Understanding the dynamics of field theories far from equilibrium

Project: CGCglasmaQGP

Kirill Boguslavski

XIIIth Quark Confinement and the Hadron Spectrum

August 2, 2018

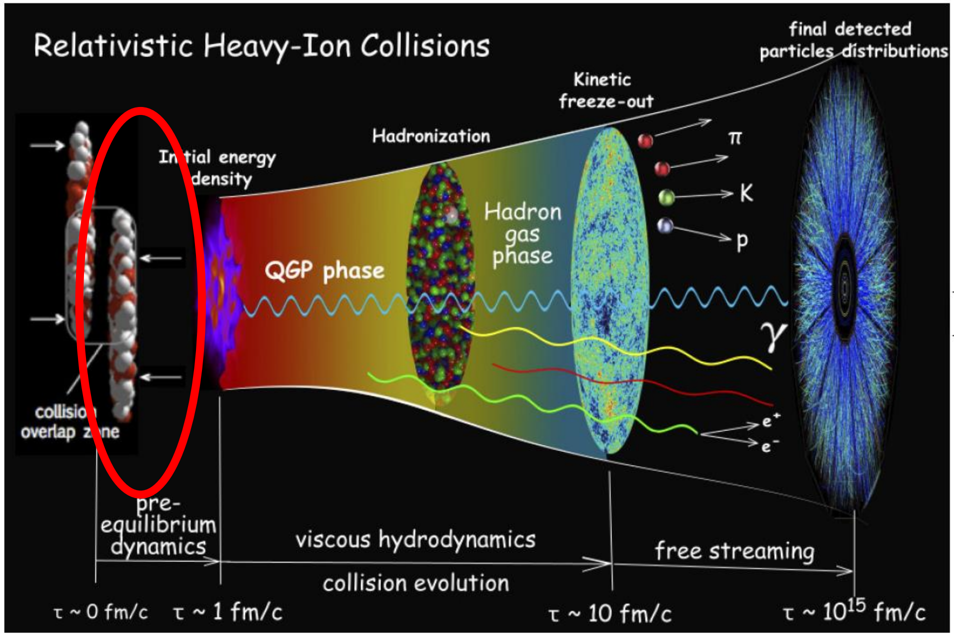
Motivation

Pre-hydro evolution

Hydrodynamical studies suggest:

- Quick onset of hydrodynamics
- Nearly ideal fluid

Romatschke, Romatschke (2007); Song, Heinz (2008); Schenke, Jeon, Gale (2011); Schenke, Tribedy, Venugopalan (2012); Gale, Jeon. Schenke, Tribedy, Venugopalan (2013)



Little Bang by P. Sorensen and C. Shen

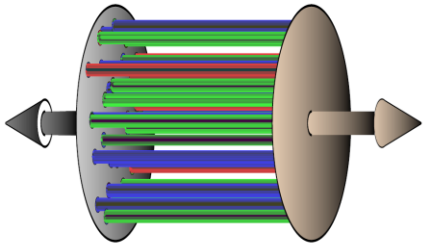
How does the created QCD matter evolve?

- Overview over the thermalization process in weak-coupling picture
- Focus on early phase: Nonthermal fixed points, universality with scalar theory
- Next step: Spectral functions

Thermalization dynamics at early times

Initial state: Glasma

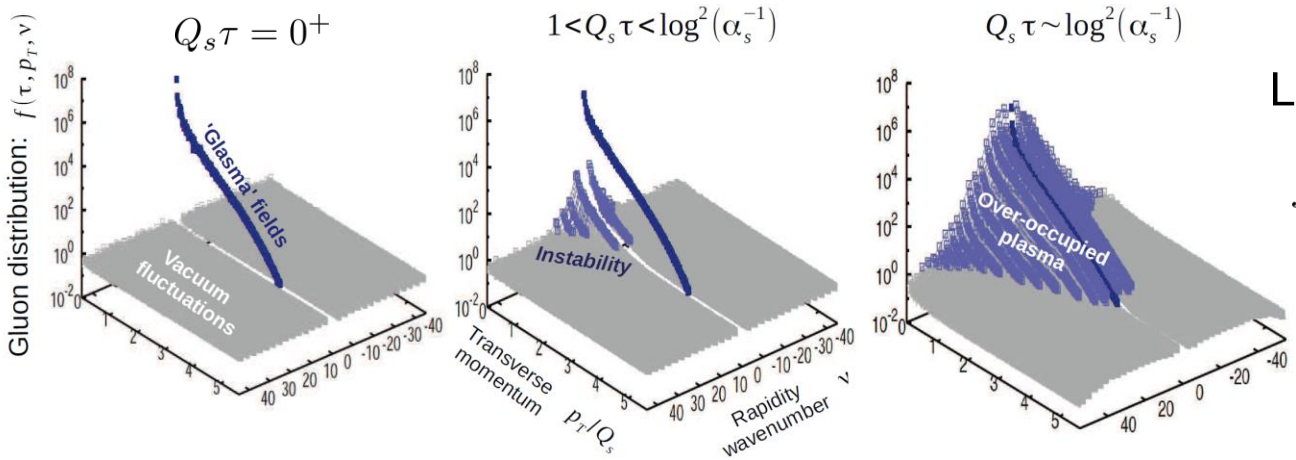
McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); Krasnitz, Nara, Venugopalan (2001, 2003); Lappi (2003, 2006, 2011); Lappi, McLerran (2006); …



Gelis, Iancu, Jalilian-Marian, Venugopalan, *Ann. Rev. Nucl. Part. Sci 60, 463 (2010)*

Plasma instabilities
(at early times)

Mrowczynski (1993); Arnold, Lenaghan, Moore (2003); Romatschke, Strickland (2003); Romatschke, Venugopalan (2006); Attems, Rebhan, Strickland (2012); Fukushima, Gelis (2012); Berges, Schlichting (2013); Epelbaum, Gelis (2013); ...

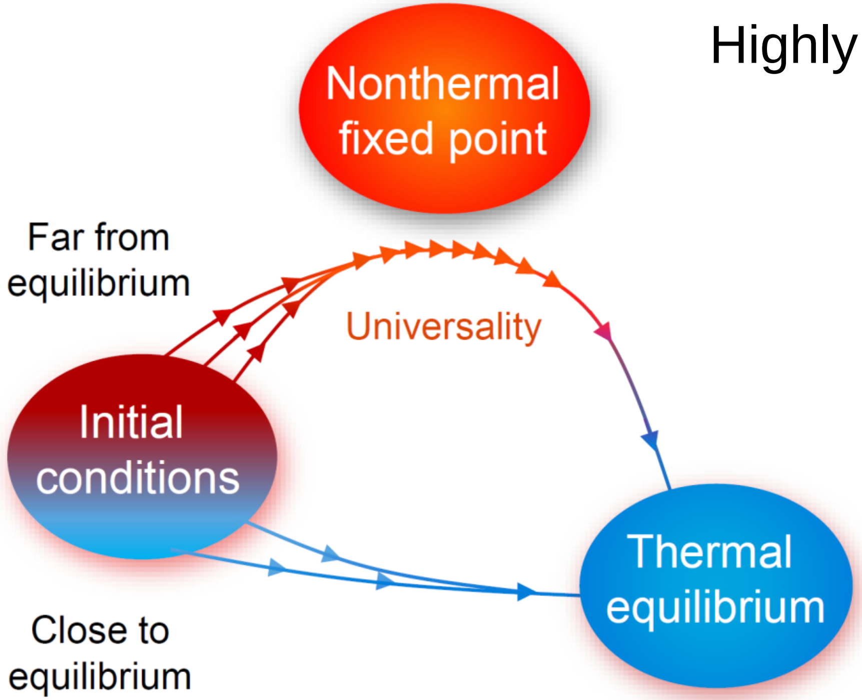


Large gluon distribution

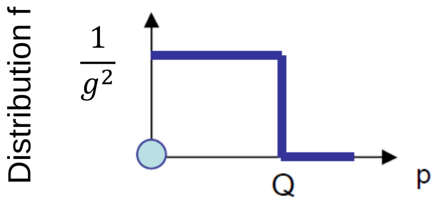
$$f = \frac{d^3N}{d^2p_T dp_z} \sim \frac{1}{\alpha_s} \gg 1$$

