# Non-Abelian Higgs Theory in a Strong Magnetic Field and Confinement

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## Abstract

We investigate non-abelian Higgs theory in a constant strong magnetic field, where the lowest-Landau-level approximation can be used.

At a critical magnetic value $eB_c = m^2$, the off-diagonal charged vector fields behave as one-dimensional massless fields and give a strong correlation along the magnetic direction, which may lead a new type of confinement caused by off-diagonal vector fields.

*Confinement 2018, Maynooth, Ireland, 2 August 2018*

## As Background and Motivation

- Dual Superconductor picture for Confinement Mechanism

- Infrared Abelian Dominance of QCD
  in the Maximally Abelian (MA) Gauge

- Large Off-diagonal Gluon Mass Generation
  in the MA gauge

# Dual Superconductor Picture for Confinement

*Historical Overview*

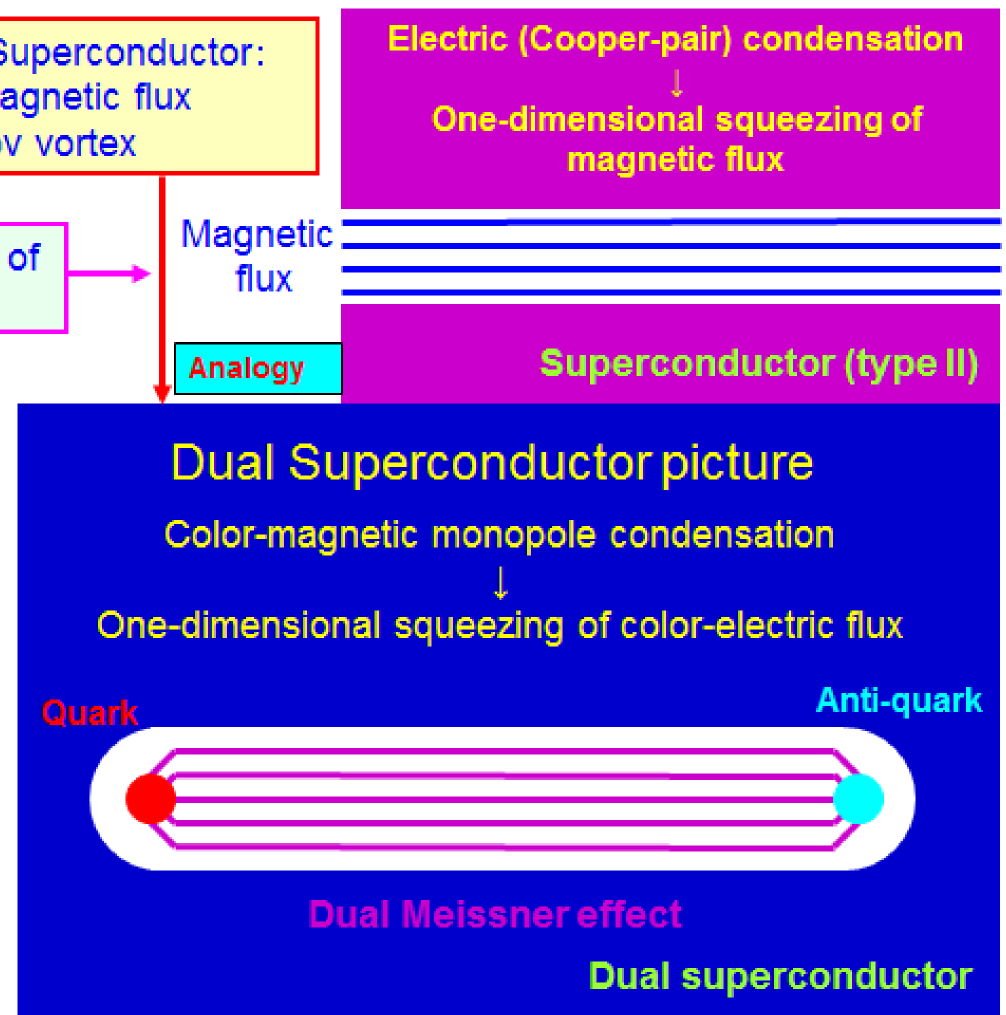
In 1970's, *Nambu, 't Hooft, Mandalstam* proposed *Dual Superconductor picture* for quark confinement, based on the analogy between Abrikosov vortex in Type-II superconductor and flux-tube/string picture for hadrons.

Meissner effect in Superconductor:
Exclusion of magnetic flux
⇒ Abrikosov vortex

Electromagnetic Duality of
Maxwell equations

$$\begin{cases} \partial_\mu F^{\mu\nu} = j^\nu \\ \partial_\mu \tilde{F}^{\mu\nu} = k^\nu \end{cases}$$

Nambu, 'tHooft, Mandelstam
(1974,75,76)




# Dual Superconductor Theory in QCD

*However, there are two large gaps between QCD and dual superconductor picture*

1. The dual superconductor is based on the Abelian gauge theory subject to the Maxwell-type equations, where electro-magnetic duality is manifest, while QCD is a non-abelian gauge theory.

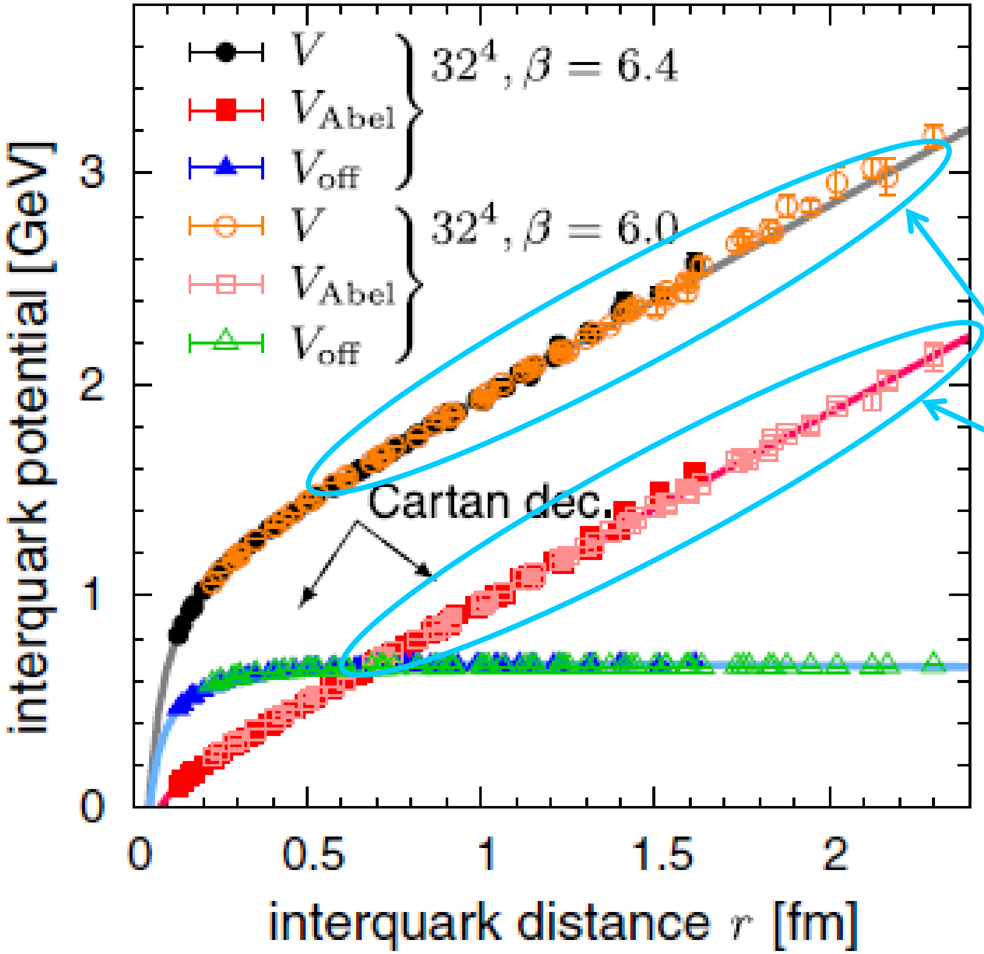
2. The dual superconductor requires condensation of color-magnetic monopoles as the key concept, while QCD does not include such a monopole as the elementary degrees of freedom.

