of $SU(2)$. Because of this factor $\sin^2\theta$, they can be different.

We can show that our claim

$$\langle \mathrm{tr}_j W_j \rangle = \frac{2}{\pi} \int_0^\pi d\theta \sin^2\theta P(\theta; C) F_j(\theta) \sim \int_0^\pi d\theta \tilde{P}(\theta; C) e^{ij\theta}$$

is equivalent to

$$P(\theta; C) \simeq \tilde{P}(\theta; C),$$

which can be called the Abelian dominance of the distribution of the Wilson loop

Generalized MA gauge

Before showing the numerical results, we introduce generalized MA gauges (c.f. Stack-Tucker-Wensley (2002)).
The gauge fixing functional of the MA gauge is the form of a mass term for the gauge fields.

$$\int \mathrm{tr}\left(A_\mu^a A_\mu^a\right) \quad (a \text{ denotes off-diagonal components})$$

In $SU(3)$ case, we can generalize it as

$$\int \left({m_1}^2\left((A_\mu^1)^2 + (A_\mu^2)^2\right) + {m_2}^2\left((A_\mu^4)^2 + (A_\mu^5)^2\right) + {m_3}^2\left((A_\mu^6)^2 + (A_\mu^7)^2\right)\right).$$

In the following we use

$$m_1 = 0,\ m_2 = m_3 = m, \text{ (GA1)}$$
$$m_1 = 2m,\ m_2 = m_3 = m. \text{ (GA2)}$$

GA1 is special because the symmetry breaking pattern is different from the MA gauge as $SU(3) \to U(2)$. Therefore we cannot use GA1 in every case, for example we can use it in fund. rep. and $\mathbf{6}^*$ and cannot use it in adj. rep.