Berges, Schenke, Schlichting, Venugopalan, *Nucl. Phys. A 931, 348 (2014)*

Nonthermal fixed point (NTFP)



Highly occupied:



$$f(p, t_0) \sim \frac{1}{g^2}\,\Theta(Q-p)$$

Nonthermal fixed point (NTFP)

Micha, Tkachev, *PRD* 70, 043538 (2004)

Berges, Rothkopf, Schmidt, *PRL 101, 041603 (2008)*

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

(Single-particle) Distribution function: $f(p_T, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_T, \tau^{\gamma} p_z)$

Since typically $f \gg 1$ ⇨ can use classical lattice simulations

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)

Next stage of thermalization dynamics

NTFP of highly occupied gauge plasma

Berges, KB, Schlichting, Venugopalan, *PRD 89, 114007 (2014); PRD 89, 074011 (2014)*

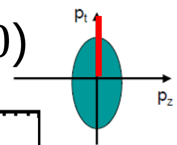
Reminder: *Self-similar evolution*

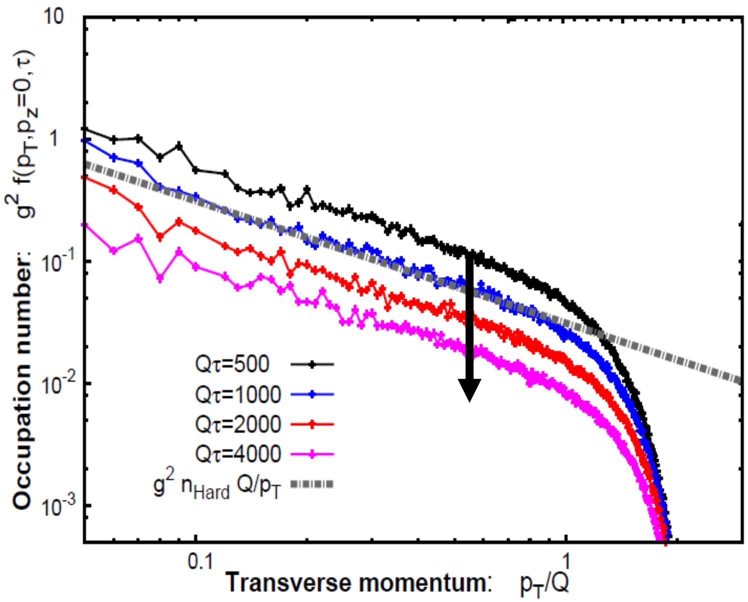
$$f(p_T, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Universal scaling exponents

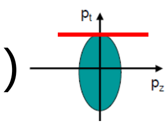
$$\alpha \simeq -2/3,\ \beta \simeq 0,\ \gamma \simeq 1/3$$

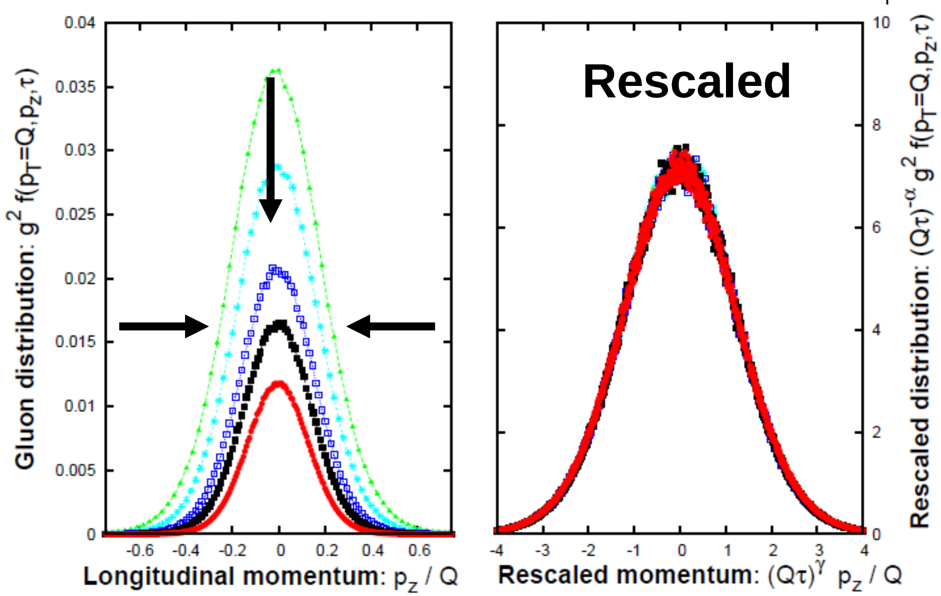
Transverse distribution $(p_z = 0)$





Longitudinal distribution $(p_T = Q)$





Comparing to a scalar $\lambda\phi^4$ field theory ($O(N)$ symmetric)

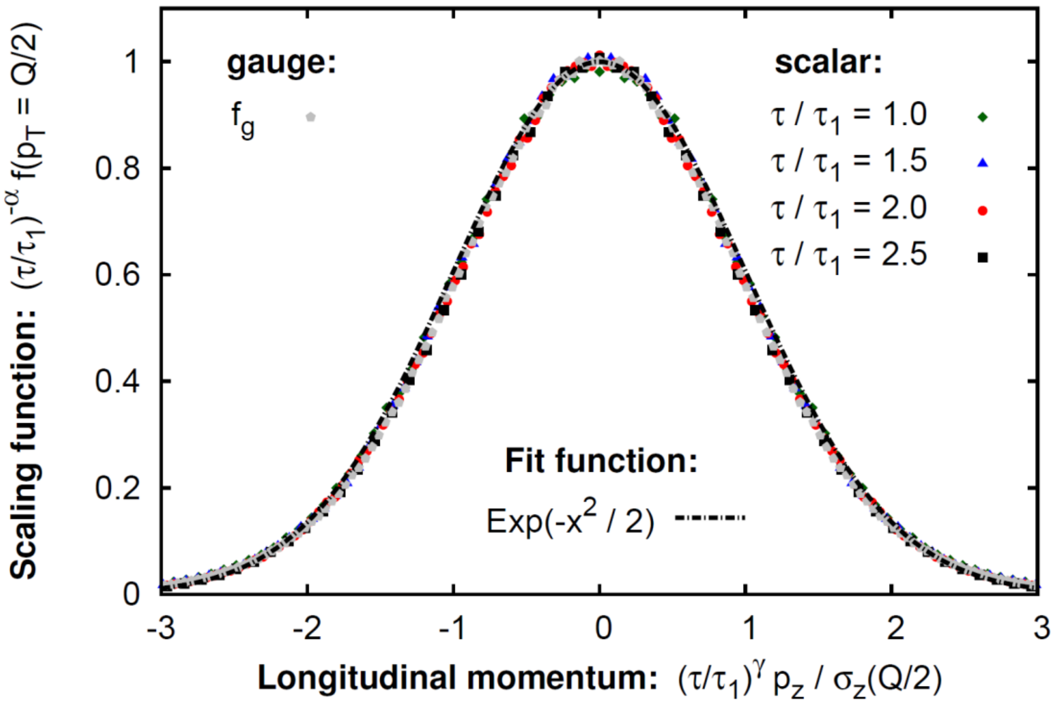
Universality with gauge theory (1. stage of bottom-up)