In 1981, *'t Hooft* gave a possible mathematical foundation of this picture by way of *Abelian Gauge Fixing in QCD*, which is a partial gauge fixing on $SU(N)/U(1)^{N-1}$, similar to Non-Abelian Higgs theory.

In particular, *Maximally Abelian (MA) Gauge* is a special successful Abelian gauge, and shows *Infrared Abelian Dominance* and suggests *Monopole Condensation*.

# Perfect Abelian Dominance for Confinement in QQbar Potential in MA gauge

N.Sakumichi and H.S., PRD90 111501(R) (2014).




SU(3) potential V(r)

Abelian part $V_{\rm Abel}(r)$

SU(3) potential and its Abelian part have almost the same slope at large distance.

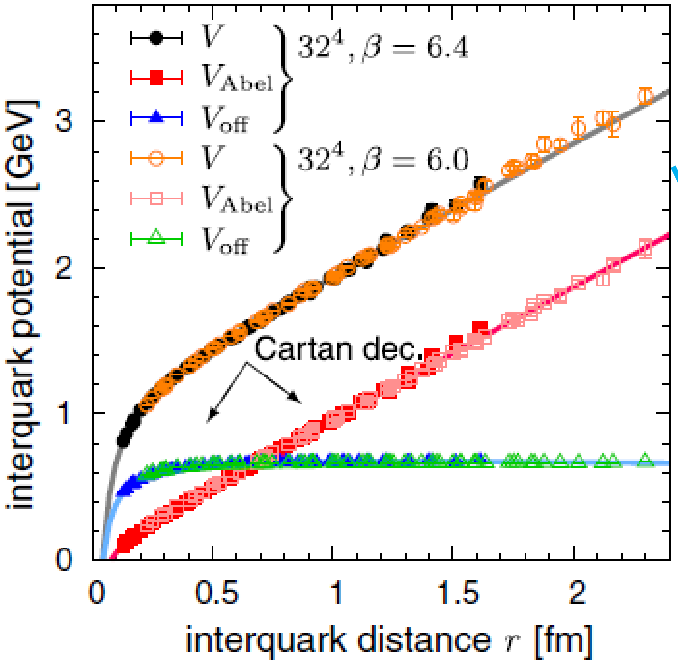
⇒

Abelian Dominance for Confinement

SU(3) potential and Abelian-projected potential in MA gauge in *large physical-volume* lattice QCD with β=6.0~6.4 and $32^4$.

# Quantitative Analysis on Abelian Dominance for Confinement in MA gauge

N.Sakumichi and H.S., PRD90 111501(R) (2014).




SU(3) potential V(r)

Abelian part $V_{Abel}(r)$

Fit analysis with
Coulomb-plus-linear Ansatz

$$V(r) = -\frac{A}{r} + \sigma r + C$$

| | $32^4, \beta = 6.4$ | | | $32^4, \beta = 6.0$ | | |
|---|---|---|---|---|---|---|
| | $\sigma$ | $A$ | $C$ | $\sigma$ | $A$ | $C$ |
| $V$ | 0.01528(12) | 0.265(3) | 0.598(1) | 0.0471(4) | 0.290(7) | 0.659(4) |
| $V_{\rm Abel}$ | 0.01550(06) | 0.056(1) | 0.167(1) | 0.0475(2) | 0.044(3) | 0.178(2) |

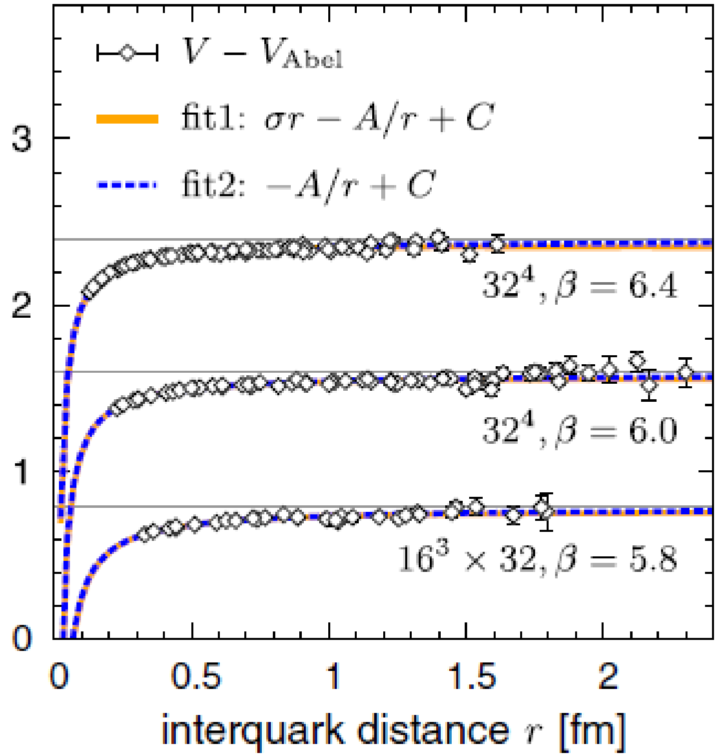
$\sigma_{Abel} = \sigma_{SU(3)}$
⇒ Perfect Abelian Dominance for Confinement

# Quantitative Analysis on Abelian Dominance for Confinement in MA gauge

N.Sakumichi and H.S., PRD90 111501(R) (2014).

## Difference between SU(3) potential $V_{SU(3)}$ and Abelian part $V_{Abel}$

$V_{SU(3)}(r) - V_{Abel}(r)$




- No string tension in the difference $V_{SU(3)}(r) - V_{Abel}(r)$.

- The difference $V_{SU(3)}(r) - V_{Abel}(r)$ can be well fitted by pure Coulomb potential.

