Confinement, infrared screening of the QCD vacuum density and small cosmological constant

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Vacuum state of the Universe

Friedmann equation

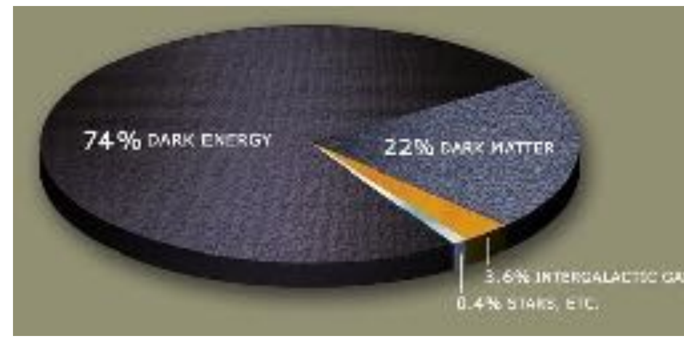
$$\boxed{H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G(\rho_M + \rho_{rad} + \rho_\Lambda + \rho_{curv})}$$

$$T_{00} = \rho(\mathbf{x})$$

$$\Omega_M = \frac{\rho_{M,0}}{\rho_c}, \quad \Omega_{rad} = \frac{\rho_{rad,0}}{\rho_c}, \quad \Omega_\Lambda = \frac{\rho_{\Lambda,0}}{\rho_c}, \quad \Omega_{curv} = \frac{\rho_{curv,0}}{\rho_c}$$

critical density of the Universe

$$\rho_c \equiv \frac{3}{8\pi G} H_0^2$$



Universe expansion is dominated by three components

$$\Omega_M = \Omega_B + \Omega_{DM}$$

Dark Energy e.o.s.

$$\boxed{\omega \equiv P/\rho = -1.13^{+0.13}_{-0.10}}$$

exactly minus one for pure vacuum!

- **Is the Dark Energy indeed the (non-trivial) vacuum state of the Universe?**
- **How has it been formed during evolution of the Universe?**
- **What is its time dependence and what will happen to our Universe in the future?**

Vacuum Catastrophe… or not?

Vacuum energy

"…the worst theoretical prediction in the history of physics" (Hobson 2006)

in Quantum Physics		in Cosmology
$\epsilon_{vac} \sim 10^{-2}\,\mathrm{GeV}^4$	Topological QCD vacuum unique strongly-coupled subsystem!	$\Lambda_{\rm cosm} \sim 10^{-47}\,\mathrm{GeV}^4$
$\sim 10^{8}\,\mathrm{GeV}^4$	Higgs condensate	"Old" CC problem: Why such small and positive? "New" CC problem: Why non-zeroth and exists at all?

$\boxed{\text{Vacuum in Quantum Physics has incredibly wrong energy scale!}}$

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle + \langle 0| : m_s \bar{s}s : |0\rangle\right)$$
$$\simeq -(5 \pm 1) \times 10^9\ \mathrm{MeV}^4.$$

Two possible approaches to this problem:

- **Let's forget about the "bare" vacuum (DE: "phantom", "quintessence", "ghost"… etc)**
  Zero vacuum density in the Minkowski limit, by (Casimir-like) definition, then (Zhitnitsky et al)

$$\Lambda_{\rm cosm} \equiv \epsilon_{\rm FLRW} - \epsilon_{\rm Mink}$$

simply imposing a cancellation of the "bare" vacuum by hands!!

- **Let's look closer at the vacuum state — why/how does it become "invisible" to gravity?**

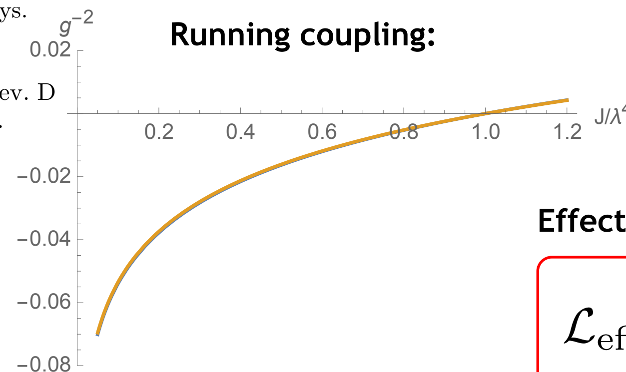
Effective YM action approach

At least, for SU(2) gauge symmetry, the all-loop and one-loop effective Lagrangians are practically indistinguishable (by FRG approach)