Berges, KB, Schlichting, Venugopalan, *PRL 114, 061601 (2015) ; PRD 92, 096006 (2015)*

Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^\alpha f_s(\tau^\beta p_T, \tau^\gamma p_z)$$

Rescaled longitudinal distribution ($p_T = Q/2$)



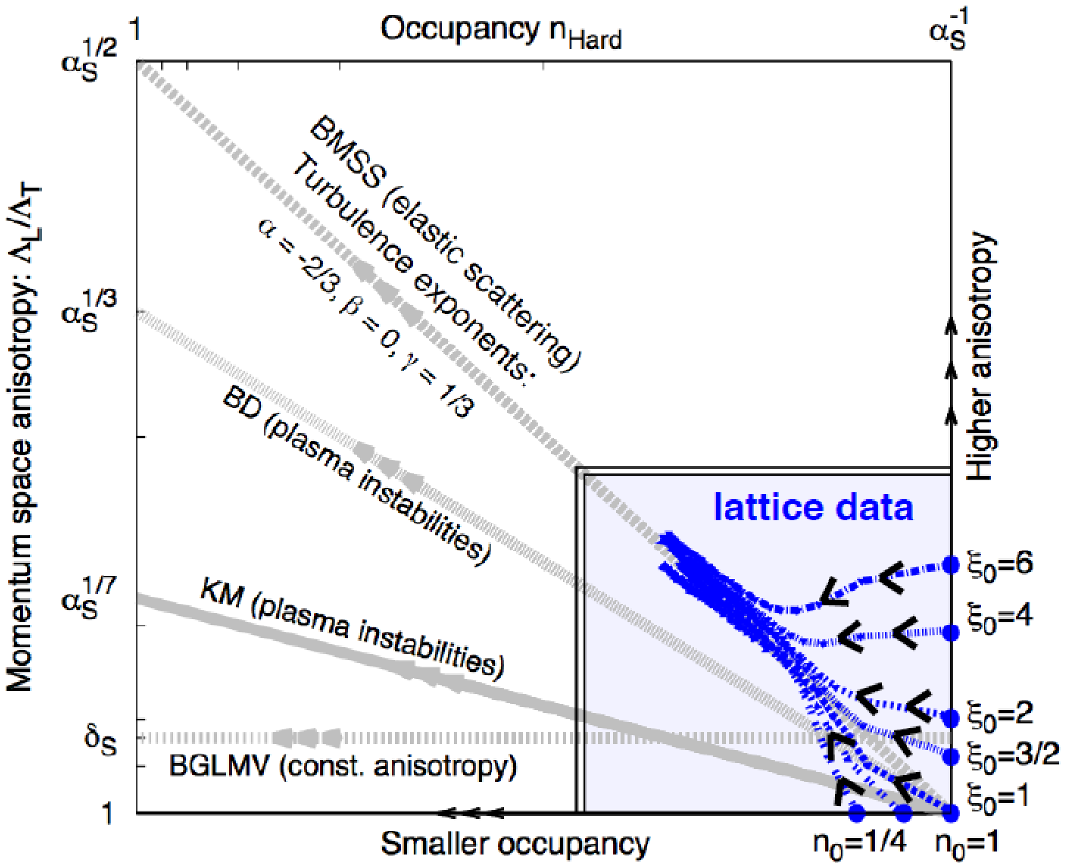
Universality:

Same scaling exponents, same f_s as in gauge theory!

$$\begin{aligned} \alpha &\simeq -2/3 \\ \beta &\simeq 0 \\ \gamma &\simeq 1/3 \end{aligned}$$

Allowed to distinguish between different kinetic scenarios

Real-time lattice simulations



Berges, KB, Schlichting, Venugopalan, *PRD 89, 074011 (2014)*

Thermalization scenarios

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Well described by “Bottom-up” (BMSS)! Baier, Mueller, Schiff, Son, *PLB 502, 51 (2001)*

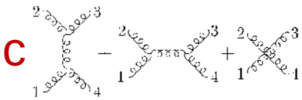
Scenario consists of 3 stages;
Self-similar evolution is 1. stage

Further studies of different stages:

Blaizot, Iancu, Mehtar-Tani, *PRL 111, 052001 (2013)*
Kurkela, Lu, *PRL 113, 182301 (2014)*
Kurkela, Zhu, *PRL 115, 182301 (2015)*

Later stages of thermalization dynamics

Onset of hydrodynamics

Bottom-up involves only elastic  and inelastic processes 

No (late-time) plasma instabilities, no condensate included

Effective kinetic theory incorporates (in)elastic processes

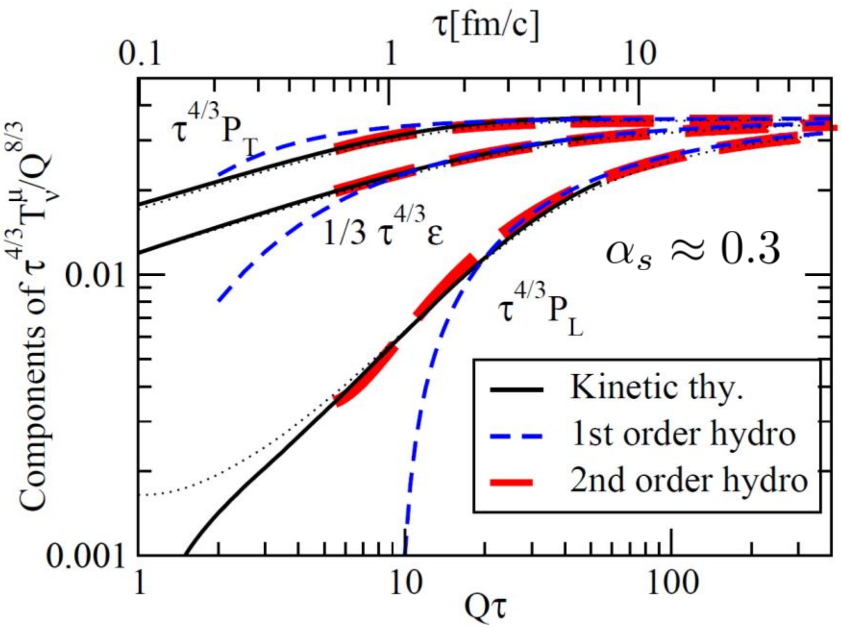
Arnold, Moore, Yaffe, *JHEP 0301, 030 (2003)*

Numerical simulations of kinetic theory

Kurkela, Zhu, *PRL 115, 182301 (2015)*

Hydrodynamics already at $\tau \sim 1\ fm/c$!

