Understanding the dynamics of field theories far from equilibrium

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

European Research Council
Established by the European Commission
Supporting top researchers from anywhere in the world

Project: CGCglasmaQGP

Kirill Boguslavski

XIIIth Quark Confinement and the Hadron Spectrum

August 2, 2018

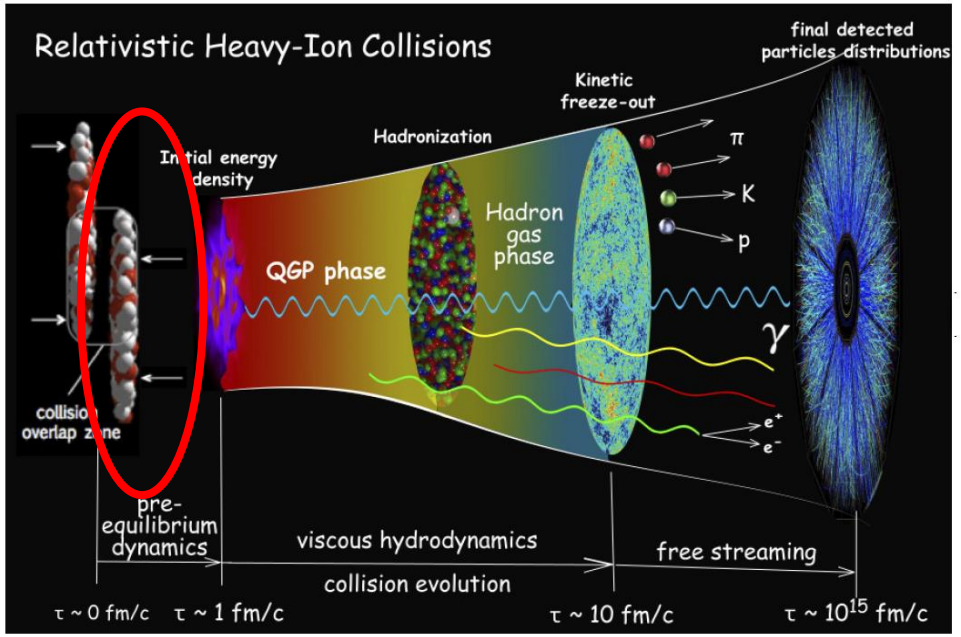
Motivation

Pre-hydro evolution

Hydrodynamical studies suggest:

- Quick onset of hydrodynamics
- Nearly ideal fluid

Romatschke, Romatschke (2007); Song, Heinz (2008); Schenke, Jeon, Gale (2011); Schenke, Tribedy, Venugopalan (2012); Gale, Jeon. Schenke, Tribedy, Venugopalan (2013)



Little Bang by P. Sorensen and C. Shen

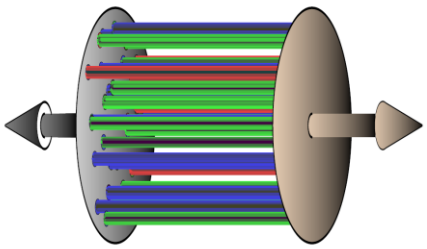
How does the created QCD matter evolve?

- Overview over the thermalization process in weak-coupling picture
- Focus on early phase: Nonthermal fixed points, universality with scalar theory
- Next step: Spectral functions

Thermalization dynamics at early times

Initial state: Glasma

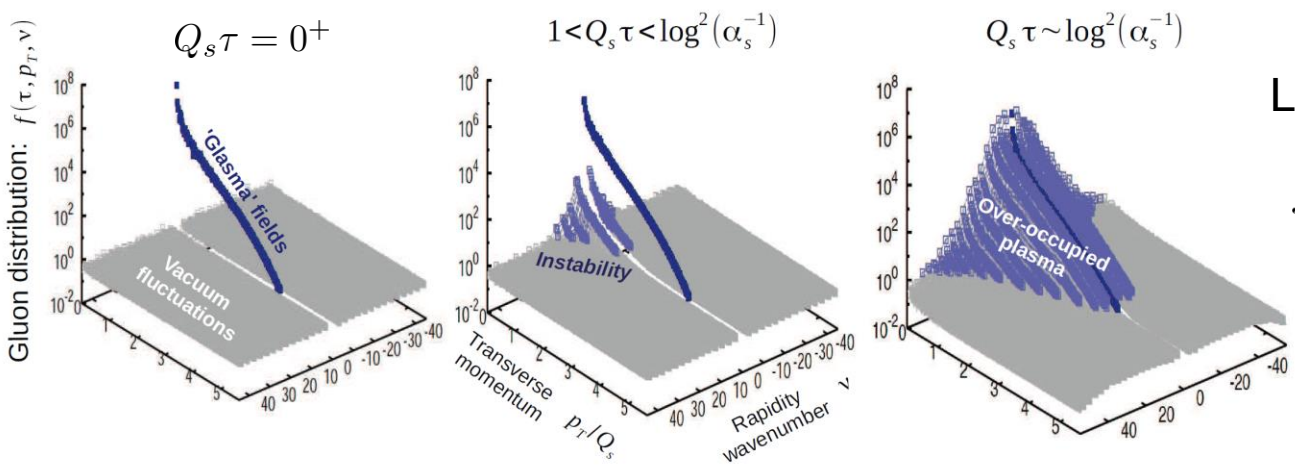
McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); Krasnitz, Nara, Venugopalan (2001, 2003); Lappi (2003, 2006, 2011); Lappi, McLerran (2006); …



Gelis, Iancu, Jalilian-Marian, Venugopalan, *Ann. Rev. Nucl. Part. Sci 60, 463 (2010)*

Plasma instabilities
(at early times)

Mrowczynski (1993); Arnold, Lenaghan, Moore (2003); Romatschke, Strickland (2003); Romatschke, Venugopalan (2006); Attems, Rebhan, Strickland (2012); Fukushima, Gelis (2012); Berges, Schlichting (2013); Epelbaum, Gelis (2013); ...

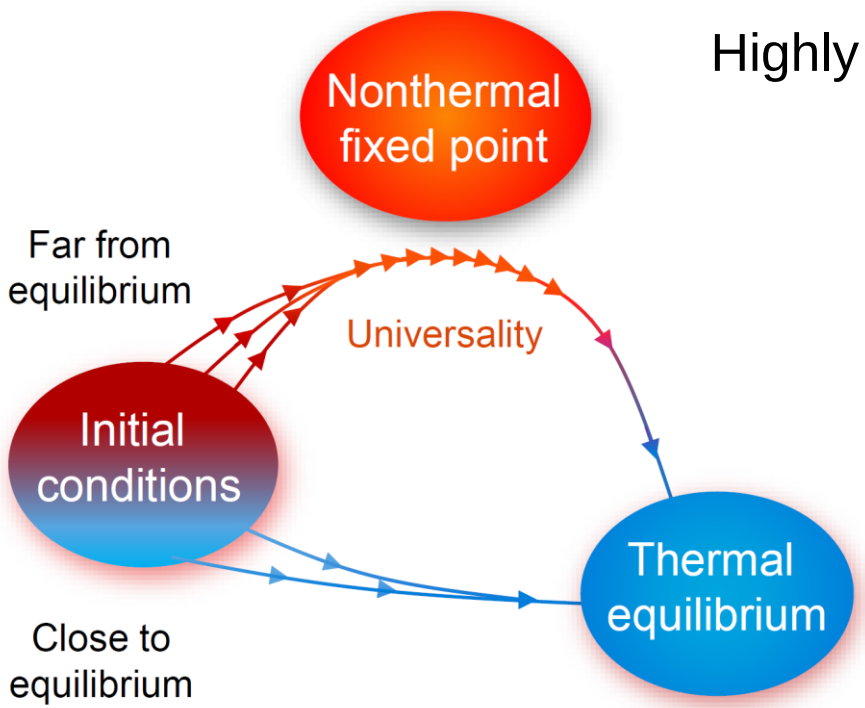


Large gluon distribution

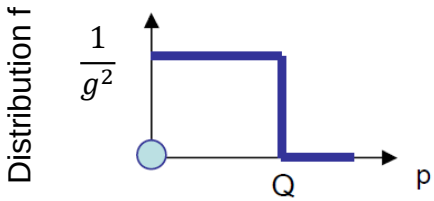
$$f = \frac{d^3N}{d^2p_T dp_z} \sim \frac{1}{\alpha_s} \gg 1$$

