University of Cyprus

Department of Physics

XIIIth Quark confinement and the hadron spectrum

"One-loop lattice study of composite bilinear operators in Supersymmetric QCD"

Marios Costa

August 3, 2018

General Outline

General Outline

1. Introduction to SQCD
2. Introduction to Perturbation Theory

General Outline

Supersymmetric gauge action with matter fields

The construction of the Lagrangian of SQCD involves chiral superfields Φ and vector superfields V.

In superspace notation the chiral superfield Φ in terms of component fields is:

$$\begin{aligned}\Phi(x;\theta,\bar\theta) &= A(y)+\sqrt{2}\,\theta\psi(y)+\theta\theta\,F(y) \qquad\qquad (y^\mu\equiv x^\mu+i\theta\sigma^\mu\bar\theta)\\ &= A(x)+\sqrt{2}\,\theta\psi(x)+\theta\theta\,F(x)+i\theta\,\sigma^\mu\,\bar\theta\,\partial_\mu A(x)\\ &\quad+\frac{i}{\sqrt{2}}\theta\theta\,\bar\theta\,\bar\sigma^\mu\,\partial_\mu\psi(x)+\frac{1}{4}\,\theta\theta\,\bar\theta\bar\theta\,\partial_\mu\partial^\mu A(x),\end{aligned}$$

and the general form of a vector superfield $V(x,\theta,\bar\theta)$ is:

$$\begin{aligned}V(x;\theta,\bar\theta) &= C(x)+i\theta\chi(x)-i\bar\theta\bar\chi(x)+\frac{i}{2}\theta\theta\left[M(x)\,+\,iN(x)\right]\\ &\quad-\frac{i}{2}\bar\theta\bar\theta\left[M(x)-iN(x)\right]-\theta\sigma^\mu\,\bar\theta\,u_\mu(x)+i\theta\theta\,\bar\theta\left[\bar\lambda(x)+\frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right]\\ &\quad-i\bar\theta\bar\theta\,\theta\left[\lambda(x)\,+\,\frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right]\\ &\quad+\frac{1}{2}\,\theta\theta\,\bar\theta\bar\theta\left[D(x)\,+\,\frac{1}{2}\partial_\mu\partial^\mu C(x)\right].\end{aligned}$$

Supersymmetric gauge action with matter fields

- A supersymmetric gauge transformation may be applied on both the chiral and vector superfields and we will require the Lagrangian to be invariant under this transformation.
- We can choose a special gauge where the components C, χ, M, N are zero. This defines the Wess-Zumino (WZ) gauge. A vector superfield in the WZ gauge reduces to the form:

$$V(x;\theta,\bar{\theta}) = -\theta\,\sigma^{\mu}\,\bar{\theta}\,u_{\mu}(x) + i\theta\theta\,\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\,\theta\lambda(x) + \frac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta}\,D(x)\ .$$

Supersymmetric gauge action with matter fields

In order to obtain a renormalizable theory, we need to construct a Lagrangian with products of superfields having dimensionality ≤ 4; in addition, we require Lorentz invariance as well as invariance under supersymmetric gauge transformations:

$$\begin{aligned} \Phi'_+ &= e^{-i\Lambda}\Phi_+ \\ \Phi'_- &= \Phi_- e^{i\Lambda} \\ e^{2g V'} &= e^{-i\Lambda^\dagger} e^{2g V} e^{i\Lambda}, \end{aligned}$$

where $\Lambda(x;\theta,\bar\theta)$ is an arbitrary chiral superfield:
$\Lambda(x;\theta,\bar\theta) = \Lambda_0(y) + \sqrt{2}\theta\Lambda_1(y) + \theta\theta\Lambda_2(y)$. The special case in which
$\Lambda_1 = \Lambda_2 = 0 \to \Lambda(x;\theta,\bar\theta) = \Lambda_0(y) = \Lambda_0(x) + i\theta\sigma^\mu\bar\theta\partial_\mu\Lambda_0(x) + \frac{1}{4}\theta\theta\,\bar\theta\bar\theta\,\Box\Lambda_0(x)$,
where $\Lambda_0(x) = \Lambda_0^\dagger(x)$, amount to ordinary gauge transformations, which do not take us out of the WZ gauge:

$$\begin{aligned} A'_+ &= G^{-1}A_+, & \psi'_+ &= G^{-1}\psi_+, & F'_+ &= G^{-1}F_+ \;\; (G(x)\equiv e^{i\Lambda_0(x)}) \\ A'_- &= A_- G, & \psi'_- &= \psi_- G, & F'_- &= F_- G \\ u'_\mu &= G^{-1}u_\mu G + \frac{i}{g}(\partial_\mu G^{-1})G, & \lambda' &= G^{-1}\lambda G, & D' &= G^{-1}DG. \end{aligned}$$

Supersymmetric gauge action with matter fields

A Lagrangian, which respects these trasformations, in terms of superfields is:

$$\begin{aligned}\mathcal{L} &= \frac{1}{16kg}\mathrm{Tr}(W^{\alpha}\, W_{\alpha}|_{\theta\theta} + \bar{W}_{\dot{\alpha}}\, \bar{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}) \\ &+(\Phi_{+}^{\dagger}\, \mathrm{e}^{2\mathrm{g}\,\mathrm{V}}\, \Phi_{+} + \Phi_{-}\, \mathrm{e}^{-2\mathrm{g}\,\mathrm{V}}\, \Phi_{-}^{\dagger})|_{\theta\theta\bar{\theta}\bar{\theta}} \\ &+m(\Phi_{-}\Phi_{+}|_{\theta\theta} + \Phi_{+}^{\dagger}\Phi_{-}^{\dagger}|_{\bar{\theta}\bar{\theta}}),\end{aligned}$$

where $\mathrm{Tr}(T^{\alpha}T^{\beta}) = k\,\delta^{\alpha\,\beta}$, $W_{\alpha} = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\, e^{-2g\,V}\,\mathcal{D}_{\alpha}\, e^{2g\,V}$ is the supersymmetric field strength, and the supersymmetric covariant derivative is defined as: $\mathcal{D}_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} + i\sigma^{\mu}_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\,\partial_{\mu}$, $\bar{\mathcal{D}}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\alpha}\,(\sigma^{\mu})_{\alpha\dot{\alpha}}\,\partial_{\mu}$.
Combining the components of Φ_{+} with Φ_{-} we can construct a 4 component Dirac Spinor (ψ_{D}).