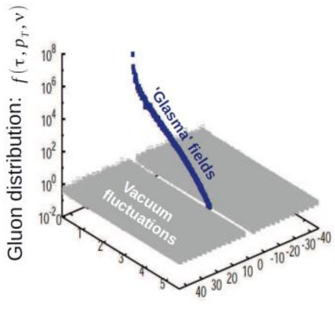
→ *See also talk by A. Mazeliauskas*

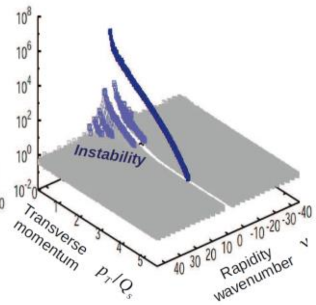


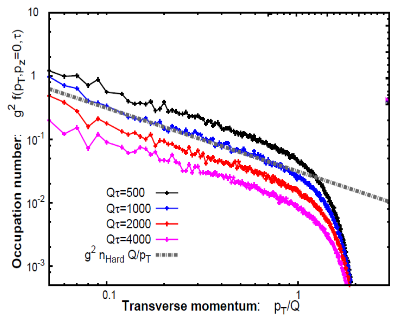
Kurkela, Zhu (2015); Keegan, Kurkela, Mazeliauskas, Teaney (2016); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney (2018), ...

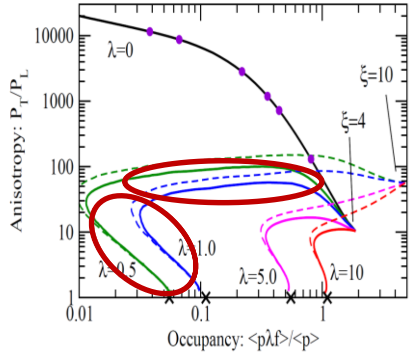
Summary:
Thermalization process in weak-coupling picture

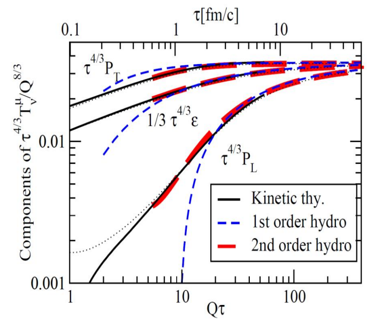
$Q_s\tau = 0^+$ | $Q_s\tau = \ln^2(\alpha_s^{-1})$ | $Q_s\tau = \alpha_s^{-3/2}$ | $Q_s\tau = \alpha_s^{-13/5}$









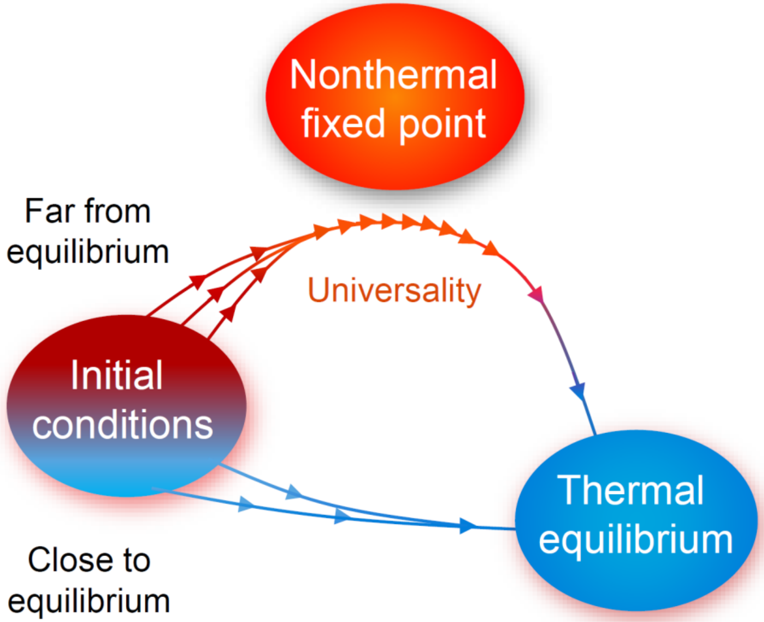


Glasma	Instabilities	**NTFP** (Bottom-up stage 1)	Bottom-up stages 2 and 3	Onset of hydrodynamics

Back to nonthermal fixed points (NTFP)

Now: *examples in other systems*

- Scalar systems
- Spin gases (experiment)
- Isotropic non-Abelian plasmas



Common approach:

- Experiment or real-time lattice simulations for *observation*
- *Understanding* with a kinetic or an effective theory
- **Often:** transport of a conserved quantity

Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

Nonthermal fixed points

In scalar systems: Dual cascade

Different theories: **Relativistic** scalars ($\lambda(\phi_a \phi_a)^n$; $O(N)$-sym.), **Nonrelativistic** scalars

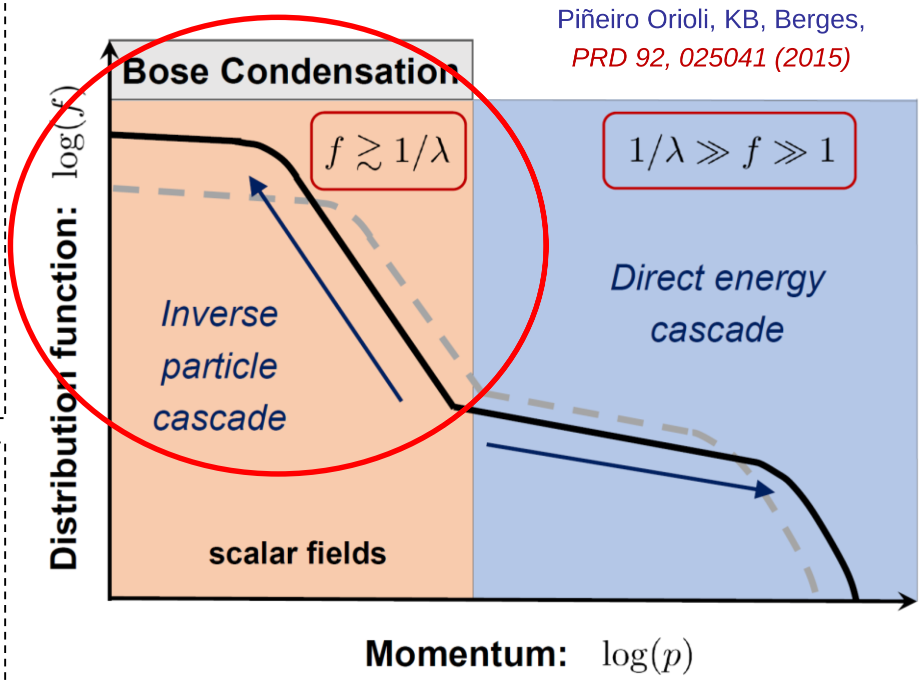
Observations:

- Two separate scaling regions: *IR*, *UV*
- *IR is universal:* same in all theories, i.e., same $\alpha \approx \frac{d}{2}$, $\beta \approx \frac{1}{2}$ and $f_s(p)$
- *Particle number conserved* in IR

Understanding:

- Large-N kinetic theory
- Low-energy effective theory

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*



Micha, Tkachev (2003); Berges, Rothkopf, Schmidt (2008); Gasenzer, Nowak, Sexty (2012); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Schachner, Piñeiro Orioli, Berges (2017); Berges, KB, Chatrchyan, Jäckel (2017); Walz, KB, Berges (2018); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018) ...

