Understanding the dynamics of field theories far from equilibrium

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XIIIth Quark
Confinement and the Hadron Spectrum

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Motivation

Pre-hydro evolution

Hydrodynamical studies suggest:

• Quick onset of hydrodynamics
• Nearly ideal fluid

Romatschke, Romatschke (2007); Song, Heinz (2008); Schenke, Jeon, Gale (2011); Schenke, Tribedy, Venugopalan (2012); Gale, Jeon. Schenke, Tribedy, Venugopalan (2013)

How does the created QCD matter evolve?

• Overview over the thermalization process in weak-coupling picture
• Focus on early phase: Nonthermal fixed points, universality with scalar theory
• Next step: Spectral functions
Thermalization dynamics at early times

Initial state: Glasma

Plasma instabilities (at early times)


Large gluon distribution

\[ f \sim \frac{d^3 N}{d^2 p_T dp_z} \sim \frac{1}{\alpha_s} \gg 1 \]

Nonthermal fixed point (NTFP)

- Partial memory loss
- Time scale independence
- Self-similar dynamics

(Single-particle) Distribution function:

\[ f(p, t_0) \sim \frac{1}{g^2} \Theta(Q - p) \]

\[ f(p_t, p_z, \tau) = \tau^\alpha f_s(\tau^\beta p_T, \tau^\gamma p_Z) \]

Since typically \( f \gg 1 \) → can use classical lattice simulations

Micha, Tkachev, PRD 70, 043538 (2004)
Berges, Rothkopf, Schmidt, PRL 101, 041603 (2008)

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)
Next stage of thermalization dynamics

NTFP of highly occupied gauge plasma

Berges, KB, Schlichting, Venugopalan, *PRD 89, 114007 (2014); PRD 89, 074011 (2014)

Reminder: *Self-similar evolution*

\[ f(p_t, p_z, \tau) = \tau^\alpha f_\text{S}(\tau^\beta p_T, \tau^\gamma p_z) \]

Universal scaling exponents

\[ \alpha \approx -2/3, \quad \beta \approx 0, \quad \gamma \approx 1/3 \]

Transverse distribution \((p_z = 0)\)

Longitudinal distribution \((p_T = Q)\)

Rescaled
Comparing to a scalar $\lambda \phi^4$ field theory ($O(N)$ symmetric)

Universality with gauge theory (1. stage of bottom-up)


Reminder: *Self-similar evolution*

$$f(p_t, p_z, \tau) = \tau^\alpha f_s(\tau^\beta p_T, \tau^\gamma p_z)$$

**Universality:**

Same scaling exponents, same $f_s$ as in gauge theory!

$$\begin{align*}
\alpha &\approx -2/3 \\
\beta &\approx 0 \\
\gamma &\approx 1/3
\end{align*}$$

Rescaled longitudinal distribution ($p_T = Q/2$)
Allowed to distinguish between different kinetic scenarios

Real-time lattice simulations

Thermalization scenarios

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Well described by “Bottom-up” (BMSS)!

Berges, KB, Schlichting, Venugopalan, *PRD* **89**, 074011 (2014)

Scenario consists of 3 stages;
Self-similar evolution is 1. stage

Further studies of different stages:

Later stages of thermalization dynamics

Onset of hydrodynamics

Bottom-up involves only elastic and inelastic processes

No (late-time) plasma instabilities, no condensate included

**Effective kinetic theory** incorporates (in)elastic processes


Numerical simulations of kinetic theory


Hydrodynamics already at $\tau \sim 1 \text{ fm}/c$!

→ See also talk by A. Mazeliauskas

Kurkela, Zhu (2015); Keegan, Kurkela, Mazeliauskas, Teaney (2016); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney (2018), ...
Summary:
Thermalization process in weak-coupling picture

\[ Q_s \tau = 0^+ \]
\[ Q_s \tau = \ln^2(\alpha_s^{-1}) \]
\[ Q_s \tau = \alpha_s^{-3/2} \]
\[ Q_s \tau = \alpha_s^{-13/5} \]

Glasma | Instabilities | **NTFP** (Bottom-up stage 1) | Bottom-up stages 2 and 3 | Onset of hydrodynamics
Back to nonthermal fixed points (NTFP)

Now: *examples in other systems*

- Scalar systems
- Spin gases (experiment)
- Isotropic non-Abelian plasmas

Common approach:

- Experiment or real-time lattice simulations for *observation*
- *Understanding* with a kinetic or an effective theory
- Often: transport of a conserved quantity
**Nonthermal fixed points**