⇒ This also suggests perfect Abelian dominance for confinement

N.Sakumichi and H.S., PRD90 111501(R) (2014).

| | $32^4, \beta = 6.4$ | | | $32^4, \beta = 6.0$ | | | $16^3 32, \beta = 5.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\sigma$ | $A$ | $C$ | $\sigma$ | $A$ | $C$ | $\sigma$ | $A$ | $C$ |
| $V$ | 0.01528(12) | 0.265(3) | 0.598(1) | 0.0471(4) | 0.290(7) | 0.659(4) | 0.0988(19) | 0.315(25) | 0.679(15) |
| $V_{\rm Abel}$ | 0.01550(06) | 0.056(1) | 0.167(1) | 0.0475(2) | 0.044(3) | 0.178(2) | 0.0988(08) | 0.039(10) | 0.183(06) |
| $V - V_{\rm Abel}$ | −0.00024(11) | 0.209(3) | 0.432(1) | −0.0005(3) | 0.247(6) | 0.481(3) | −0.0010(17) | 0.285(21) | 0.502(12) |
| $V - V_{\rm Abel}$ | 0 | 0.205(1) | 0.429(1) | 0 | 0.240(3) | 0.476(1) | 0 | 0.273(09) | 0.494(03) |

# SU(3) Lattice QCD calculation for 3Q potential

N. Sakumichi and H.S., PRD92, 034511 (2015).

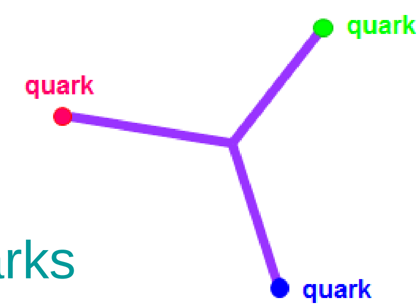
For more than 300 different patterns of 3Q systems,

(101 3Q-systems at β = 5.8, 211 3Q-systems at β = 6.0)

The 3Q potential is well described by Y-Ansatz, i.e., a sum of one-gluon-exchange (OGE) Coulomb and Y-type linear potential.

Y-Ansatz:

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{\left|\vec{r}_i - \vec{r}_j\right|} + \sigma L_{\min} + C$$




$L_{\min}$ : total length of string linking three valence quarks

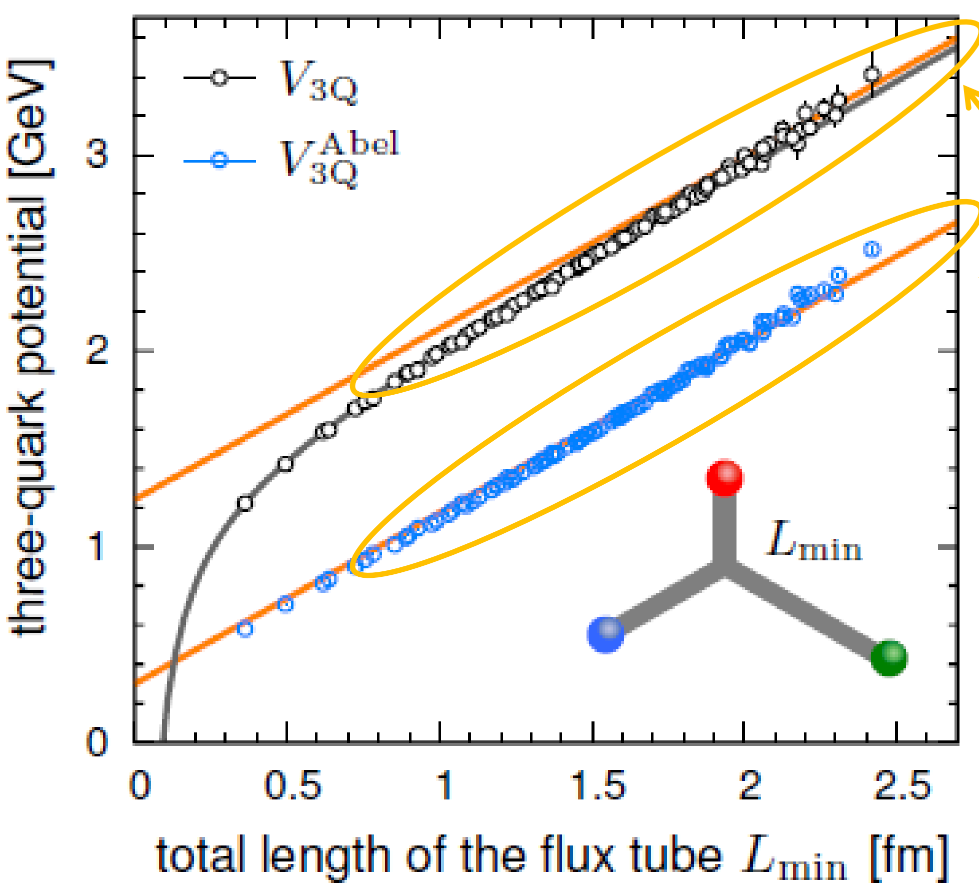
TABLE II. Fit analysis of interquark potentials in lattice units at $\beta = 5.8$ and 6.0 (i.e., $a \simeq 0.15$ and $a \simeq 0.10$ fm). The best-fit parameter sets $(\sigma, A, C)$ of the $Q\bar{Q}$ potential $V$ and the Abelian part $V^{\text{Abel}}$ are listed with the functional form (1). The best-fit parameter sets $(\sigma_{3Q}, A_{3Q}, C_{3Q})$ of the 3Q potential $V_{3Q}$ and the Abelian part $V_{3Q}^{\text{Abel}}$ are listed with the Y-ansatz (2). The label of (equi. triangle) means the fit analysis only with the lattice data of equilateral-triangle 3Q configurations. $N_Q$ is the number of different patterns of $Q\bar{Q}$ or 3Q systems. The string tension ratio $\sigma^{\text{Abel}}/\sigma$ is listed at the last column.

| | | | SU(3) | | | Abelian part | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\beta$ | | $N_Q$ | $\sigma$ | $A$ | $C$ | $\sigma^{\text{Abel}}$ | $A^{\text{Abel}}$ | $C^{\text{Abel}}$ | $\sigma^{\text{Abel}}/\sigma$ |
| 5.8 | $Q\bar{Q}$ | 26 | 0.099(2) | 0.30(3) | 0.67(2) | 0.098(1) | 0.043(12) | 0.187(7) | 0.99(3) |
| | 3Q (equi. triangle) | 5 | 0.097(1) | 0.118(3) | 0.93(1) | 0.098(3) | −0.001(8) | 0.19(2) | 1.01(3) |
| | 3Q | 101 | 0.0997(4) | 0.109(1) | 0.905(4) | 0.0967(5) | 0.006(2) | 0.213(5) | 0.97(1) |
| 6.0 | $Q\bar{Q}$ | 39 | 0.0472(6) | 0.289(10) | 0.658(5) | 0.0457(2) | 0.050(3) | 0.183(2) | 0.97(1) |
| | 3Q (equi. triangle) | 8 | 0.0471(10) | 0.121(3) | 0.936(9) | 0.0455(12) | 0.014(4) | 0.233(12) | 0.97(3) |
| | 3Q | 211 | 0.0480(3) | 0.113(1) | 0.917(3) | 0.0456(2) | 0.013(1) | 0.232(2) | 0.95(1) |

# 3Q Potential and Abelian Dominance in MA gauge

N. Sakumichi and H.S., PRD92, 034511 (2015).

(b) MA projection of the 3Q potential




3Q potential $V_{3Q}(r)$

Abelian part $V_{3Q}^{Abel}(r)$

3Q potential and its Abelian part have almost the same slope at large distance.