Supersymmetric gauge action with matter fields

We conclude that the SQCD Lagrangian for $\mathcal{N} = 1$ supersymmetry in 4 dimensions contains, for each flavor of matter fields, two complex scalars (squarks) A_+, A_-, a Dirac spinor (quark) $\{\psi_+, \psi_-\}$, and two auxiliary complex scalars F_+, F_-; in addition, the Lagrangian contains a gauge field (gluon) u_μ, a Majorana spinor (gluino) λ and one further real auxiliary field D. Taking the corresponding components of the superfields (appropriate powers of θ and $\bar{\theta}$), the continuum Lagrangian in the WZ gauge is:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SQCD}} = & -\frac{1}{4}u^{(\alpha)}_{\mu\nu}u^{\mu\nu(\alpha)} + \frac{1}{2}D^{(\alpha)}D^{(\alpha)} - i\bar{\lambda}^{(\alpha)}\bar{\sigma}^\mu\mathcal{D}_\mu\lambda^{(\alpha)} \\
& -\mathcal{D}_\mu A_+^\dagger\mathcal{D}^\mu A_+ - \mathcal{D}_\mu A_-^\dagger\mathcal{D}^\mu A_- - i\bar{\psi}_+\bar{\sigma}^\mu\mathcal{D}_\mu\psi_+ - i\bar{\psi}_-\bar{\sigma}^\mu\mathcal{D}_\mu\psi_- + F_+^\dagger F_+ \\
& +F_-^\dagger F_- + i\sqrt{2}g\big(A_+^\dagger\lambda^{(\alpha)}T^{(\alpha)}\psi_+ - \bar{\psi}_+\bar{\lambda}^{(\alpha)}T^{(\alpha)}A_+ + A_-\bar{\lambda}^{(\alpha)}T^{(\alpha)}\bar{\psi}_- \\
& -\psi_-\lambda^{(\alpha)}T^{(\alpha)}A_-\big) + g\big(A_+^\dagger D^{(\alpha)}T^{(\alpha)}A_+ - A_-D^{(\alpha)}T^{(\alpha)}A_-^\dagger\big) \\
& +m\big(A_-F_+ + F_-A_+ - \psi_-\psi_+ + A_+^\dagger F_-^\dagger + F_+^\dagger A_-^\dagger - \bar{\psi}_+\bar{\psi}_-\big)\,.
\end{aligned}
$$

Supersymmetric gauge action with matter fields

$\mathcal{L}_{\rm SQCD}$ is invariant, up to a total derivative, under the supersymmetric transformations (ξ is a Majorana spinor parameter):

$$
\begin{aligned}
\delta_\xi A_+ &= \sqrt{2}\xi\psi_+ \,, \\
\delta_\xi A_- &= \sqrt{2}\psi_-\xi \,, \\
\delta_\xi \psi_{+a} &= i\sqrt{2}\sigma^\mu_{a\dot{b}}\bar{\xi}^{\dot{b}}\mathcal{D}_\mu A_+ + \sqrt{2}\xi_a F_+ \,, \\
\delta_\xi \psi^a_- &= -i\sqrt{2}\bar{\xi}_{\dot{b}}\bar{\sigma}^{\dot{b}a\mu}\mathcal{D}_\mu A_- + \sqrt{2}F_-\xi^a \,, \\
\delta_\xi F_+ &= i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\mathcal{D}_\mu\psi_+ + 2igT^{(\alpha)}A_+\bar{\xi}\bar{\lambda}^{(\alpha)} \,, \\
\delta_\xi F_- &= -i\sqrt{2}\mathcal{D}_\mu\psi_-\sigma^\mu\bar{\xi} - 2igA_-T^{(\alpha)}\bar{\xi}\bar{\lambda}^{(\alpha)} \,, \\
\delta_\xi u^{(\alpha)}_\mu &= -i\bar{\lambda}^{(\alpha)}\bar{\sigma}^\mu\xi + i\bar{\xi}\bar{\sigma}^\mu\lambda^{(\alpha)} \,, \\
\delta_\xi \lambda^{(\alpha)} &= \sigma^{\mu\nu}\xi u^{(\alpha)}_{\mu\nu} + i\xi\, D^{(\alpha)} \,, \\
\delta_\xi D^{(\alpha)} &= -\xi\sigma^\mu\mathcal{D}_\mu\bar{\lambda}^{(\alpha)} - \mathcal{D}_\mu\lambda^{(\alpha)}\sigma^\mu\bar{\xi}.
\end{aligned}
$$

Supersymmetric gauge action with matter fields

- The auxiliary fields may now be eliminated, either by applying their equations of motion (classical case), or by functionally integrating over them (quantum case).
- The Lagrangian can be rewritten in 4 dimensions in Dirac notation and in the Weyl basis.
- The construction of the Euclidean action can be done after a Wick rotation.
- Introduction of a gauge-fixing term in the Lagrangian, along with a compensating Faddeev-Popov ghost term to avoid additional infinities which would appear upon functionally integrating over gauge orbits.

Supersymmetric gauge action with matter fields

- In our lattice calculation, we extend Wilson's formulation of the QCD action, to encompass SUSY partner fields as well.
- In this standard discretization quarks, squarks and gluinos live on the lattice sites, and gluons live on the links of the lattice: $U_\mu(x) = e^{igaT^\alpha u_\mu^\alpha(x+a\hat{\mu}/2)}$; α is a color index in the adjoint representation of the gauge group.
- In our ongoing investigation we plan to address also improved actions, so that we can check to what extent some of the SUSY breaking effects can be alleviated.

Supersymmetric gauge action with matter fields

For Wilson-type quarks (ψ) and gluinos (λ), the Euclidean action $\mathcal{S}^L_{\rm SQCD}$ on the lattice becomes ($A_\pm$ are the squark field components):

$$
\begin{aligned}
&\mathcal{S}^L_{\rm SQCD} = \\
&a^4 \sum_x \Big[\frac{N_c}{g^2} \sum_{\mu,\,\nu} \left(1 - \frac{1}{N_c}\,{\rm Tr} U_{\mu\,\nu}\right) + \sum_\mu {\rm Tr}\left(\bar\lambda_M \gamma_\mu \mathcal{D}_\mu \lambda_M\right) - a\frac{r}{2}{\rm Tr}\left(\bar\lambda_M \mathcal{D}^2 \lambda_M\right) \\
&+ \sum_\mu \left(\mathcal{D}_\mu A_+^\dagger \mathcal{D}_\mu A_+ + \mathcal{D}_\mu A_- \mathcal{D}_\mu A_-^\dagger + \bar\psi_D \gamma_\mu \mathcal{D}_\mu \psi_D\right) - a\frac{r}{2}\bar\psi_D \mathcal{D}^2 \psi_D \\
&+ i\sqrt{2} g \Big(A_+^\dagger \bar\lambda^\alpha_M T^\alpha P_+ \psi_D - \bar\psi_D P_- \lambda^\alpha_M T^\alpha A_+ \\
&\qquad + A_- \bar\lambda^\alpha_M T^\alpha P_- \psi_D - \bar\psi_D P_+ \lambda^\alpha_M T^\alpha A_-^\dagger \Big) \\
&+ \frac{1}{2} g^2 (A_+^\dagger T^\alpha A_+ - A_- T^\alpha A_-^\dagger)^2 - m(\bar\psi_D \psi_D - m A_+^\dagger A_+ - m A_- A_-^\dagger) \Big]
\end{aligned}
$$

- A standard "measure" term must be added to the action, in order to accound for the Jacobian in the change of integration variables: $U_\mu \to u_\mu$
- We have calculated the renormalization factors of the coupling constant (Z_g) and of the quark (Z_ψ), gluon (Z_u), gluino (Z_λ), squark ($Z_{A_\pm}$), ghost (Z_c) fields, quark mass (Z_{m_ψ}) and squark mass (Z_{m_A}) on the lattice. We also computed the critical values of the gluino, quark and squark masses.

M. Costa and H. Panagopoulos, "Supersymmetric QCD on the lattice: An exploratory study", Phys. Rev. D **96**, 034507 (2017).

Perturbation theory

- In perturbation theory the true dynamical variables are the gluonic fields u_μ's.
- In fact, when a link, in our action, is written in terms of the u_μ's, using: $U_\mu(x) = 1 + ig_0 a u_\mu(x + a\hat{\mu}/2) - \frac{1}{2!} g_0^2 a^2 u_\mu^2(x + a\hat{\mu}/2) + \cdots$, it becomes much more complicated.
- We can rewrite the action in two parts. A part which includes quadratic terms and a part with higher order ("interaction") terms.

$$S^{Total} = S^{Quadratic} + S^{Interactions}$$

- $S^{Interactions}$ consists of an infinite number of terms, which give rise to an infinite number of interaction vertices.
- Fortunately, only a finite number of vertices is needed at any given order in g_0.