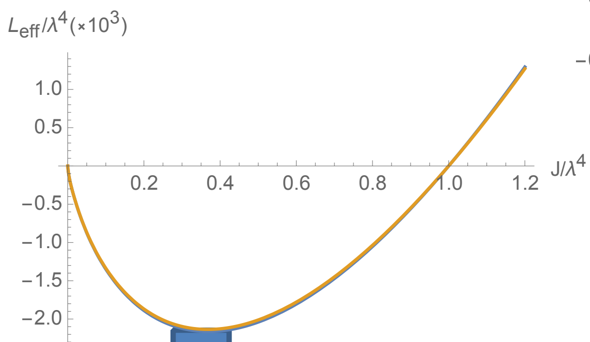
P. Dona, A. Marciano, Y. Zhang and C. Antolini, Phys. Rev. D **93** (2016) no.4, 043012.

A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D **83** (2011) 045014 [Phys. Rev. D **83** (2011) 069903].

Running coupling:



Effective Lagrangian:



chromoelectric (CE) condensate (Savvidy vacuum) $\qquad \mathcal{J}^* > 0$

G. K. Savvidy, Phys. Lett. **71B**, 133 (1977)

H. Pagels and E. Tomboulis, Nucl. Phys. B **143**, 485 (1978).

Classical YM Lagrangian:

$$\mathcal{L}_{\rm cl} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\rm YM} f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{A}^a_\mu \equiv g_{\rm YM} A^a_\mu$$

Effective YM Lagrangian: $\quad \mathcal{F}^a_{\mu\nu} \equiv g_{\rm YM} F^a_{\mu\nu}$

$$\boxed{\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2(\mathcal{J})}, \qquad \mathcal{J} = -\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a}$$

The energy-momentum tensor:

$$T^\nu_\mu = \frac{1}{\bar{g}^2}\left[\frac{\beta(\bar{g}^2)}{2} - 1\right]\left(\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{J}\right) - \delta^\nu_\mu \frac{\beta(\bar{g}^2)}{8\bar{g}^2}\mathcal{J}$$

Equations of motion:

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2}\left(1 - \frac{\beta(\bar{g}^2)}{2}\right)\right] = 0,$$

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \equiv \left(\delta^{ab}\overrightarrow{\partial}_\nu - f^{abc}\mathcal{A}^c_\nu\right),$$

trace anomaly:

$$T^\mu_\mu = -\frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{J}$$

NOTE: the RG equation $\quad \dfrac{d\ln|\bar{g}^2|}{d\ln|\mathcal{J}|/\mu_0^4} = \dfrac{\beta(\bar{g}^2)}{2}\quad$ **appears to be invariant under**

$$\boxed{\begin{gathered}\mathcal{J} \longleftrightarrow -\mathcal{J} \\ \bar{g}^2 = \bar{g}^2(|\mathcal{J}|)\end{gathered}}$$

CE condensate on non-stationary (FLRW) background

$$\mathcal{J} = \frac{2}{\sqrt{-g}} \sum_a (\mathbf{E}_a \cdot \mathbf{E}_a - \mathbf{B}_a \cdot \mathbf{B}_a) \equiv \frac{2}{\sqrt{-g}} (\mathbf{E}^2 - \mathbf{B}^2)$$

FLRW metric in conformal time:

$$g \equiv \det(g_{\mu\nu}),\; g_{\mu\nu} = a(\eta)^2 \mathrm{diag}(1,\, -1,\, -1,\, -1)$$

$$\sqrt{-g} = a^4(\eta)\,, \qquad t = \int a(\eta) d\eta$$

- **Basic qualitative features on the non-perturbative YM action are noticed already at one loop**

Einstein-YM equations of motion for the effective YM theory:

$$\frac{1}{\varkappa}\left(R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R\right) = \bar{\epsilon}\delta^\nu_\mu + \frac{b}{32\pi^2}\frac{1}{\sqrt{-g}}\left[\left(-\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{F}^a_{\sigma\lambda}\mathcal{F}^{\sigma\lambda}_a\right)\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}^{\alpha\beta}_a|}{\sqrt{-g}\,\lambda^4} - \frac{1}{4}\delta^\nu_\mu\, \mathcal{F}^a_{\sigma\lambda}\mathcal{F}^{\sigma\lambda}_a\right],$$