⇒

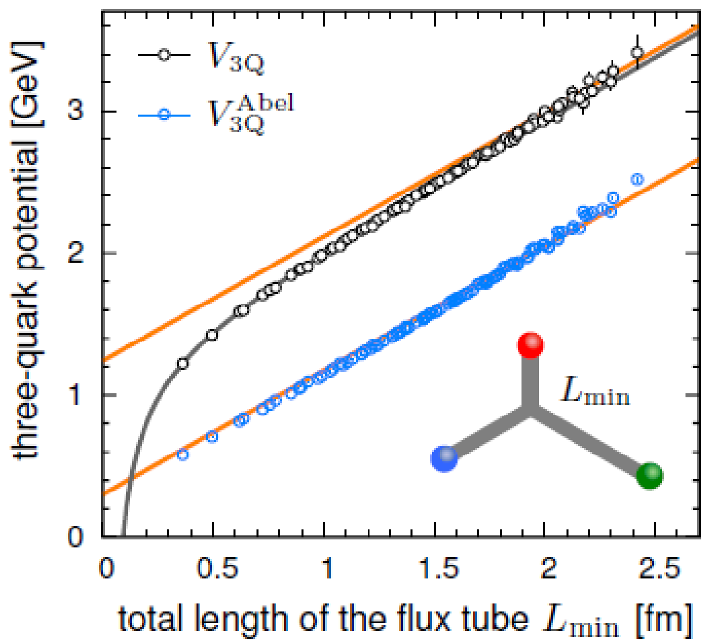
Abelian Dominance for Confinement

3Q potential and Abelian-projected 3Q potential plotted against minimal linking length $L_{min}$ in MA gauge in SU(3) lattice QCD with β=5.8, $16^3 32$.

# Quantitative Analysis on Abelian Dominance for 3Q Confinement in MA gauge

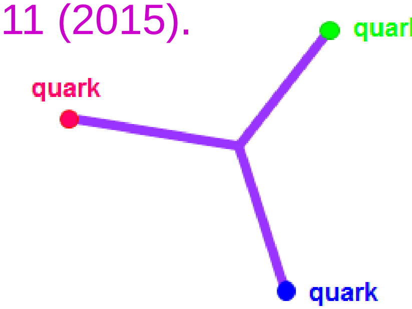
N.Sakumichi and H.S., PRD92, 034511 (2015).

(b) MA projection of the 3Q potential




3Q potential $V_{3Q}(r)$

Abelian part $V_{3Q}^{\rm Abel}(r)$




Fit analysis with Y-Ansatz for 3Q potential

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma L_{\rm min} + C$$

$L_{\rm min}$ : total length of string linking three valence quarks

| $\beta$ | | $N_Q$ | SU(3) $\sigma$ | SU(3) $A$ | SU(3) $C$ | Abelian part $\sigma^{\rm Abel}$ | Abelian part $A^{\rm Abel}$ | Abelian part $C^{\rm Abel}$ | $\sigma^{\rm Abel}/\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 5.8 | $Q\bar{Q}$ | 26 | 0.099(2) | 0.30(3) | 0.67(2) | 0.098(1) | 0.043(12) | 0.187(7) | 0.99(3) |
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| | 3Q | 211 | 0.0480(3) | 0.113(1) | 0.917(3) | 0.0456(2) | 0.013(1) | 0.232(2) | 0.95(1) |

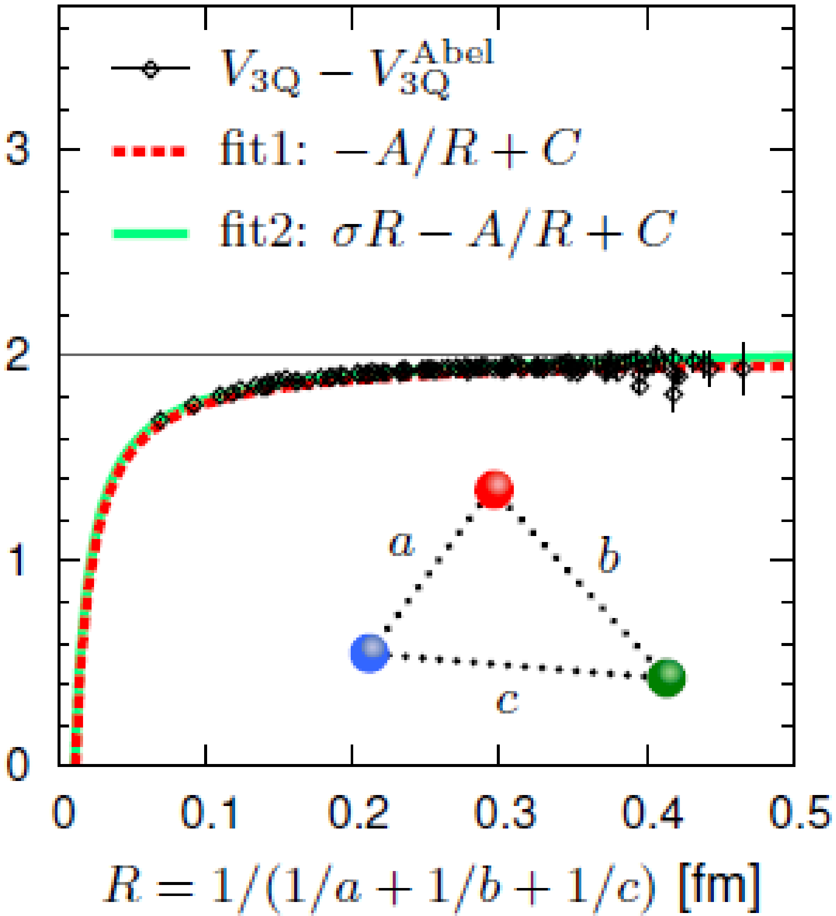
$\sigma^{\rm Abel} \fallingdotseq \sigma \Rightarrow$ Perfect Abelian Dominance for 3Q Confinement

# Quantitative Analysis on Abelian Dominance for 3Q Confinement in MA gauge

N. Sakumichi and H.S., PRD92, 034511 (2015).

## Difference between 3Q potential $V_{3Q}$ and Abelian part $V_{3Q}^{Abel}$

$V_{3Q}(r) - V_{3Q}^{Abel}(r)$




- No string tension in the difference $V_{3Q}(r) - V_{3Q}^{Abel}(r)$.

- The difference $V_{3Q}(r) - V_{3Q}^{Abel}(r)$ can be well fitted by 2-body pure Coulomb potential.

⇒ This also suggests perfect Abelian dominance for 3Q confinement

This plot was proposed by N. Brambilla et al., PRD 81, 054031 (2010); PRD 87, 074014 (2013).