In scalar systems: Dual cascade

**Observations:**
- Two separate scaling regions: **IR**, **UV**
- **IR is universal:** same in all theories, i.e., same $\alpha \approx \frac{d}{2}$, $\beta \approx \frac{1}{2}$ and $f_s(p)$
- **Particle number conserved** in IR

**Understanding:**
- Large-N kinetic theory
- Low-energy effective theory

**Different theories:** **Relativistic** scalars $(\lambda(\phi_a\phi_a)^n; O(N)$-sym.), **Nonrelativistic** scalars

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Micha, Tkachev (2003); Berges, Rothkopf, Schmidt (2008); Gasenzer, Nowak, Sexty (2012); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Schachner, Piñeiro Orioli, Berges (2017); Berges, KB, Chatrchyan, Jäckel (2017); Walz, KB, Berges (2018); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018) ...
Nonthermal fixed point in a spin gas (ultracold atoms)

First experimental observation of a NTFP


Self-similar evolution

\[ f(p, t) = t^\alpha f_s(t^\beta p) \]

Observation also

\[ \alpha \approx \frac{d}{2}, \quad \beta \approx \frac{1}{2} \]

Same as for IR in scalar systems!

Piñeiro Orioli, KB, Berges, PRD 92, 025041 (2015)

In \( d = 1 \), system prepared highly occupied
Next step: spectral functions

Go beyond distribution functions

- To *better understand microscopic dynamics*
- To *test quasiparticle assumptions* underlying kinetic theories
- To extract *transport and diffusion* properties

A typical spectral function

- $\rho(\omega, p)$ includes *all possible excitations*
- *Quasiparticles* emerge as Lorentz peaks
- *Dispersion* $\omega(p)$ is energy of “on-shell” particles
- *Damping rate* $\gamma(p)$ is inverse of their life time
- More complicated structures can also emerge (cuts, extra poles, etc.)
Next step: Spectral functions
Linear response theory on a class. lattice

• Classical field simulations for background
• Source $j$ at time $t'$
• Response in linear fluctuations $a_j$ for $t > t'$

$\Delta t = t - t'$

$\langle a_j(t, p) \rangle = \int dt' G_{R,jk}(t, t', p) j^k(t', p)$, obtain ret. propagator $G_{R,jk}$ from response

• Spectral function: $G_{R,jk} = \theta(t - t') \rho_{jk}$
• Distinguish polarizations

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)
Kurkela, Lappi, Peuron, EUJC 76 (2016) 688
**First application: to NTFP**

In an isotropic Yang-Mills system

\[ f(p, t) = t^\alpha f_s(t^\beta p) \]

*Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014)*

**Observations:**

- Self-similar, cascade to UV
- *Scale separation grows* with time

\[ m/\Lambda \sim (Q t)^{-2/7} \ll 1 \]

Asymptotic mass:

\[ m^2 \sim g^2 \int d^3 p \frac{f(t,p)}{p} \]

**Understanding:** *Arnold, Moore, Yaffe (2003); York, Kurkela, Lu, Moore (2014)*

- *Effective kinetic theory* (AMY)

**Scale separation allows usage of**

- *Hard-thermal Loop* (HTL) *Braaten, Pisarski (1990); Blaizot, Iancu (2002)*
Next step: Spectral functions

Transverse spectral function $\rho_T$

$\rho_T$ as function of $\Delta t = t - t'$ (left) or frequency $\omega$ (right) at late time $t, t' \gg \Delta t$

- Lorentzian peaks: existence of quasi-particles
- for $|\omega| \leq p$: Landau cut
- black dashed lines: Hard-thermal Loop (HTL) at LO

✓ Good agreement with HTL!
✓ System dominated by quasiparticles with relatively narrow width!
Conclusion

a. Thermalization process in weak-coupling picture in HIC:
   Glasma \( \rightarrow \) Instabilities \( \rightarrow \) Bottom-up \( \rightarrow \) Hydrodynamics

b. Nonthermal fixed points (NTFP) commonly emerge far from equilibrium. Examples: gluonic systems, scalars, spin gases (ultracold atoms), …

c. Non-perturbative numerical approach developed for spectral functions
   \( \Rightarrow \) more information on microscopic dynamics accessible

Outlook: The technique to study spectral functions can be applied, e.g., to:

- Transport coefficients, jet quenching, diffusion
- Anisotropy, plasma instabilities, Glasma

Thank you for your attention!
BACKUP SLIDES
Computational method
Classical-statistical lattice simulations