Perturbation theory

- The Functional integral is a Gaussian integral after writing:

$$e^{-S^{Total}} = e^{-S^{Quatratic}}(1 - S^{Interactions} + \frac{(S^{Interactions})^2}{2!} + \cdots)$$

- Lattice perturbation theory is much more complicated than continuum perturbation theory: there are more fundamental vertices and more diagrams.
- The propagators and vertices, with which one builds the Feynman diagrams, are more complicated on the lattice than they are in the continuum, which can lead to expressions containing a huge number of terms.

Perturbation theory

- Perturbation theory involves an expansion in the coupling constant g_0, and is well-justified in high energy where the QCD coupling constant is small.
- It fails completely when the coupling is large and higher order corrections are larger than lower orders in the perturbative series.
- Lattice perturbative calculations are useful and necessary for the determinations of the:
  - Multiplicative renormalization factors of composite operators.
  - Multiplicative renormalization of the bare parameters of the Lagrangian.
  - Operator mixings on the lattice under renormalization.

  They also lead to useful results, when combined with non-perturbative investigations, for:
  - Additive renormalizations (critical mass).
  - Vacuum expectation values (expectation value of the plaquette).

Study of Composite Operators

In studying the properties of physical states, the main observables would be:

- Green's functions of operators made of squark fields $\mathcal{O}^A(x) = A^\dagger(x) A(x)$, etc.
- Green's functions of operators made of quark fields, having the form $\mathcal{O}_i^\psi(x) = \bar{\psi}(x)\Gamma_i\psi(x)$, where Γ_i denotes all possible distinct products of Dirac matrices. Different values of the index "i" lead to the following possibilities for the Γ matrices: (scalar) $\Gamma_S = 1$, (pseudoscalar) $\Gamma_P = \gamma_5$, (vector) $\Gamma_V = \gamma_\mu$, (axial vector) $\Gamma_{AV} = \gamma_5\gamma_\mu$ and tensor $\Gamma_T = [\gamma_\mu, \gamma_\nu]/2$.

Mixing of operators

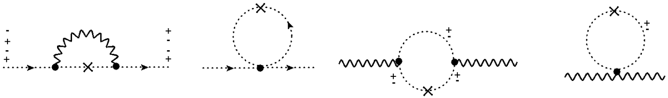
The bilinear operators, being composite, could in principle mix with three types of operators having the same quantum numbers.
The three types are as follows:

- Type I are gauge invariant operators.
- Type II are operators which are not gauge invariant but are the BRST variation of some other operators.
- Type III operators vanish by the equations of motion.

In this work we focus on extracting the mixing coefficients between quark, squark, gluino and gluon bilinears, which are relevant for physical external states and thus we do not take into account the ghost bilinears.

Mixing of Squark Bilinear operators

In order to calculate the one-loop rerormalizations and mixing coefficients relevant to the squark- and quark-bilinear operators, we must evaluate the corresponding Green's functions (=Feynman diagrams).



One-loop Feynman diagrams leading to the renormalization of dimension-2 squark bilinear operators and to the potential mixing coefficients among themselves and/or with gluon bilinears.

A cross corresponds to squark operators. A dotted line corresponds to squarks. A wavy line represents gluons. Squark lines are further marked with a $+(-)$ sign, to denote an A_+ (A_-) field. A squark line arrow entering (exiting) a vertex denotes a A_+ ($A_+^\dagger$) field; the opposite is true for A_- ($A_-^\dagger$) fields.

Renormalization and Mixing Patterns for Squark bilinears

- There are four squark operators which we denote as $O^A_{\pm\pm}$:

$$
\begin{aligned}
\mathcal{O}^A_{++}(x) &= A^\dagger_+(x)\,A_+(x)\\
\mathcal{O}^A_{+-}(x) &= A^\dagger_+(x)\,A^\dagger_-(x)\\
\mathcal{O}^A_{-+}(x) &= A_-(x)\,A_+(x)\\
\mathcal{O}^A_{--}(x) &= A_-(x)\,A^\dagger_-(x)
\end{aligned}
$$

- The renormalization condition is:

$$
\langle \tilde{A}^R_a \mathcal{O}^A_{b\,c}{}^R \tilde{A^R_d}^\dagger \rangle = (Z_A)^{-1/2}_{a\,a'} Z_{b'\,bcc'} \langle \tilde{A}^B_{a'} \mathcal{O}^A_{b'\,c'}{}^B \tilde{A^B_{d'}}^\dagger \rangle (Z_A)^{-1/2}_{d'\,d}
$$
$$
\mathcal{O}^A_{a\,b}{}^R = Z_{a'\,a\,b\,b'} \mathcal{O}^A_{a'\,b'}{}^B,
$$

where the indices $a, b, c, d, a', b', c', d'$ can take the values $+$ and $-$.In order to determine the renormalization factors for the squark bilinear operators, $\mathcal{O}^A_{\pm\pm}(x)$, we use the renormalization condition, requiring that the lhs be finite.

DR results of the amputated 1PI 2-pt Green's functions with external squark fields

We present the amputated 1PI 2-pt Green's functions with external squark fields in the continuum, where we regularize the theory in D dimensions ($D = 4 - 2\,\epsilon$):

$$\langle \tilde{A}_+(q)\mathcal{O}^A_{++}\tilde{A}^\dagger_+(q')\rangle_{DR} = \langle \tilde{A}^\dagger_-(q)\mathcal{O}^A_{--}\tilde{A}_-(q')\rangle_{DR} =$$

$$(2\pi)^4\delta(q-q')\left\{1+\frac{g^2\,C_F}{16\,\pi^2}\left[\frac{\alpha-1}{\epsilon}-2(\alpha-3)+(\alpha-1)\log\left(\frac{\bar{\mu}^2}{m^2}\right)+2(\alpha-3)\frac{m^2}{q^2}\log\left(1+\frac{q^2}{m^2}\right)\right]\right\}$$

$$\langle \tilde{A}_+(q)\mathcal{O}^A_{+-}\tilde{A}_-(q')\rangle_{DR} = \langle \tilde{A}^\dagger_-(q)\mathcal{O}^A_{-+}\tilde{A}^\dagger_+(q')\rangle_{DR} =$$

$$(2\pi)^4\delta(q-q')\left\{1+\frac{g^2\,C_F}{16\,\pi^2}\left[\frac{\alpha+1}{\epsilon}-2(\alpha-3)+(\alpha+1)\log\left(\frac{\bar{\mu}^2}{m^2}\right)+2(\alpha-3)\frac{m^2}{q^2}\log\left(1+\frac{q^2}{m^2}\right)\right]\right\}$$

At one-loop order, using the previous conditions, we find for the renormalization factors that:

$$Z^{DR,\overline{\mathrm{MS}}}_{++++} = Z^{DR,\overline{\mathrm{MS}}}_{----} = 1 + \frac{g^2\,C_F}{16\,\pi^2}\frac{2}{\epsilon}, \qquad Z^{DR,\overline{\mathrm{MS}}}_{++--} = Z^{DR,\overline{\mathrm{MS}}}_{--++} = 1$$

Thus in the 't Hooft-Veltman regularization scheme, with $\overline{\mathrm{MS}}$ renormalization, the operators $\mathcal{O}^A_{+-}$ and $\mathcal{O}^A_{-+}$ receive no corrections up to one loop; furthermore, there is no mixing between any of these operators.