Berges, Schenke, Schlichting, Venugopalan, *Nucl. Phys. A 931, 348 (2014)*

Nonthermal fixed point (NTFP)



Highly occupied:



$$f(p,t_0) \sim \frac{1}{g^2}\,\Theta(Q-p)$$

Nonthermal fixed point (NTFP)

Micha, Tkachev, *PRD* 70, 043538 (2004)

Berges, Rothkopf, Schmidt, *PRL 101, 041603 (2008)*

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

(Single-particle) Distribution function: $f(p_t, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_T, \tau^{\gamma} p_z)$

Since typically $f \gg 1$ ⇨ can use classical lattice simulations *Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)*

Next stage of thermalization dynamics

NTFP of highly occupied gauge plasma

Berges, KB, Schlichting, Venugopalan, *PRD 89, 114007 (2014); PRD 89, 074011 (2014)*

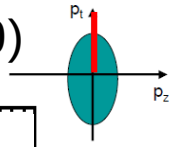
Reminder: *Self-similar evolution*

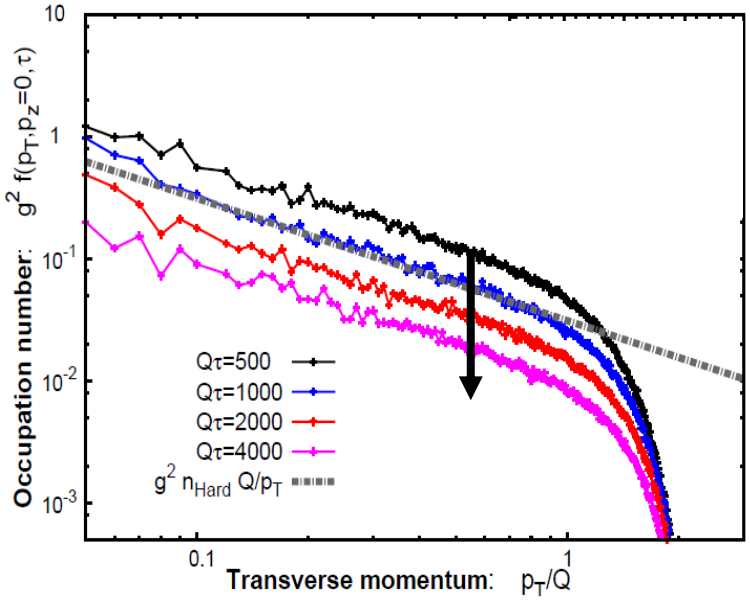
$$f(p_t, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Universal scaling exponents

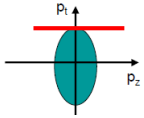
$$\alpha \simeq -2/3, \;\; \beta \simeq 0, \;\; \gamma \simeq 1/3$$

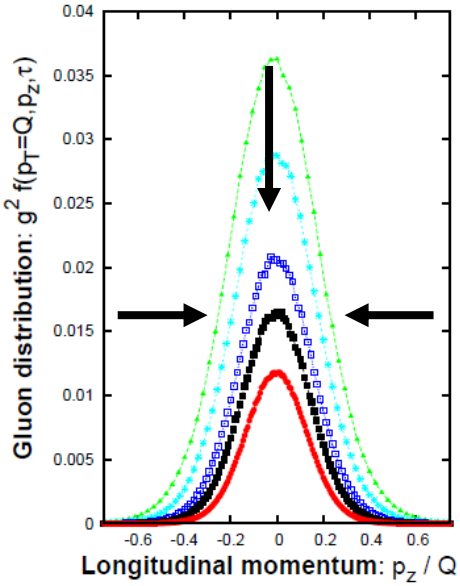
Transverse distribution $(p_z = 0)$

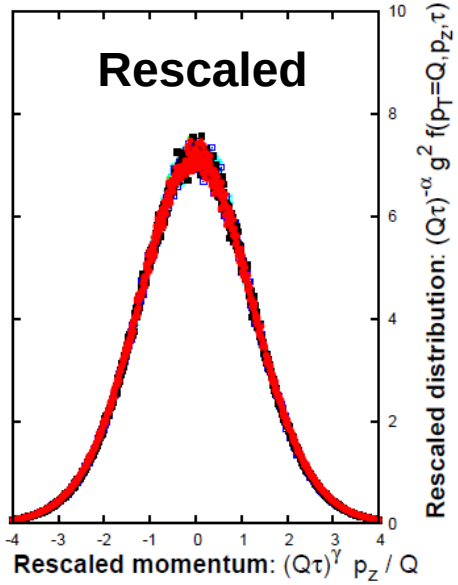




Longitudinal distribution $(p_T = Q)$







Comparing to a scalar $\lambda\phi^4$ field theory ($O(N)$ symmetric)

Universality with gauge theory (1. stage of bottom-up)

Berges, KB, Schlichting, Venugopalan, *PRL 114, 061601 (2015) ; PRD 92, 096006 (2015)*

Reminder: *Self-similar evolution*

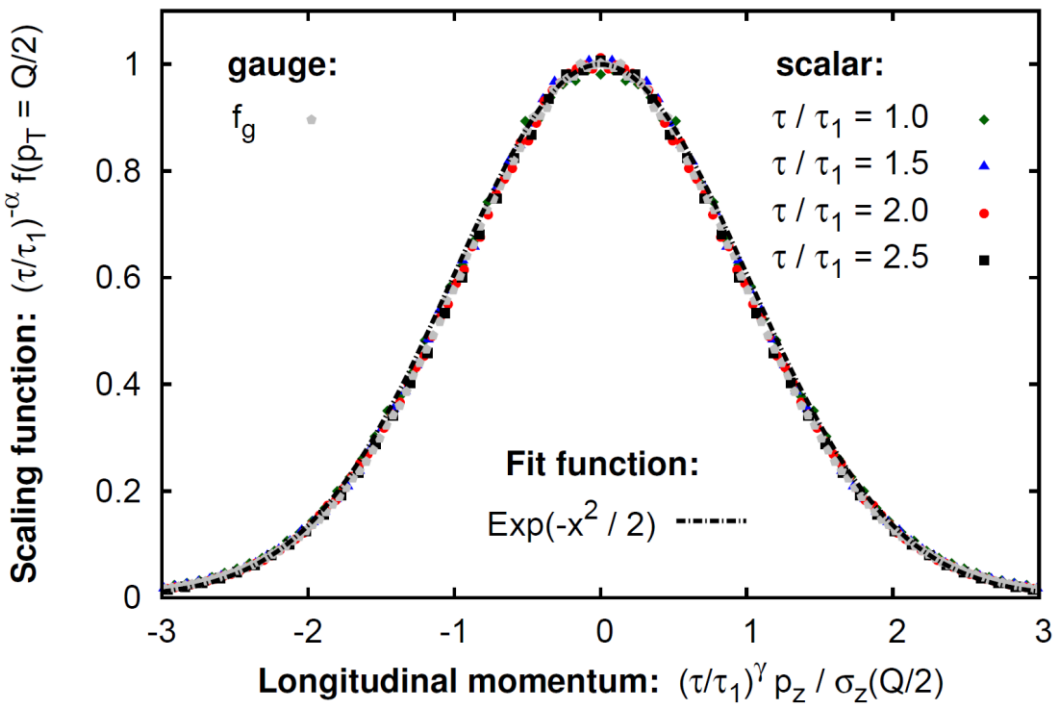
$$f(p_t, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Universality:

Same scaling exponents, same f_s as in gauge theory!