Nonthermal fixed point in a spin gas (ultracold atoms)

First experimental observation of a NTFP

Prüfer, Kunkel, Strobel, Lanning, Linnemann, Schmied,
Berges, Gasenzer, Oberthaler, *arXiv:1805.11881*

Self-similar evolution

$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

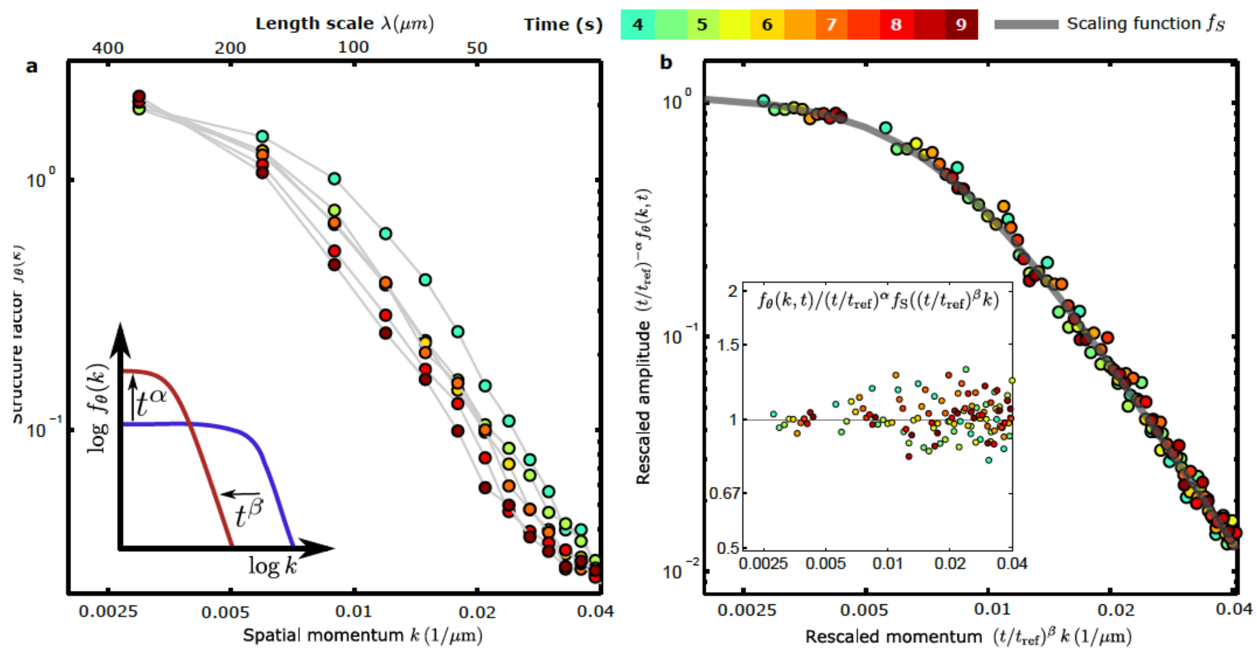
Observation also

$$\alpha \approx \frac{d}{2}, \qquad \beta \approx \frac{1}{2}$$

Same as for IR in scalar systems!

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

Original distribution **Rescaled** distribution



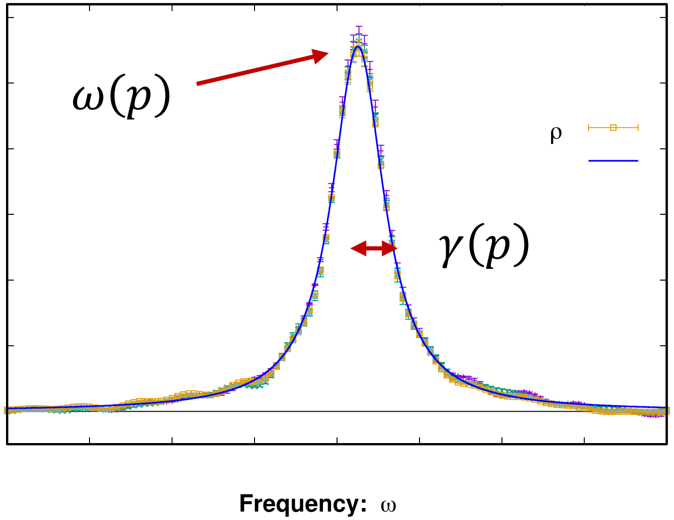
In $d = 1$, system prepared highly occupied

Next step: spectral functions

Go beyond distribution functions

- To *better understand microscopic dynamics*
- To *test quasiparticle assumptions* underlying kinetic theories
- To extract *transport and diffusion* properties

A typical spectral function

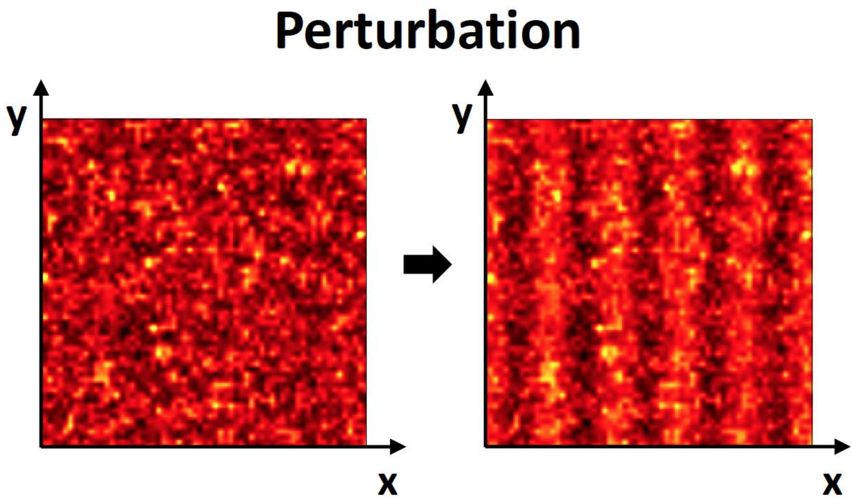


- $\rho(\omega, p)$ includes *all possible excitations*
- *Quasiparticles* emerge as Lorentz peaks
- *Dispersion* $\omega(p)$ is energy of “on-shell” particles
- *Damping rate* $\gamma(p)$ is inverse of their life time
- More complicated structures can also emerge (cuts, extra poles, etc.)

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

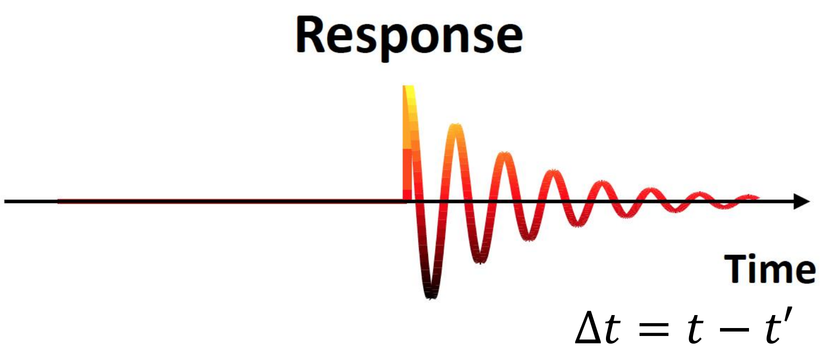
Next step: Spectral functions

Linear response theory on a class. lattice



- Classical field simulations for background
- Source j at time t'
- Response in linear fluctuations a_j for $t > t'$

Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*



- $\langle a_j(t, \boldsymbol{p}) \rangle = \int dt' G_{R,jk}(t, t', \boldsymbol{p})\, j^k(t', \boldsymbol{p})$, obtain ret. propagator $G_{R,jk}$ from response
- Spectral function: $G_{R,jk} = \theta(t - t')\, \rho_{jk}$
- Distinguish polarizations