# Large off-diagonal gluon mass and infrared Abelian dominance in MA gauge in SU(2) Lattice QCD

Analysis of gluon propagators in MA gauge in SU(2) QCD

↓

Large effective-mass generation of off-diagonal gluons in MA gauge

↓ $M_{\rm off} \fallingdotseq$ 1GeV

Infrared inactiveness of off-diagonal gluons in MA gauge

↓

**Infrared Abelian Dominance**

infrared quantities such as confinement and chiral symmetry breaking would be well described only with diagonal gluons in MA gauge

K. Amemiya, H.S., PRD 60 (1999) 114509.

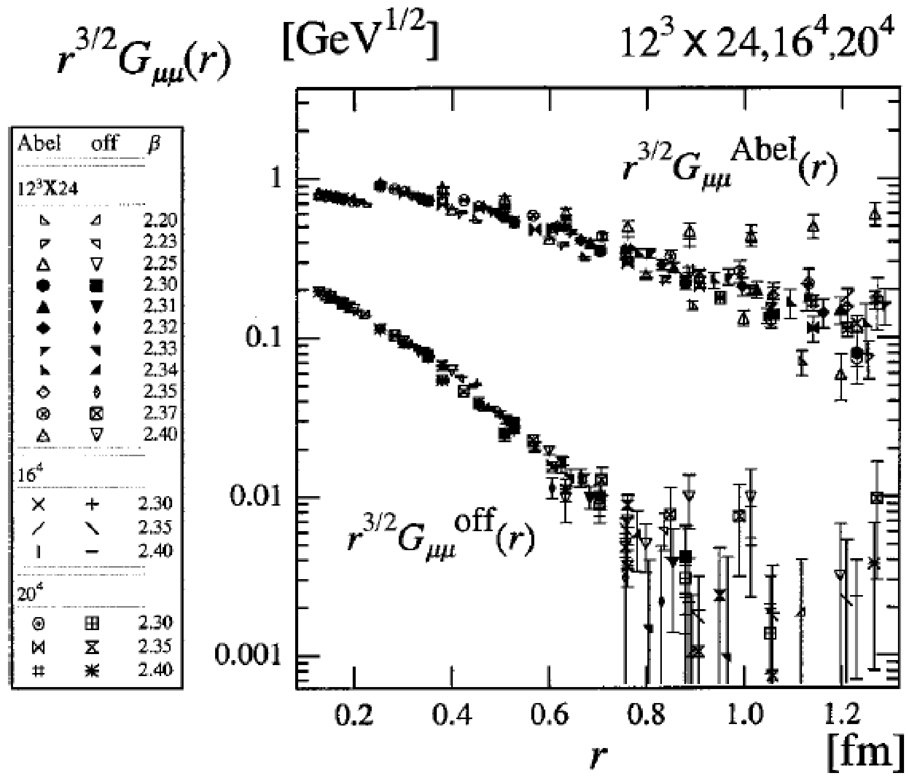
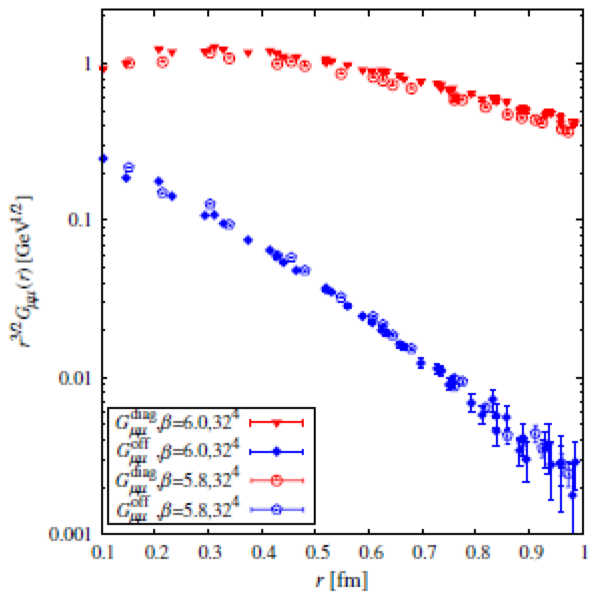



FIG. 3. The logarithmic plot of $r^{3/2}G_{\mu\mu}^{\rm off}(r)$ and $r^{3/2}G_{\mu\mu}^{\rm Abel}(r)$ as the function of the distance $r$ in the MA gauge with the $U(1)_3$ Landau gauge fixing, using the SU(2) lattice QCD with $12^3\times 24$ $(2.2 \leqslant \beta \leqslant 2.4)$, $16^4$ and $20^4$ $(2.3 \leqslant \beta \leqslant 2.4)$. The off-diagonal gluon propagator behaves as the Yukawa-type function $G_{\mu\mu}^{\rm off}(r) \sim [\exp(-M_{\rm off}r)]/r^{3/2}$ with $M_{\rm off} \simeq 1$ GeV for $r \gtrsim 0.2$ fm. Therefore, the off-diagonal gluon seems to have a large mass $M_{\rm off} \simeq 1$ GeV in the MA gauge.

# Large off-diagonal gluon mass and infrared Abelian dominance in MA gauge in SU(3) Lattice QCD

S. Gongyo, T. Iritani, H.S., PRD 86 (2012) 094018, PRD 87 (2013) 074506.

We investigate gluon propagators in MA gauge with $U(1)^2$ Landau gauge fixing in SU(3) quenched QCD, and find a *large off-diagonal gluon mass* of about 1GeV, which leads to Infrared inactiveness of off-diagonal gluons and *infrared Abelian dominance*, similar to low-energy Abelianization in the Weinberg-Salam model.




diagonal gluon propagator

off-diagonal gluon propagator

The diagonal and off-diagonal gluon propagators (log plot of $r^{3/2}$ G(r)) in SU(3) lattice QCD in the MA gauge with the $U(1)^2$ Landau gauge fixing.

| Lattice size | $\beta$ | $a$[fm] | $M_{\rm off}$[GeV] | $M_{\rm diag}$[GeV] |
|---|---|---|---|---|
| $32^4$ | 5.8 | 0.152 | 1.1 | 0.3 |
| | 6.0 | 0.104 | 1.1 | 0.3 |

The lattice QCD result of effective masses of off-diagonal and diagonal gluons in the MA gauge. The off-diagonal gluon has a large mass of about 1GeV.

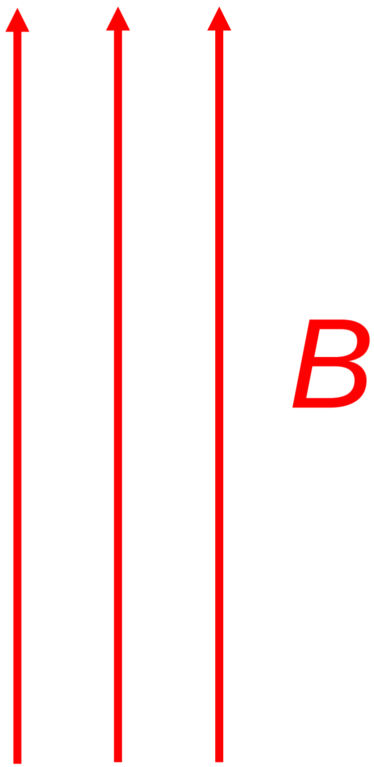
Thus, QCD in Maximally Abelian Gauge exhibits
the Off-diagonal Inactiveness and then
Infrared Abelian Dominance

This situation is similar to
low-energy Abelianization
in the *Non-Abelian Higgs Theory*
such as the *Weinberg-Salam model*

Actually, *Non-Abelian Higgs Theory*
gives Off-diagonal gauge-boson mass,
which leads to Off-diagonal Inactiveness,
and low-energy Abelianization

However, this situation can be drastically changed
in the presence of a *Strong Magnetic Field* for
*Non-Abelian Higgs Theory*.