$\overline{\mathrm{MS}}$-renormalized Green's functions

The continuum calculations are necessary:

- to compute the $\overline{\mathrm{MS}}$-renormalized Green's functions; the latter are relevant for the ensuing calculation of the corresponding Green's functions using lattice regularization and $\overline{\mathrm{MS}}$ renormalization. In this scheme, renormalization factors are simply defined in such a way as to only remove the pole parts.
- to check that the bare lattice Green's functions contain terms which diverge in the limit $a \to 0$; these divergent terms have a form similar to the continuum Green's functions, with two differences:
  - $\frac{1}{\epsilon} \to -\log(a^2)$
  - There are additional $\mathcal{O}\left(\frac{1}{a^2}\right)$ or $\mathcal{O}\left(\frac{1}{a}\right)$ contributions.

DR results of the amputated 1PI 2-pt Green's functions with external gluon fields

In order to check that there is no mixing with other Lorentz scalar dimension-2 gluon operators, we calculate the diagrams of the squark bilinear operators with external gluons. By studying the corresponding Green's functions we find that indeed this case receives no mixing, and thus flavor singlet squark operators cannot mix with the gluon bilinear $u_\mu u^\mu$. In particular, there was a cancelation of the pole part in the diagrams with external gluons leading to the expected result.
Indeed, the 2-pt Green's functions of these operators with external gluons turn out to be finite and transverse. Their numerical expressions are:

$$\langle \tilde{u}^\alpha_\mu(q)\mathcal{O}^A_{++}\tilde{u}^\beta_\nu(q')\rangle_{DR} = \langle \tilde{u}^\alpha_\mu(q)\tilde{\mathcal{O}}^A_{--}\tilde{u}^\beta_\nu(q')\rangle_{DR} =$$
$$-(2\pi)^4\delta(q+q')\delta^{\alpha\beta}\frac{g^2}{16\,\pi^2}\left(q^2\delta_{\mu\nu}-q_\mu q_\nu\right)\left[\frac{2}{q^2}-2\frac{\sqrt{4m^2+q^2}}{q^3}\log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\right]$$

Since $u_\mu u^\mu$ is not type I, II or III, the lattice regulator also does not allow mixing with this operator. We check the above, calculating the same quantities on the lattice. Our results coincide with those of the continuum.

Lattice results of the amputated 1PI 2-pt Green's functions with external squark fields

Regarding the lattice Green's functions of squark bilinear operators with external squarks, our results are:

$$\langle \tilde{A}_+(q)\mathcal{O}^A_{++}\tilde{A}^\dagger_+(q')\rangle_L = \langle \tilde{A}^\dagger_-(q)\mathcal{O}^A_{--}\tilde{A}_-(q')\rangle_L =$$

$$(2\pi)^4\delta(q-q')\left\{1+\frac{g^2\,C_F}{16\,\pi^2}\left[4+1.7920(\alpha-1)-(\alpha-1)\log\left(m^2\,a^2\right)+2(\alpha-3)\frac{m^2}{q^2}\log\left(1+\frac{q^2}{m^2}\right)\right]\right\}$$

$$\langle \tilde{A}_+(q)\mathcal{O}^A_{+-}\tilde{A}_-(q')\rangle_L = \langle \tilde{A}^\dagger_-(q)\mathcal{O}^A_{-+}\tilde{A}^\dagger_+(q')\rangle_L =$$

$$(2\pi)^4\delta(q-q')\left\{1+\frac{g^2\,C_F}{16\,\pi^2}\left[8+1.7920(1+\alpha)-(1+\alpha)\log\left(m^2a^2\right)+2(\alpha-3)\frac{m^2}{q^2}\log\left(1+\frac{q^2}{m^2}\right)\right]\right\}$$

Requiring that the above bare lattice Green's functions, upon renormalization, lead to the same expressions as the continuum ones, we arrive at the following lattice renormalization factors:

$$Z^{L,\overline{\rm MS}}_{++++} = Z^{L,\overline{\rm MS}}_{----} = 1-\frac{g^2\,C_F}{16\,\pi^2}\left(12.5586+2\log(a^2\bar{\mu}^2)\right)$$

$$Z^{L,\overline{\rm MS}}_{++--} = Z^{L,\overline{\rm MS}}_{--++} = 1-\frac{g^2\,C_F}{16\,\pi^2}\,(20.1462)$$

Mixing of Quark Bilinear operators

We list the type I operators in the next tables for the case of quark bilinears.

Operators	Category
$\bar{\psi}\psi$	S
$\bar{\psi}\gamma_5\psi$	P
$\bar{\psi}\gamma_\mu\psi$	V
$\bar{\psi}\gamma_5\gamma_\mu\psi$	AV
$\frac{1}{2}\bar{\psi}[\gamma_\mu, \gamma_\nu]\psi$	T
$A_+^\dagger A_+$	S
$A_- A_-^\dagger$	S
$A_+^\dagger A_-^\dagger + A_- A_+$	S
$A_+^\dagger A_-^\dagger - A_- A_+$	P
$mA_+^\dagger A_+$	S
$mA_- A_-^\dagger$	S
$m(A_+^\dagger A_-^\dagger + A_- A_+)$	S
$m(A_+^\dagger A_-^\dagger - A_- A_+)$	P
$A_+^\dagger D_\mu A_+$	V
$A_- D_\mu A_-^\dagger$	V
$A_+^\dagger D_\mu A_-^\dagger + A_- D_\mu A_+$	V
$A_+^\dagger D_\mu A_-^\dagger - A_- D_\mu A_+$	AV

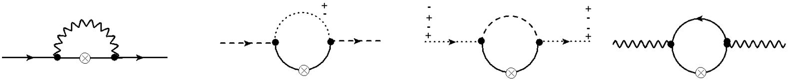
Quark bilinears and other operators with which these can mix, in the flavor non-singlet case. Operators with lower dimensionality will mix on the lattice.

Mixing of Quark Bilinear operators

Operators	Category
$\mathbb{1}$	S
$\bar{\lambda}\lambda$	S
$\bar{\lambda}\gamma_5\lambda$	P
$\bar{\lambda}\gamma_\mu\lambda$	V
$\bar{\lambda}\gamma_5\gamma_\mu\lambda$	AV
$\frac{1}{2}\bar{\lambda}[\gamma_\mu, \gamma_\nu]\lambda$	T

Additional operators which can mix with quark bilinears in the flavor singlet case.

Renormalization and Mixing Patterns for Quark bilinears



One-loop Feynman diagrams leading to the renormalization of dimension-3 quark bilinear operators and to the potential mixing coefficients with gluino, squark and gluon bilinears.

A circled cross corresponds to quark operators.

A dotted (dashed) line corresponds to squarks (gluinos). A wavy (solid) line represents gluons (quarks). Squark lines are further marked with a $+(-)$ sign, to denote an A_+ (A_-) field. A squark line arrow entering (exiting) a vertex denotes a A_+ ($A_+^\dagger$) field; the opposite is true for A_- ($A_-^\dagger$) fields.

We compute both in the continuum and on the lattice the matrix elements for quark bilinear operators. From these matrix elements we provide the renormalization of the quark bilinears and the mixing coefficients with gluino bilinear operators, as well as with operators made of gluon and of squark fields.