Self-similar evolution

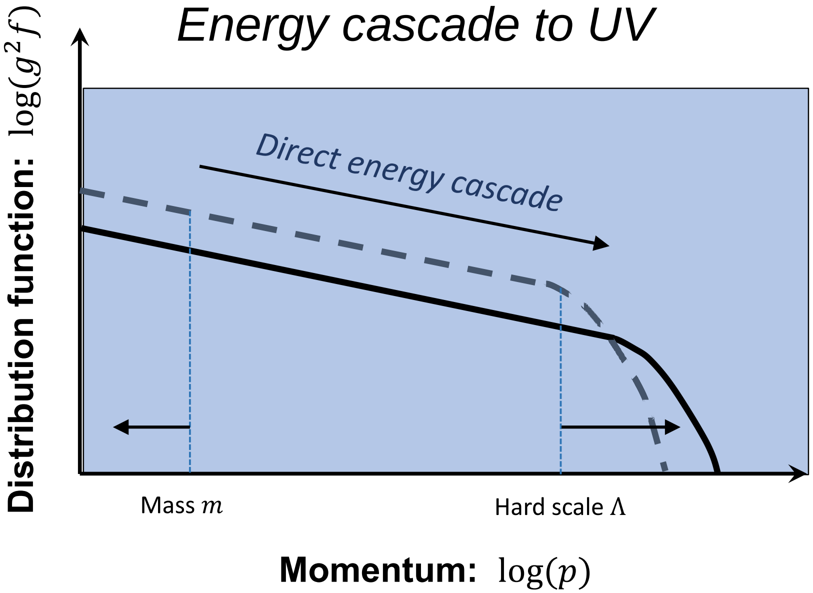
$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

First application: to NTFP

In an isotropic Yang-Mills system

Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014)

Energy cascade to UV



Observations:

- Self-similar, cascade to UV
- *Scale separation grows* with time

$$m/\Lambda \sim (Qt)^{-2/7} \ll 1$$

Asymptotic mass: $m^2 \sim g^2 \int \mathrm{d}^3 p \, \frac{f(t,p)}{p}$

Understanding: Arnold, Moore, Yaffe (2003); *York, Kurkela, Lu, Moore (2014)*

- *Effective kinetic theory* (AMY)

Scale separation allows usage of

- *Hard-thermal Loop* (HTL) *Braaten, Pisarski (1990); Blaizot, Iancu (2002)*

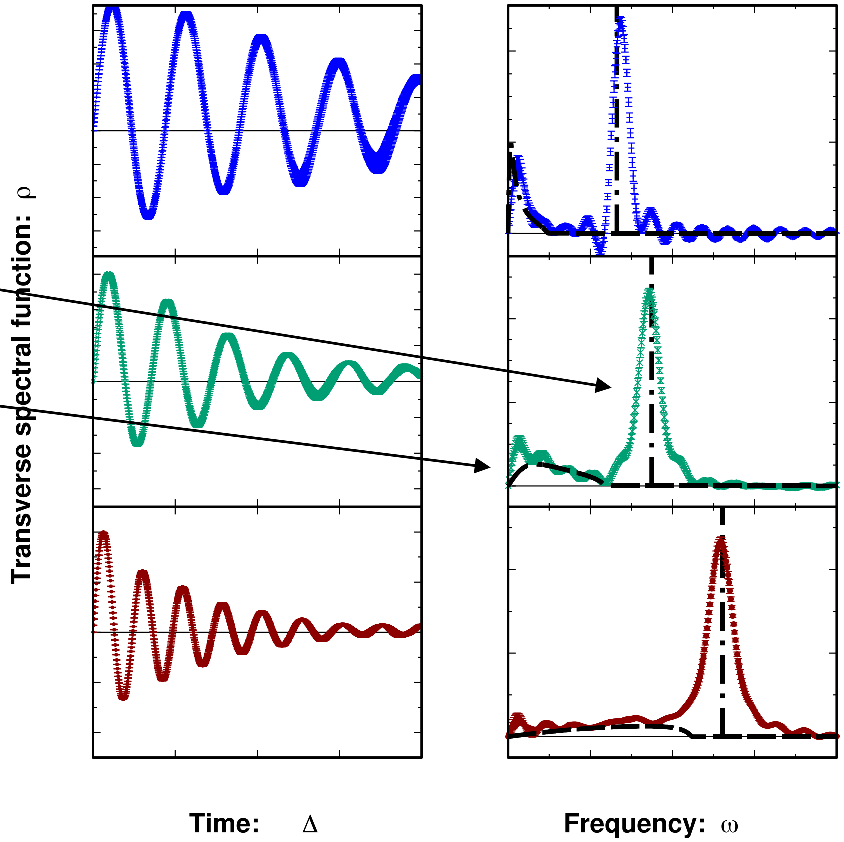
Next step: Spectral functions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Transverse spectral function ρ_T

ρ_T as function of $\Delta t = t - t'$ (left) or frequency ω (right) at late time $t, t' \gg \Delta t$

- Lorentzian peaks:
  existence of quasi-particles
- for $|\omega| \leq p$: *Landau cut*
- black dashed lines:
  Hard-thermal Loop (HTL) at LO



✓ Good agreement with HTL!

✓ System dominated by quasiparticles with relatively narrow width!

Conclusion

a. Thermalization process in weak-coupling picture in HIC:
   Glasma → Instabilities → Bottom-up → Hydrodynamics

b. Nonthermal fixed points (NTFP) commonly emerge far from equilibrium.
   Examples: gluonic systems, scalars, spin gases (ultracold atoms), …

c. Non-perturbative numerical approach developed for spectral functions
   ⇒ more information on microscopic dynamics accessible

Outlook: The technique to study spectral functions can be applied, e.g., to:

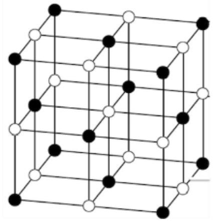
- *Transport* coefficients, jet quenching, diffusion
- *Anisotropy*, plasma instabilities, Glasma

Thank you for your attention!

BACKUP SLIDES

Computational method

Classical-statistical lattice simulations



- $SU(N_c)$ *gauge theory* with $N_c = 2$ in temporal $A_0 = 0$ gauge
- Large occupancies $f(p \sim \Lambda) \gg 1$, weak coupling, real time
  $\Rightarrow$ *Classical* approximation for *field dynamics* applicable

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)

- Fields are link and chromo-electric fields U_i, E_i on 3D spatial lattice
- Initialization:

$$\langle |A(t=0,\boldsymbol{p})|^2 \rangle \sim \frac{f(t=0,p)}{p}, \qquad \langle |E(t=0,\boldsymbol{p})|^2 \rangle \sim p\, f(t=0,p)$$

then $A_i(0,\boldsymbol{p}) \to U_i(0,\boldsymbol{x})$, $E_i(0,\boldsymbol{p}) \to E_i(0,\boldsymbol{x})$ and restore Gauss law