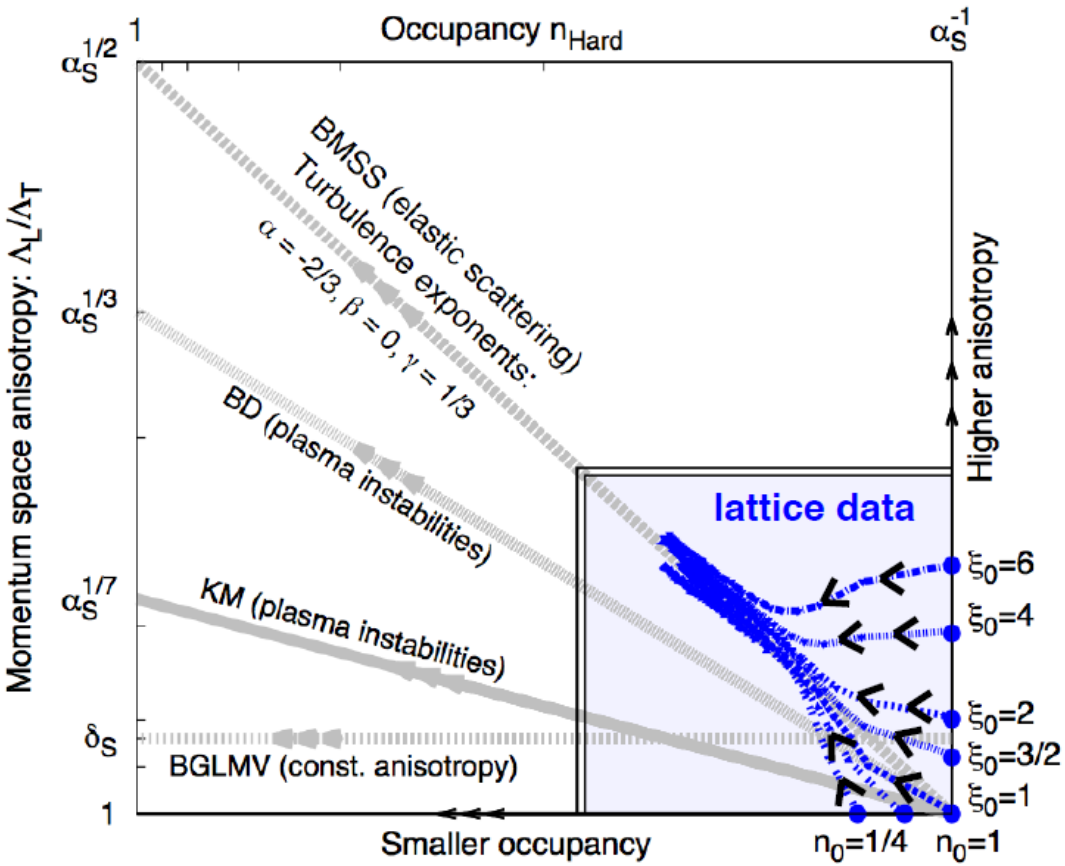
$$\begin{aligned} \alpha &\simeq -2/3 \\ \beta &\simeq 0 \\ \gamma &\simeq 1/3 \end{aligned}$$

Rescaled longitudinal distribution ($p_T = Q/2$)



Allowed to distinguish between different kinetic scenarios

Real-time lattice simulations



Berges, KB, Schlichting, Venugopalan, *PRD 89, 074011 (2014)*

Thermalization scenarios

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Well described by "Bottom-up" (BMSS)! Baier, Mueller, Schiff, Son, *PLB 502, 51 (2001)*

Scenario consists of 3 stages; Self-similar evolution is 1. stage

Further studies of different stages:

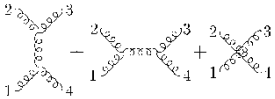
Blaizot, Iancu, Mehtar-Tani, *PRL 111, 052001 (2013)*

Kurkela, Lu, *PRL 113, 182301 (2014)*

Kurkela, Zhu, *PRL 115, 182301 (2015)*

Later stages of thermalization dynamics

Onset of hydrodynamics

Bottom-up involves only elastic  and inelastic processes 

No (late-time) plasma instabilities, no condensate included

Effective kinetic theory incorporates (in)elastic processes

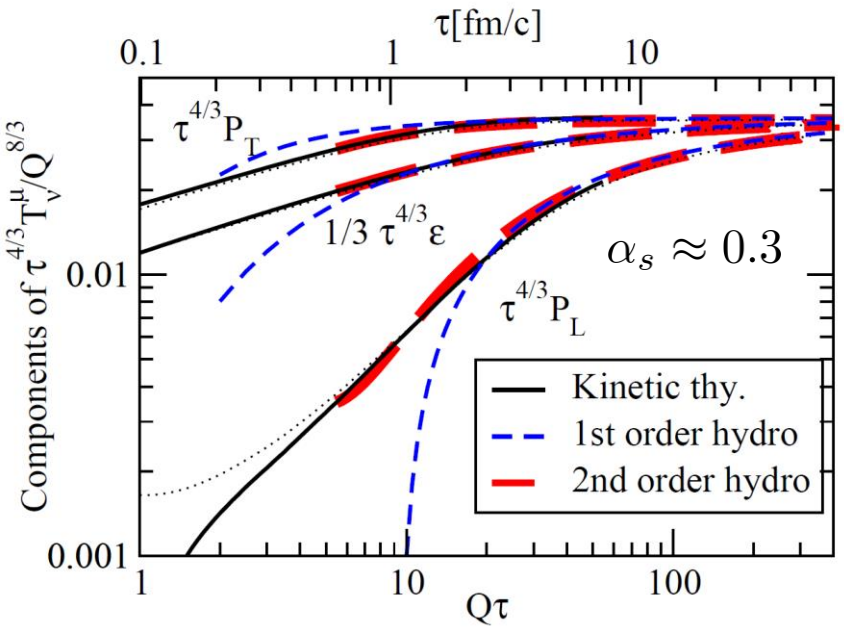
Arnold, Moore, Yaffe, *JHEP 0301, 030 (2003)*

Numerical simulations of kinetic theory

Kurkela, Zhu, *PRL 115, 182301 (2015)*

Hydrodynamics already at $\tau \sim 1\ fm/c$!

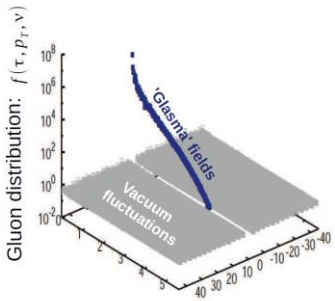
→ *See also talk by A. Mazeliauskas*



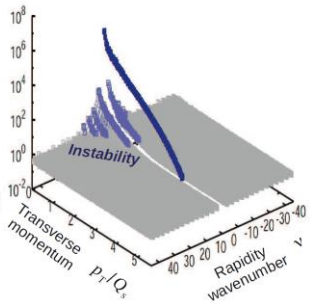
Kurkela, Zhu (2015); Keegan, Kurkela, Mazeliauskas, Teaney (2016); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney (2018), ...