Renormalization and Mixing Patterns for Quark bilinears

We compute both in the continuum and on the lattice the matrix elements for quark bilinear operators. From these matrix elements we provide the renormalization of the quark bilinears and the mixing coefficients with gluino bilinear operators, as well as with operators made of gluon and of squark fields.

- From the diagram with external quarks we calculate the renormalization factors Z_i of the quark bilinear operators $\mathcal{O}_i^{\psi}$.

Renormalization and Mixing Patterns for Quark bilinears

We compute both in the continuum and on the lattice the matrix elements for quark bilinear operators. From these matrix elements we provide the renormalization of the quark bilinears and the mixing coefficients with gluino bilinear operators, as well as with operators made of gluon and of squark fields.

- From the diagram with external quarks we calculate the renormalization factors Z_i of the quark bilinear operators $\mathcal{O}_i^{\psi}$.
- The other diagrams contribute to the mixing coefficients z_i^{λ}, $z_i^{\pm\,\pm}$, $z_i^{\pm D \pm}$, $z_i^{m\pm\pm}$, z_i^{u}, with gluino, squark (involving zero or one derivatives, or one power of the mass) and gluon bilinear operators, respectively.

The expressions relevant for the mixing of each quark bilinear assume the following forms:

$$
\begin{aligned}
\mathcal{O}_S^{\psi\,R} &= Z_S \mathcal{O}_S^{\psi\,B} + z_S^{\lambda} \mathcal{O}_S^{\lambda\,B} + z_S^{++} \mathcal{O}^A_{+\,+}{}^B + z_S^{+-} (\mathcal{O}^A_{+\,-}{}^B + \mathcal{O}^A_{-\,+}{}^B) + z_S^{--} \mathcal{O}^A_{-\,-}{}^B \\
&\quad + z_S^{m++} m\mathcal{O}^A_{+\,+}{}^B + z_S^{m+-} (m\mathcal{O}^A_{+\,-}{}^B + m\mathcal{O}^A_{-\,+}{}^B) + z_S^{m--} m\mathcal{O}^A_{-\,-}{}^B \\
\mathcal{O}_P^{\psi\,R} &= Z_P \mathcal{O}_P^{\psi\,B} + z_P^{\lambda} \mathcal{O}_P^{\lambda\,B} + z_P^{+-} (\mathcal{O}^A_{+\,-}{}^B - \mathcal{O}^A_{-\,+}{}^B) + z_P^{m+-} (m\mathcal{O}^A_{+\,-}{}^B - m\mathcal{O}^A_{-\,+}{}^B) \\
\mathcal{O}_V^{\psi\,R} &= Z_V \mathcal{O}_V^{\psi\,B} + z_V^{\lambda} \mathcal{O}_V^{\lambda\,B} + z_V^{+D+} A_+^\dagger D_\mu A_+ + z_V^{-D-} A_- D_\mu A_-^\dagger + z_V^{+D-} (A_+^\dagger D_\mu A_-^\dagger + A_- D_\mu A_+) \\
&\quad + z_V^{u} u_\mu \partial_\nu u_\nu \\
\mathcal{O}_{AV}^{\psi\;\,R} &= Z_{AV} \mathcal{O}_{AV}^{\psi\;\,B} + z_{AV}^{\lambda} \mathcal{O}_{AV}^{\lambda\;\,B} + z_{AV}^{+D-} (A_+^\dagger D_\mu A_-^\dagger - A_- D_\mu A_+) + z_{AV}^{u} \epsilon_{\mu\,\nu\,\rho\,\sigma} u_\nu \partial_\rho u_\sigma \\
\mathcal{O}_T^{\psi\,R} &= Z_T \mathcal{O}_T^{\psi\,B} + z_T^{\lambda} \mathcal{O}_T^{\lambda\,B}
\end{aligned}
$$

On the rhs of these Equations there appear all operators that can possibly mix with those on the lhs; the tree-level Green's functions of these mixing operators naturally show up in the results for the one-loop Green's functions of the quark operators, thus allowing us to deduce the corresponding mixing coefficients.

Renormalization and Mixing Patterns for Quark bilinears

- The determination of the renormalization factors can be achieved by imposing the renormalization conditions:

$$\langle \tilde{\psi}^R \mathcal{O}_i^{\psi\,R} \bar{\tilde{\psi}}^R \rangle = Z_i Z_\psi^{-1} \langle \tilde{\psi}^B \mathcal{O}_i^{\psi\,B} \bar{\tilde{\psi}}^B \rangle \,,$$

and demanding the lhs to be finite.

- In order to calculate the mixing coefficient, we concentrate on the Green's functions of $\mathcal{O}_i^{\psi}$ with external gluino, squark and gluon fields. Taking as an example the scalar quark operator with external squark fields, the renormalized Green's function will be given by:

$$\langle \tilde{A}^R \mathcal{O}_S^{\psi\,R} \tilde{A}^{R\dagger} \rangle = Z_S Z_A^{-1/2} \langle \tilde{A}^B \mathcal{O}_S^{\psi\,B} \tilde{A}^{B\dagger} \rangle Z_A^{-1/2} - \sum_{\alpha,\beta=+,-} z_S^{\alpha\,\beta} Z_A^{-1/2} \langle \tilde{A}^B \mathcal{O}^A_{\alpha\,\beta} \tilde{A}^{B\dagger} \rangle_{tree} Z_A^{-1/2}$$

Skip Results

Continuum Results

We first calculate the renormalization factors by computing the Green's functions with external quarks. The following expressions are our results in *DR*:

$$\begin{aligned}
\langle\tilde{\psi}^B(q)\mathcal{O}_S^{\psi\,B}\bar{\tilde{\psi}}^B(q')\rangle^{DR}_{\rm amp} &= (2\pi)^4\delta(q-q')\Bigg\{\mathbb{1}\Bigg[1+\frac{g^2\,C_F}{16\,\pi^2}\Bigg(\frac{3+\alpha}{\epsilon}+4+2\alpha+(3+\alpha)\log\left(\frac{\bar{\mu}^2}{m^2}\right)\\
&- (3+\alpha)\left(1+3\frac{m^2}{q^2}\right)\log\left(1+\frac{q^2}{m^2}\right)\Bigg)\Bigg]+4i\frac{g^2\,C_F}{16\,\pi^2}\alpha\,\not{q}\left(\frac{m}{q^2}+\frac{m^3}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\right)\Bigg\}\\
\langle\tilde{\psi}^B(q)\mathcal{O}_P^{\psi\,B}\bar{\tilde{\psi}}^B(q')\rangle^{DR}_{\rm amp} &= (2\pi)^4\delta(q-q')\gamma_5\Bigg[1+\frac{g^2\,C_F}{16\,\pi^2}\Bigg(\frac{3+\alpha}{\epsilon}+4+2\alpha+(3+\alpha)\log\left(\frac{\bar{\mu}^2}{m^2}\right)\\
&- (3+\alpha)\left(1+\frac{m^2}{q^2}\right)\log\left(1+\frac{q^2}{m^2}\right)\Bigg)\Bigg]\\
\langle\tilde{\psi}^B(q)\mathcal{O}_V^{\psi\,B}\bar{\tilde{\psi}}^B(q')\rangle^{DR}_{\rm amp} &= (2\pi)^4\delta(q-q')\Bigg\{\gamma_\mu\Bigg[1+\frac{g^2\,C_F}{16\,\pi^2}\alpha\Bigg(\frac{1}{\epsilon}+1+\log\left(\frac{\bar{\mu}^2}{m^2}\right)-\log\left(1+\frac{q^2}{m^2}\right)\\
&\quad -\frac{m^2}{q^2}+\frac{m^4}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\Bigg)\Bigg]\\
&\quad +q_\mu\,\not{q}\frac{g^2\,C_F}{16\,\pi^2}\alpha\left(4\frac{m^2}{q^2}-2\frac{1}{q^2}-4\frac{m^4}{q^6}\log\left(1+\frac{q^2}{m^2}\right)\right)\\
&\quad -i\frac{g^2\,C_F}{16\,\pi^2}q_\mu(6+2\alpha)\left(\frac{m}{q^2}-\frac{m^3}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\right)\Bigg\}
\end{aligned}$$