# Non-Abelian Higgs Theory in a Strong Magnetic Field




# Color-Magnetic property in Non-Abelian Gauge Theories

$SU(2)_{color}$ QCD is formally divided into diagonal gluon (photon) and off-diagonal gluon (charged vector)

$$\begin{cases} A_\mu \equiv A_\mu^3 & \text{diagonal gluon (photon)} \\ A_\mu^\pm \equiv \dfrac{1}{\sqrt{2}}(A_\mu^1 \pm i A_\mu^2) & \text{off-diagonal gluon (charged vector)} \end{cases}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \qquad D_\mu \equiv \partial_\mu + ieA_\mu \qquad D_\mu^* = \partial_\mu - ieA_\mu$$

U(1)-field strength U(1)-covariant derivative

$$L_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}(D_\mu^* A_\nu^- - D_\nu^* A_\mu^-)(D_\mu A_\nu^+ - D_\nu A_\mu^+) + \underline{ieF^{\mu\nu} A_\mu^- A_\nu^+}$$

$$+ \frac{1}{2} e^2 [(A_\mu^- A_\mu^+)^2 - (A_\mu^- A_\mu^-)(A_\nu^+ A_\nu^+)]$$

→ gyromagnetic ratio: *g*=2

*Anomalous Magnetic Moment*

of off-diagonal gluons

# Non-Abelian Higgs (NAH) Theory

## SU(2) Higgs theory with Higgs condensation:

diagonal gauge field → massless photon

off-diagonal gauge field → massive charged vector

$$\begin{cases} A_\mu \equiv A_\mu^3 & \text{diagonal massless photon} \\ A_\mu^\pm \equiv \dfrac{1}{\sqrt{2}}(A_\mu^1 \pm i A_\mu^2) & \text{off-diagonal charged vector} \end{cases}$$

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ U(1)-field strength

$D_\mu \equiv \partial_\mu + ieA_\mu$ U(1)-covariant derivative

SU(2) NAH with triplet Higgs $\Phi_3 = v + \sigma \rightarrow M \equiv ev$ : charged-vector mass

$$L_{NAH} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \frac{1}{2} D_\mu^{SU(2)} \Phi^a D_{SU(2)}^\mu \Phi^a - \frac{\lambda}{8}(\Phi^a \Phi^a - v^2)^2$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}(D_\mu^* A_\nu^- - D_\nu^* A_\mu^-)(D_\mu A_\nu^+ - D_\nu A_\mu^+) + \underline{ieF^{\mu\nu} A_\mu^- A_\nu^+}$$

→ gyromagnetic ratio: *g*=2

$$+\frac{1}{2} e^2 [(A_\mu^- A_\mu^+)^2 - (A_\mu^- A_\mu^-)(A_\nu^+ A_\nu^+)] + \underline{(M + e\sigma)^2 A_\mu^+ A_-^\mu} + \frac{1}{2}(\partial_\mu \sigma)^2 - V(\sigma)$$

$$V(\sigma) = \frac{\lambda}{8}(2v\sigma + \sigma^2)^2 = \frac{1}{2}\lambda v^2 \sigma^2 + \frac{\lambda v}{2}\sigma^3 + \frac{\lambda}{8}\sigma^4$$

For simplicity, the gauge coupling *e* is taken to be small.

# General argument in Constant Strong Magnetic Field

For general charged field with gyromagnetic ratio *g*

in the constant magnetic field $B \equiv B_z = F_{xy}$ along *z*-axis,

the system is *1-dim HO* in *x, y* directions and trivial in *t, z* directions

*B*

$$[(\hat{p} - e\hat{A})^2 - m^2 + geBs_z]\phi = (\hat{p}_0^2 - \hat{p}_z^2 - \hat{\Pi}_x^2 - \hat{\Pi}_y^2 - m^2 + geBs_z)\phi = 0$$

$$\hat{\Pi}_x \equiv \hat{p}_x - e\hat{A}_x \qquad \hat{\Pi}_y \equiv \hat{p}_y - e\hat{A}_y$$

$$[\hat{\Pi}_x, \hat{\Pi}_y] = ie(\partial_x A_y - \partial_y A_x) = ieF_{xy} = ieB$$

$$\hat{Q} \equiv \frac{1}{\sqrt{eB}}\hat{\Pi}_x \quad \hat{P} \equiv \frac{1}{\sqrt{eB}}\hat{\Pi}_y \quad \rightarrow \quad [\hat{Q}, \hat{P}] = i \qquad \hat{\Pi}_x^2 + \hat{\Pi}_y^2 = 2eB \cdot \frac{1}{2}(\hat{Q}^2 + \hat{P}^2)$$

*1-dim HO*

$$\hat{a} \equiv \frac{1}{\sqrt{2}}(\hat{Q} + i\hat{P}) \qquad [\hat{a}, \hat{a}^\dagger] = 1$$

$$[\hat{p}_0^2 - \hat{p}_z^2 - 2eB(\hat{a}^\dagger\hat{a} + \frac{1}{2}) - m^2 + geBs_z]\phi = 0$$

# Non-Abelian Higgs Theory in a Strong Magnetic Field

Constant Magnetic Field in $z$-direction: $F_{12} = B$

$$L_{NAH} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (D_\mu^* A_\nu^- - D_\nu^* A_\mu^-)(D_\mu A_\nu^+ - D_\nu A_\mu^+) + ieF^{\mu\nu} A_\mu^- A_\nu^+ + M^2 A_\mu^+ A_-^\mu$$
$$+ \frac{1}{2} e^2 [(A_\mu^- A_\mu^+)^2 - (A_\mu^- A_\mu^-)(A_\nu^+ A_\nu^+)] + Neutral\ Higgs$$

Photon: $A_\mu \equiv A_\mu^3$

U(1)-covariant derivative: $D_\mu \equiv \partial_\mu + ieA_\mu$

U(1)-field strength: $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

Charged-vector propagator:

$$\hat{D}^{\mu\nu} = \hat{D} \tilde{g}^{\mu\nu}$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} + \hat{p}^\mu \hat{p}^\nu f(\hat{p}^2)$$ :tensor factor

gauge-dep.

Charged-vector propagator apart from tensor factor in constant magnetic field:

$$\hat{D}^{-1} = \hat{p}_0^2 - \hat{p}_z^2 - 2eB(\hat{a}^\dagger \hat{a} + \frac{1}{2} - \hat{s}_z) - M^2$$

*physical*, *gauge-invariant*

gyromagnetic ratio: *g=2*

$$\hat{a} \equiv \frac{1}{\sqrt{2eB}} [(\hat{p}_x - e\hat{A}_x) + i(\hat{p}_y - e\hat{A}_y)]$$

# Charged-vector propagation in Strong Magnetic Field

$$\hat{D}^{-1} = \hat{p}_0^2 - \hat{p}_z^2 - 2eB(\hat{a}^\dagger \hat{a} + \frac{1}{2} - \hat{s}_z) - M^2$$

diagonal basis $\quad \hat{a}^\dagger \hat{a} \left| n \right\rangle = n \left| n \right\rangle$

$$D^{-1}(p_0, p_z, n, s_z) \equiv \left\langle p_0 p_z n s_z \right| \hat{D}^{-1} \left| p_0 p_z n s_z \right\rangle = p_0^2 - p_z^2 - 2eB(n + \frac{1}{2} - s_z) - M^2$$

Coordinate-space representation of Charged-vector propagator:

$$D(x, x') \equiv \left\langle x \right| \hat{D} \left| x' \right\rangle = \sum_{p_0, p_z, n, s_z} \left\langle x \middle| p_0 p_z n s_z \right\rangle D(p_0, p_z, n, s_z) \left\langle p_0 p_z n s_z \middle| x' \right\rangle$$

$$= \int \frac{dp_0}{2\pi} \int \frac{dp_z}{2\pi} \sum_{n=0}^{\infty} \sum_{s_z=-s}^{s} \psi_{p_0 p_z n s_z}(x) \frac{1}{p_0^2 - p_z^2 - 2eB(n + \frac{1}{2} - s_z) - M^2} \psi^\dagger_{p_0 p_z n s_z}(x')$$

$$\psi_{p_0 p_z n s_z}(x) \equiv \left\langle x \middle| p_0 p_z n s_z \right\rangle = \left\langle t \middle| p_0 \right\rangle \left\langle z \middle| p_z \right\rangle \left\langle xy \middle| n \right\rangle \chi_{s_z} = e^{-ip_0 t} e^{ip_z z} \varphi_n(x, y) \chi_{s_z}$$

spin-w.f.