Summary:
Thermalization process in weak-coupling picture

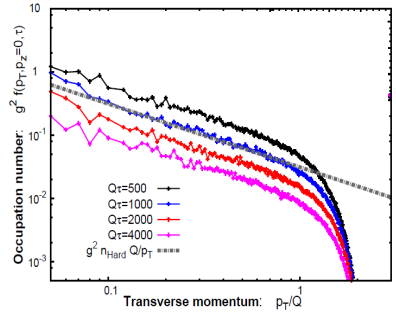
$Q_s\tau = 0^+$



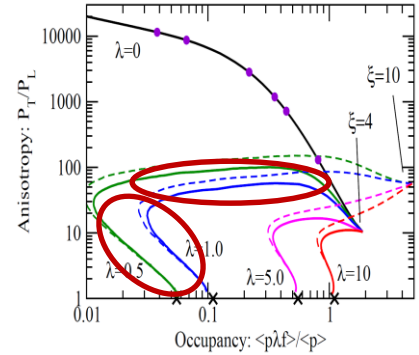
$Q_s\tau = \ln^2(\alpha_s^{-1})$

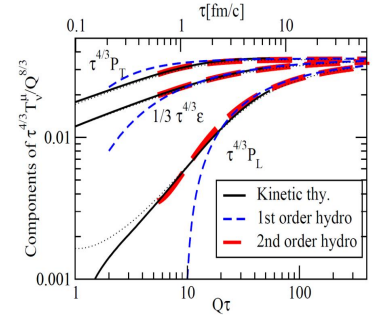


$Q_s\tau = \alpha_s^{-3/2}$



$Q_s\tau = \alpha_s^{-13/5}$



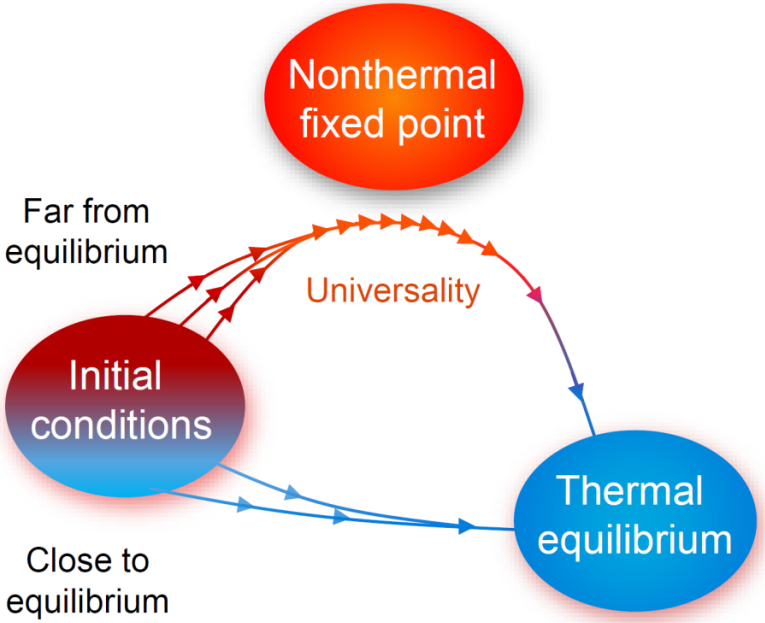


Glasma	Instabilities	**NTFP** (Bottom-up stage 1)	Bottom-up stages 2 and 3	Onset of hydrodynamics

Back to nonthermal fixed points (NTFP)

Now: *examples in other systems*

- Scalar systems
- Spin gases (experiment)
- Isotropic non-Abelian plasmas



Common approach:

- Experiment or real-time lattice simulations for *observation*
- *Understanding* with a kinetic or an effective theory
- **Often:** transport of a conserved quantity

Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

Nonthermal fixed points

In scalar systems: Dual cascade

Different theories: **Relativistic** scalars ($\lambda(\phi_a\phi_a)^n$; $O(N)$-sym.), **Nonrelativistic** scalars

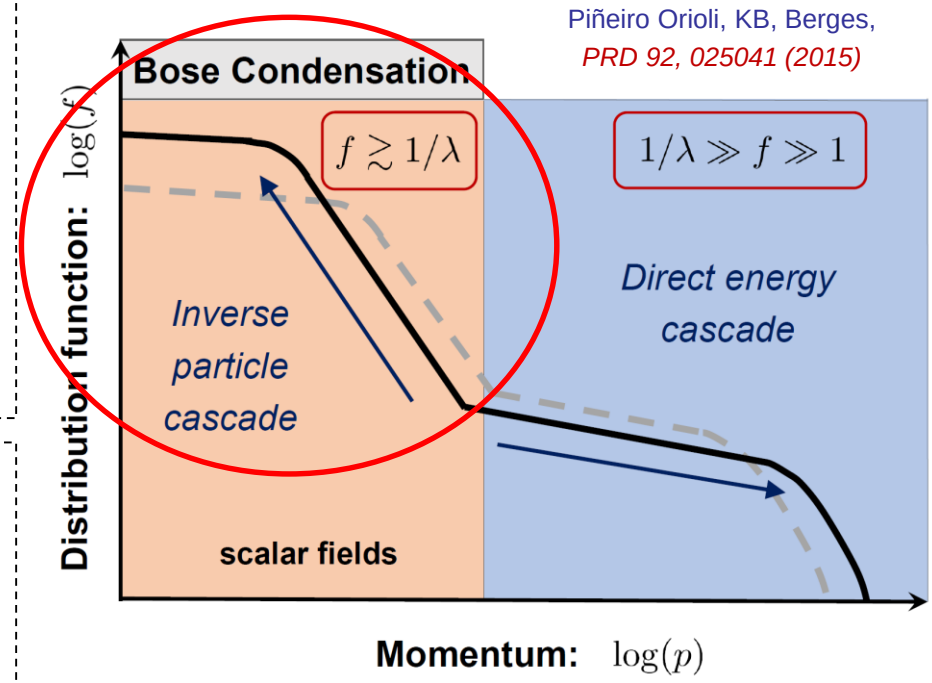
Observations:

- Two separate scaling regions: *IR*, *UV*
- *IR is universal:* same in all theories, i.e., same $\alpha \approx \frac{d}{2}$, $\beta \approx \frac{1}{2}$ and $f_s(p)$
- *Particle number conserved* in IR

Understanding:

- Large-N kinetic theory
- Low-energy effective theory

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*



Micha, Tkachev (2003); Berges, Rothkopf, Schmidt (2008); Gasenzer, Nowak, Sexty (2012); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Schachner, Piñeiro Orioli, Berges (2017); Berges, KB, Chatrchyan, Jäckel (2017); Walz, KB, Berges (2018); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018) ...