Continuum Results

$$
\begin{aligned}
\langle\tilde{\psi}^B(q)\mathcal{O}^{\psi}_{AV}{}^B\tilde{\bar{\psi}}^B(q')\rangle^{DR}_{\mathrm{amp}} \quad = \quad & (2\pi)^4\delta(q-q')\Bigg\{\gamma_5\gamma_\mu\Bigg[1+\frac{g^2\,C_F}{16\,\pi^2}\Bigg(\frac{\alpha}{\epsilon}+4+\alpha+\alpha\log\left(\frac{\bar{\mu}^2}{m^2}\right)-\alpha\log\left(1+\frac{q^2}{m^2}\right) \\
& -(2-\alpha)\frac{m^2}{q^2}+(2-\alpha)\frac{m^4}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\Bigg)\Bigg] \\
& +i\frac{g^2\,C_F}{16\,\pi^2}\gamma_5 q_\mu\left(2(1-\alpha)\frac{m}{q^2}-2(1-\alpha)\frac{m^3}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\right) \\
& -i\frac{g^2\,C_F}{16\,\pi^2}\gamma_5\gamma_\mu\,\not{q}\left(2(1-\alpha)\frac{m}{q^2}-2(1-\alpha)\frac{m^3}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\right) \\
& -\frac{g^2\,C_F}{16\,\pi^2}\gamma_5 q_\mu\,\not{q}\Bigg(2\alpha\frac{1}{q^2}-4(2-\alpha)\frac{m^2}{q^4}+4(1-\alpha)\frac{m^2}{q^4}\log\left(1+\frac{q^2}{m^2}\right) \\
& +4(2-\alpha)\frac{m^4}{q^6}\log\left(1+\frac{q^2}{m^2}\right)\Bigg)\Bigg\}
\end{aligned}
$$

Continuum Results

$$
\begin{aligned}
\langle \tilde{\psi}^B(q) \mathcal{O}_T^{\psi\,B} \tilde{\bar{\psi}}^B(q') \rangle^{DR}_{\rm amp} \quad = \quad & (2\pi)^4 \delta(q-q') \Bigg\{ \frac{1}{2}[\gamma_\mu, \gamma_\nu] \Bigg[1 + \frac{g^2\,C_F}{16\,\pi^2}\,(\alpha-1) \left(\frac{1}{\epsilon} + 2\frac{m^2}{q^2} + \log\left(\frac{\bar{\mu}^2}{m^2}\right) \right. \\
- \quad & \left. \left(1 + 2\frac{m^2}{q^2} + 2\frac{m^4}{q^4}\right) \log\left(1 + \frac{q^2}{m^2}\right) \right) \Bigg] \\
+ \quad & 4i \frac{g^2\,C_F}{16\,\pi^2} (\gamma_\mu q_\nu - \gamma_\nu q_\mu) \left(\frac{m}{q^2} - \frac{m^3}{q^4} \log\left(1 + \frac{q^2}{m^2}\right) \right) \\
- \quad & 4i \frac{1}{2} [\gamma_\mu, \gamma_\nu] \not{q} \left(\frac{m}{q^2} - \frac{m^3}{q^4} \log\left(1 + \frac{q^2}{m^2}\right) \right) \\
+ \quad & \frac{g^2\,C_F}{16\,\pi^2} (\gamma_\mu q_\nu - \gamma_\nu q_\mu) \not{q} \left(4(1-\alpha)\frac{m^2}{q^4} - 2(1-\alpha) \left(\frac{m^2}{q^4} + 2\frac{m^4}{q^6} \right) \log\left(1 + \frac{q^2}{m^2}\right) \right) \Bigg\}
\end{aligned}
$$

Continuum Results

The continuum renormalization factors for the quark bilinears in the $\overline{\mathrm{MS}}$ scheme are:

$$
\begin{aligned}
{Z_S}^{DR,\overline{\mathrm{MS}}} &= 1 - \frac{g^2\,C_F}{16\,\pi^2}\frac{1}{\epsilon} \\
{Z_P}^{DR,\overline{\mathrm{MS}}} &= 1 - \frac{g^2\,C_F}{16\,\pi^2}\frac{1}{\epsilon} \\
{Z_V}^{DR,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\,C_F}{16\,\pi^2}\frac{2}{\epsilon} \\
{Z_{AV}}^{DR,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\,C_F}{16\,\pi^2}\frac{2}{\epsilon} \\
{Z_T}^{DR,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\,C_F}{16\,\pi^2}\frac{3}{\epsilon}.
\end{aligned}
$$

Continuum Results

Similarly, taking into account the potential mixing with $\mathcal{O}_i^{\lambda}$, which appears only in the flavor singlet case, and the corresponding tree-level Green's functions we can determine z_i^{λ}. The expressions we obtain for $\langle \lambda^B \mathcal{O}_i^{\psi\,B} \bar{\lambda}^B \rangle_{\rm amp}^{DR}$ are shown here:

$$\langle \tilde\lambda^B(q) \mathcal{O}_S^{\psi\,B} \bar{\tilde\lambda}^B(q') \rangle_{\rm amp}^{DR} = (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} \left[i\not{q} \left(\frac{2m}{q^2} - \frac{8m^3}{q^3\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m} \right) \right) \right]$$

$$\langle \tilde\lambda^B(q) \mathcal{O}_P^{\psi\,B} \bar{\tilde\lambda}^B(q') \rangle_{\rm amp}^{DR} = 0$$

$$\begin{aligned} \langle \tilde\lambda^B(q) \mathcal{O}_V^{\psi\,B} \bar{\tilde\lambda}^B(q') \rangle_{\rm amp}^{DR} &= (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} \left[\gamma_\mu \left(1 + \frac{1}{2\epsilon} + \frac{1}{2} \log\left(\frac{\bar\mu^2}{m^2} \right) \right. \right. \\ &- \left. \frac{4m^2+q^2}{q\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m} \right) \right) \\ &+ \left. \not{q} q_\mu \left(-\frac{1}{q^2} + \frac{4m^2}{q^3\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m} \right) \right) \right] \end{aligned}$$

Continuum Results

$$\begin{aligned}
\langle \tilde\lambda^B(q) \mathcal{O}_{AV}^{\psi\, B} \bar{\tilde\lambda}^B(q') \rangle_{\rm amp}^{DR} &= (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} \Bigg[\gamma_5 \gamma_\mu \Bigg(-2 - \frac{1}{2\epsilon} - \frac{1}{2} \log\left(\frac{\bar\mu^2}{m^2}\right) \\
&+ \frac{8m^2+q^2}{q\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right) \Bigg) \\
&+ \gamma_5\, \not{q} q_\mu \left(\frac{1}{q^2} - \frac{4m^2}{q^3\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\right) \Bigg] \\
\langle \tilde\lambda^B(q) \mathcal{O}_{T}^{\psi\, B} \bar{\tilde\lambda}^B(q') \rangle_{\rm amp}^{DR} &= (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} \Bigg[\frac{1}{2}[\gamma_\mu, \gamma_\nu] i \not{q} \left(\frac{2m}{q^2} - \frac{8m^3}{q^3\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\right) \\
&- i(\gamma_\mu q_\nu - \gamma_\nu q_\mu) \left(\frac{2m}{q^2} - \frac{8m^3}{q^3\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\right) \Bigg]
\end{aligned}$$