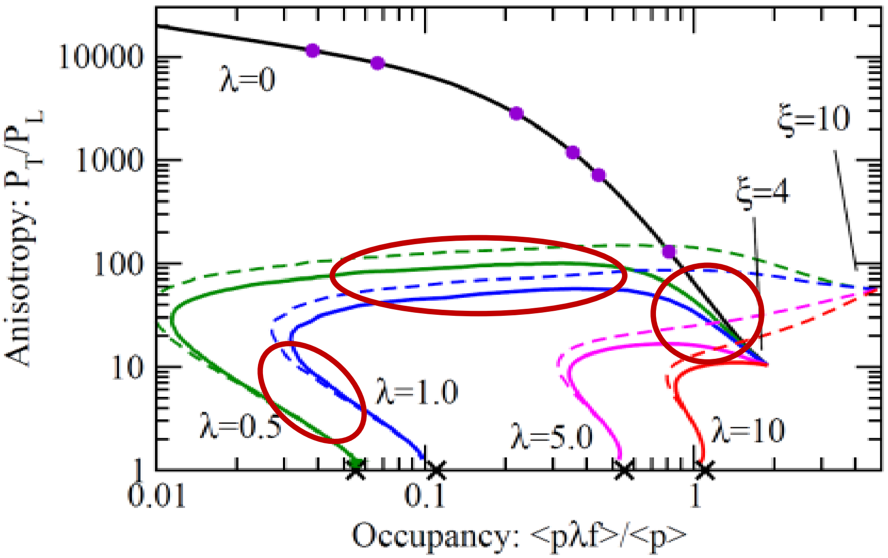
Thermalization dynamics at early times

‚Bottom-up‘ picture and onset of hydrodynamics

Kurkela, Zhu, *PRL 115, 182301 (2015)*

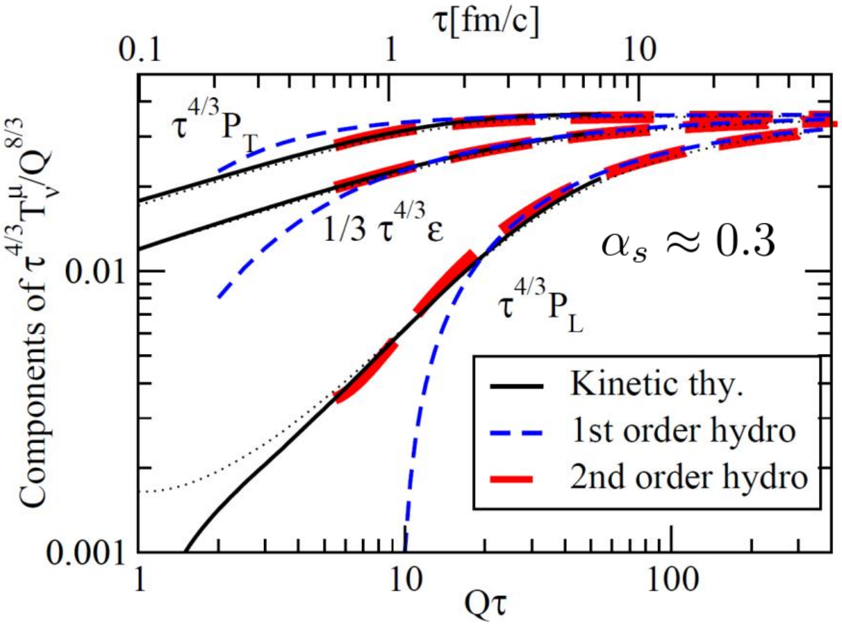
Full ‚bottom-up‘ evolution

($\lambda = 4\pi\alpha_s N_c$ extrapolated to moderate values = 5, 10)



3 stages: i) classical scaling; ii) anisotropy freezes; iii) radiational breakup

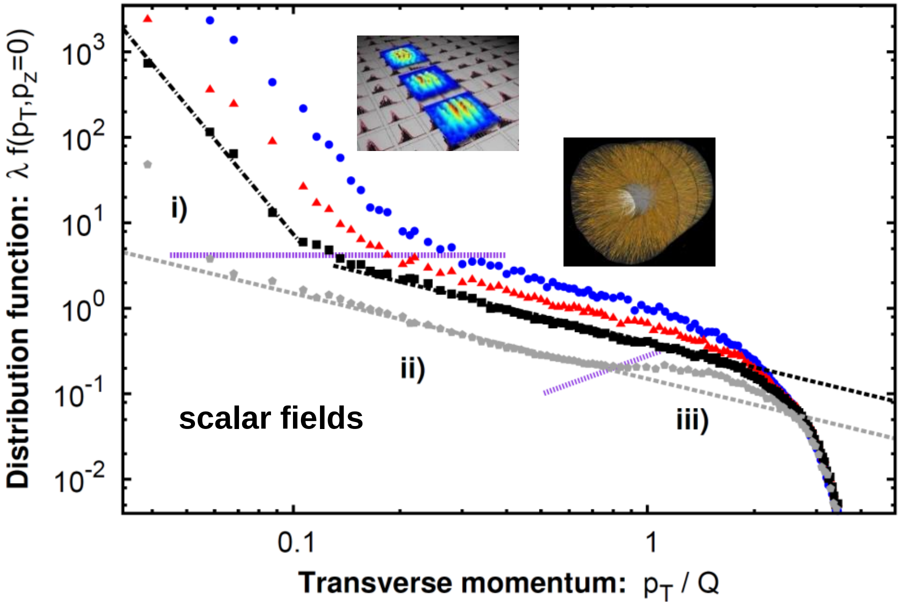
Onset of hydrodyncamics



Hydrodynamics already at $\tau \lesssim 1 fm/c$!

Universality classes

The attractor in longitudinally expanding scalars



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)$$

Berges, KB, Schlichting, Venugopalan,
PRL 114, 061601 (2015) ;
PRD 92, 096006 (2015)

Fixed points	α	β	γ	λf_S
i)	1	$2/3$	$2/3$	$\left((p/b)^{-1/2} + (p/b)^{-5}\right)^{-1}$
ii)	$-2/3$	0	$1/3$	$p_T^{-1}\ e^{-p_z^2/2\sigma_z^2}$
iii)	$-1/2$	0	$1/2$	$\mathrm{sech}(p_z/\sigma_z)$

Nonthermal fixed points

Scalars: Inverse particle cascade to IR

Piñeiro Orioli, KB, Berges,
PRD 92, 025041 (2015)

Self-similar evolution

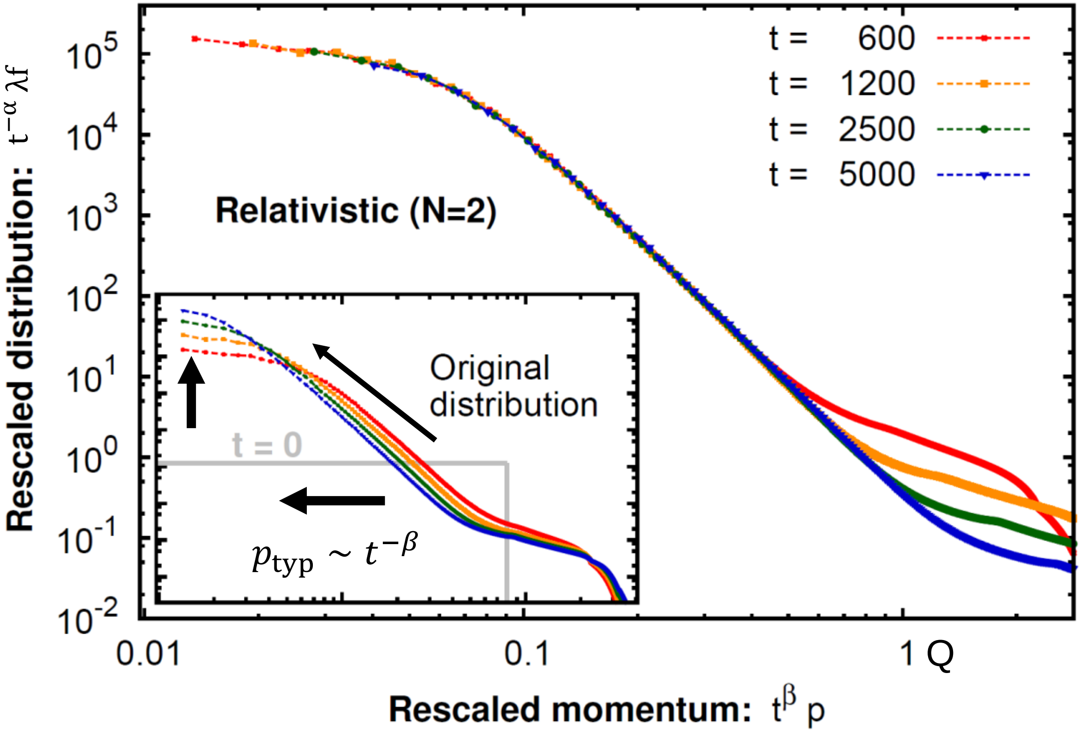
$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