Nonthermal fixed point in a spin gas (ultracold atoms)

First experimental observation of a NTFP

Prüfer, Kunkel, Strobel, Lanning, Linnemann, Schmied, Berges, Gasenzer, Oberthaler, *arXiv:1805.11881*

Self-similar evolution

$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

Observation also

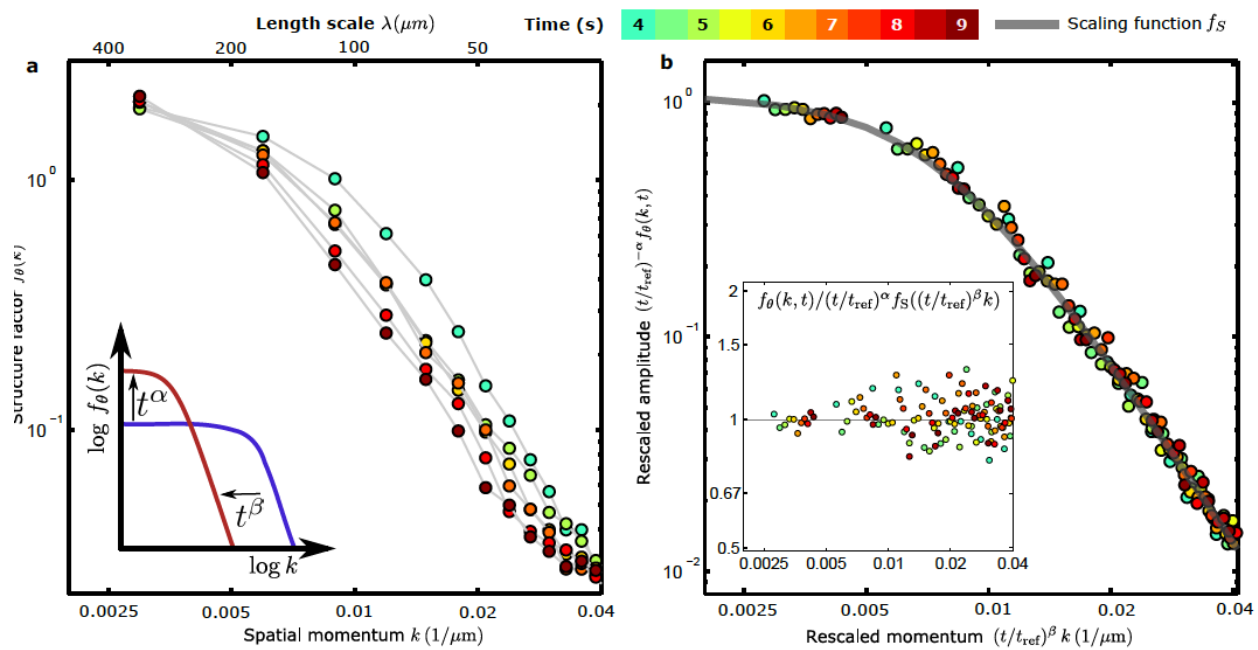
$$\alpha \approx \frac{d}{2}, \qquad \beta \approx \frac{1}{2}$$

Same as for IR in scalar systems!

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

Original distribution

Rescaled distribution



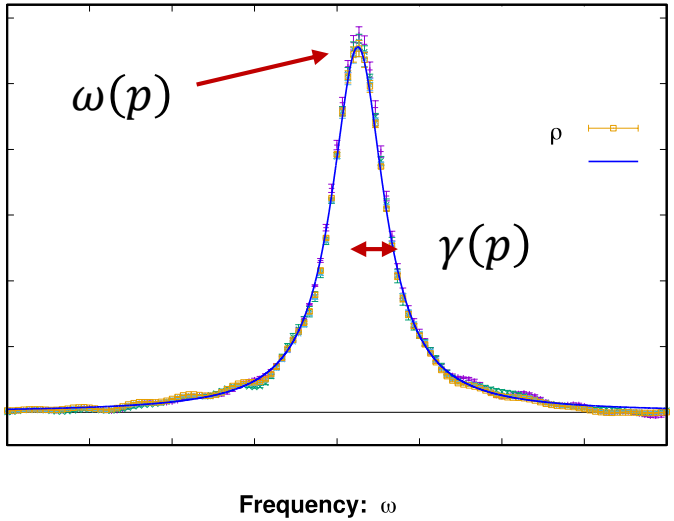
In $d = 1$, system prepared highly occupied

Next step: spectral functions

Go beyond distribution functions

- To *better understand microscopic dynamics*
- To *test quasiparticle assumptions* underlying kinetic theories
- To extract *transport and diffusion* properties

A typical spectral function

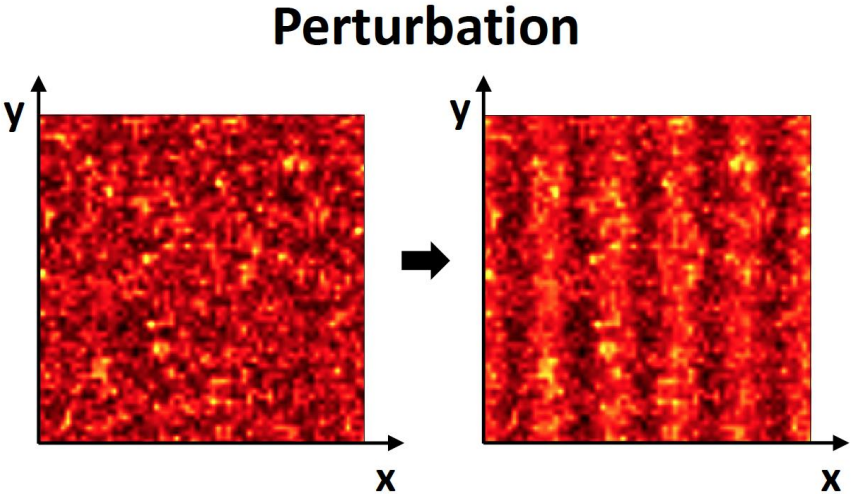


- $\rho(\omega, p)$ includes *all possible excitations*
- *Quasiparticles* emerge as Lorentz peaks
- *Dispersion* $\omega(p)$ is energy of "on-shell" particles
- *Damping rate* $\gamma(p)$ is inverse of their life time
- More complicated structures can also emerge (cuts, extra poles, etc.)

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

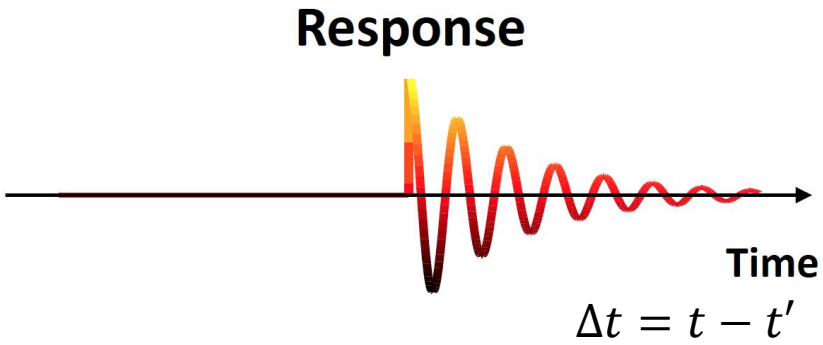
Next step: Spectral functions

Linear response theory on a class. lattice



- Classical field simulations for background
- Source j at time t'
- Response in linear fluctuations a_j for $t > t'$

Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*



- $\langle a_j(t,\boldsymbol{p})\rangle = \int dt' G_{R,jk}(t,t',\boldsymbol{p})\, j^k(t',\boldsymbol{p})$, obtain ret. propagator $G_{R,jk}$ from response
- Spectral function: $G_{R,jk} = \theta(t-t')\,\rho_{jk}$
- Distinguish polarizations