Continuum Results

The Green's functions of each quark bilinear with external squarks, according to the choice of squark components, A_+ or A_-, are:

$$
\begin{aligned}
\langle \tilde{A}_+^B(q) \mathcal{O}_S^{\psi\,B} \tilde{A}_+^{\dagger\,B}(q') \rangle_{\rm amp}^{DR} &= \langle \tilde{A}_-^{\dagger\,B}(q) \mathcal{O}_S^{\psi\,B} \tilde{A}_-^B(q') \rangle_{\rm amp}^{DR} = (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} \Bigg[24m + 16m \Bigg(\frac{1}{\epsilon} + \log\left(\frac{\bar\mu^2}{m^2}\right) \\
&- \log\left(1 + \frac{q^2}{m^2}\right) - \frac{m^2}{q^2} \log\left(1 + \frac{q^2}{m^2}\right) \Bigg) \Bigg] \\
\langle \tilde{A}_+^B(q) \mathcal{O}_S^{\psi\,B} \tilde{A}_-^B(q') \rangle_{\rm amp}^{DR} &= \langle \tilde{A}_-^{\dagger\,B}(q) \mathcal{O}_S^{\psi\,B} \tilde{A}_+^{\dagger\,B}(q') \rangle_{\rm amp}^{DR} = (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} (-8m) \\
\langle \tilde{A}_+^B(q) \mathcal{O}_P^{\psi\,B} \tilde{A}_+^{\dagger\,B}(q') \rangle_{\rm amp}^{DR} &= \langle \tilde{A}_-^{\dagger\,B}(q) \mathcal{O}_P^{\psi\,B} \tilde{A}_-^B(q') \rangle_{\rm amp}^{DR} = 0 \\
\langle \tilde{A}_+^B(q) \mathcal{O}_P^{\psi\,B} \tilde{A}_-^B(q') \rangle_{\rm amp}^{DR} &= -\langle \tilde{A}_-^{\dagger\,B}(q) \mathcal{O}_P^{\psi\,B} \tilde{A}_+^{\dagger\,B}(q') \rangle_{\rm amp}^{DR} = (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2} (-8m) \\
\langle \tilde{A}_+^B(q) \mathcal{O}_V^{\psi\,B} \tilde{A}_+^{\dagger\,B}(q') \rangle_{\rm amp}^{DR} &= -\langle \tilde{A}_-^{\dagger\,B}(q) \mathcal{O}_V^{\psi\,B} \tilde{A}_-^B(q') \rangle_{\rm amp}^{DR} = (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2}\, i\, q_\mu \Bigg[\frac{32}{3} + \frac{8}{\epsilon} - 8\frac{m^2}{q^2} \\
&+ 8\log\left(\frac{\bar\mu^2}{m^2}\right) - 8\log\left(1 + \frac{q^2}{m^2}\right) + 8\frac{m^4}{q^4} \log\left(1 + \frac{q^2}{m^2}\right) \Bigg] \\
\langle \tilde{A}_+^B(q) \mathcal{O}_V^{\psi\,B} \tilde{A}_-^B(q') \rangle_{\rm amp}^{DR} &= \langle \tilde{A}_-^{\dagger\,B}(q) \mathcal{O}_V^{\psi\,B} \tilde{A}_+^{\dagger\,B}(q') \rangle_{\rm amp}^{DR} = (2\pi)^4 \delta(q-q') \frac{g^2}{16\,\pi^2}\, i\, q_\mu \left(-\frac{8}{3}\right)
\end{aligned}
$$

Continuum Results

$$\langle \tilde{A}_{+}^{B}(q)\mathcal{O}_{AV}^{\psi\ B}\tilde{A_{+}^{\dagger}}^{B}(q')\rangle_{\rm amp}^{DR} = (2\pi)^4\delta(q-q')\frac{g^2}{16\,\pi^2}\,i\,q_\mu\Bigg[-16-\frac{8}{\epsilon}-8\frac{m^2}{q^2}-8\log\left(\frac{\bar{\mu}^2}{m^2}\right)+8\log\left(1+\frac{q^2}{m^2}\right) + 16\frac{m^2}{q^2}\log\left(1+\frac{q^2}{m^2}\right)+8\frac{m^4}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\Bigg]$$

$$\langle \tilde{A_{-}^{\dagger}}^{B}(q)\mathcal{O}_{AV}^{\psi\ B}\tilde{A}_{-}^{B}(q')\rangle_{\rm amp}^{DR} = (2\pi)^4\delta(q-q')\frac{g^2}{16\,\pi^2}\,i\,q_\mu\Bigg[-16-\frac{8}{\epsilon}-8\frac{m^2}{q^2}-8\log\left(\frac{\bar{\mu}^2}{m^2}\right)+8\log\left(1+\frac{q^2}{m^2}\right) + 16\frac{m^2}{q^2}\log\left(1+\frac{q^2}{m^2}\right)+8\frac{m^4}{q^4}\log\left(1+\frac{q^2}{m^2}\right)\Bigg]$$

$$\langle \tilde{A}_{+}^{B}(q)\mathcal{O}_{AV}^{\psi\ B}\tilde{A}_{-}^{B}(q')\rangle_{\rm amp}^{DR} = \langle \tilde{A_{-}^{\dagger}}^{B}(q)\mathcal{O}_{AV}^{\psi\ B}\tilde{A_{+}^{\dagger}}^{B}(q')\rangle_{\rm amp}^{DR} = 0$$

$$\langle \tilde{A}_{+}^{B}(q)\mathcal{O}_{T}^{\psi\ B}\tilde{A_{+}^{\dagger}}^{B}(q')\rangle_{\rm amp}^{DR} = \langle \tilde{A_{-}^{\dagger}}^{B}(q)\mathcal{O}_{T}^{\psi\ B}\tilde{A}_{-}^{B}(q')\rangle_{\rm amp}^{DR} = 0$$

$$\langle \tilde{A}_{+}^{B}(q)\mathcal{O}_{T}^{\psi\ B}\tilde{A}_{-}^{B}(q')\rangle_{\rm amp}^{DR} = \langle \tilde{A_{-}^{\dagger}}^{B}(q)\mathcal{O}_{T}^{\psi\ B}\tilde{A_{+}^{\dagger}}^{B}(q')\rangle_{\rm amp}^{DR} = 0$$