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\overrightarrow{\partial}_\nu \sqrt{-g} - f^{abc}\mathcal{A}^c_\nu\right)\left(\frac{\mathcal{F}^{\mu\nu}_b}{\sqrt{-g}}\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}^{\alpha\beta}_a|}{\sqrt{-g}\,\lambda^4}\right) = 0$$

temporal (Hamilton) gauge $\quad A^a_0 = 0 \qquad e^a_i A^a_k \equiv A_{ik} \qquad e^a_i e^a_k = \delta_{ik} \qquad e^a_i e^b_i = \delta_{ab}$

due to local SU(2) ~ SO(3) isomorphism $\quad A_{ik}(t,\vec{x}) = \delta_{ik} U(t) + \widetilde{A}_{ik}(t,\vec{x})$

The resulting equations:

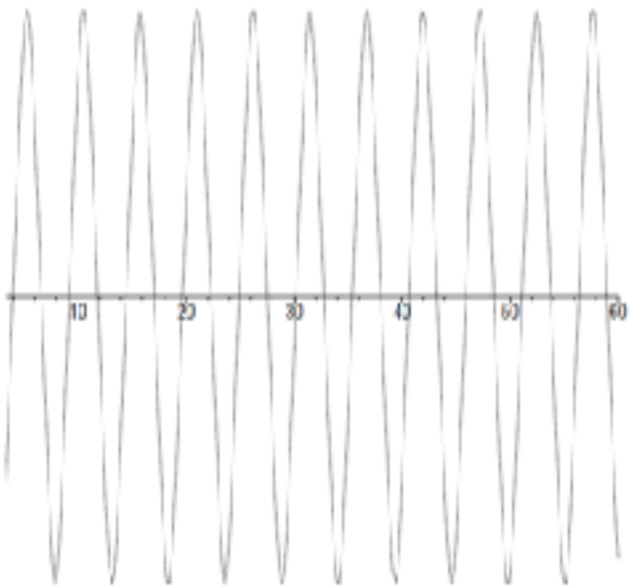
$$\frac{6}{\varkappa}\frac{a''}{a^3} = 4\bar{\epsilon} + T^{\mu,\mathrm{U}}_\mu\,, \qquad T^{\mu,\mathrm{U}}_\mu = \frac{3b}{16\pi^2 a^4}\left[(U')^2 - \frac{1}{4}U^4\right],$$

$$\frac{\partial}{\partial\eta}\left(U' \ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4\lambda^4}\right) + \frac{1}{2}U^3 \ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4\lambda^4} = 0$$

CE condensate on non-stationary (FLRW) background

$$Q \equiv \frac{32}{11}\pi^2 e(\xi\Lambda_{\rm QCD})^{-4} T^{\mu}_{\mu}[U]$$
$$= 6e\left[(U')^2 - \frac{1}{4}U^4\right] a^{-4}(\xi\Lambda_{\rm QCD})^{-4}$$

Classical YM condensate



"Radiation" medium

$$\epsilon_{\rm YM} \propto 1/a^4$$

Unstable solution!

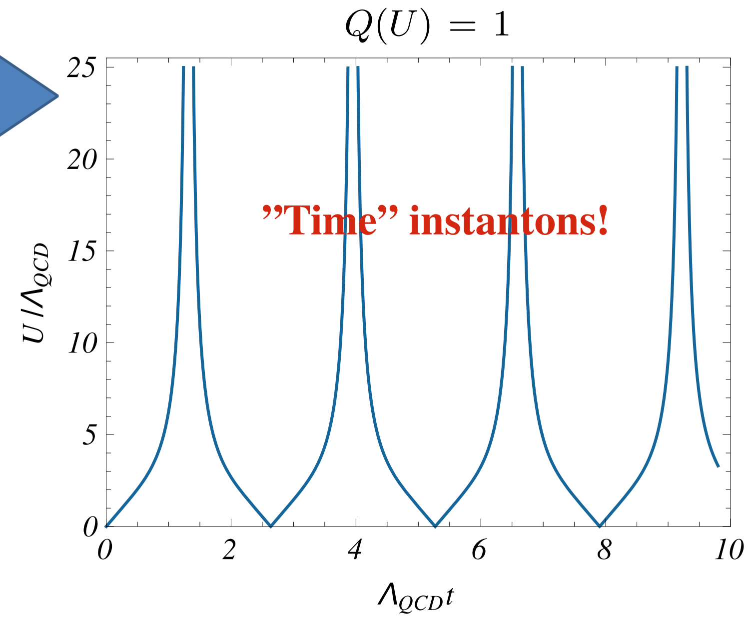
Exact partial solution:

$$|Q| = 1$$

Quantum corrections

QCD vacuum:
a ferromagnetic undergoing spontaneous magnetisation (Pagels&Tomboulis)