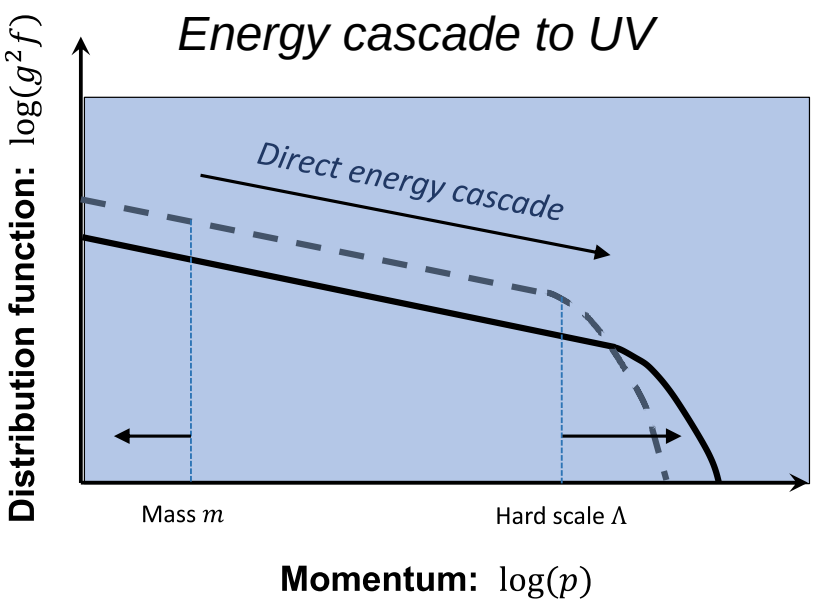
Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

First application: to NTFP

In an isotropic Yang-Mills system

Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014)



Observations:

- Self-similar, cascade to UV
- *Scale separation grows* with time

$$m/\Lambda \sim (Qt)^{-2/7} \ll 1$$

Asymptotic mass: $m^2 \sim g^2 \int \mathrm{d}^3 p \, \frac{f(t,p)}{p}$

Understanding: Arnold, Moore, Yaffe (2003); *York, Kurkela, Lu, Moore (2014)*

- *Effective kinetic theory* (AMY)

Scale separation allows usage of

- *Hard-thermal Loop* (HTL) *Braaten, Pisarski (1990); Blaizot, Iancu (2002)*

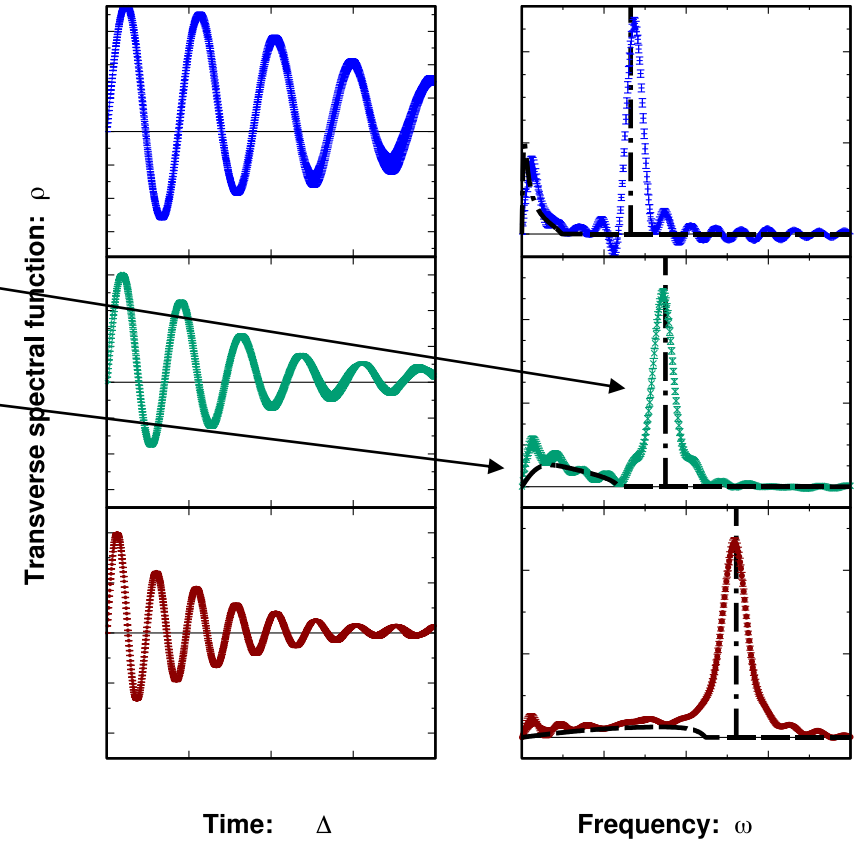
Next step: Spectral functions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Transverse spectral function ρ_T

ρ_T as function of $\Delta t = t - t'$ (left) or frequency ω (right) at late time $t, t' \gg \Delta t$

- Lorentzian peaks:
  existence of quasi-particles
- for $|\omega| \leq p$: *Landau cut*
- black dashed lines:
  Hard-thermal Loop (HTL) at LO



- ✓ Good agreement with HTL!
- ✓ System dominated by quasiparticles with relatively narrow width!

Conclusion

a. Thermalization process in weak-coupling picture in HIC:
   Glasma → Instabilities → Bottom-up → Hydrodynamics

b. Nonthermal fixed points (NTFP) commonly emerge far from equilibrium.
   Examples: gluonic systems, scalars, spin gases (ultracold atoms), …

c. Non-perturbative numerical approach developed for spectral functions
   ⇒ more information on microscopic dynamics accessible

Outlook: The technique to study spectral functions can be applied, e.g., to:

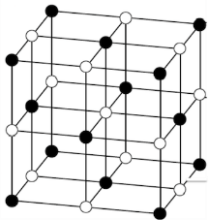
- *Transport* coefficients, jet quenching, diffusion
- *Anisotropy*, plasma instabilities, Glasma

Thank you for your attention!

BACKUP SLIDES

Computational method

Classical-statistical lattice simulations



- *SU*(N_c) *gauge theory* with $N_c = 2$ in temporal $A_0 = 0$ gauge
- Large occupancies $f(p \sim \Lambda) \gg 1$, weak coupling, real time
  $\Rightarrow$ *Classical* approximation for *field dynamics* applicable

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)

- Fields are link and chromo-electric fields U_i, E_i on 3D spatial lattice
- <u>Initialization:</u>

$$\langle |A(t=0,\boldsymbol{p})|^2 \rangle \sim \frac{f(t=0,p)}{p}, \qquad \langle |E(t=0,\boldsymbol{p})|^2 \rangle \sim p\, f(t=0,p)$$

then $A_i(0,\boldsymbol{p}) \to U_i(0,\boldsymbol{x})$, $E_i(0,\boldsymbol{p}) \to E_i(0,\boldsymbol{x})$ and restore Gauss law