*HO-w.f.* $\quad \varphi_n(x, y) \equiv \left\langle xy \middle| n \right\rangle = \sqrt{\frac{eB}{2\pi}} P_n(x, y) e^{-\frac{x^2 + y^2}{4l^2}} \qquad l \equiv \frac{1}{\sqrt{eB}}$

polynomial

*symmetric gauge*

# Charged-vector propagation in Strong Magnetic Field

In a strong magnetic field, lowest Landau level (*LLL*: *n*=0, $s_z$=1) is dominant.

→ *LLL projection*

$$\varphi_0(x,y) \equiv \langle xy|0\rangle = \sqrt{\frac{eB}{2\pi}}e^{-\frac{x^2+y^2}{4l^2}} \in R \qquad l \equiv \frac{1}{\sqrt{eB}}$$

*symmetric gauge*

Coordinate-space representation of Charged-vector propagator:

$$D(x,x') \simeq D_{LLL}(x,x') \equiv \sum_{p_0,p_z,n,s} \langle x|p_0 p_z n s_z\rangle D(p_0,p_z,n,s_z)\langle p_0 p_z n s_z|x'\rangle \delta_{n0}\delta_{s_z 1}$$

$$= \int\frac{dp_0}{2\pi}\int\frac{dp_z}{2\pi}e^{-ip_0(t-t')}e^{ip_z(z-z')}\varphi_0(x,y)\varphi_0(x',y')\frac{1}{p_0^2-p_z^2-\mu^2}$$

$$= \frac{eB}{2\pi}e^{-\frac{x^2+y^2}{4l^2}}e^{-\frac{x'^2+y'^2}{4l^2}}\int\frac{dp_0}{2\pi}\int\frac{dp_z}{2\pi}e^{-ip_0(t-t')}e^{ip_z(z-z')}\frac{1}{p_0^2-p_z^2-\mu^2}$$

*Landau degeneracy*

Quasi 1+1 dimensional propagation with mass $\mu$

$$\mu^2 \equiv M^2 - eB$$

# Non-Abelian Higgs Theory in Strong Magnetic Field

Charged-vector propagator ~ Quasi 1+1 dimensional propagation with mass $\mu$

$$D(x,x') \simeq D_{LLL}(x,x') = \frac{eB}{2\pi} e^{-\frac{x^2+y^2}{4l^2}} e^{-\frac{x'^2+y'^2}{4l^2}} \int \frac{dp_0}{2\pi} \int \frac{dp_z}{2\pi} e^{-ip_0(t-t')} e^{ip_z(z-z')} \frac{1}{p_0^2 - p_z^2 - \mu^2}$$

↓

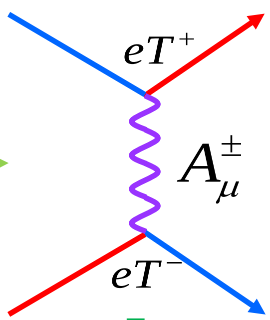
Strong correlation along *B*-direction for small mass $\mu$

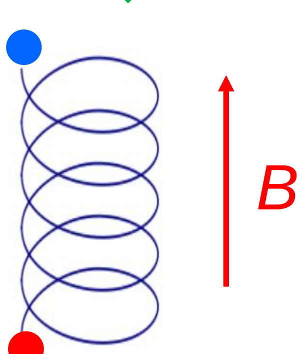
$$\mu^2 \equiv M^2 - eB$$

*Fermionic matter field coupled with SU(2) gauge field* $A^{\mu}_{SU(2)}$

$$L_{matter} = \bar{\psi}(i\not{D}_{SU(2)} - m)\psi = \bar{\psi}(i\not{\partial} - e\not{A}_{SU(2)} - m)\psi$$

$$= \bar{\psi}(i\not{\partial} - e\not{A}_3 T_3 - m)\psi - \underline{e\bar{\psi}\not{A}_{\pm} T_{\mp}\psi}$$




Coupling with 3-dim photon

↓

Landau motion of fermion

*Coupling with quasi 1-dim charged vector*

↓

*Strong correlation along B*

The charged fermion does Landau motion *affecting with strong correlation along B induced by charged vector*

# Non-Abelian Higgs Theory in Strong Magnetic Field

Charged-vector propagator ~ Quasi 1+1 dimensional propagation with mass $\mu$

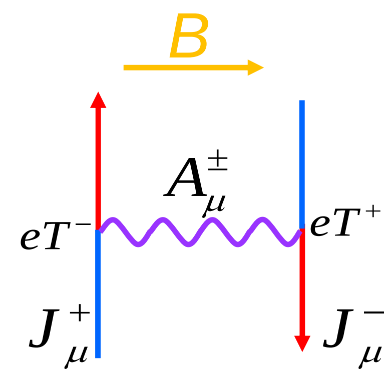
$$D(x,x') \simeq D_{LLL}(x,x') = \frac{eB}{2\pi} e^{-\frac{x^2+y^2}{4l^2}} e^{-\frac{x'^2+y'^2}{4l^2}} \int \frac{dp_0}{2\pi} \int \frac{dp_z}{2\pi} e^{-ip_0(t-t')} e^{ip_z(z-z')} \frac{1}{p_0^2 - p_z^2 - \mu^2}$$

$$\mu^2 \equiv M^2 - eB$$

Strong correlation along $B$-direction for small mass $\mu$

$J_\mu^{\pm}$ :current coupled with charged vector $A_\mu^{\pm}$ $\quad \partial^\mu J_\mu^{\pm} = 0$

(*e.g.* $J_\mu^{\pm} = -e\bar{\psi}\gamma_\mu T^{\pm}\psi$ for charged fermion)