Continuum Results

Lastly, we compute the gluon matrix elements of the quark bilinears:

$$\begin{aligned}
\langle \tilde{u}^B_\sigma(q) \mathcal{O}^{\psi\,B}_S \tilde{u}^B_\nu(q')\rangle^{DR}_{\rm amp} &= (2\pi)^4 \delta(q+q') \frac{g^2}{16\,\pi^2} \Bigg[\delta_{\sigma\,\nu} \Bigg(-4m + 16 \frac{m^3}{q\,\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\Bigg) \\
&+ q_\sigma q_\nu \left(4\frac{m}{q^2} - 16 \frac{m^3}{q^3\,\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\right)\Bigg] \\
\langle \tilde{u}^B_\sigma(q) \mathcal{O}^{\psi\,B}_V \tilde{u}^B_\nu(q')\rangle^{DR}_{\rm amp} &= (2\pi)^4 \delta(q+q') \frac{g^2}{16\,\pi^2} \Bigg[i \frac{q_\sigma \delta_{\nu\,\mu} - q_\nu \delta_{\sigma\,\mu}}{2} \Bigg(8 + 8\frac{m^2}{q^2} \\
&- 32 \left(\frac{m^2}{q\,\sqrt{4m^2+q^2}} + \frac{m^4}{q^3\,\sqrt{4m^2+q^2}} \right) \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\Bigg)\Bigg] \\
\langle \tilde{u}^B_\sigma(q) \mathcal{O}^{\psi\,B}_{AV} \tilde{u}^B_\nu(q')\rangle^{DR}_{\rm amp} &= (2\pi)^4 \delta(q+q') \frac{g^2}{16\,\pi^2} \Bigg[\epsilon_{\sigma\,\nu\,\mu\,\rho} i q_\rho \Bigg(2 + \frac{4}{\epsilon} + 4\log\left(\frac{\bar{\mu}^2}{m^2}\right) \\
&- \frac{16m^2+8q^2}{q\,\sqrt{4m^2+q^2}} \log\left(\frac{q+\sqrt{4m^2+q^2}}{2m}\right)\Bigg)\Bigg] \\
\langle \tilde{u}^B_\sigma(q) \mathcal{O}^{\psi\,B}_P \tilde{u}^B_\nu(q')\rangle^{DR}_{\rm amp} &= \langle \tilde{u}^B_\sigma(q) \mathcal{O}^{\psi\,B}_T \tilde{u}^B_\rho(q')\rangle^{DR}_{\rm amp} = 0
\end{aligned}$$

Continuum Results

Following the renormalization condition, and using the definition of Z_i and the renormalization of each field, we determine all mixing coefficients z_i^{λ}, $z_i^{\pm\pm}$ ($z_i^{\pm D \pm}$, $z_i^{m\pm\pm}$):

$$
\begin{aligned}
z_S^{\lambda} &= z_P^{\lambda} = z_T^{\lambda} = 0,\ z_V^{\lambda} = \frac{g^2}{16\,\pi^2}\frac{1}{\epsilon},\ z_{AV}^{\lambda} = -\frac{g^2}{16\,\pi^2}\frac{1}{\epsilon} \\
z_S^{+-} &= z_S^{-+} = z_S^{++} = z_S^{--} = z_P^{+-} = z_P^{-+} = 0 \\
z_V^{+D-} &= z_V^{-D+} = 0,\ z_V^{+D+} = \frac{g^2}{16\,\pi^2}\frac{8}{\epsilon},\ z_V^{-D-} = -\frac{g^2}{16\,\pi^2}\frac{8}{\epsilon} \\
z_{AV}^{+D-} &= z_{AV}^{-D+} = 0,\ z_{AV}^{+D+} = -\frac{g^2}{16\,\pi^2}\frac{8}{\epsilon},\ z_{AV}^{-D-} = -\frac{g^2}{16\,\pi^2}\frac{8}{\epsilon} \\
z_S^{m+-} &= z_S^{m-+} = z_P^{m+-} = 0,\ z_S^{m++} = z_S^{m--} = \frac{g^2}{16\,\pi^2}\frac{16}{\epsilon} \\
z_V^{u} &= z_{AV}^{u} = 0
\end{aligned}
$$

Lattice Preliminary Results

Having calculated the above quantities in the continuum, we proceed with the computation of the lattice Green's functions in order to extract the renormalization and mixing coefficients of to the quark bilinears. We present here the renormalization factors:

$$
\begin{aligned}
{Z_S}^{L,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\, C_F}{16\,\pi^2}\left(-13.1105 + \log(a^2\bar{\mu}^2)\right) \\
{Z_P}^{L,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\, C_F}{16\,\pi^2}\left(-22.7536 + \log(a^2\bar{\mu}^2)\right) \\
{Z_V}^{L,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\, C_F}{16\,\pi^2}\left(-20.7759 - 2\log(a^2\bar{\mu}^2)\right) \\
{Z_{AV}}^{L,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\, C_F}{16\,\pi^2}\left(-15.9544 - 2\log(a^2\bar{\mu}^2)\right) \\
{Z_T}^{L,\overline{\mathrm{MS}}} &= 1 + \frac{g^2\, C_F}{16\,\pi^2}\left(-17.1762 - 3\log(a^2\bar{\mu}^2)\right)
\end{aligned}
$$

Lattice Preliminary Results

The lattice one-loop expressions for the mixing coefficients are presented here.

$$
\begin{aligned}
z_S^{\lambda} &= z_P^{\lambda} = z_T^{\lambda} = 0,\ z_V^{\lambda} = \frac{g^2}{16\,\pi^2}\left(4.4839 - \log\left(a^2\bar{\mu}^2\right)\right),\ z_{AV}^{\lambda} = \frac{g^2}{16\,\pi^2}\left(5.7087 + \log\left(a^2\bar{\mu}^2\right)\right)\\
z_S^{+\,-} &= z_S^{-\,+} = \frac{g^2\,C_F}{16\,\pi^2}\,\frac{1}{a}8.9274r,\ z_S^{+\,+} = z_S^{-\,-} = -\frac{g^2\,C_F}{16\,\pi^2}\,\frac{1}{a}23.8429r\\
z_S^{m+\,+} &= z_S^{m-\,-} = -\frac{g^2\,C_F}{16\,\pi^2}\left(52.8968 + 16\log\left(a^2\bar{\mu}^2\right)\right),\ z_S^{m+\,-} = z_S^{m-\,+} = -\frac{g^2\,C_F}{16\,\pi^2}7.27797\\
z_P^{+\,-} &= -z_P^{-\,+} = \frac{g^2\,C_F}{16\,\pi^2}\,\frac{1}{a}32.7704r\\
z_P^{m+\,-} &= -z_P^{m-\,+} = \frac{g^2\,C_F}{16\,\pi^2}29.7772\\
z_V^{+D-} &= -z_V^{-D+} = \frac{g^2\,C_F}{16\,\pi^2}0.8693,\ z_V^{+D+} = -z_V^{-D-} = -\frac{g^2\,C_F}{16\,\pi^2}\left(5.6888 + 8\log\left(a^2\bar{\mu}^2\right)\right)\\
z_{AV}^{+D-} &= z_{AV}^{-D+} = 0,\ z_{AV}^{+D+} = -z_{AV}^{-D-} = \frac{g^2\,C_F}{16\,\pi^2}\left(14.6168 + 8\log\left(a^2\bar{\mu}^2\right)\right)\\
z_S^{u} &= -\frac{g^2}{16\,\pi^2}\,\frac{1}{a}3.8984r,\ z_V^{u} = z_{AV}^{u} = 0
\end{aligned}
$$

Conclusions – Summary of Results

- Calculation of the one-loop renormalization factors and mixing coefficients for local quark operators and dimension-2 squark operators both in the continuum and on the lattice.

- Construction of improved versions of the operators, but also removal $\mathcal{O}(g^2\, a^0)$ contributions (fine tuning/ counterterms) from future non-perturbative data.

The above perturbative estimates of the renormalization factors Z_i and of the mixing coefficients z_i can be used for the determination of the properly renormalized $\mathcal{O}_i^{\psi}$ and $\mathcal{O}^A$ matrix elements.

M. Costa and H. Panagopoulos, "Supersymmetric QCD: Renormalization and Mixing of Composite Operators", in preparation.

The END

Thanks for your attention!
Any Questions?