• $SU(N_c)$ gauge theory with $N_c = 2$ in temporal $A_0 = 0$ gauge
• Large occupancies $f(p \sim \Lambda) \gg 1$, weak coupling, real time
  $\Rightarrow$ Classical approximation for field dynamics applicable

• Fields are link and chromo-electric fields $U_i, E_i$ on 3D spatial lattice
• Initialization:
  \[
  \langle |A(t = 0, p)|^2 \rangle \sim \frac{f(t = 0, p)}{p}, \quad \langle |E(t = 0, p)|^2 \rangle \sim p f(t = 0, p)
  \]
  then $A_i(0, p) \rightarrow U_i(0, x)$, $E_i(0, p) \rightarrow E_i(0, x)$ and restore Gauss law

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)
Thermalization dynamics at early times

'BOTTOM-UP' picture and onset of hydrodynamics


Full 'bottom-up' evolution

Onset of hydrodynamics

(\( \lambda = 4\pi\alpha_s N_c \) extrapolated to moderate values = 5, 10)

3 stages: i) classical scaling; ii) anisotropy freezes; iii) radiational breakup

Hydrodynamics already at \( \tau \lesssim 1\, fm/c \) !
Universality classes

The attractor in longitudinally expanding scalars

Reminder: Self-similar evolution

\[ f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z) \]

Berges, KB, Schlichting, Venugopalan,

PRL 114, 061601 (2015);
PRD 92, 096006 (2015)

<table>
<thead>
<tr>
<th>Fixed points</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \lambda f_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i)</td>
<td>1</td>
<td>2/3</td>
<td>2/3</td>
<td>((p/b)^{-1/2} + (p/b)^{-5})^{-1}</td>
</tr>
<tr>
<td>ii)</td>
<td>-2/3</td>
<td>0</td>
<td>1/3</td>
<td>(p_T^{-1} e^{-p_z^2/2\sigma_z^2})</td>
</tr>
<tr>
<td>iii)</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
<td>(\text{sech}(p_z/\sigma_z))</td>
</tr>
</tbody>
</table>
Nonthermal fixed points

Scalars: Inverse particle cascade to IR

Self-similar evolution

\[ f(p, t) = t^\alpha f_S(t^\beta p) \]

Scaling exponents

\[ \alpha \approx \frac{3}{2}, \quad \beta \approx \frac{1}{2} \]

Particle conservation

\[ n = \int_{IR} \frac{d^3p}{(2\pi)^3} f(p) \approx \text{const} \]

Piñeiro Orioli, KB, Berges, PRD 92, 025041 (2015)
Nonthermal fixed points
Scalars: Universality in IR

- **Same** $\alpha, \beta$ and scaling function
  \[ \lambda f_s \approx \frac{a}{(|p|/b)^{\kappa<} + (|p|/b)^{\kappa>}} \]
  with $\kappa_< \approx 0 - 0.5$ and $\kappa_> \approx 4 - 4.5$

Across relativistic (different $N$), nonrelativistic

- **New large-$N$ kinetic theory** describes it quantitatively, shows that $\kappa_< \to 0, \kappa_> \to 4$
  
  *Piñeiro Orioli, KB, Berges (2015); Walz, KB, Berges (2017)*

- (Systematically derived in $1/N$, resums vertex)

02.08.2018 | University of Jyväskylä, Finland | Kirill Boguslavski | 23
Extracted spectral function vs. HTL predictions

Extracted dispersion relations $\omega_{T,L}(p)$

- Extracted from peak position (for $\omega_L$ after subtracting HTL Landau cut)
- **Similar to HTL predictions:** $\omega_{T,L}^{HTL}(p)$
- Deviations at small $p$, for finite $m/\Lambda$?
- ”$\omega_L(p)$” deviates at $p \sim m$ because peak is smaller than Landau cut, harder to measure

Remark: $\omega_T(p)$ also compatible with $\omega_T^{rel} = \sqrt{m_\infty^2 + p^2}$

Momentum: $p / m_{HTL}$

Just transverse
Extracted spectral function vs. HTL predictions

**Extracted damping rates** $\gamma_{T,L}(p)$

- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2 Q)$ and **beyond HTL at LO**, it may contain non-perturbative contributions (**magnetic scale**)
- Here **first determination** of $\gamma_{T,L}(p)$!
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{HTL}(p = 0)$
- “Isotropic” $\gamma_T \approx \gamma_L$ for $p \lesssim m$

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

Braaten, Pisarski, *PRD 42, 2156 (1990)*