Tree-level current-current correlation:

$$L_{JJ} = \int d^4x d^4y \, J_\mu^+(x) D(x-y) J_-^\mu(y)$$

cf : off-diagonal gluon exchange between quarks in QCD

Inter-charge potential along $B$

$$V(r) = -e^2 \frac{eB}{4\pi} \frac{1}{\mu} e^{-\mu r}$$

$\mu \to 0$ i.e. $eB_c = M^2$

$$V(r) = e^2 \frac{eB}{4\pi} r$$

apart from a constant

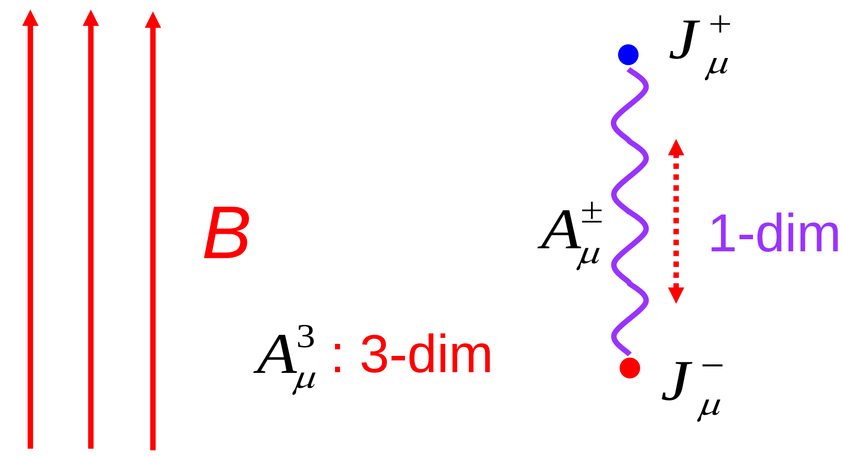
$r$ : inter-charge distance

Charges are located along $B$

Linear confinement potential along $B$

(Landau degeneracy $\frac{eB}{4\pi}$ gives dimension of string tension)

# Non-Abelian Higgs Theory in Strong Magnetic Field




Quasi 1-dimensional
strong correlation along *B*
between off-diagonal charges
coupled with charged vector $A_\mu^\pm$

$A_\mu^3$ 3-dimensional → background magnetic field and Coulomb

$A_\mu^\pm$ 1-dimensional → quasi-linear potential along *B*

In particular, at the critical magnetic value $eB_c = M^2$, off-diagonal charged-vector exchange leads to a linear potential between off-diagonal charges along the magnetic direction at tree level.

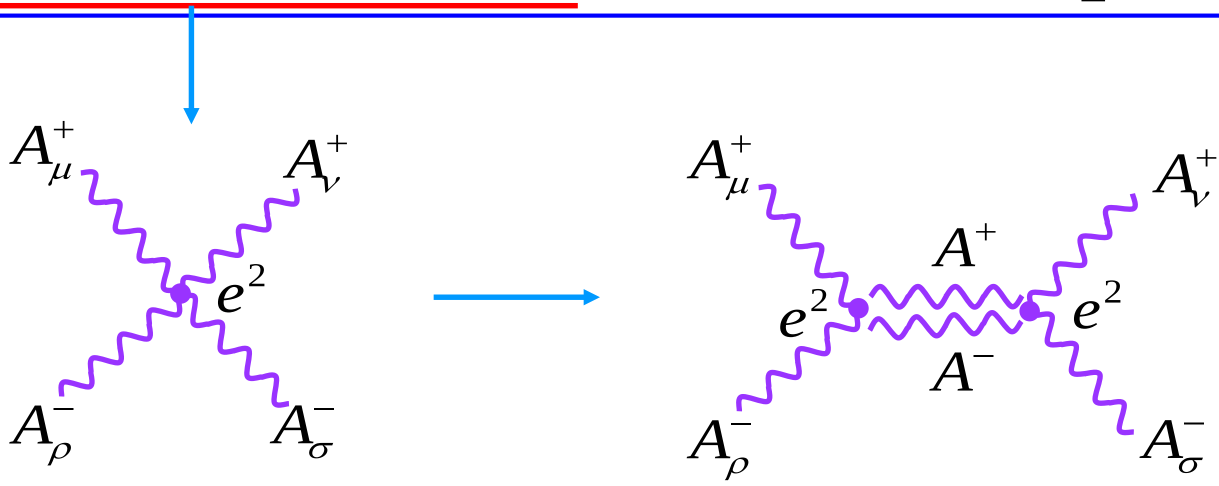
(cf. Nielsen-Olesen instability)

S. Bolognesi, C. Chatterjee, S.B. Gudnason, K.Konishi, JHEP 1410 (2014) 101.

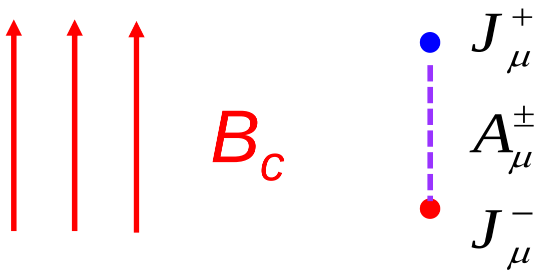
# Non-Abelian Higgs Theory in Strong Magnetic Field

As for off-diagonal charged vectors,
they may also feel some correlation along *B*, because of their self-interaction.
But, this effect is higher order of small gauge coupling *e*.
(In our situation, *eB* is large but *e* is small.)

$$
\begin{aligned}
L_{NAH} &= -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \frac{1}{2} D_\mu^{SU(2)} \Phi^a D^\mu_{SU(2)} \Phi^a - \frac{\lambda}{8} (\Phi^a \Phi^a - v^2)^2 \\
&= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (D^*_\mu A^-_\nu - D^*_\nu A^-_\mu)(D_\mu A^+_\nu - D_\nu A^+_\mu) + ieF^{\mu\nu} A^-_\mu A^+_\nu \\
&\quad + \underline{\frac{1}{2} e^2 [(A^-_\mu A^+_\mu)^2 - (A^-_\mu A^-_\mu)(A^+_\nu A^+_\nu)]} + (M + g\sigma)^2 A^+_\mu A^\mu_- + \frac{1}{2} (\partial_\mu \sigma)^2 - V(\sigma)
\end{aligned}
$$



# Non-Abelian Higgs Theory in the critical magnetic field $eB_c=M^2$




1-dimensional
strong correlation along *B*

$A_\mu^\pm$ 1-dimensional → tree-level linear potential along *B*

At critical magnetic field $eB_c=M^2$, off-diagonal charged vectors behave as 1-dimensional massless fields along *B*, and leads to tree-level linear potential between off-diagonal charges coupled with charged vector.

This leads to the "off-diagonal-charge confinement", similar to 1+1 QED, even though there appear screening effects due to charged pair creation.

This may be new-type of "off-diagonal-dominant confinement" between off-diagonal charges, caused by off-diagonal vector $A_\mu^\pm$

## Non-Abelian Higgs Theory

$$\begin{cases} A_\mu \equiv A_\mu^3 \\ A_\mu^\pm \equiv \dfrac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2) \end{cases}$$

3-dimensional massless

3-dimensional massive

→ Infrared Abelian Dominant System

⇓ Strong constant magnetic

## NAH Theory in strong magnetic field $B_c$

$$\begin{cases} A_\mu \equiv A_\mu^3 \\ A_\mu^\pm \equiv \dfrac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2) \end{cases}$$

3-dimensional massless

1-dimensional massless

→ Off-diagonal Dominant System along *B*

# Non-Abelian Higgs Theory in a Strong Magnetic Field and Confinement


Summary

We investigate non-abelian Higgs theory in a constant strong magnetic field, where the lowest-Landau-level approximation can be used.

At the critical magnetic value $eB_c = m^2$, the off-diagonal charged vector fields behave as one-dimensional massless fields and give a strong correlation along the magnetic direction.

This may lead a new type of confinement caused by off-diagonal vector fields.


Thank you!