Savvidy (CE) vacuum



Asymptotic tracker solution!

$$\epsilon_{\rm CE} \to +{\rm const} \quad t \to \infty$$

Stable solution!

- **In fact, both chromoelectric and chromomagnetic condensates are stable on non-stationary (FLRW) background of expanding Universe**

“Mirror” symmetry of the ground state

In a vicinity of the ground state, the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2} \qquad \mathcal{J} \simeq \mathcal{J}^*$$

is invariant under

$$\mathbb{Z}_2: \qquad \mathcal{J}^* \longleftrightarrow -\mathcal{J}^*, \quad \bar{g}^2(\mathcal{J}^*) \longleftrightarrow -\bar{g}^2(\mathcal{J}^*), \quad \beta(\bar{g}_*^2) \longleftrightarrow -\beta(\bar{g}_*^2)$$

For pure gluodynamics at one-loop: $\beta_{(1)} = -\frac{bN}{48\pi^2}\,\bar{g}^2_{(1)} \qquad b = 11$

$$\alpha_s = \frac{\bar{g}^2}{4\pi} \qquad \alpha_{\rm s}(\mu^2) = \frac{\alpha_{\rm s}(\mu_0^2)}{1 + \beta_0\,\alpha_{\rm s}(\mu_0^2)\ln(\mu^2/\mu_0^2)} \qquad \mu^2 \equiv \sqrt{|\mathcal{J}|}$$

Choosing the ground state value of the condensate $\mu_0^2 \equiv \sqrt{|\mathcal{J}^*|}$ **as the physical scale**

we observe that the mirror symmetry, indeed, holds provided

$$\mathcal{J} \simeq \mathcal{J}^* \qquad \alpha_s(\mu_0^2) \longleftrightarrow -\alpha_s(\mu_0^2)$$

i.e. in the ground state only!

Heterogenic quantum YM ground state: two-scale vacuum

The running coupling at one-loop

$$\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1+\frac{bN}{96\pi^2}\bar{g}_1^2(\mu_0^4)\ln(|\mathcal{J}|/\mu_0^4)} = \frac{96\pi^2}{bN\ln(|\mathcal{J}|/\lambda_\pm^4)}$$

$$\mathcal{L}_{\rm eff}^{(1)} = \frac{bN}{384\pi^2}\mathcal{J}\ln\left(\frac{|\mathcal{J}|}{\lambda_\pm^4}\right)$$

with two energy scales

$$\boxed{\lambda_\pm^4 \equiv |\mathcal{J}^*|\exp\left[\mp\frac{96\pi^2}{bN|\bar{g}_1^2(\mathcal{J}^*)|}\right] \qquad |\mathcal{J}^*| = \lambda_+^2\lambda_-^2}$$

CE vacuum: $\beta(\bar{g}_*^2) \;=\; 2$

CM vacuum: $\beta(\bar{g}_*^2) \;=\; -2$

Mirror symmetry

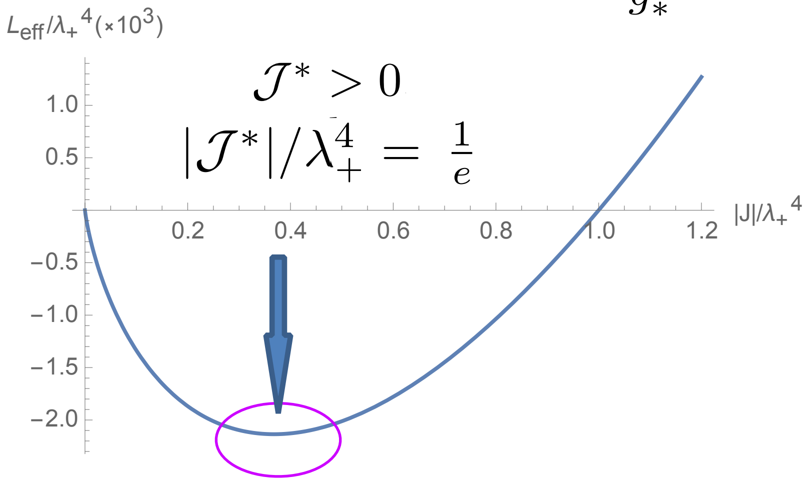
e.o.m. is automatically satisfied!