Scaling exponents

$$\alpha \approx \frac{3}{2}, \qquad \beta \approx \frac{1}{2}$$

Particle conservation

$$n = \int_{IR} \frac{d^3p}{(2\pi)^3} f(p) \approx const$$



Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

Nonthermal fixed points

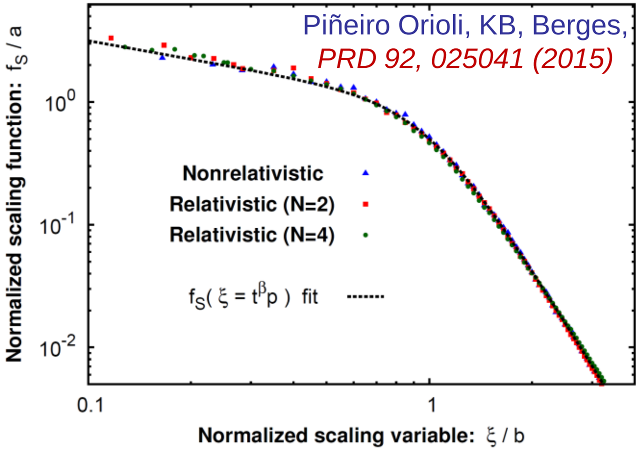
Scalars: Universality in IR

- *Same α, β and scaling function*

$$\lambda f_S \simeq \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}$$

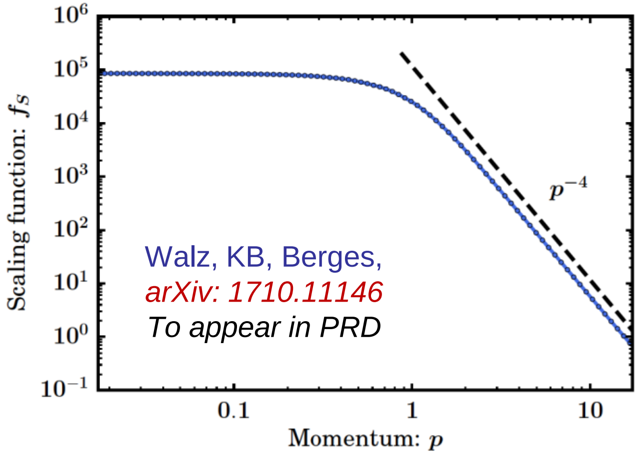
with $\kappa_< \simeq 0 - 0.5$ and $\kappa_> \simeq 4 - 4.5$

Across relativistic (different N), nonrelativistic

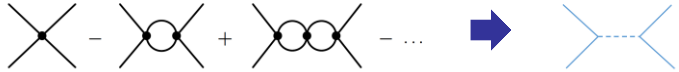


- New *large-N kinetic theory* describes it quantitatively, shows that $\kappa_< \to 0$, $\kappa_> \to 4$

  Piñeiro Orioli, KB, Berges (2015); Walz, KB, Berges (2017)



- (Systematically derived in $1/N$, resums vertex)

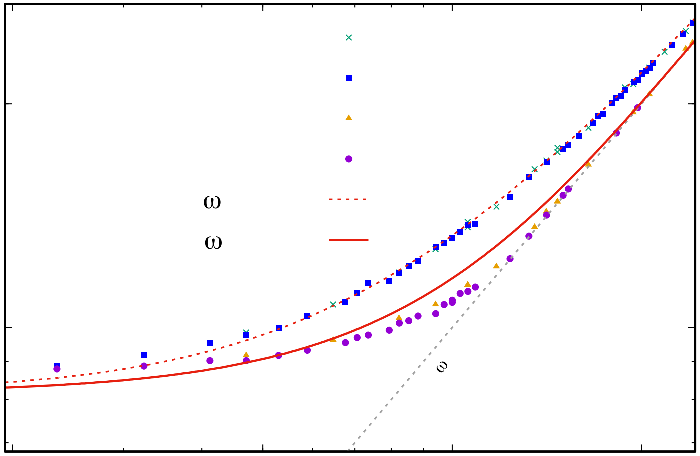


Extracted spectral function vs. HTL predictions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Extracted dispersion relations $\omega_{T,L}(p)$

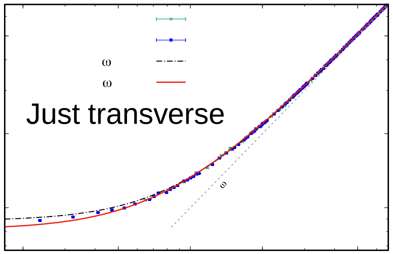
$\omega_{T,L} \,/\, m_{\mathrm{HTL}}$



Momentum: $p \,/\, m_{\mathrm{HTL}}$

- Extracted from peak position (for ω_L after subtracting HTL Landau cut)
- *Similar to HTL* predictions: $\omega_{T,L}^{\mathrm{HTL}}(p)$
- Deviations at small p, for finite m/Λ?
- "$\omega_L(p)$" deviates at $p \sim m$ because peak is smaller than Landau cut, harder to measure

Remark: $\omega_T(p)$ also compatible with $\omega_T^{\mathrm{rel}} = \sqrt{m_\infty^2 + p^2}$ →

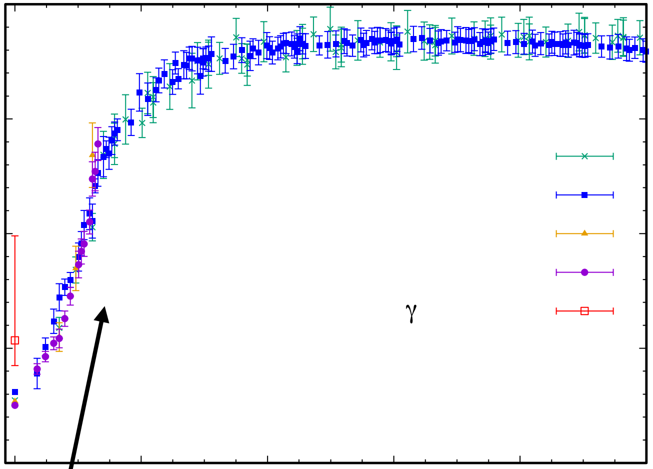


Extracted spectral function vs. HTL predictions

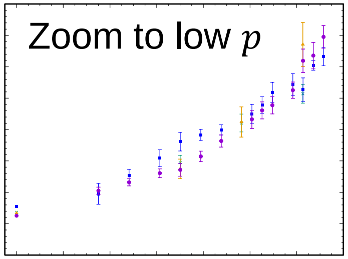
KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Extracted damping rates $\gamma_{T,L}(p)$

$\gamma_{T,L}(p)\ /\ Q$



Momentum:



- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2 Q)$ and *beyond HTL at LO*, it may contain non-perturbative contributions (*magnetic scale*)
- Here *first determination* of $\gamma_{T,L}(p)$!
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{\mathrm{HTL}}(p=0)$ Braaten, Pisarski, *PRD 42, 2156 (1990)*
- "Isotropic" $\gamma_T \approx \gamma_L$ for $p \lesssim m$