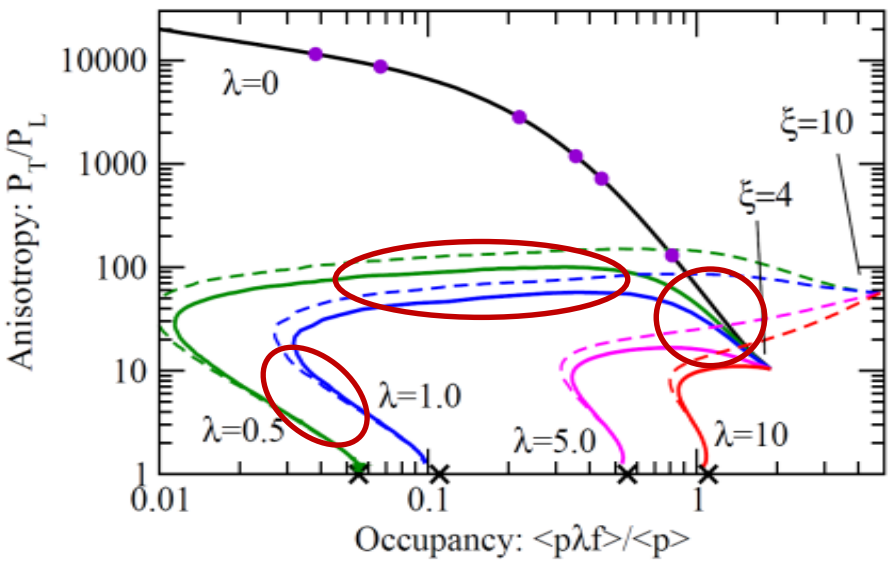
Thermalization dynamics at early times

‚Bottom-up' picture and onset of hydrodynamics

Kurkela, Zhu, *PRL 115, 182301 (2015)*

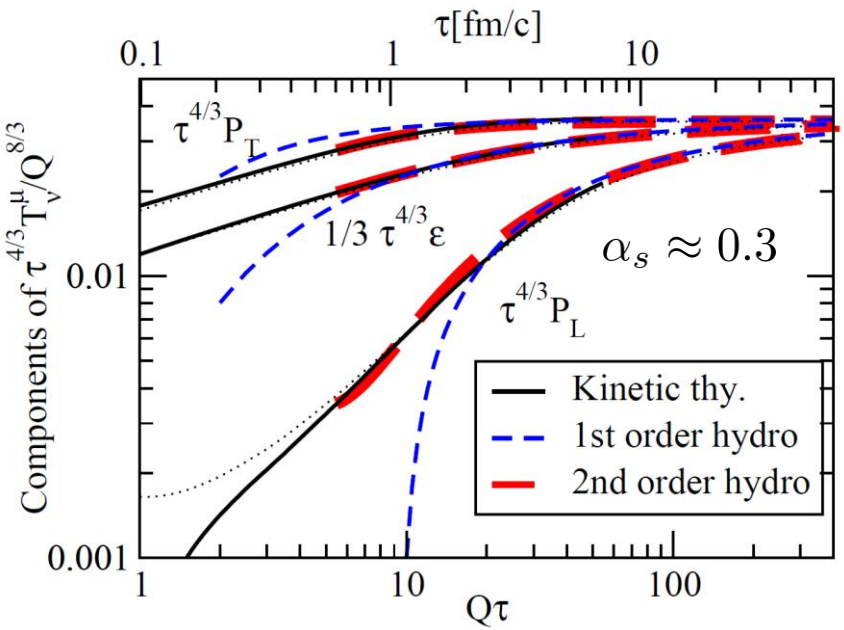
Full ‚bottom-up' evolution

($\lambda = 4\pi\alpha_s N_c$ extrapolated to moderate values = 5, 10)



3 stages: i) classical scaling; ii) anisotropy freezes; iii) radiational breakup

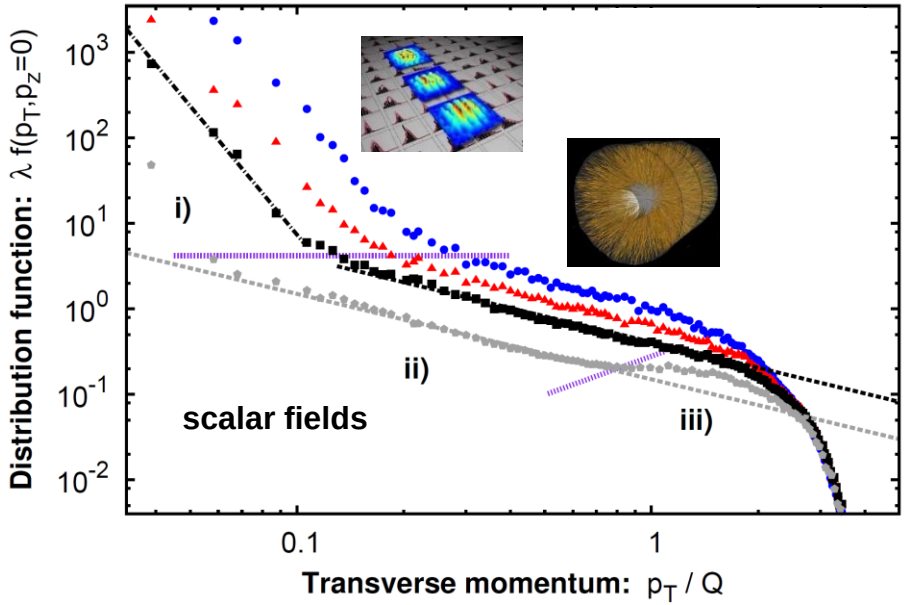
Onset of hydrodyncamics



Hydrodynamics already at $\tau \lesssim 1 fm/c$!

Universality classes

The attractor in longitudinally expanding scalars



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Berges, KB, Schlichting, Venugopalan,
PRL 114, 061601 (2015) ;
PRD 92, 096006 (2015)

Fixed points	α	β	γ	λf_S
i)	1	$2/3$	$2/3$	$\left((p/b)^{-1/2} + (p/b)^{-5}\right)^{-1}$
ii)	$-2/3$	0	$1/3$	$p_T^{-1}\ e^{-p_z^2/2\sigma_z^2}$
iii)	$-1/2$	0	$1/2$	$\mathrm{sech}(p_z/\sigma_z)$

Nonthermal fixed points

Scalars: Inverse particle cascade to IR

Piñeiro Orioli, KB, Berges,
PRD 92, 025041 (2015)

Self-similar evolution

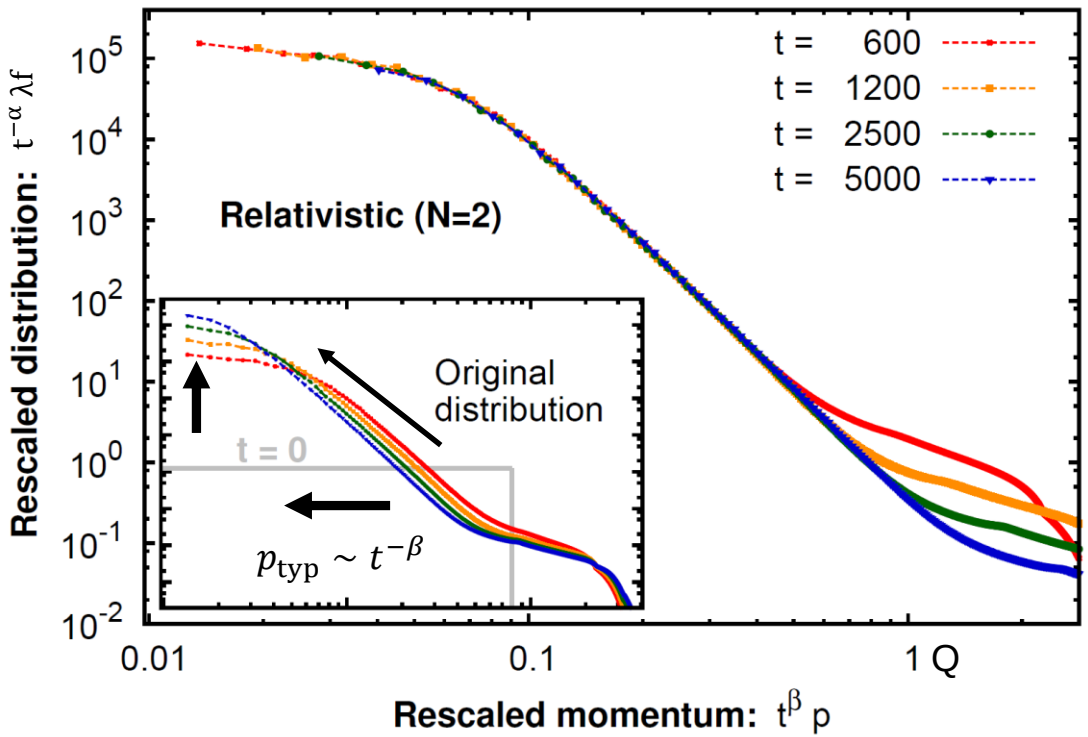
$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