Reduces to the standard YM e.o.m. discussed in e.g. in instanton theory

$$\overrightarrow{\mathcal{D}}_\nu^{ab}\left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2}\right] = 0, \quad \bar{g}^2 \simeq \bar{g}_*^2$$

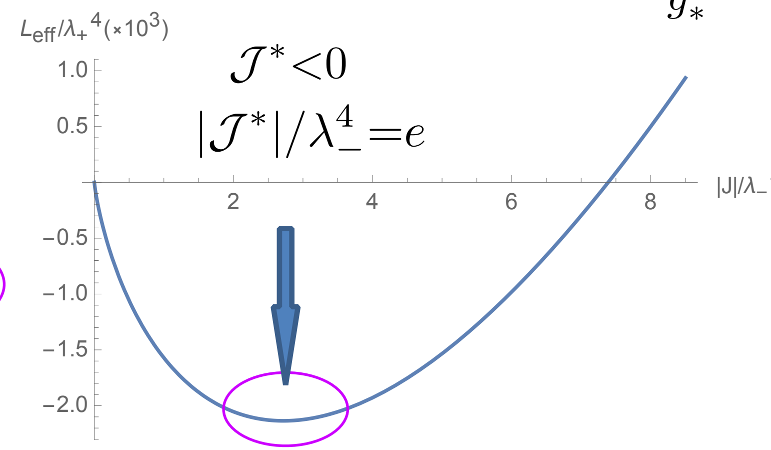
Trace anomaly: $T^{\mu}_{\mu,{\rm CE}} = -\frac{1}{\bar{g}_*^2}\mathcal{J}^*$

Trace anomaly: $T^{\mu}_{\mu,{\rm CM}} = +\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



One-loop:

$$\lambda_+^2/\lambda_-^2 = e$$

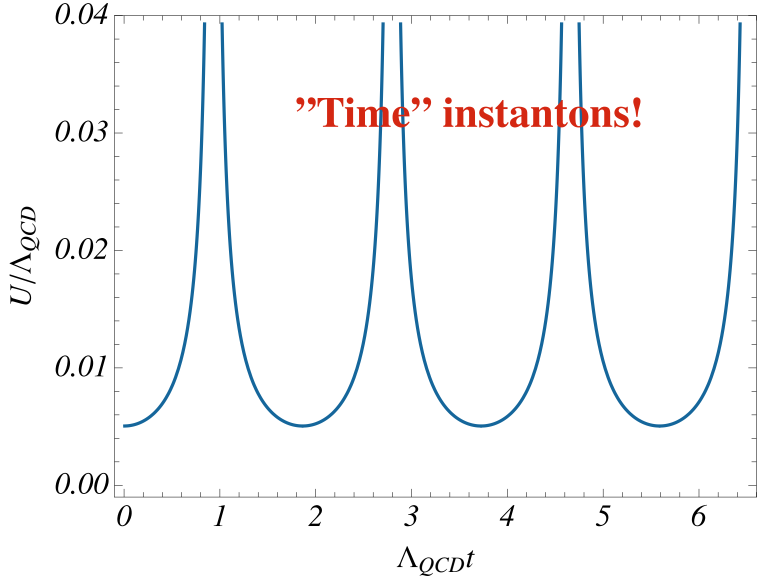


Cosmological CE attractor

Cosmological CM attractor

Cosmological evolution and vacua cancellation

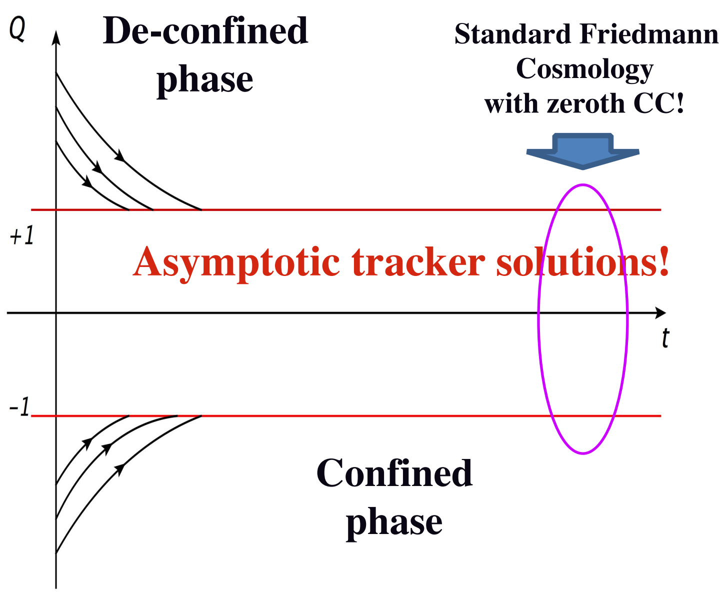
$Q(U) = -1$



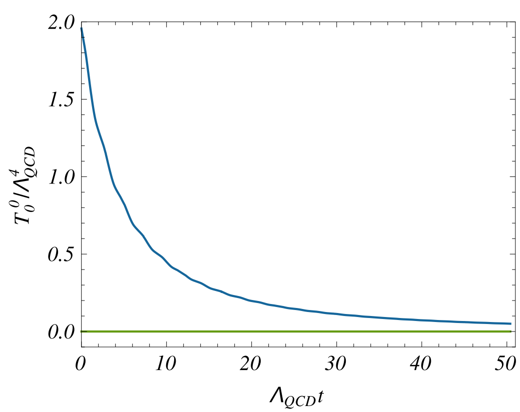
$$\epsilon_{\rm vac} \equiv \frac{1}{4}\langle T^{\mu}_{\mu}\rangle_{\rm vac} = \mp \mathcal{L}_{\rm eff}(\mathcal{J}^*)$$

$$\boxed{\epsilon^{\rm CE}_{\rm vac}\Big|_{\mathcal{J}^*>0} + \epsilon^{\rm CM}_{\rm vac}\Big|_{\mathcal{J}^*<0} \equiv 0}$$

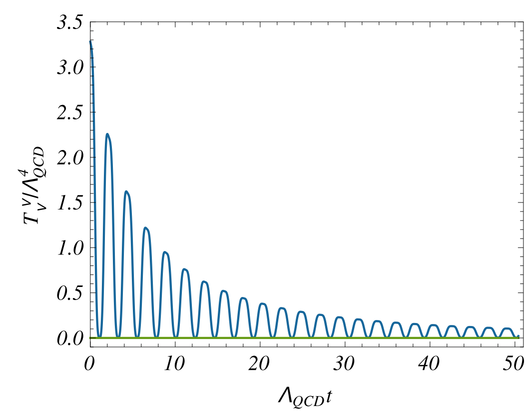
Exact compensation of CM and CE vacua as soon as the cosmological attractor is achieved!



CE energy density



CE EMT trace



System with very unusual dynamical properties!

Summary

- No ghost problem associated with negative coupling due to:
  (i) only gauge invariant quantities are used
  (ii) local loss of Lorentz (e.g. rotational) invariance

- Nielsen-Olsen proof of instability of CE condensate on a rigid Minkowski in NOT in contradiction with our results: we consider YM evolution on a dynamical (FLRW) spacetime while equilibrium is achieved only asymptotically.

- A possible decay of CE condensate into an anisotropic vacuum after a cosmological relaxation time would be exponentially suppressed and is practically never realised

- Even starting from an initial non-zero energy-density, the evolution of localised 3-space "pockets" of the CE and CM condensates trigger a mutual screening, flowing towards a zero-energy density attractor and accompanying by a formation of the domain walls corresponding to an asymptotic restoration of the Z2 (Mirror) symmetry and effectively protecting the "false" CE vacua pockets from further decay

- The vacua cancellation mechanism seems to naturally marry the existing confinement pictures related to a formation of a network of t'Hooft monopoles or chromovortices. In this approach, the scalar kink profile may correspond the J-invariant whose change may be related to the presence of monopole or vortex solutions localise inside the space-time domain walls. This implies the existence of space-time solitonic objects of a new type.