Scaling exponents

$$\alpha \approx \frac{3}{2}, \qquad \beta \approx \frac{1}{2}$$

Particle conservation

$$n = \int_{IR} \frac{\mathrm{d}^3 p}{(2\pi)^3} f(p) \approx const$$



Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

Nonthermal fixed points

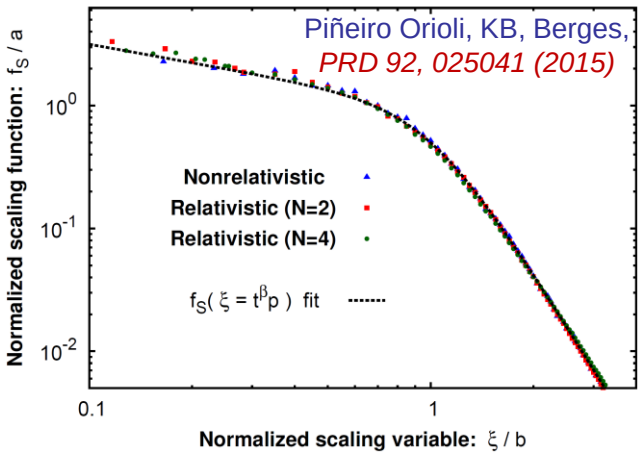
Scalars: Universality in IR

- *Same α, β and scaling function*

$$\lambda f_S \simeq \frac{a}{(|\boldsymbol{p}|/b)^{\kappa_<} + (|\boldsymbol{p}|/b)^{\kappa_>}}$$

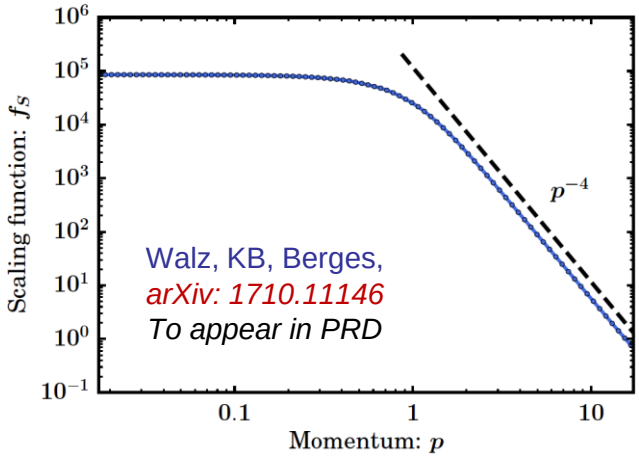
with $\kappa_< \simeq 0 - 0.5$ and $\kappa_> \simeq 4 - 4.5$

Across relativistic (different N), nonrelativistic

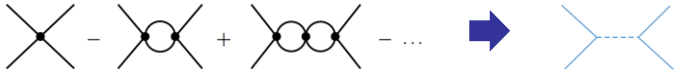


- New *large-N kinetic theory* describes it quantitatively, shows that $\kappa_< \to 0$, $\kappa_> \to 4$

Piñeiro Orioli, KB, Berges (2015); Walz, KB, Berges (2017)



- (Systematically derived in $1/N$, resums vertex)

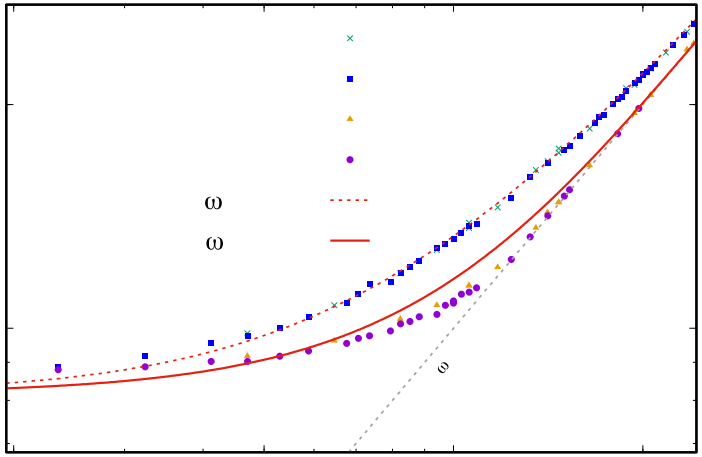


Extracted spectral function vs. HTL predictions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Extracted dispersion relations $\omega_{T,L}(p)$

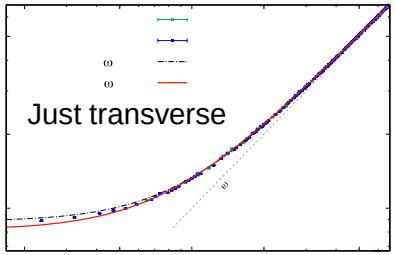
$\omega_{T,L}$ / m_{HTL}



Momentum: p / m_{HTL}

- Extracted from peak position (for ω_L after subtracting HTL Landau cut)
- *Similar to HTL* predictions: $\omega_{T,L}^{\mathrm{HTL}}(p)$
- Deviations at small p, for finite m/Λ?
- "$\omega_L(p)$" deviates at $p \sim m$ because peak is smaller than Landau cut, harder to measure

<u>Remark</u>: $\omega_T(p)$ also compatible with $\omega_T^{\mathrm{rel}} = \sqrt{m_\infty^2 + p^2}$ →

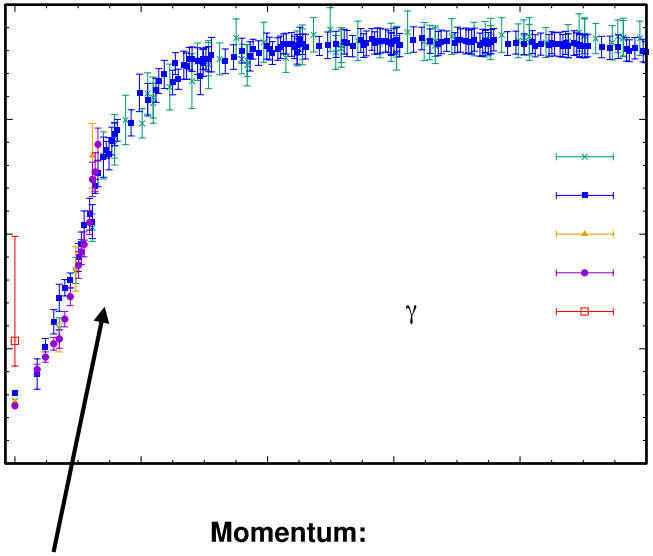


Extracted spectral function vs. HTL predictions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Extracted damping rates $\gamma_{T,L}(p)$

$\gamma_{T,L}(p)\ /\ Q$



- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2 Q)$ and *beyond HTL at LO*, it may contain non-perturbative contributions (*magnetic scale*)
- Here *first determination* of $\gamma_{T,L}(p)$!
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{\mathrm{HTL}}(p=0)$

Braaten, Pisarski,
PRD 42, 2156 (1990)

- "Isotropic" $\gamma_T \approx \gamma_L$ for $p \lesssim m$